A stellar ranging scheme based on the second-order correlation measurement

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Abstract

The stellar ranging is the basis for stellar dynamics research and in-depth research on astrophysics. Parallax method is the most widely used and important basic method for stellar ranging. However, it needs to perform high-precision measurement of the parallax angle and the baseline length together. We aim to propose a new stellar ranging scheme based on second-order correlation that does not require a parallax angle measurement. We hope our solution to be as basic as the parallax method. We propose a new stellar ranging scheme by using the offset of second-order correlation curve signals. The optical path difference between the stars and different base stations is determined by the offset of the second-order correlation curve signals. Then the distance of the stars could be determined by the geometric relation. With the distance to stars out to 10kpc away, our astrometric precision can be better compared to Gaia by simulation. We also design an experiment and successfully prove the feasibility of this scheme. This stellar ranging scheme makes it possible to make further and more accurate stellar ranging without using any prior information and angle measurement.

Key words: astrometry — techniques: miscellaneous — methods: observational — methods: miscellaneous — instrumentation: miscellaneous

1 INTRODUCTION

The determination of star distance is the foundation for studying the size, structure and morphology of galaxies. It is also the basis for stellar dynamics research and in-depth research on astrophysics. The development of astronomy is inseparable from the development of ranging. The trigonometric parallax method is undoubtedly the most basic ranging method [Reid et al. 2014; Zhang et al. 2017; Mignard 2019; Reid et al. 2009]. As the Earth revolves around the Sun, the observer can see that the star draws a circle or ellipse or line within a year on the celestial sphere depending on the position of the star. The annual parallax of the star can be calculated based on the position of the star separated by two measurements. On the basis of the trigonometric parallax methods such as the luminosity parallax and the mechanical parallax are successively developed. Last century, the Hipparcos satellite released the catalogue which
contains the position, parallax and proper motion of 117 955 stars with a precision of 0.001 arc
seconds and the stellar distances extending out to more than 300 light-years. Today, with an
astrometric precision of up to 0.00001 arc seconds, Gaia will determine distances to stars out
30 000 light-years away – one hundred times farther than Hipparcos[Smith & Eichhorn 1996;
Maíz Apellániz et al. 2018; Gaia Collaboration et al. 2021; Madore & Freedman 1998].
In 1956, Hanbury Brown and Richard Quentin Twiss introduced the second-order correlation
of the light field in the measurement of stellar angular diameter[Hanbury Brown 1956].
Since then, people have studied the second-order correlation of light field deeply, which opened
the field of quantum optics research. In recent years, some researchers have studied the rang-
ing technology of emitting pseudo-thermal light and measuring the second order correlation
2004; Zhu et al. 2013; Zhu et al. 2012]. Just as laser ranging technology needs to emit laser,
this method needs observer to emit pseudo-thermal light. This method has advantages of high
precision, no measured dead zone, strong anti-noise ability, etc. However, this method can only
be used for close range and cannot be used for stellar ranging, which is similar to laser ranging
technology.
In this paper, we first propose a stellar ranging method based on measuring the second-order
correlation of the light field. We can get the distance difference between multi-observers and
the light source by measuring the characteristic peak or dip of \( g^{(2)} \) curve, and get the baseline
distance by laser ranging, finally get the light source distance, as long as the light source does
not emit laser, such as single photon, thermal light and entangled light. Most stars emit thermal
light, and the full width at half maximum (FWHM) of \( g^{(2)} \) curve is only dozens of femtoseconds,
leading to high accuracy of time measurement[Boitier et al. 2009]. Recently, the ghost image
using sun as a light source was done by measuring the \( g^{(2)} \) of sun, showing the feasibility of
measuring the \( g^{(2)} \) of star[Karmakar et al. 2011; Liu et al. 2014]. Simulation results show that our
method has a longer range and higher measurement accuracy than the trigonometric parallax
method. In our principle experiment, we produced thermal light, single photon and entangled
light based on SPDC process[Chang et al. 2014; Burnham & Weinberg 1970; Kwiat et al. 1995;
Ling et al. 2008] and carried out the principle demonstration and the verification experiment
of distance difference measurement, which showed that our results with three kinds of light
sources had good consistency in measuring distance, and the measurement accuracy was only
constrained by the system’s time measurement accuracy.

