

# Dynamical Evolution of Gamma-Ray Burst Remnants with Evolving Radiative Efficiency

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**Abstract** In previous works, a generic dynamical model has been suggested by Huang et al., which is shown to be correct for both adiabatic and radiative blast-waves, in both ultra-relativistic and non-relativistic phases. In deriving their equations, Huang et al. have assumed that the radiative efficiency of the fireball is constant. They then applied their model directly to realistic cases where the radiative efficiency evolves with time. In this paper, we abandon the above assumption and re-derive a more accurate dynamical equation for gamma-ray burst remnants. Numerical results show that the model presented by Huang et al. is accurate enough in general cases.

**Key words:** gamma rays: bursts — hydrodynamics — radiation mechanisms: nonthermal

## 1 INTRODUCTION

Although the progenitors of gamma-ray bursts (GRBs) are still controversial (Cheng & Dai 2001; Cheng & Lu 2001b; Lu et al. 2000a, b), it is generally believed that energetic fireballs should be involved, where baryons are eventually accelerated to ultra-relativistic speed (Wu et al. 2001). After the main burst phase, the thin baryonic shell expands at ultra-relativistic speed into the surrounding matter, producing afterglows in soft bands (Cheng, Huang & Lu 2001; Mao & Wang 2001a, b; Gou et al. 2001a, b; Huang, Yang & Lu 2001; Zhang & Mészáros 2002). For good recent reviews on afterglow observations and theories, see van Paradijs, Kouveliotou & Wijers (2000) and Cheng & Lu (2001a).

The dynamics of the gamma-ray burst remnants is different in two cases in which the remnant expansion is either adiabatic or highly radiative (Blandford & McKee 1976, 1977). However, the conditions under which the remnant dynamics may be considered adiabatic or radiative are far from unambiguous and are crucially dependent on poorly known questions about postshock energy exchange between protons and electrons (Mészáros, Rees & Wijers

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1998). Furthermore, a partially radiative regime with decreasing radiative efficiency may exist in realistic fireballs (Dai, Huang & Lu 1999). So, it is necessary to construct a dynamical model that is able to describe a realistic fireball, i.e., a fireball with evolving radiative efficiency.

The dynamics of gamma-ray burst remnants has been studied extensively (Sari 1997; Cohen, Piran & Sari 1998; Panaitescu, Mészáros & Rees 1998; Wei & Lu 1998; Chiang & Dermer 1999; Rhoads 1999; Panaitescu & Mészáros 1999; Kobayashi, Piran & Sari 1999; Huang et al. 1998a, b, c, 1999a, b, 2000a, b, c, 2002; Dermer & Humi 2001). Especially, a generic dynamical model was proposed by Huang, Dai & Lu (1999a, hereafter HDL99), which is shown to be applicable to both ultra-relativistic and non-relativistic blastwaves, whether adiabatic or highly radiative. In their derivation, Huang, Dai and Lu implicitly assumed that the radiative efficiency of the fireball,  $\epsilon$ , is a constant during the deceleration. They then generalized their model to discuss realistic blastwaves, where  $\epsilon$  evolves with time (Huang et al. 2000a, b). In this work we will make a careful inspection of their generalization. We first repeat the derivation of HDL99, but without the  $\epsilon \equiv \text{const}$  assumption. We then compare our result with that of HDL99 numerically. It is found that Huang et al.'s generic model can be applied to realistic remnants satisfactorily.

## 2 DYNAMICS

We assume that after the initial GRB phase, the total energy left in the fireball is comparable to the radiation energy emitted in gamma-rays, i.e.,  $E_0 \sim 10^{51}\text{--}10^{52}$  erg. Denote the mass of the contaminating baryons as  $M_0$ , then the fireball continues to expand at a Lorentz factor of  $\eta = E_0/(M_0c^2)$ . Subsequently, at a radius  $R_0$ , the expansion of the fireball starts to be significantly influenced by the swept-up medium and external shock may form (Rees & Mészáros 1992). As usual,  $R_0$  is supposed to be

$$R_0 = \left( \frac{3E_0}{4\pi n m_p c^2 \eta^2} \right)^{1/3}, \quad (1)$$

where  $n$  is the number density of the interstellar medium,  $m_p$  is the mass of a proton.

