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Effect of Conversion Efficiency on Gamma-Ray Burst Energy *

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Abstract Beaming effect makes it possible that gamma-ray bursts have a standard energy, but the gamma-ray energy release is sensitive to some parameters. Our attention is focused on the effect of the gamma ray conversion efficiency (η_γ), which may range between 0.01 and 0.9, and which probably has a random value for different GRBs under certain conditions. Making use of the afterglow data from the literature, we carried out a complete correction to the conical opening angle formula. Within the framework of the conical jet model, we ran a simple Monte Carlo simulation for random values of η_γ, and found that the gamma-ray energy release is narrowly clustered, whether we use a constant value of η_γ or random values for different gamma-ray bursts.

Key words: gamma rays: bursts — ISM: jets

1 INTRODUCTION

Gamma-ray bursts (GRBs) are believed to originate from internal shocks that arise in irregular relativistic winds. The kinetic energy of the resulting shells is converted into internal energy by the relativistic shocks. Electrons are heated by the shocks, and the internal energy is radiated via synchrotron and inverse Compton emission, with broken power law spectra. (Sari, Piran & Narayan 1998)

One of the keys to understanding the progenitors of gamma-ray bursts and the physics of their central engines lies in determining the energetics of the explosion. The true energy release depends sensitively on such parameters as the geometry of the ejecta, the circumburst medium density and the efficiency of gamma-ray conversion.

Jets are common in most gamma-ray bursts (Sari et al. 1998). If GRB explosions are conical (as opposed to spherical) then the true energy release is significantly below that inferred by assuming isotropy. Frail et al. (2001) presented a complete sample of 17 GRBs with good afterglow data and known redshifts, corrected for the geometry within the conical jet model, and found that the released gamma-ray energy is narrowly clustered around 5×10^{50} erg. Bloom, Frail & Kulkarni (2003) presented a sample of 29 GRBs for which it has been possible to determine temporal breaks (of limits) from their afterglow light curves, and found a

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similarly tightly-clustered energy around 1.21×10^{51} erg. They interpreted the breaks with several simplifying assumptions: constant ambient density and constant gamma-ray conversion efficiency. They took a constant value of the conversion efficiency $\eta_\gamma = 0.2$ for all GRBs, and a circumburst density $n = 0.1\,\mathrm{cm}^{-3}$ (Frail et al. 2001) or $n = 10\,\mathrm{cm}^{-3}$ (Bloom et al. 2003) if the circumburst density is unknown. In addition, they derived the jet opening angle using an approximate formula.

A number of recent papers (Panaitescu & Kumar 2002; Guetta, Spada & Waxman 2001; Kumar 1999; Kobayashi 2001) have argued that internal shocks under certain conditions have different conversion efficiencies. Considering their results and estimates, we assume that η_γ may range between 0.01 and 0.9. Furthermore, we assume that different GRBs should have different energy conversion efficiencies.

In Sect. 2, we derive a completely corrected formula of the jet opening angle, and carry out an analysis of the effect of η_γ. In Sect. 3, we run a simple Monte Carlo simulation, and obtain a narrowly distributed energy release corresponding to assuming the value of η_γ to be constant or random for different GRBs. In Sect. 4 we present a brief discussion and conclusions.

2 MODEL AND PARAMETERS

In the relativistic fireball model, if we are along the jet's axis and the Lorentz factor γ is larger than the inverse of the jet's half opening angle θ_j, then we shall have similar emissions from a spherically expanding shell and a jet. When γ drops below θ_j^{-1}, a break appears in the light curve of the afterglow. For spherical adiabatic evolution, we have (Rhoads 1999; Sari, Piran & Halpern 1999)

$$\gamma(t) \approx 6(E_{52}/n)^{1/8} t_\mathrm{day}^{-3/8}, \quad (1)$$

where $E_\mathrm{iso,kin} = E_{52} \times 10^{52}$ erg is the isotropic-equivalent kinetic energy of the ejecta during the afterglow, i.e., the inferred energy assuming isotropic expansion, n is the surrounding ISM particle density in cm^{-3}, and t_day is the time in units of day. The break should appear at

$$t_\mathrm{j} \approx 6.2(E_{52}/n)^{1/3}(\theta_\mathrm{j}/0.1)^{8/3}\mathrm{hr}. \quad (2)$$

