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Thermal Bremsstrahlung Radiation in a Two-Temperature Plasma *

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Abstract In normal one-temperature plasma the motion of ions is usually neglected when calculating the Bremsstrahlung radiation of the plasma. We calculate the Bremsstrahlung radiation of a two-temperature plasma by taking into account of the motion of ions. Our results show that the total radiation power is always lower if the motion of ions is considered. We also apply the two-temperature Bremsstrahlung radiation mechanism for an analytical Advection-Dominated Accretion Flow (ADAF) model; we find the two-temperature correction to the total Bremsstrahlung radiation for ADAF is negligible.

Key words: plasmas — radiation mechanisms: thermal

1 INTRODUCTION

In a plasma, electrons are constantly accelerated during their collisions with ions, leading to Bremsstrahlung radiation. Usually, when calculating the radiation power of the plasma, the motion of ions is neglected, because the ion's mass is much higher than the electron mass. However, the motion of ions may not be neglected if the ion temperature is much higher than that of electrons, such as happens in the ADAF model, in which the temperature of ions $\sim 10^{11}\,\mathrm{K}$ may be much higher than that of electrons ($10^8 \sim 10^9\,\mathrm{K}$) (Narayan & Yi 1995). Under such condition, the velocities of the ions are comparable to or even higher than that of electrons, and the motion of ions must be taken into account when calculating the thermal Bremsstrahlung radiation of the two-temperature plasma.

Our calculations show that the radiation power is reduced significantly if the ion temperature is much higher than the electron temperature. However, when applying the two-temperature Bremsstrahlung radiation to the ADAF model, we find the total Bremsstrahlung radiation emissivity is not significantly different from the one-temperature Bremsstrahlung radiation, because most of the Bremsstrahlung radiation is produced at large radii where the ion's temperature is not significantly different from the electron temperature.

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2 THERMAL BREMSSTRAHLUNG RADIATION

We begin by considering an individual scattering event between an electron and an ion in which an electron with a velocity v and an impact parameter b is scattered by an ion; we assume the electron's motion is non-relativistic. Then the radiation power emitted at a specific frequency by this electron is given by (Padmanabhan 2000)

$$W_1(\omega) = \frac{8Z^2e^6}{3\pi c^3 m_{\rm e}^2 v^2 b^2}. \tag{1}$$

Integrating over b between the limits $b_{\rm min} = Ze^2/m_{\rm e}v^2$ and $b_{\rm max} = v/\omega$ (You 1998), we obtain

$$P_1(\nu) = 2\pi P_1(\omega) = 4\pi^2 N_z v \int_{b_{\rm min}}^{b_{\rm max}} W_1(\omega) b \, {\rm d}b = \frac{32\pi N_Z Z^2 e^6}{3c^3 m_e^2 v} \ln \frac{m_e v^3}{2\pi Z e^2 \nu}, \tag{2}$$

where N_Z is the number density of the ions. Integrating it over all electrons and assuming the electrons follow the non-relativistic Maxwellian velocity distribution, we obtain the specific emissivity,

$$j_1(\nu) = \frac{128\pi^2 Z^2 e^6}{3c^3 m_e^2} N_Z N_e \Big(\frac{m_e}{2\pi kT}\Big)^{3/2} \int_0^\infty \exp\Big(-\frac{m_e v^2}{2kT}\Big) v \ln \frac{m_e v^3}{2\pi Z e^2 \nu} \, {\rm d}v, \tag{3}$$

where N_e is the number density of the electrons.

3 TWO-TEMPERATURE BREMSSTRAHLUNG RADIATION

Let the temperature of ions and electrons be T_Z and $T_{\rm e}$, respectively. We again begin by considering an individual scattering event. The radiation power of an electron in the rest-frame of an ion is similar to Eq. (1),

$$W_2(\omega) = \frac{8Z^2e^6}{3\pi c^3 m_e^2 v^2 b^2}, \tag{4}$$

except that v is the relative velocity between the ion and the electron,

$$v = (v_e^2 + v_Z^2 - 2 v_e v_Z \cos\theta)^{1/2}.$$

Integrating over b, we obtain

$$P_2(\nu) = 2\pi P_2(\omega) = 4\pi^2 \int_0^\infty {\rm d}\boldsymbol{v}_Z \int_{b_{\rm min}}^{b_{\rm max}} W_2(\omega) N(\boldsymbol{v}_Z) v b \, {\rm d}b. \tag{5}$$

