A Two-Temperature Supernova Fallback Disk Model for Anomalous X-ray Pulsars

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Abstract We present a case study of the relevance of the radially pulsational instability of a two-temperature accretion disk around a neutron star to anomalous X-ray pulsars (AXPs). Our estimates are based on the approximation that such a neutron star disk with mass in the range of $10^{-6} - 10^{-5} M_\odot$ is formed by supernova fallback. We derive several peculiar properties of the accretion disk instability: a narrow interval of X-ray pulse periods; lower X-ray luminosities; a period derivative and an evolution time scale. All these results are in good agreement with the observations of the AXPs.

Key words: pulsars: general — pulsars — stars: neutron — X-rays: stars — accretion disk — instability

1 INTRODUCTION

Anomalous X-ray Pulsars (AXPs) are sources of pulsed X-ray emission with relatively low persistent X-ray luminosities, $L_X \sim 10^{35} - 10^{36} \text{ erg s}^{-1}$, and soft spectra that are well fitted by a combination of blackbody and power-law contributions, with effective temperatures and photon indices in the range of $T_e \sim 0.3 - 0.4 \text{ keV}$ and $\Gamma \sim 3 - 4$, respectively. They have relatively long spin periods of about $P \sim 6 - 12 \text{ s}$, which increase steadily with time. Their characteristic ages are about $P/2\dot{P} \sim 10^3 - 10^5 \text{ yr}$. The energy source of the AXPs is not understood, hence their designation as anomalous (Chatterjee & Hernquist 2000). Unlike binary X-ray pulsars, no binary companions have been detected for these objects, and observations have placed strong constraints on companion masses (Mereghetti, Israel & Stella 1998; Wilson et al. 1999). So the energy cannot be supplied by accretion of matter from a companion star.

At present, two models are generally invoked to explain the properties of AXPs: accretion from a large disk left over from the birth process (Mereghetti & Stella 1995; van Paradijs, Taam & van den Heuvel 1995; Ghosh, Angelini & White 1997), or decay of a very strong magnetic field ($10^{15} \text{ G}$) associated with a ‘magnetar’ (Heyl & Hernquist 1997a, 1997b; Thompson & Duncan 1996; Hulleman, van Kerkwijk & Kulkarni 2000). More recently, Chatterjee, Hernquist & Narayan (2000, hereafter CHN) developed another accretion model in which AXPs are neutron

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stars with standard magnetic fields that are accreting from a disk formed after fallback of material from a supernova explosion. It has been observed that at least three AXPs have been associated with young supernova remnants, which limit their ages to $10 - 20$ kyr (Chatterjee & Hernquist 2000). The evolution of supernova fallback accretion disk was first proposed by Michel (1988) in connection with the evolution of a young neutron star. Some of the fallback (fallback is a standard ingredient of contemporary core-collapse supernova scenarios) material will have a specific angular momentum in excess of the Keplerian value at the surface of the central compact object and therefore are expected to settle into a disk as it cools. Alpar (2001) also presented an accretion scenario for AXPs. Menou, Perna & Hernquist (2001, hereafter MPH) considered the stability and evolution of supernova fallback disks in more detail, with particular application to AXPs and young radio pulsars.

In this paper we propose an alternative scenario that will reproduce the inferred distributions of AXP spin periods, ages, and their luminosities for reasonable and broad distributions of the model parameters. We argue that the observed characteristics of the AXP and its associated SNR can be explained in terms of a neutron star with a weak magnetic field ($B \sim 10^8$ G) surrounded by a two-temperature disk.

It is known that accretion from the disk onto a neutron star can occur if $r_A \leq r_c$ (Lamb, Pethick & Pines 1973), where $r_A$ is the Alfvén radius at which the magnetic pressure originating from the magnetic field of the compact star exceeds the gas pressure in the disk, $r_c$ is defined as the radius at which the Kepler period equals the period of rotation of the compact object. Shapiro, Lightman & Eardley (1976, hereafter SLE) pointed out that a hot two-temperature accretion disk around a black hole could lie within the “inner region” of the cooling disk (Shakura & Sunyaev 1973). This kind of a two-temperature accretion disk around a black hole could be extended to the case of a disk around a neutron star according to an early work by Lightman (1974), if $r_A \leq r_c$. We construct such a two-temperature accretion disk around neutron star from the fallback of supernova ejecta, similar to the neutron star disk model of Michel (1988). Such a disk evolves for thousands of years.