The jet break time t_j is determined from the afterglow light curves. We convert the jet break time t_j to the opening angle of the conical blast wave, using the formula of Sari et al. (1999),

$$\theta_\mathrm{j} = \frac{1}{6} E_{52}^{-1/8} n^{1/8} t_\mathrm{day}^{3/8}. \quad (3)$$

With the present canonical values for the cosmological parameters, $\Omega_\Lambda = 0.7,\ \Omega_\mathrm{M} = 0.3,\ H_0 = 65\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, the luminosity distance at redshift z is given by

$$D_\mathrm{L} = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_\mathrm{M}(1+z')^3}}. \quad (4)$$

The total isotropic prompt energy release in a certain bandpass can be determined by

$$E_{\mathrm{iso},\gamma} = F_\gamma \frac{4\pi D_\mathrm{L}^2}{1+z} k, \quad (5)$$

where F_γ is the fluence received in the bandpass. The quantity k is a multiplicative correction of order unity relating the observed bandpass to the standard rest-frame bandpass.

We specially note that

$$E_{\rm iso,kin} = E_{\rm iso,exp} - E_{\rm iso,\gamma} = \frac{E_{\rm iso,\gamma}}{\eta_\gamma}(1-\eta_\gamma). \quad (6)$$

Then we can obtain the formula of $\theta_{\rm j}$,

$$\theta_{\rm j} = 0.105 \left(\frac{t_{\rm j}}{1\,{\rm day}}\right)^{3/8} \left(\frac{1+z}{2}\right)^{-3/8} \left(\frac{E_{\rm iso,\gamma}}{10^{53}\,{\rm erg}}\right)^{-1/8} \left[\frac{\eta_\gamma}{0.2\,(1-\eta_\gamma)}\right]^{1/8} \left(\frac{n}{10\,{\rm cm}^{-3}}\right)^{1/8}. \quad (7)$$

Obviously, the opening angle $\theta_{\rm j}$ is a function of the jet break time $t_{\rm j}$, isotropic-equivalent gamma-ray energy $E_{\rm iso,\gamma}$, ambient number density n, and gamma-ray conversion efficiency η_γ.

The beaming fraction is $f_{\rm b} = 1 - \cos\theta_{\rm j}$. After the beaming-correction, the true gamma-ray energy release is

$$E_{\rm j,\gamma} = E_{\rm iso,\gamma} f_{\rm b} = E_{\rm iso,\gamma}(1-\cos\theta_{\rm j})\,. \quad (8)$$

If the opening angle is small enough, i.e., $1 - \cos\theta_{\rm j} = 2\sin^2(\frac{\theta_{\rm j}}{2}) \simeq \frac{\theta_{\rm j}^2}{2}$, then the gamma-ray energy release is

$$E_{\rm j,\gamma} = E_{\rm iso,\gamma}\frac{\theta_{\rm j}^2}{2}\,. \quad (9)$$

Assuming $\eta_\gamma = 0.2$, and neglecting the factor $1-\eta_\gamma$ (i.e., adopting $1-\eta_\gamma \approx 1$), Frail et al. (2001) and Bloom et al. (2003) obtained the formula of $\theta_{\rm j}$,

$$\theta_{\rm j} = 0.057 \left(\frac{t_{\rm j}}{1\,{\rm day}}\right)^{3/8} \left(\frac{1+z}{2}\right)^{-3/8} \left(\frac{E_{\rm iso,\gamma}}{10^{53}\,{\rm erg}}\right)^{-1/8} \left(\frac{\eta_\gamma}{0.2}\right)^{1/8} \left(\frac{n}{0.1\,{\rm cm}^{-3}}\right)^{1/8}. \quad (10)$$

Frail et al. (2001) adopted $n = 0.1\,{\rm cm}^{-3}$ for bursts whose circumburst density n is unknown. Panaitescu & Kumar (2002) found in most cases the density of the external medium is between $0.1\,{\rm cm}^{-3}$ and $100\,{\rm cm}^{-3}$. To find an actual value of E_γ, Bloom et al. (2003) suggested that a more realistic estimate of the medium density n should be taken by broadband modelling of the afterglow light curves. The modelling gave estimates in the range $0.1\,{\rm cm}^{-3} < n < 30\,{\rm cm}^{-3}$, and thus Bloom et al. (2003) adopted a new canonical value of $10\,{\rm cm}^{-3}$. Here we take $n = 10\,{\rm cm}^{-3}$ as the canonical value.