### 2.1 Basic Dynamical Equations

In HDL99, a generic dynamical model that is applicable in both ultra-relativistic and non-relativistic phases of GRB afterglows has been proposed. The key point of the model is a differential equation

$$\frac{d\gamma}{dm} = - \frac{\gamma^2 - 1}{M_0 + \epsilon m + 2(1 - \epsilon)\gamma m}, \quad (2)$$

where  $\gamma$  is the bulk Lorentz factor of the fireball,  $m$  is the swept-up mass. The equation can be derived as follows. Global conservation of energy implies that

$$d[\gamma(M_0c^2 + mc^2 + U)] = dmc^2 + \gamma dU_{\text{rad}}. \quad (3)$$

Here  $U$  is the co-moving internal energy with rest-mass excluded,  $U_{\text{rad}}$  is the internal energy that is radiated from the fireball. If a fraction  $\epsilon$  of swept-up kinetic energy is instantaneously radiated from the fireball, then  $dU_{\text{rad}} = -\epsilon(\gamma - 1)dm c^2$ . The internal energy  $U$  in the fireball changes because of the change of the kinetic energy of the swept-up matter, due to expansion of the fireball and the energy loss through radiation. Thus, we assume  $U = (1 - \epsilon)U_{\text{ex}}$ , where  $U_{\text{ex}}$  is the internal energy produced in this expansion. It is usually assumed that  $dU_{\text{ex}} = (\gamma - 1)dm c^2$ .

However, the jump conditions (Blandford & Mckee 1976) at the forward shock imply that  $U_{\text{ex}} = (\gamma - 1)mc^2$ , so the correct expression for  $dU_{\text{ex}}$  under thin shell approximation should be  $dU_{\text{ex}} = d[(\gamma - 1)mc^2] = (\gamma - 1)dmc^2 + mc^2d\gamma$ . Assuming  $\epsilon \simeq \text{const}$ , then from Equation (3) we can obtain Equation (2).

It is worth noting that in the expression of  $dU_{\text{ex}} = (\gamma - 1)dmc^2 + mc^2d\gamma$ , the term  $mc^2d\gamma$  is negative when the fireball is decelerating. This term, in fact, represents the loss of internal energy due to volume expansion of the fireball, i.e., the adiabatic loss term ( $dU_{\text{adi}}$ ) defined by Dermer and Humi (2001). This can be clearly seen from equation (13) of Dermer and Humi (2001). Under thin shell approximation, their equation can be approximately simplified as  $mc^2d\gamma$ .

In the above derivation,  $\epsilon$  is assumed to be constant during the deceleration. However, in realistic fireballs,  $\epsilon$  is expected to evolve from 1 to 0 owing to the changes in the relative importance of synchrotron-induced and expansion-induced loss of energy (Dai, Huang & Lu 1999). Equation (2) has been simply generalized to the case that  $\epsilon$  evolves with time (Huang et al. 1999b, 2000a, b, c). However, this might induce some errors. Below, we will abandon the constant  $\epsilon$  assumption and derive the equations that are strictly applicable for fireballs with evolving radiative efficiency.

The assumption that  $U = (1 - \epsilon)U_{\text{ex}}$  overestimates the true internal energy, because at late stages  $\epsilon$  is near 0, but at early stages it is about 1. Instead of using  $U = (1 - \epsilon)U_{\text{ex}}$ , we use the expression  $dU = (1 - \epsilon)dU_{\text{ex}}$ . Substituting it into Equation (3), we obtain another differential equation describing the evolution of the fireball

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_0 + m + U/c^2 + (1 - \epsilon)\gamma m}, \quad (4)$$

with

$$dU = (1 - \epsilon)dU_{\text{ex}} = (1 - \epsilon)[(\gamma - 1)dmc^2 + mc^2d\gamma]. \quad (5)$$

In the highly radiative case ( $\epsilon \simeq 1$ , and  $U = 0$ ), Equation (4) reduces to the case of Blandford & McKee (1976)

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_0 + m}. \quad (6)$$

While in the fully adiabatic case ( $\epsilon \simeq 0$ , and  $U = U_{\text{ex}} = (\gamma - 1)mc^2$ ), Equation (4) reduces to the adiabatic case of HDL99

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_0 + 2\gamma m}. \quad (7)$$

In fact, taking  $\epsilon \equiv \text{const}$ , Equation (4) exactly reduces to the generic model of HDL99. If  $\epsilon$  evolves with time, however, we would expect that the fireball described by Equation (4) will decelerate more rapidly than one described by Equation (2).