A two-temperature disk is fruitful to model the properties of AXPs from the following considerations: First, in the two-temperature disk, the electron temperature is about $10^9$ K and the ion temperature is one or two orders higher. Therefore, the two-temperature disk has a much higher temperature than the cooling optically thick disk. Even though the electron temperature is about $10^9$ K and the ion temperature is even as high as $10^{11}$ K in the two-temperature region of the present disk model, hard X-ray and/or even gamma-ray photons must undergo a number of scattering in the cool, “middle” and “inner” regions of the disk outside the two-temperature region before escape. As argued by SLE, this comptonization process is expected to lead to soft X-ray emission, and can produce the observed X-ray spectrum. Secondly, pulsational instability is possible in a two-temperature accretion disk because, in some regions, the efficiency of turbulent angular momentum transport can increase to local compression. In this case, if a compressional normal mode of oscillation is established, the thermal energy generation arising from viscous dissipation of the shear flow increases during compression and can result in amplification of the oscillation. This effect is entirely analogous to the way by which nuclear energy generation can cause stellar pulsations to be overstable (Kato 1978). Previous studies of the instability of a two-temperature accretion disk around a compact object have indicated that there is a pulsational instability (inertial-acoustic mode instability, Wu 1997). The period of the acoustic mode has been examined by Yang et al. (1997) in the case of a magneto-viscous disk. We use the pulsational instability to explain the feature
of persistent X-ray emission in SGR 1806-20. In Section 2 we discuss the main features, the thermal ionization instability and the radially pulsational instability of the supernova fallback disk. In Section 3 we model the X-ray pulse observed in AXPs. In Section 4 we present a discussion and our conclusions.

2 STRUCTURE AND INSTABILITY OF A TWO-TEMPERATURE SUPERNOVA FALLBACK DISK

2.1 Physical Assumptions and Model Parameters of a Two Temperature Disk

Our system consists of a neutron star (with magnetic field $B \sim 10^8$ G and mass $M = 1.4M_\odot$) and a surrounding equipartition accretion disk. One crucial, but complicated feature of neutron star accretion is the presence of a strong magnetic field extending from the stellar surface outward into the accretion disk (Lamb, Pethick & Pines 1973). However, far from the neutron star the magnetic field has little influence on the accreting flow. This condition specifies the characteristic radius of the magnetospheric boundary, $r_A$, called the Alfvén radius:

$$r_A = 1.6 \times 10^6 M_\odot^{-2/7} B_8^{4/7} R_6^{12/7} \left( \frac{M}{1.4M_\odot} \right)^{-1/7} \text{ cm},$$

(1)

at which the magnetic pressure equals the gas pressure. Here $R_6$ is the neutron-star radius in units of $10^6$ cm. If accretion occurs at radius $r > r_A$, then the disk flow solutions for the black hole-disk system remain valid for the neutron star-disk case. Due to the secular instability (Lightman & Eardley 1974), the probable formation of a two-temperature disk occurs within the inner region of the standard disk (Shakura & Sunyeav 1973). It takes place at radius $r_0$ according to the relation (SLE)

$$r_0^{21/8} \zeta^{-2}(r_0) \approx 10^4 \alpha^{1/4} M_*^{-7/4} \dot{M}^2,$$

(2)

where $M_* = M/3M_\odot$, $\dot{M}_* = \dot{M}/(10^{17}$ g s$^{-1}$) and $r_*= r/GM/c^2$, $\zeta$ expresses the boundary condition that the viscous stress must vanish at the inner edge of the disk, and $\zeta = 1 - (r_0/r_*)^{-1/2}$. Thus, for a given $M$ and $\alpha$, $r_0$ is a function of $\dot{M}$ determined by the luminosity of the disk. If we assume that most of the radiation generated in the two-temperature inner region is observed, the integrated luminosity from the inner edge of the disk to $r_0$ is

$$L_{r_0} = \left[ 1 + 2 \left( \frac{r_0}{6r_0} \right)^{-3/2} - 18/r_0 \right] \dot{M} c^2/12.$$

(3)

Combining Eq.(2) and Eq.(3), the condition $L_{r_0} \sim 4 \times 10^{35}$ erg s$^{-1}$ yields

$$\left[ 1 + 2 \left( \frac{r_0}{6} \right)^{-3/2} - 18/r_0 \right] r_0^{21/16} \left[ 1 - \left( \frac{r_0}{6} \right)^{-1/2} \right]^{-1} = 2.7 \lambda ,$$