Assuming that the ions and electrons all follow the non-relativistic Maxwellian velocity distribution, and integrating $P_2(\omega)$ over all the electrons, we obtain the specific emissivity,

$$j_2(\nu) = \frac{256\pi^3 Z^2 e^6}{3c^3 m_e^2} N_Z N_e \Big(\frac{m_e}{2\pi kT_{\rm e}}\Big)^{3/2} \Big(\frac{m_Z}{2\pi kT_{\rm Z}}\Big)^{3/2} \int_0^\infty \exp\Big(-\frac{m_e v_e^2}{2kT_{\rm e}}\Big) v_e^2 \, {\rm d}v_e$$
$$\int_0^\infty \exp\Big(-\frac{m_Z v_Z^2}{2kT_{\rm Z}}\Big) v_Z^2 \, {\rm d}v_Z \int_0^\pi \ln \frac{m_e v^3}{2\pi Z e^2 \nu} \frac{1}{v} \sin\theta \, {\rm d}\theta. \tag{6}$$

Because we are only considering the non-relativistic regime, the radiation power observed in the rest-frame of the ion is the same as that in the laboratory frame.

4 RESULTS

We make numerical calculations of the two-temperature plasma radiation; all the ions are assumed to be protons. The results are shown in Figs. 1–3. We can see that in all cases the two-temperature Bremsstrahlung radiation emissivity is lower than the one-temperature case, and the difference is greater for higher electron and/or ion temperatures.

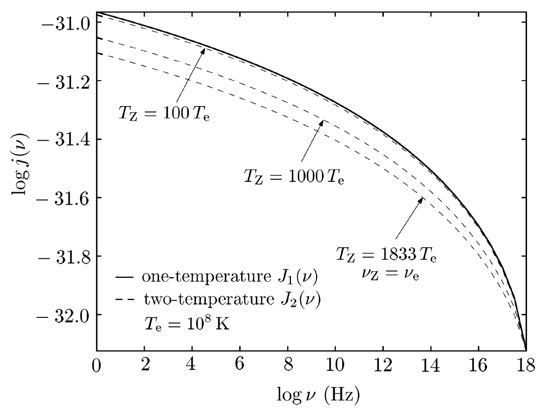

Fig. 1 Specific emissivity of the one- temperature and two-temperature Bremsstrahlung radiation. $T_e = 10^8$ K. If $T_Z = 1833T_e$, corresponding to the same average velocities of ions and electrons, the radiation power of the two-temperature Bremsstrahlung radiation is about 75% of the one-temperature Bremsstrahlung radiation.

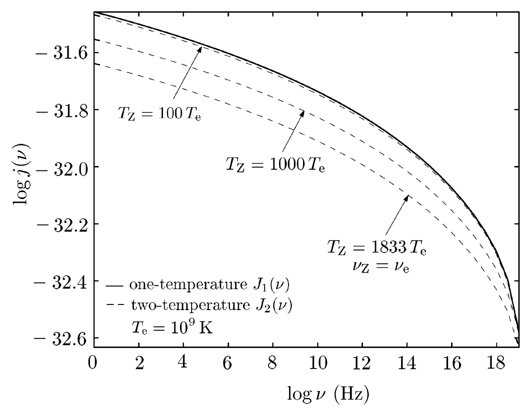

Fig. 2 Specific emissivity of the one- temperature and two-temperature Bremsstrahlung radiation. $T_e = 10^9$ K. If $T_Z = 1833T_e$, the radiation power of the two-temperature Bremsstrahlung radiation is about 68% of the one-temperature Bremsstrahlung radiation.