The gamma-ray conversion efficiency has been discussed in many earlier works. Mochkovitch, Maitia & Marqueeset (1995) considered a simple model in which an outflow of ultra-relativistic matter is represented by a succession of shells with random Lorentz factor values, and concluded that the efficiency of this process is low ($< 10\%$) if the spread in the Lorentz factor is small. While Kobayashi, Piran & Sari (1997) found a $\sim 10\%$ efficiency for a uniform Lorentz factor distribution with a maximum to minimum Lorentz factor ratio ≈ 10; Kumar (1999) argued that the conversion efficiency from bulk motion to gamma radiation is as low as 1%.

However, Kobayashi et al. (1997) showed that if the spread in the Lorentz factors of the ultra-relativistic matter is larger, a higher efficiency could be achieved. The most efficient case is the one in which the inner engine produces shells with comparable energy but very different Lorentz factors. Beloborodov (2000) demonstrated that the efficiency may approach 100% for non-uniform distributions with a wide range of Lorentz factors. Guetta, Spada & Waxman (2001) found that, a significant fraction of the wind kinetic energy, on the order of 20%, can be converted to radiation, if the Lorentz factor within the wind spreads over the range $10 - 10^3$. Kobayashi & Sari (2001) similarly found $\sim 60\%$ efficiency for a uniform distribution of the logarithm of the Lorentz factor over the range $1 - 4$.

Panaitescu & Kumar (2002) have made broadband modelling of 10 GRB afterglows within a specific framework, which revealed several properties of GRB jets. By their modelling, the conversion efficiency η_γ was found to vary over a wide range from 0.11 to 0.82 for different GRBs.

Based on the above results, and using the estimates of Kumar (1999) and Kobayashi & Sari (2001), we assume that η_γ may range between 0.01 and 0.9 and that η_γ has a random value for different GRBs. This is possible because the central engines eject randomly relativistic shells with some unknown distribution of Lorentz factors.

3 NUMERICAL SIMULATION FOR ENERGY CONVERSION EFFICIENCY

Assuming a constant conversion efficiency, $\eta_\gamma = 0.2$, for all GRBs, Frail et al. (2001) found the gamma-ray energy release is clustered around 5×10^{50} erg, Bloom et al. (2003) obtained a clustered energy release around 1.21×10^{51} erg, with a small scatter 0.08 dex (≈ 0.20 mag). Confining to the GRBs with measured t_j, z and n, Bloom et al. found the energy is clustered around 1.33×10^{51} erg $\pm$ 0.07 dex.

Table 2 lists the observed and modelled data for the GRB energy determination, taken from Bloom et al. (2003). There are 35 GRBs in this table, but only 24 GRBs of which have both t_j and z. Using the formula in Sect. 2, we can derive the gamma-ray energy release for these 24 GRBs.

In fact, the energy conversion efficiency is such a complex parameter that it can change over a wide range. It is determined by many parameters, such as ξ_e, the ratio of the accelerated electron energy density to the total thermal energy density of the shocked medium. However, ξ_e is not constant for all GRBs. Panaitescu & Kumar (2002) have modelled 10 GRBs, and found that the values of η_γ of these 10 GRBs are different, ranging from 0.11 to 0.82. So, we believe that η_γ may be random for different GRBs.

We ran simulations for two specific cases: in one, η_γ is assumed to be a constant for all the GRBs; in the other, random for different GRBs.