## 2.2 Radiative Efficiency

According to Blandford & McKee (1976), the electron number density ( $n'$ ) and energy density ( $e'$ ) of the shocked medium in the frame co-moving with the fireball can be written as (also see Huang et al. 1998b)

$$n' = \frac{\hat{\gamma}\gamma + 1}{\hat{\gamma} - 1}n, \quad (8)$$

$$e' = \frac{\hat{\gamma}\gamma + 1}{\hat{\gamma} - 1}(\gamma - 1)nm_p c^2, \quad (9)$$

where  $\hat{\gamma}$  is the adiabatic index of the shocked medium, which is generally between 4/3 and 5/3. Equations (8) and (9) are appropriate for both relativistic and non-relativistic blastwaves. From the definition of  $\hat{\gamma}$  (Blandford & McKee 1976), Dai, Huang & Lu (1999) gave a simple and useful approximate expression for  $\hat{\gamma}$ :  $\hat{\gamma} \simeq (4\gamma + 1)/(3\gamma)$ . It can be seen from this approximation that  $\hat{\gamma} \simeq 4/3$  for an extremely relativistic blastwave and  $\hat{\gamma} \simeq 5/3$  for a non-relativistic shock.

As usual, we assume that the magnetic density in the co-moving frame is a fixed fraction  $\epsilon_B$  of the internal energy density, viz.,  $B' = (8\pi\epsilon_B e')^{1/2}$ , and that the shock-accelerated electrons behind the blastwave carry a fraction  $\epsilon_e$  of the internal energy (Huang et al. 2000a, b). This implies that the minimum Lorentz factor of the random motion of electrons in the co-moving frame is  $\gamma_{e, \min} = \epsilon_e(\gamma - 1)m_p/m_e + 1$ . We here consider only synchrotron emission from these electrons, and neglect the contribution of inverse Compton emission because the latter emission is of minor importance particularly at late times of the evolution (Waxman 1997; Dai & Lu 1998). The energy of a typical accelerated electron behind the blastwave is lost both through synchrotron radiation and through expansion of the fireball, thus the radiative efficiency of this single electron is given by  $t'_{\text{syn}}^{-1}/(t'_{\text{syn}}^{-1} + t'_{\text{ex}}^{-1})$  (Dai & Lu 1998; Dai, Huang & Lu 1999), where  $t'_{\text{syn}}$  is the synchrotron cooling time,  $t'_{\text{syn}} = 6\pi m_e c/(\sigma_T B'^2 \gamma_{e, \min})$ , and  $t'_{\text{ex}} = R/(\gamma c)$  is the co-moving frame expansion time. Here  $R$  is the radius of the blastwave. Since all of the accelerated electrons behind the blastwave carry only a fraction  $\epsilon_e$  of the internal energy, the radiative efficiency of the fireball can be given by (Dai, Huang & Lu 1999)

$$\epsilon = \epsilon_e \frac{t'_{\text{syn}}^{-1}}{t'_{\text{syn}}^{-1} + t'_{\text{ex}}^{-1}}. \quad (10)$$

In the highly radiative case,  $\epsilon_e \simeq 1$  and  $t'_{\text{syn}} \ll t'_{\text{ex}}$ , we have  $\epsilon \simeq 1$ . The early evolution of the remnants is likely to be in this regime. For an adiabatic expansion,  $\epsilon_e \ll 1$  or  $t'_{\text{syn}} \gg t'_{\text{ex}}$ , we obtain  $\epsilon \simeq 0$ , which regime we believe to apply to the late evolution. In realistic case, the radiative efficiency of the fireball ( $\epsilon$ ) evolves from about 1 to 0 (Huang et al. 2000a).

### 2.3 Numerical Results

The evolution of the radius and swept-up mass are described by (Huang et al. 1998a, 2000a, b)

$$dm = 4\pi R^2 n m_p dR, \quad (11)$$

$$dR = \beta c \gamma (\gamma + \sqrt{\gamma^2 - 1}) dt, \quad (12)$$

where  $t$  is the time measured in the observer's frame. Then Equations (4) and (5) can be solved numerically.

Figure 1 compares the evolution of the Lorentz factor calculated according to Equations (2) and (4). In our calculations, we take  $E_0 = 10^{52}$  erg,  $n = 1 \text{ cm}^{-3}$ ,  $M_0 = 2 \times 10^{-5} M_\odot$ ,  $\epsilon_e = 1.0$ ,  $\epsilon_B = 0.01$ . In both cases, Equation (10) is used to describe the evolution of  $\epsilon$ . We see that, as expected above, the bulk Lorentz factor of the fireball ( $\gamma$ ) calculated by Equation (4) (the solid line) declines more rapidly than that of Equation (2) (the dashed line), but we notice that the difference here is slight. Figure 2 shows the time dependence of the blastwave radius ( $R$ ). Figure 3 shows the evolution of the radiative efficiency of the realistic fireball ( $\epsilon$ ).