2 THEORY

2.1 Second-order correlation function

The second-order correlation function defined in quantum optics is[Glauber 1963c,b,a; Mandel
& Wolf 1995]

\[
G^{(2)}(r_1, r_2; t_1, t_2) = tr\{E^-_1(r_1, t_1)E^+_2(r_2, t_2)E^+_2(r_2, t_2)E^-_1(r_1, t_1)\}
\]

(1)

Where \( E^+(r, t) \), \( E^-(r, t) \) represent the positive and negative frequency components of the light
field respectively. Normalize it, we get the following equation:

\[
g^{(2)}((r_1, t_1), (r_2, t_2)) = \frac{tr\{\rho E^-_1(r_1, t_1)E^-_2(r_2, t_2)E^+_2(r_2, t_2)E^+_1(r_1, t_1)\}}{\sqrt{tr\{\rho E^-_1(r_1, t_1)E^+_1(r_1, t_1)\}tr\{\rho E^-_2(r_2, t_2)E^+_2(r_2, t_2)\}}}
\]

(2)

By analyzing the second-order correlation function of multi-mode thermal light field, the fol-
lowing equation can be obtained:

\[
g^{(2)}(\tau^+) = 1 + e^{-(\tau^2)^2}
\]

(3)
In which \( \tau^\dagger = \Delta - \tau \). \( \Delta \) is the optical path difference from the source to \( r_1 \) and \( r_2 \), \( \tau = t_1 - t_2 \) is the time delay. In general, if \( g^{(2)}(\tau^\dagger) > 1 \), it is shown as the bunching effect of photons. In other words, photons in a thermal state tend to come in pairs.

For particle number state \( |n\rangle \),

\[
g^{(2)}(\tau^\dagger = 0) = 1 - \frac{1}{n}
\]

(4)

Obviously it’s less than 1. Therefore, the particle number states have obvious anti-bunching effect. In particular, when the number of photons \( n=1 \), that is, the single photon state

\[
g^{(2)}(\tau^\dagger = 0) = 0
\]

(5)

The laser always has the following equation:

\[
g^{(2)}(\tau^\dagger) = 1
\]

(6)

The \( g^{(2)} \) curve of the light field could be measured by coincidence counting [Valencia et al. 2002; Gatti et al. 2003].

2.2 Ranging principle

When \( \tau^\dagger = 0 \), \( g^{(2)} \) of thermal light (or a non-classical light) will be the maximum (or minimum), therefore, we can use the peak or dip position of the \( g^{(2)}(\tau) \) to calibrate \( \Delta \), the distance of the optical path difference (shown in Fig. 1). An example is as follows:

Assuming that in the base station A and B, we detected the light signal from the star respectively and measure \( g^{(2)} \). The results of measurement will be shown in Fig. 2. AA signal is the \( g^{(2)} \) result of numerical simulation of two observers both located at base station A, with no optical path difference and time difference. AB signal is showing the \( g^{(2)} \) of two observers respectively located at base station A and B.

As shown in the Fig.2, when an observer moves from base station A to base station B, the peak position of \( g^{(2)} \) is shifted from position of AA (\( \tau = 0 \)) to position of AB. In Fig. 2, the peak shifts by 6\( \mu \)s, that is to say, the arrival time difference of optical signal is 6\( \mu \)s. Thus, we obtain the optical path difference \( \Delta = c\tau \) between the star and stations based on the shift of \( g^{(2)} \) curve’s characteristic peak.

From this, we conclude that the abscissa of \( g^{(2)} \) curve peak (or dip) is 0 only if the arrival time difference between the optical signals of two observers is 0. When there is a difference in the flight time of the optical signal, it is reflected in that the \( g^{(2)} \) peak (or dip) shifting left or right. Therefore, we can get the optical path difference between the star to each base station from the offset distance of the abscissa of peak or dip, and then get the relationship between the optical distance difference and the distance based on the geometric relationship, so as to achieve the stellar ranging. Notably, this method of ranging is not limited by the coherence length. That is because the optical path difference can be compensated using time delay. Therefore, we could set the base station at any place. It is also worth noting that this method does not require accurate angle measurements or parallax calculations, and only requires each base station to collect photons from the stars.