3.1 Simulation with a Constant Energy Conversion Efficiency

First, we reproduce the result of Bloom et al. (2003). We assume that the gamma-ray conversion efficiency η_γ is constant for all GRBs, and η_γ ranges from 0.01 to 0.9. For the 24 GRBs with measured z, t_j, and n ($n = 10\,\mathrm{cm}^{-3}$ if n is unknown), the logarithmically-weighted mean total energy release is 1.27×10^{51} erg ± 0.07 dex, with a median energy of 1.35×10^{51} erg. Compared with Bloom's result in the case of $\eta_\gamma = 0.2$, our result is larger because of the $(1-\eta_\gamma)^{-1/4}$ factor in Eqs. (7) and (9). While η_γ ranges from 0.01 to 0.9 the gamma-ray energy release is very narrowly clustered, and becomes larger. In Table 1, we show the median energy of 24 GRBs when η_γ is 0.01, 0.2 and 0.9. Figure 1 also clearly shows a comparison with the result calculated from Eq. (10).

Table 1 Effect of η_γ on the Clustered Energy for all GRBs Following the Corrected Formula

Conversion efficiency η_γ	$E_{\mathrm{j},\gamma}$ $\times 10^{51}$erg
0.01	0.6 ± 0.09 dex
0.2	1.35 ± 0.09 dex
0.9	3.29 ± 0.09 dex

3.2 Simulation with Random Distribution of Energy Conversion Efficiency

To acquire the distribution of the beaming-corrected gamma-ray energy release, we next run a simple Monte Carlo simulation. The value of η_γ is taken to be random between 0.01 and 0.9 for the 24 GRBs. We calculate each GRB's gamma-ray energy release, and find that the median energy release of 24 GRBs is clustered.

Table 2 Observed and Modelled Data for GRB Energy Determination (taken from Bloom et al. 2003)

GRB	z	S_γ $\times 10^{-6}$ erg cm^{-2}	Bandpass keV	k	$t_{\rm jet}$ day	n cm^{-3}
970228	0.6950	11.70 ± 2.00	1.5, 700	0.830 ± 0.041		
970508	0.8349	3.17	20, 2000	0.814 ± 0.041	25.00 ± 5.00	1.00
970828	0.9578	96.00	20, 2000	0.823 ± 0.036	2.20 ± 0.40	
971214	3.4180	9.44	20, 2000	0.804 ± 0.057	> 2.5	
980326		0.92	20, 2000		< 0.4	
980329		55.10	20, 2000		< 1.0	20.00 ± 10.00
980425	0.0085	3.87	20, 2000	1.002 ± 0.000		
980519		10.30	20, 2000		0.55 ± 0.17	$0.14^{+0.32}_{-0.03}$
980613	1.0969	1.71 ± 0.25	20, 2000	0.863 ± 0.110	> 3.1	
980703	0.9662	22.60	20, 2000	0.940 ± 0.041	3.40 ± 0.50	28.00 ± 10.00
981226		0.40 ± 0.10	40, 700		> 5.0	
990123	1.6004	268.00	20, 2000	0.720 ± 0.052	2.04 ± 0.46	
990506	1.3066	194.00	20, 2000	0.873 ± 0.054		
990510	1.6187	22.60	20, 2000	1.026 ± 0.055	1.20 ± 0.08	$0.29^{+0.11}_{-0.15}$
990705	0.8424	93.00 ± 2.00	40, 700	1.279 ± 0.098	1.00 ± 0.20	
990712	0.4331	6.50 ± 0.30	40, 700	1.387 ± 0.132	> 47.7	
991208	0.7055	100.00	25, 10000	0.746 ± 0.206	< 2.1	$18.00^{+18.00}_{-6.00}$
991216	1.0200	194.00	20, 2000	0.877 ± 0.042	1.20 ± 0.40	$4.70^{+6.80}_{-1.80}$
000131	4.5110	41.80	20, 2000	0.646 ± 0.074	< 3.5	
000210	0.8463	61.00 ± 2.00	40, 700	1.278 ± 0.097	> 1.7	
000301C	2.0335	4.10	25, 1000	0.928 ± 0.094	7.30 ± 0.50	27.00 ± 5.00
000418	1.1181	20.00	15, 1000	0.997 ± 0.018	25.00 ± 5.00	$27.00^{+250.00}_{-14.00}$
000630		2.00	25, 100		> 4.0	
000911	1.0585	230.00	15, 8000	0.508 ± 0.063	< 1.5	
000926	2.0369	6.20	25, 100	3.912 ± 1.328	1.80 ± 0.10	27.00 ± 3.00
010222	1.4768	120.00 ± 3.00	2, 700	1.115 ± 0.004	$0.93^{+0.15}_{-0.06}$	1.70
010921	0.4509	15.40 ± 0.20	8, 400	1.475 ± 0.289	33.00 ± 6.50	
011121	0.3620	24.00	25, 100	4.996 ± 2.006	> 7.0	
011211	2.1400	5.00	40, 700	1.068 ± 0.084	1.77 ± 0.28	
020124		3.00	8, 85			
020405	0.6899	38.00 ± 4.00	50, 700	1.318 ± 0.096	1.67 ± 0.52	
020331		0.40	8, 40			
020813	1.2540	38.00	25, 100	4.336 ± 1.682	0.43 ± 0.06	
021004	2.3320	3.20	7, 400	1.188 ± 0.098	7.60 ± 0.30	$30.00^{+270.00}_{-27.00}$
021211	1.0060	1.00	8, 40	12.345 ± 6.462		