The relation between the radius ( $R$ ) and the fireball momentum ( $P = (\gamma^2 - 1)^{1/2}$ ) is shown in Figure 4. The solid line is the case when  $\epsilon$  evolves according to Equation (10). The dashed line is the adiabatic case, i.e.,  $\epsilon \equiv 0$ . The dotted line is the highly radiative case, viz.,  $\epsilon \equiv 1$ . We can see that, at early times when the realistic fireball is ultra-relativistic and highly radiative,

the solid line approximately satisfies  $P \propto R^{-3}$ . At late times when the fireball is non-relativistic and adiabatic, the deceleration is approximately  $P \propto R^{-3/2}$ , consistent with the Sedov limit.

We emphasize that for the  $\epsilon \equiv \text{const}$  cases, the results are precisely the same in the two models characterized by Equation (2) and Equation (4).

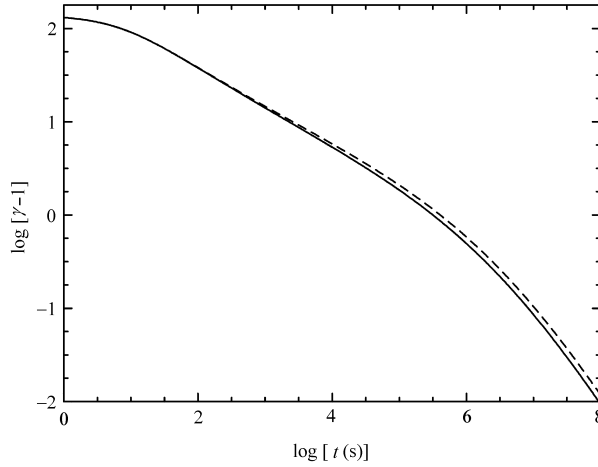


Fig. 1 Evolution of the bulk Lorentz factor ( $\gamma$ ). The dashed line corresponds to Eq. (2). The solid line is drawn according to Eq. (4). Parameters:  $E_0 = 10^{52}$  erg,  $n = 1 \text{ cm}^{-3}$ ,  $M_0 = 2 \times 10^{-5} M_\odot$ ,  $\epsilon_e = 1.0$  and  $\epsilon_B = 0.01$ .

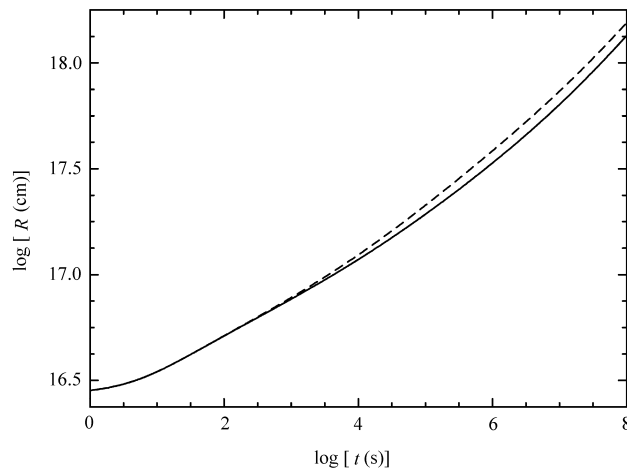


Fig. 2 Evolution of the shock radius ( $R$ ). Parameters and line styles are the same as in Fig. 1.

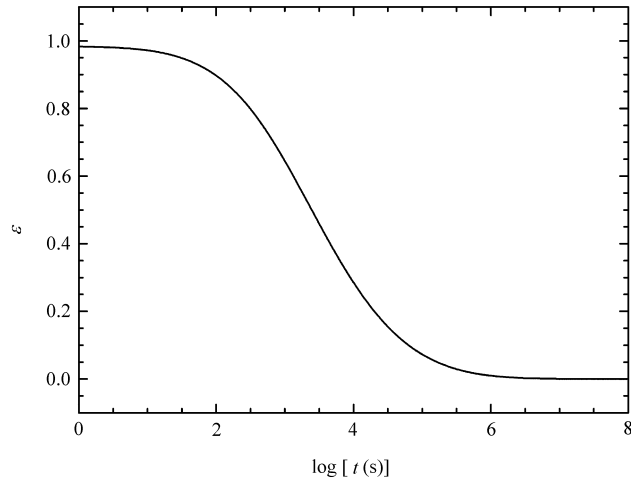


Fig. 3 Evolution of the radiative efficiency of the fireball ( $\epsilon$ ). Parameters are the same as in Fig. 1.