(4)

where $\lambda \equiv (\alpha M_*^{-7})^{1/8}$. Since $\lambda$ is insensitive to $\alpha$, $r_0$, and $\dot{M}$ are uniquely determined for a given $M$, by Eq.(3). Given the value of $\alpha$, we obtain

$$\alpha = 3 \times 10^{-4}, \quad r_0 = 7.41, \quad \dot{M} = 5.67 \times 10^{17}$ g s$^{-1}$. $$

From the above calculation and discussion, it can be noted that the innermost radius, $r_0$, satisfies the condition $r_0 \geq r_A$. So, the disk flow solutions for a two-temperature disk ($r_A < r < r_0$)
examined by SLE are valid in the present case. The existence of such a disk requires that the radius of the central neutron star should be smaller than $r_0 \sim 15$ km. A neutron star with this requirement is also consistent with realistic modern equations of state for dense matter (Wiringa, Fiks & Fabrocinii 1988). We assume that the disk is geometrically thin and optically thin and the two-temperature region is a narrow region ($\Delta r/r \ll 1$) which probably occurs within the cooling disk inner region. In this paper, the “outer” and “middle” regions of the cool disk (Shakura & Sunyaev 1973) describe the outer portions of the present disk model. The inner region of the cool disk, whose outer boundary lies at the point where $P_G = P_R$ extends inwards to the radius $r_0$, where $P_R = 3P_G$. $P_R$ and $P_G$ are the radiation pressure and gas pressure, respectively. The radius $r_0$ marks the outer boundary of “the two-temperature inner region”, which then extends inward to the innermost radius $r_A$. The present disk is therefore identical to the standard disk (Shakura & Sunyaev 1973) for $r \geq r_0$, and is described by the two-temperature structure equations for $r < r_0$. Following SLE, we outline the main features and basic equations of a two-temperature equipartition disk in terms of the ion and electron temperatures, $T_i$ and $T_e$. The following equations describe separately the five disk structure variables, the density $\rho$, the disk thickness $h$, the pressure $p$, ion and electron temperatures $T_i$, and $T_e$:

$$T_e = 7 \times 10^8 (M_\ast M_*^{-1} \alpha^{-1} \zeta^{-1/6} r_*^{1/4}) \ K,$$

$$T_i = 5 \times 10^{11} M_*^{5/6} \alpha^{-7/6} \zeta^{-5/6} r_*^{-5/4} \ K,$$

$$h/r \sim 0.2 M_*^{-5/12} \alpha^{-7/12} \zeta^{5/12} r_*^{-1/8},$$

$$\rho = 5 \times 10^{-5} M_*^{-3/4} \alpha^{-1/4} \zeta^{-1/4} r_*^{-8/9} \ g \ cm^{-1},$$

$$P = 2 \times 10^{15} M_*^{-19/12} \alpha^{-5/12} \zeta^{7/12} r_*^{-19/8} \ dyn \ cm^{-2}.$$  

If the two-temperature accretion disk is assumed to be a magnetic equipartition disk (Lightman & Eardley 1974; Sakimoto & Coroniti 1981), then important quantities may be derived from the above equations, e.g.,

$$B_d = 10^8 M_*^{7/24} M_*^{-19/24} \alpha^{-5/24} \zeta^{7/24} r_*^{-19/16} \ G,$$

where $B_d$ is the magnetic field of the disk.

### 2.2 Radially Pulsational Instability

It is known that the two-temperature accretion disk of SLE is thermally unstable, this was first shown by Piran (1978), and has since been referred to from time to time by other authors (Abramowicz et al. 1995; Narayan & Yi 1995b; Chen 1996; Kato, Abramowicz & Chen 1996; Wu & Li 1996). Recent studies of the global stability and thermal ionization instability for supernova fallback disks are discussed by MPH (2001). In the view of MPH, the relevance of the instability to supernova fallback disks probably means that their power-law evolution breaks down when they first become neutral. When the accretion rate $\dot{M}(R)$ of the fallback disk is higher than a critical value $\dot{M}_{\text{crit}}^+(R)$ everywhere, it is ionized and thermally (as well as viscously) stable (MPH). This criterion argued by MPH is applied to the two-temperature fallback disk in this paper. We emphasize that the pulsational instability of the two-temperature fallback disk with higher accretion rate can operate before it first becomes neutral. The stability properties of the pulsational instability (also called inertial acoustic mode instability) were first addressed.
in viscous accretion disks by Kato (1978), and later studied in detail by Blumenthal, Yang & Lin (1984). It has been suggested that the acoustic instability may account for the observed quasi-period oscillations (QPO) in some Galactic black hole candidates (Chen & Taam 1995). For a purely viscous disk, the period of the pulsational instability (Wu et al. 1995) is about \((\alpha \Omega)^{-1}\), where \(\Omega^{-1}\) is the local Keplerian time scale and \(\Omega^{-1} \approx \sqrt{r^3/GM}\), \(r\) being the radius at which the instability occurs. This period of pulsational instability is also confirmed by Yang, Henning, Lu & Wu (1997). As discussed above, the period of the pulsational instability is