2.3 Ranging method

After the location of the base stations and the second-order correlation information between the base stations are known, the distance and ranging error of the star can be analyzed by using the geometric relationship. We design a simplified model to show how it works and to discuss the ranging capability of this scheme.
According to the principle that mentioned in 2.2, we propose to establish three base stations A, B and C on the same baseline. The length of line segment AB and BC is the distance between each other’s base stations respectively, and determined by their locations. But for the convenience of simplified models and simulation, we equalize them and set the value as a in this paper. The distance from the star to the base station A(B or C) is SA(SB or SC). Then the distance difference is \( L_B = SB - SA, L_C = SC - SA \). According to the cosines theorem, the distance from the star to the base station A, SA=L can be calculated by:

\[
L = \frac{2a^2 + 2L_B^2 - L_C^2}{2L_C - 4L_B} \tag{7}
\]

In general, baseline distance a is much less than star distance L. The baseline distance \( a = \frac{1}{2}c \tau_a \) is measured by laser ranging. \( L_B = c \tau_B, L_C = c \tau_C \) can be calculated by measuring the shift of \( g^{(2)} \) curve’s characteristic peak. After getting these information, the equation (7) could be written as

\[
L = \frac{t_a^2 + 2t_b^2 - t_c^2}{4t_c - 8t_b} \times c \tag{8}
\]

The above equation indicates that by using the three-station ranging method, only \( t_a, t_b, t_c \) need to be measured to achieve stellar ranging, with no need for measuring viewing angle difference between the base stations.
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Fig. 2: An example to illustrate the shifting of $g^{(2)}$ curve peak. The red curve describes the $g^{(2)}$ curve two observers are both in base station A. The $g^{(2)}$ peak is at 0 on the time axis because there is no optical path difference between them. The blue one describes the $g^{(2)}$ curve when observers are relatively in A and B. The $g^{(2)}$ peak is at $-6\mu$s on the time axis, that is to say, the path difference between A and B with respect to the star is $c \cdot (-6\mu s)$.

2.4 Error analysis

Assuming that the measurement error of $t_a, \tau_b, \tau_c$ totally depend on the precision of time measuring $\Delta t$, the following error measurement formula can be obtained:

$$\Delta L = \left( \frac{t_a}{2\tau_c - 4\tau_b} - \frac{t_a^2 + 2\tau_b^2 - \tau_c^2}{4 \cdot (\tau_c - 2\tau_b)^2} - \frac{1}{2} \cdot c \cdot \Delta t \right)^2$$

According to formula (9), error estimation can be made via the actual measurement values of $t_a, \tau_b, \tau_c$ and the precision of time measuring $\Delta t$.

In order to analyze the relationship between the relative error of ranging and various influencing factors, we drew Fig. 4 according to the formula (9).

Fig. 4 shows the numerical simulation results of stellar distance $L$ and relative error with different baseline lengths $a$. We should mention that Logarithmic coordinate is used to analyse relative error and the measurement accuracy is set as $\Delta t = 10^{-18} s$. The upper panel is a three-dimensional diagram and the panel (b) shows the distribution between relative error and stellar distance when the baseline length is 2000km, 10000km and 40000km respectively. The stellar distance extending out to 10kpc. We can see the relative error increase along with the stellar distance $L$, and decrease with the increase of baseline length $a$. When the star is 10kpc...
Fig. 3: Three stations ranging method. A, B and C are three base station on the same baseline where the length of line segment AB and BC are both equal to a. SA(SB or SC) is the distance from the star to the base station A(B or C). $L_B = SB - SA, L_C = SC - SA$ are the distance differences. If we know $a, L_B$ and $L_C$, then SA can be determined by calculation.

away and the baseline distance is 2000km, the ranging error is about 232pc, the relative error is within 3%. The error can get lower by extending the baseline length. As mentioned before, Gaia can determine distances to stars out to 30 000, i.e. 10kpc, with the precision of up to 0.00001 arc seconds. and we can know the relative error is approximately 8%.

Fig. 5 shows the trend of the relative error with the stellar distance($L$) and $\Delta t$. Baseline length($a$) is set as 20000km. We can see relative error will increase by one order of magnitude with each order of increase in the stellar distance($L$). This result is consistent with that shown Fig.4(b).

