

LETTERS

Wavelet Analysis of Several Important Periodic Properties in the Relative Sunspot Numbers *

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Abstract We investigate the wavelet transform of yearly mean relative sunspot number series from 1700 to 2002. The curve of the global wavelet power spectrum peaks at 11-yr, 53-yr and 101-yr periods. The evolution of the amplitudes of the three periods is studied. The results show that around 1750 and 1800, the amplitude of the 53-yr period was much higher than that of the the 11-yr period, that the ca. 53-yr period was apparent only for the interval from 1725 to 1850, and was very low after 1850, that around 1750, 1800 and 1900, the amplitude of the 101-yr period was higher than that of the 11-yr period and that, from 1940 to 2000, the 11-yr period greatly dominates over the other two periods.

Key words: Sun: sunspots — Sun: activity — methods: data analysis

1 INTRODUCTION

Many papers have been devoted to the study of cyclic behaviors in the relative sunspot numbers or in the sunspot group series. Romany et al. (1994) used Fourier analysis to analyze the periods in Wolf number series. O Chadlick et al. (1993) first used the wavelet transform to analyze the solar cycles, known as the Schwabe cycles and Gleissberg cycles. Frick et al. (1997) applied the same technique to analyze the solar activity recorded by sunspot group numbers. Fligge et al. (1999) used wavelet transform to determine the solar cycle length variations using several parameters including the sunspot number, sunspot area, plage area and ¹⁰Be records. Recently Prabhakaran et al. (2002) examined the periodic properties with periods less than 16 years of several parameters including the sunspot number, solar wind and geomagnetic indices. Feng et al. (1998) and Han et al. (2002a, 2002b) also made study on cyclic behavior of the sunspot relative numbers using wavelet transform. Usoskin et al. (2000, 2001) used delayed component technique to check the cyclic behavior during the Maunder minimum. Hathaway et al. (2002) studied the relationship between sunspot, sunspot groups, 10.7-cm radio flux and total sunspot area, and also addressed the question as to which parameter can better represent solar activity. The properties of the Schwabe cycles have been checked in detail, but the properties

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of the Gleissberg cycles have not been so well researched, especially as regards the correlation between the amplitudes of the Gleissberg and the Schwabe cycles. In this paper we mainly examine the periodic properties in relative sunspot numbers by using wavelet transform of the yearly mean relative numbers for the time span from 1749 to 2002. Moreover, we shall compare the global wavelet power spectra and amplitudes of the classical Schwabe 11-yr cycle and the classical Gleissberg 101-yr cycle, as well as also with the 53-yr cycle. Our results show that the amplitudes of the 11-yr period of the relative sunspot number series are not always higher than the amplitudes of the two periods of 53-yr and 101-yr.

2 DATA AND ANALYSIS

The relative sunspot number used in this paper is the yearly mean sunspot number shown in Fig. 1. According to Fig. 1 the yearly mean relative sunspot number reaches its highest value, 190.3, in 1957 for the whole time span from 1700 to 2002.

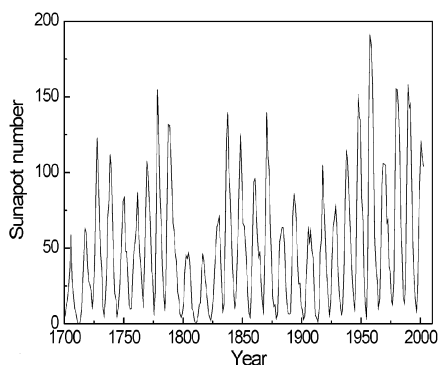


Fig. 1 Yearly mean sunspot number from 1700 to 2002.

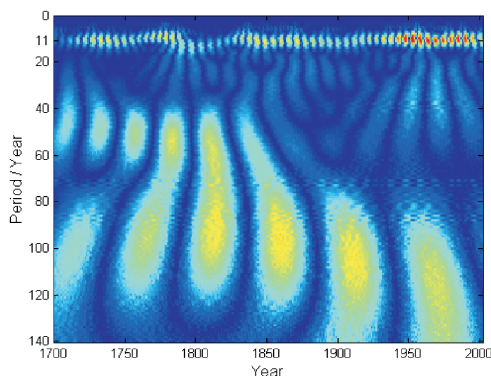


Fig. 2 Wavelet transformation of the yearly mean relative sunspot number

The analyzing wavelet which is particularly well adapted to the sunspot time sequence was initially proposed by Morlet (1982) and later reintroduced by Daubechies (1992). The wavelet transform $W(a, t)$ of signal $f(t)$ is

$$\omega(a, t) = C_{\psi}^{-1/2} a^{-1/2} \int_{-\infty}^{+\infty} \psi^* \left(\frac{t-t'}{a} \right) f(t') dt', \quad (1)$$

where a is known as *scale* and t denotes time. Here, we choose Morlet Wavelet as the analyzing wavelet $\psi(t)$, and $\psi(t) = \exp(-t^2/2) \cos(5t)$ is the Morlet Wavelet which satisfies $\int_{\mathbb{R}} \psi(t) dt = 0$, C_{ψ} is given by the formula,

$$C_{\psi} = \int_{-\infty}^{+\infty} |\omega|^{-1} |\widehat{\psi}(\omega)| d\omega, \quad (2)$$

and $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$

$$\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty} \psi(t) e^{-i\omega t} dt. \quad (3)$$