\[ P \approx (\alpha \Omega)^{-1}. \]  

(11)

For the case of neutron star-disk systems, the pulsational instabilities are gradually damped down in time. From Eq.(11), the period derivative \(\dot{P}\) is

\[ \dot{P} = \frac{3}{2} P \frac{1}{r} \frac{\partial r}{\partial t}. \]  

(12)

Using \(v_r = \frac{\partial r}{\partial t}\) and \(\dot{M} = 4\pi \rho hr v_r\), we can rewrite Eq.(11) as

\[ \dot{P} = \frac{3}{2} P \frac{\dot{M}}{M_d} (h/r)^{-1}, \]  

(13)

where \(M_d\) is the mass of the disk at any time \(t_n\), and \(M_d = 4\pi \rho hr^2\). However, the disk structure depends significantly on the radius, which implies that disk instability may be quite different from one radius to another. As a consequence, the values of the pulsational instability period and the period derivative vary with different models. We define an effective radius, \(r_{\text{eff}}\), to be the radius where the disk is unstable against radial pulsation and list the parameters of our calculated models in Table 1. According to the above discussion, we start the numerical calculation from the two-temperature disk given by SLE, corresponding to a given accretion rate \(\dot{M}\) (Eq.(2)) at a radius \(r_0\) for a given \(\alpha\). When the observed pulsed X-ray luminosity is given, the pulsational instability radius \(r_{\text{eff}}\) is determined by Eq.(3).

### Table 1

<table>
<thead>
<tr>
<th>Model</th>
<th>(\alpha)</th>
<th>(L_{r0})</th>
<th>(r_{\text{eff}})</th>
<th>(P)</th>
<th>(\dot{P})</th>
<th>(M_d)</th>
<th>(M)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1.09 \times 10^{-5}</td>
<td>4 \times 10^{35}</td>
<td>6.91</td>
<td>6</td>
<td>3 \times 10^{-11}</td>
<td>2.24 \times 10^{-5}</td>
<td>2.96 \times 10^{17}</td>
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<td>2</td>
<td>9.4 \times 10^{-6}</td>
<td>4 \times 10^{35}</td>
<td>6.88</td>
<td>8</td>
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<td>3.15 \times 10^{-6}</td>
<td>3.14 \times 10^{17}</td>
</tr>
<tr>
<td>3</td>
<td>6.57 \times 10^{-6}</td>
<td>4 \times 10^{35}</td>
<td>6.86</td>
<td>10</td>
<td>3 \times 10^{-11}</td>
<td>4.14 \times 10^{-5}</td>
<td>3.29 \times 10^{17}</td>
</tr>
<tr>
<td>4</td>
<td>5.47 \times 10^{-6}</td>
<td>4 \times 10^{35}</td>
<td>6.84</td>
<td>12</td>
<td>3 \times 10^{-11}</td>
<td>5.17 \times 10^{-4}</td>
<td>3.42 \times 10^{17}</td>
</tr>
</tbody>
</table>

It is known that the mechanism of exciting the disk radially pulsational instability involves the thermal and dynamical processes of the viscosity in the disk (Kato 1978). The bulk of thermal energy is supplied via viscous dissipation. For the four models listed, we find the last time scale of the radially pulsational instability of the accretion disk, \(\tau = M_d/\dot{M}\),

\[ \tau = 7200 \text{ yr}, \quad \text{(with model 1)} \]  

(14)

\[ \tau = 950 \text{ yr}, \quad \text{(with model 2)} \]  

(15)

\[ \tau = 1200 \text{ yr}, \quad \text{(with model 3)} \]  

(16)

\[ \tau = 1.44 \times 10^4 \text{ yr}, \quad \text{(with model 4)} \]  