For example, we run the simulation for 10000 times. For each time, by taking a random sample of η_γ for 24 GRBs, we obtain the total sum of the highest bar and its adjacent two bars, written as $N_{\rm cluster}$, which represents the dispersion of the cluster. The larger the sum, the smaller the dispersion becomes. The clustered distribution is shown in Fig. 2, which shows that the dispersion becomes larger than the one shown in Fig. 3. We also calculate the median gamma-ray energy release of 24 GRBs after beaming-correction.

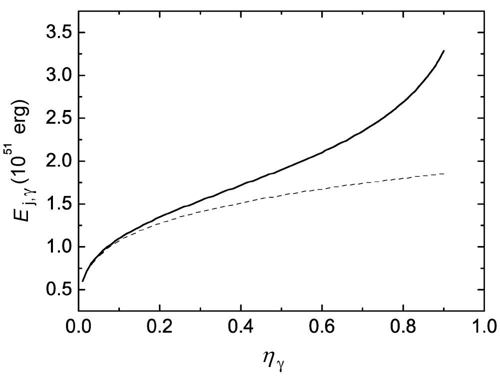

Fig. 1 η_γ ranges from 0.01 to 0.9. With η_γ increasing, the clustered energy becomes larger. The solid curve is our result following Eqs. (7) and (9), and the dashed curve is the result following Eq. (10). Obviously, after (1-η_γ) factor corrected, the clustered energy is larger.

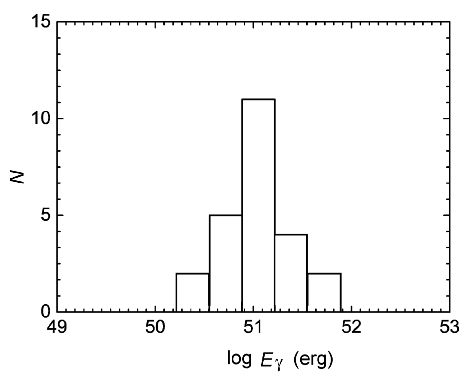

Fig. 2 Histogram of GRB energies (E_γ) with three equal logarithmic spacings per decade when η_γ is generated randomly for 24 GRBs. It shows a narrow distribution of GRB energies about the energy 1.67×10^{51} erg, with an error of $\sigma = 0.22$ dex. This is just an example.

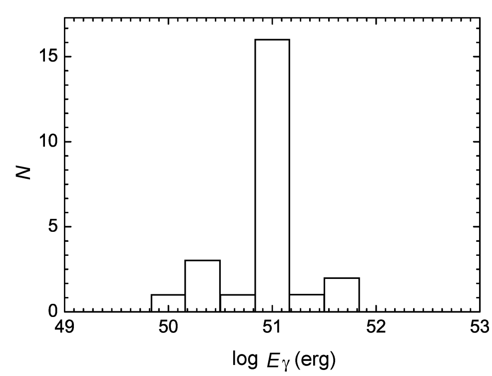

Fig. 3 Histogram of GRB energies (E_γ) with three equal logarithmic spacings per decade when η_γ= 0.2. It shows a narrow distribution of GRB energies about the standard energy 1.35×10^{51} erg, with an error of $\sigma = 0.09$ dex.