So our ranging method has the advantage of higher measuring accuracy. Meanwhile, because it is only restricted by time accuracy, it can theoretically measure farther distances. Currently, the best time measurement accuracy is $\Delta t = 1\text{s}$. Assuming we select three base stations on the earth at the distance of 2000 km and measure the distance to Betelgeuse, which is 640 light years away from us. The ranging error is only 0.29 light years and relative error is within 0.045%. If we can extend the baseline length (like geosynchronous orbit(40000km)), and improve the time measurement accuracy to $\Delta t=1\text{fs}$, the relative error can be lower and within 0.11%.

Above all, the measurement can be improved by a couple of orders of magnitude by changing a better clock, expanding the baseline length or just using more base stations. It is worth mentioning that clocks with an accuracy of $10^{-19}\text{s}$ have been prepared[Campbell et al. 2017].The huge development potential is inestimable.In practical situations, existing base stations can be selected, as long as the relative position relationship between them is known. In addition, the influence of other error sources on ranging accuracy can be reduced as much as possible by selecting the appropriate time to do the range.
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Fig. 4: (a). The distribution of the relative error, stellar distance (L) and baseline length (a). The measurement accuracy is set as $\Delta t = 10^{-18}$s). Logarithmic coordinates are used for relative errors. (b). The trend of relative error with stellar distance when the baseline length is 2000 km, 10000 km and 40000 km respectively. The relative error decreases with the length of the base station and increases with the distance from the star. When the star is 10 kpc away and the baseline distance (a) is 2000 km, the relative error is within 3%.
Fig. 5: The trend of the ratio of the relative error to the precision of time measuring $\Delta t$ with different $\Delta t$. When the baseline length($a$) and the stellar distance($L$) are determined, the relative error of ranging is proportional to the precision of time measuring.

2.5 Other ranging methods

In 2.3, we put forward a ranging method based on the assumption that the station distribution is strictly equidistant, and analyzed its ranging accuracy in 2.4. In practical work, we can choose existing base stations for the ranging measurement, as long as that these base stations can carry out second-order correlation measurement.

Assuming that there are $N$ base stations selected for the ranging measurement, the connection between these base stations can form a series of baselines $L$, the number of baselines is $N*(N-1)/2$. Generally, the locations of these base stations are known, so we don’t have to measure the length of these baselines. For a particular baseline $L_{ij}$ with base station $i$ and base station $j$ as endpoints, a time delay $\tau_{ij}$ can be measured by using second-order correlation measurement. For the $N*(N-1)/2$ baselines, the number of independent time delays obtained by measuring $g^{(2)}$ is $n$, $n \leq N*(N-1)/2$, $n$ will not always be equal to the baseline number $N*(N-1)/2$, because some time delays can be determined from the time delay getting from other baselines. Using these independent time delays, we can know the optical path difference between the star to each base station. Thus, we can use the base station locations to calculate the distance to the stars. Noting that this method can be used to measure not only the distance of stars, but also the position of stars. Each independent time delay will reduce one degree of freedom of the star’s position. When the star’s position is fully determined, the remaining independent time delay can also be added to the calculation to reduce the uncertainty of the position. Similar
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Fig. 6: Experimental schematic diagram. A heralded single photon source was used. TCSPC was used to measure the second order correlation $g^{(2)}$. By adding additional fiber to B2, the optical path difference between B1 and B2 can be changed. B1 and B2 represent two base stations. From the offset distance of the abscissa of $g^{(2)}$ curve peak, we can get the optical path difference between B1 and B2 caused by the additional fiber. According to the ranging principle, we could range the star as long as we know the optical path difference between the base stations.

methods of position calculation using time delays are widely used in VLBI [Sekido & Fukushima 2006; Liao et al. 2014] and other astronomical observation.