(17)
3 COMPARISON WITH OBSERVATIONS OF AXPs

In this section we compare the observed features of AXPs with our two temperature equipartition disk model. Persistent X-ray sources and emission periods \( P \) were discovered to be associated with the AXPs. The period derivative, \( \dot{P} \), of the pulses can be found by \( P / t_{\text{snr}} \), where \( t_{\text{snr}} \) is the age of the supernova remnant (SNR). The observed properties of the persistent X-ray emission for AXPs are summarized as follows:

1. The observed pulse periods of the persistent X-ray emission in AXPs narrowly range from 6 s to 12 s, while the calculated pulsational instability periods listed in Table 1 fall in this range. Thus, the pulse periods of the X-ray emission in AXPs may be reproduced by our model.

2. The period derivative of the pulses in AXPs is in agreement with our model, when the mass of the accretion disk is \( \sim 5 \times 10^{-6} M_\odot \) to \( \sim 5 \times 10^{-4} M_\odot \) and the viscous parameter is in a certain range.

4 DISCUSSION AND CONCLUSIONS

We have shown how the pulsational instability operating in the two temperature accretion disk can be used to model the pulse periods and period derivatives of the persistent X-ray pulse seen in the AXPs. We stress here that these estimates of the model are only approximate, because we do not know the exact mass of the fallback material that makes the disk, therefore, there are still some uncertainties on the accretion rates, the age and the size at which supernova fallback disks are expected to become neutral under the action of the thermal ionization instability. We ignore its precise stability properties, and the pulsational instability can operate in the fallback two-temperature disk depending on the model parameters. Adjusting the initial physical parameters of fallback disk, the pulsational instability can operate before the two-temperature fallback disk become neutral. Our main results are:

1. For a given central object mass \( M \) and viscous parameter (\( \alpha \)), the period of the pulsational instability (\( \text{cf. Eq.}(11) \)) is determined by the effective radius \( r_{\text{eff}} \) where the instability occurs. For the effective radii roughly ranging from 6.84 to 6.91, the observed pulse periods and amplification magnitudes of the AXPs may be reproduced by our model. Because of the very small size inferred for the disk, the optical emission is likely to be dominated by viscous dissipation, so that no optical pulsations would be expected in this AXPs model, as discussed by MPH.

2. The accretion disk mass \( M_d \) ranging from \( 1 \times 10^{-6} M_\odot \) to \( 1 \times 10^{-4} M_\odot \) is chosen so as to be consistent with the observed period derivative \( \dot{P} \) of the X-ray pulse for AXPs. This neutron star accretion disk can be constructed from a supernova fallback. In fact, Michel (1988) examined a simple model of gravitational fallback of supernova ejecta forming a neutron star disk with a mass of the order of \( 10^{-5} M_\odot \), and this disk would requires thousands of years to be removed. The period is longer than the observed X-ray luminosity produced by the cooling of the neutron star. These features are in good agreement with our model.

3. For most low-mass X-ray binaries (LMXBs), it is believed that the persistent LMXBs, such as Cyg X-2 and Sco X-1, have accretion rates (luminosities) comparable to those in the persistent X-ray emission of SGRs. However, the observed pulse periods and X-ray luminosities of LMXBs are absent in our model. An important difference between AXPs and LMXBs is
that the mass $M_d$ of accretion disk is very different. In our model the observed limited period derivative range reflects that the mass range of AXP accretion disks is limited (cf. Table 1). It is known that the mass of the accretion disk in a LMXB is $\sim 0.1 M_\odot$, which is much larger than the $M_d \sim 10^{-6} M_\odot$ in AXP systems. Because $\alpha \sim \Sigma^{-8/3}$ in the inner region of the disk (Lightman 1974), where $\Sigma = 2\rho h$, this means that the more material is accreted, the less the viscosity is, for a given accretion rate and accretion time. Considering the condition of constructing a two temperature disk given by SLE, we find that the inequality $\alpha^{1/4} \dot{M}_d^2 M_*^{-7/4} \geq 0.6$, cannot be satisfied in the case of LMXBs. Thus, a two-temperature equipartition disk cannot exist in LMXBs, and hence the observed properties of AXPs cannot occur in LMXBs. Furthermore, LMXBs are binaries system, while AXPs have several striking similarities with a new group discovered in 1980’s; called soft Gamma ray bursts (Mereghetti & Stella 1995; Van Paradijs et al. 1995; Duncan & Thompson 1995), which are very different from normal binary X-ray pulsars. This provides evidence against a binary companion.

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