After 10 000-times simulation, we find that the total sum $N_{\rm cluster}$ and the median energy satisfy the Gaussian distribution, shown in Figs. 4 and 5. Clearly, among the 10 000-times simulation, it is only very few times that $N_{\rm cluster}$ is smaller than 17, showing a good cluster. We find the median energy is also clustered, and acquire a new standard energy: 1.82×10^{51} erg.

In both cases above, the beaming-corrected gamma-ray energy release is clustered and the dispersion is smaller if η_γ is constant. In other cases, such as the Gaussian distribution of η_γ, it puts greater weight on some η_γ, so that the beaming-corrected jet energy is still clustered, with a smaller dispersion than for the random distribution of η_γ. In addition, in any other distribution, we can obtain a similar conclusion.

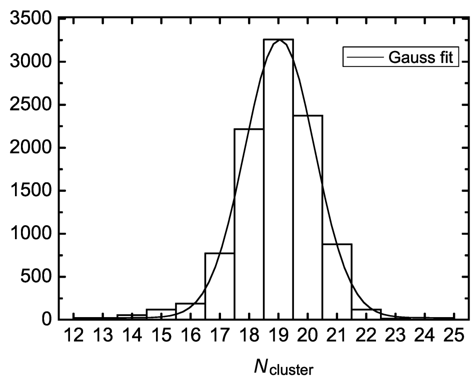

Fig. 4 Dispersion distribution from Monte Carlo simulations. Here η_γ is generated randomly for 24 GRBs. It shows that for only few times $N_{\rm cluster}$ is smaller than 17. The gamma-ray energy release is well clustered for most times in our simulation.

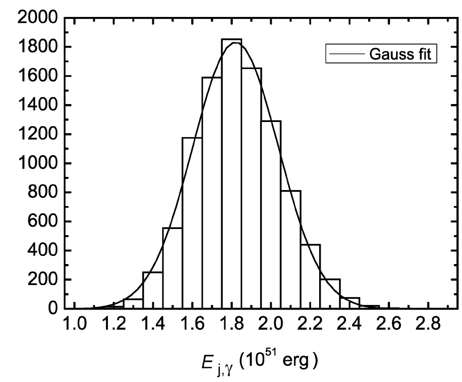

Fig. 5 Median energy (of 24 GRBs after beaming-correction) distribution from Monte Carlo simulations. Here η_γ is generated randomly for 24 GRBs. It shows the median energy is clustered around 1.82×10^{51} erg.

4 DISCUSSION AND CONCLUSIONS

Our work is based on the following assumptions:

(1) GRBs explode in a constant density medium and any sharp break in the afterglow light curve is attributed to sideways expansion of a relativistic jet.

(2) The jet is uniform, with an energy per solid angle independent of direction within the jet. In general, the observer is assumed to be located on the jet axis.

Our analysis is focused on the effect of η_γ. In fact, there are some other factors affecting the jet energy. For example, since the total energy $E_{\rm j,exp} = E_{\rm j,\gamma}/\eta_\gamma \propto n^{1/4}\eta_\gamma{}^{-3/4}$ (which follows from Eq. (7)), the gamma-ray energy release is also sensitive to the circumburst density. In our analysis, we have taken the medium densities from broadband modelling of some afterglows but used $n = 10\,{\rm cm}^{-3}$ for other afterglows. We have not considered wind environments, which may appear in the vicinity of GRBs associated with massive stars (Dai & Lu 1998; Chevalier & Li 1999, 2000).

Our analysis shows that under the assumptions above, if η_γ is constant for different GRBs, the gamma-ray energy release is tightly clustered; if η_γ is randomly distributed, the gamma-ray energy release is still clustered, but not so tightly. We also deduce that the gamma-ray energy release is clustered in any other η_γ distribution. In the latter case, we can take a new standard energy 1.82×10^{51} erg for GRBs. The GRB standard energy makes possible to use GRBs and their afterglows for cosmography. Our results may be tested by the upcoming GRB data.

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