If $C_\psi < \infty$, then the wavelet transform can be inverted (Grossmann & Morlet 1984) to give

$$f(t) = C_\psi^{-1/2} \int_0^\infty \int_0^\infty a^{-1/2} \psi\left(\frac{t-t'}{a}\right) \omega(a, t') \frac{dt' da}{a^2} . \tag{4}$$

If we just consider the value $|\omega(a, t)|$, then we obtain the results shown in Fig. 2. We can clearly see that there are mainly two pronounced periods in the relative sunspot number. One is the Schwabe cycle and the other is the Gleissberg cycle. Besides these two pronounced cycles, periods around 50-yr are also apparent for the interval from 1750 to 1850 but they fade away after 1850. From Fig. 2 we find that the amplitudes of the Schwabe cycles are very low around 1800 and around 1900.

The global wavelet power spectrum, i.e., the energy contained in all wavelet coefficients of the same scale a , as a function of a , can be written as

$$M(a) = \int |\omega(a, t)|^2 dt . \tag{5}$$

The resulting global wavelet power spectrum as a function of the period is shown in Fig. 3. We can see from Fig. 3 that the curve shows three peaks, one located at 11 yr (the Schwabe cycle), one at 101 yr (the Gleissberg cycle), and one at 53 yr.

Figure 4 shows how the amplitude of each of three periods varies in time. We can find that, around 1750 and 1800, the amplitude with the 53-yr period is stronger than that with the 11-yr period, that the amplitude of the signal with the 53-yr period is very low after 1850, and that the amplitude of the signal with the 101-yr period is always high and around 1750, 1800, and 1900, is higher than the amplitudes of the 11-yr period.

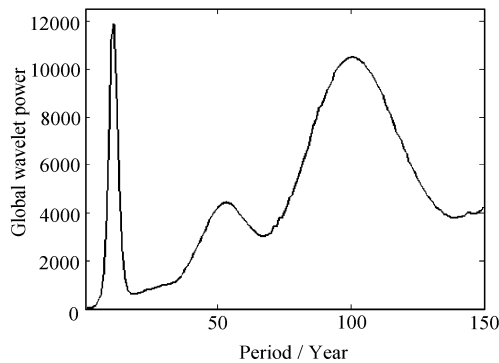


Fig. 3 Global wavelet power spectrum of the yearly mean relative sunspot numbers.

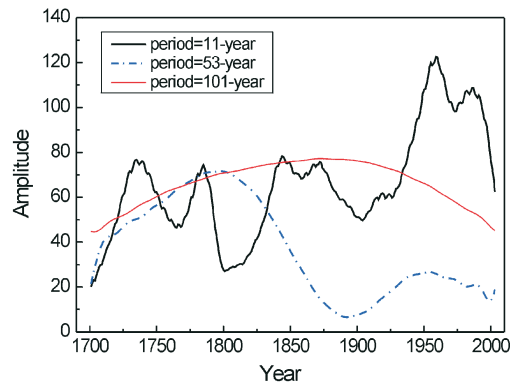


Fig. 4 Time Evolution of the amplitudes of the 11-yr, 53-yr and 101-yr periods.

3 CONCLUSIONS AND DISCUSSION

In this paper, we study several important periodic properties by using wavelet transform of the yearly mean relative numbers. The main results are summarized as follows:

- (1) There are mainly three periods in the relative sunspot number series. the two pronounced periods are the Schwabe cycles and the Gleissberg cycles, the weaker third period around 53 yr is apparent only in the interval from 1725 to 1820.

(2) Around 1800, the amplitudes of the 101-yr and 53-yr periods are much higher than that of the 11-yr period. Around 1750, 1800 and 1900, the amplitude of the 101-yr period is much higher than that of the 11-yr period. The amplitude of the 53-yr period is consistently very low after 1850.

(3) From 1940 to 2000, the amplitude of the 11-yr period is much higher than those of the 53-yr period and the 101-yr period.

The amplitude of the Schwabe cycle is very low around 1800 and 1900, as can be seen in Fig. 4. During both these times the level of sunspot activity was also very low. At present the amplitude of the Schwabe cycle is decreasing very sharply, as shown in Fig. 4. The amplitudes of the 101-yr period are never very low for the whole interval from 1700 to 2002. This may mean that this period is a relative stable cycle. It may be very useful for the prediction of the solar activity. The time span from 1800 to 1900 is 100 years, from 1900 to the present is just over 100 years. The two properties of the relative sunspot numbers may mean that the amplitude for Solar Cycle 24 will be much lower than those of Solar Cycles 21, 22 and 23, and this coincides with the results predicted by Wang et al. (2002) and Duhau (2003). Is this predicted trend right? Observational data of the relative sunspot number in the future may soon give an answer.

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