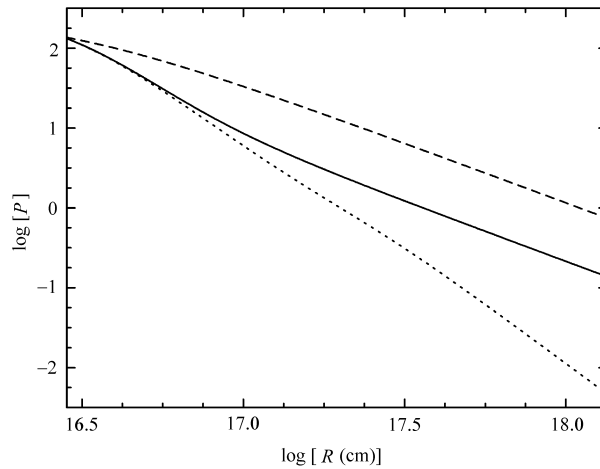


Fig. 4 Evolution of the fireball momentum ( $P$ ) under different assumptions for the radiative efficiency of the fireball ( $\epsilon$ ). The solid line is the case when  $\epsilon$  evolves according to Equation (10). The dashed line is the adiabatic case, i.e.,  $\epsilon \equiv 0$ . The dotted line is the highly radiative case, viz.,  $\epsilon \equiv 1$ . Parameters are the same as in Fig. 1.

### 3 LIGHT CURVE

In Section 2, the dynamical evolution of a postburst fireball has been calculated numerically. As in Dai, Huang & Lu (1999), we calculate the light curves of optical afterglows. The results

are shown in Figure 5. Here the solid line is drawn by using the dynamics of Equation (4) and the dashed line, by using Equation (2). We see that the difference between the two curves is not large. Flux densities on the dashed curve are higher by about 2 after the peak, but the slopes of the two curves are identical.

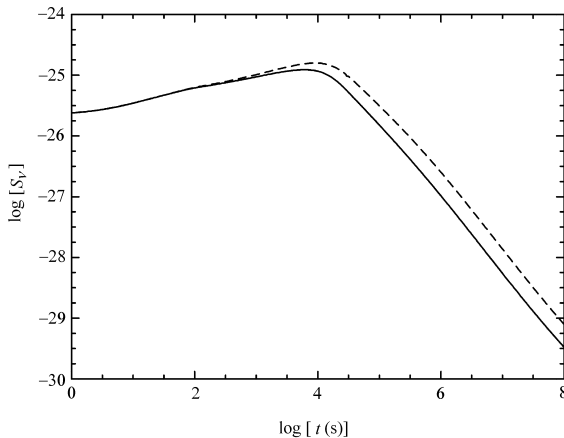


Fig. 5 Predicted afterglow light curves in fixed frequency  $\nu = 10^{15}$  Hz.  $S_\nu$  is in units of  $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$ . The dashed line corresponds to the generic model of HDL99, and the solid line corresponds to Eq. (4). Parameters adopted:  $E_0 = 10^{52}$  erg,  $n = 1 \text{ cm}^{-3}$ ,  $M_0 = 2 \times 10^{-5} M_\odot$ ,  $\epsilon_e = 1.0$ ,  $\epsilon_B = 0.01$ ,  $p = 2.1$ , and  $D = 1 \text{ Gpc}$ . Note that after  $\sim 10^4$  s, the two curves have nearly the same slope.

#### 4 DISCUSSION AND CONCLUSIONS

The generic model of HDL99 is applicable to both radiative and adiabatic fireballs, and during both ultra-relativistic and non-relativistic phases. A problem is whether this model is correct or not when the radiative efficiency of the blastwave ( $\epsilon$ ) evolves with time. We have shown that in this case, for the evolution of  $\gamma$  and  $R$ , the errors induced in the generic model are almost negligible. The errors in the optical light curves are slightly amplified due to the strong dependence of flux density on the Lorentz factor, but the results are still acceptable. We suggest that the generic model in its simple form of equation (7) in HDL99 could be safely used when  $\epsilon$  varies during the deceleration.

A dynamical model that is applicable to both relativistic and non-relativistic expansion has been established for quasars and active galactic nuclei by Blandford & McKee (1977). Their dynamics is most convenient for either adiabatic or highly radiative blastwaves, even allowing for steady injection of energy into the remnant from the central engine. However, for partially radiative blastwaves, especially blastwaves with an evolving efficiency, the simple generic dynamical model of HDL99 is still more convenient.

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