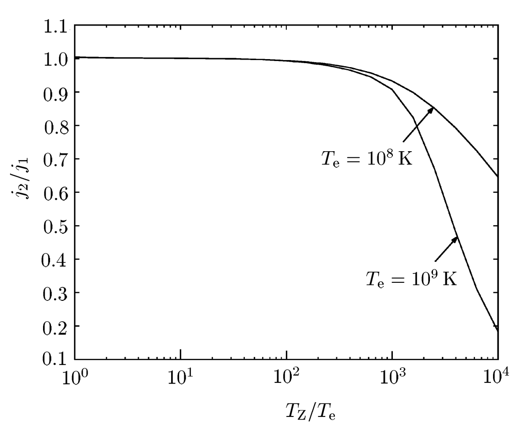

Fig. 3 Ratio between the two types of radiation emissivity (j_2/j_1) as a function of the ratio between the ion temperature to the electron temperature (T_Z/T_e). If T_Z/T_e is less than 100, j_2/j_1 is very close to unity. When T_Z/T_e is more than 1000, the difference between j_2 and j_1 becomes significant.

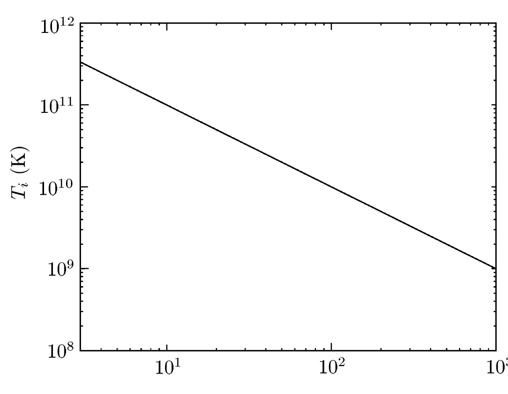

Fig. 4 Ion temperature profile. The horizontal axis denotes the dimensionless radius r ($R = rR_{\rm Schw} = r\frac{2GM}{c^2}$) of the accretion disk, ranging from 3 to 1000.

We then calculate the total luminosity following an analytical ADAF model (Mahadevan 1997), assuming spherical accretion and with all the self-similar equations as showed in Mahadevan (1997). The electron temperature T_e is around 10^9 K, and is assumed to be constant for $r < 10^3$, where r is the dimensionless radius of the accretion disk, defined in $R = rR_{\rm Schw} = r\frac{2GM}{c^2}$. The ion temperature given by Mahadevan (1997) is approximated by

$$T_i = 9.99 \times 10^{11} r^{-1} \text{ K}. \tag{7}$$

The temperature profile is shown in Fig. 4. We simply assume an optically thin accretion disk model and integrate the specific emissivity over all radii to obtain the total luminosity. The ratio between the luminosity of two-temperature and one-temperature Bremsstrahlung radiation L_2/L_1 is about 0.964 for $T_e = 10^9$ K and 0.950 for $T_e = 10^8$ K, respectively. The small difference between the two cases is due to the small difference between the ion temperature and the electron temperature in most of the accretion flow volume; only at small radii could T_Z/T_e exceed 1000 where the two types of Bremsstrahlung radiation become significantly different. We therefore conclude that in the ADAF model the correction due to the two-temperature Bremsstrahlung is negligible.

5 DISCUSSION

In a plasma with a high electron temperature, the bremsstrahlung from electron-positron (e^+e^-), electron-electron (ee), positron-positron (e^+e^+) collisions may become important (Svensson 1982). In our non-relativistic case, e^+e^- pair creation and annihilation can be neglected, so we only need to consider the ee bremsstrahlung, in addition to the e-proton bremsstrahlung we have calculated above. For $T_e < 10^9$ K, we compute the cooling rates of electron-ion bremsstrahlung ($q_{\rm ei}$) and ee bremsstrahlung ($q_{\rm ee}$) according to Svensson (1982) and Narayan & Yi (1995), and obtain that $q_{\rm ee}/q_{\rm ei} < 0.3$. Therefore, the electron-ion bremsstrahlung dominates the radiation power and the ee bremsstrahlung can also be neglected in non-relativistic cases.

In summary, our results show that the two-temperature Bremsstrahlung radiation power is significantly lower than the one-temperature Bremsstrahlung radiation if the ion temperature is more than $1000\,T_e$. Although the temperature difference in the ADAF model could exceed this critical value, the luminosity correction due to this effect is still negligible because of the rapid decrease of the ion temperature at large radii. However, if in some more extreme astrophysical environment the ion temperature is significantly higher than the electron temperature, the two-temperature Bremsstrahlung radiation calculated in this work should be taken into account.

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