3 PRINCIPAL EXPERIMENT

We have completed the demonstrative experiment to prove the feasibility of measuring distance difference. The experimental setup is illustrated in Fig. 6. The 405 nm pump light came from the laser passes through HWP and PBS successively. The coherence length of our laser is more than 3 meters. Then we adjust power so as to convert the photon into a horizontally polarized state $|H\rangle$. After focusing light onto a PPKTP crystal, we successfully obtained the 810nm entangled photon pair with orthogonal polarization state by using SPDC process where polarization states are horizontal polarization $|H\rangle$ and vertical polarization $|V\rangle$. By using LongPass filter to filter
Fig. 7: \(g^{(2)}\) between B1 and B2 (without additional fiber).

out the pump light of 405 nm, finally we have a pair of 810 nm photons produced at the same
time. Therefore, our device could be used as a source of entangled photon pairs.

The photon pair are divided in two ways (A and B), according to polarization after passing
through PBS, and are received respectively by observers at A and B. As we mentioned before,
entangled photons must appear simultaneously. So if observer A receives a photon at a time,
B will receive the other photon accordingly. In this way, we successfully prepared a heralded
single photon source.

If we do not detect single photon observer A and only use B to receive signal light, now ob-
server B receives thermal light instead of heralded single photon source. Thus, our device
could prepare the thermal light, single photon and entangled photon pair. Since the \(g^{(2)}\) functions
of three sources are not equal to 1, we can determine the distance difference according to the
previous theory. We propose our ranging scheme based on that.

We divide signal light into B1 and B2 by BS, then use the single photon detector to detect
the signal. Counting B2 and B1 signals as the START and STOP signal of time-correlated
single-photon counting (TCSPC) respectively [Phillips et al. 1985]. The statistical distribution
of the optical field signal we obtained is shown below.

If we extend the optical path difference between B1 and B2 by setting up a roughly 159
cm section of fiber on B2, we can gain the following results after repeating the above operation
with TCSPC. As we can see, the peak of \(g^{(2)}\) curve shifts from 157.77 bin to 280.21 bin. The refractive index of the fiber we used is \(n_g = 1.4735\).

So it is easy to calculate the length of the fiber:

\[
l = (280.21 - 157.77) \text{bin} \times 64 \text{ps} \times 2.034 \times 10^8 \text{m/s} \approx 159.39 \text{cm}.
\] (10)
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Fig. 8: $g^{(2)}$ between B1 and B2 (with additional fiber). Compared with Fig. 7, the second-order correlation function is shifted to the right by 122.44 bins, which corresponds to the optical path difference introduced by the additional fiber.

In this experiment, the time resolution of TCSPC is 64 ps, so the upper limit of theoretical resolution is

$$64\text{ps} \times 2.034 \times 10^8 \text{m/s} \approx 1.30 \text{cm}.$$  \hspace{1cm} (11)

Compared with the actual length of the fiber, we demonstrate the optical path difference can be measured from the $g^{(2)}$ curve shifting.

In addition, if the source is single photon source or entangled source, we can also achieve the measurement of fiber length difference, and result is consistent with the above conclusion. In other words, the scheme we proposed could realize the measurement of the distance difference between light source and observers as long as the source does not emit laser which $g^{(2)}$ equals to 1. After measuring the distance difference between the source and observers ($L_B, L_C$), we can calculate the distance to the star (L) by using formula (7).

This experiment demonstrates the feasibility of the theoretical scheme for measuring the optical path difference and shows that the brightness of the star does not need to be too high because the optical path difference of a single photon can still be detected in the experiment.

4 CONCLUSIONS

In this paper, we propose the stellar ranging scheme based on second order correlation measurement for the first time, applying the second order correlation theory of light field to stellar ranging, and carrying out the primary demonstration and the verification experiment based on SPDC light source. Compared with the traditional method of trigonometric parallax ranging, we confirm that our scheme has many advantages.
This method is based on second order correlation theory, and its accuracy only depends on the accuracy of time measurement. So it does not require a parallax angle measurement. Then the objects of this scheme can be thermal sources or other sources that emit non-classical light. From the summary above, we can say that the scheme has overcome the limitation of emission power on the measurement distance. So the range of it is much larger than active ranging. Apart from this, it also has good resistance to external noise, random signal interference and requires lower cost and shorter time. Furthermore, since the error of stellar ranging is directly proportional to the time accuracy and inversely proportional to the square of the distance between the observers, we may improve the measurement range and accuracy by changing a better clock and extending base line length in the future. As long as the light emitted by star can be received by observers, we can actualize it.

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