Resonant Heating of Ions by Parallel Propagating Alfvén Waves in Solar Coronal Holes *

Tian-Xi Zhang¹,², Jing-Xiu Wang¹ and Chi-Jie Xiao¹

¹ National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012; wangjx@bao.ac.cn
² Center for Space Plasma and Aeronomic Research, The University of Alabama in Huntsville, Huntsville, AL 35899, USA; zhangt@cspar.uah.edu

Received 2004 December 14; accepted 2005 March 5

Abstract Resonant heating of H, O⁺⁵, and Mg⁺⁹ by parallel propagating ion-cyclotron Alfvén waves in solar coronal holes at a heliocentric distance is studied using the heating rate derived from the quasilinear theory. It is shown that the particle-Alfvén-wave interaction is a significant microscopic process. The temperatures of the ions are rapidly increased up to the observed order in only microseconds, which implies that simply inserting the quasilinear heating rate into the fluid/MHD energy equation to calculate the radial dependence of ion temperatures may cause errors as the time scales do not match. Different species ions are heated by Alfvén waves with a power law spectrum in approximately a mass order. To heat O⁺⁵ over Mg⁺⁹ as measured by the Ultraviolet Coronagraph Spectrometer (UVCS) in the solar coronal hole at a region \( \sim 1.9R_\odot \), the energy density of Alfvén waves with a frequency close to the O⁺⁵-cyclotron frequency must be at least double of that at the Mg⁺⁹-cyclotron frequency. With an appropriate wave-energy spectrum, the heating of H, O⁺⁵ and Mg⁺⁹ can be consistent with the UVCS measurements in solar coronal holes at a heliocentric distance.

Key words: acceleration of particles – plasmas – waves – solar wind – Sun: corona

1 INTRODUCTION

Ultraviolet Coronagraph Spectrometer (UVCS) measurements from the Solar and Heliospheric Observatory (SOHO) have shown that O⁺⁵ ions are enormously heated in solar coronal holes (Kohl et al. 1998; Dodero et al. 1998; Cranmer et al. 1999; Antonucci et al. 2000; Miralles, Cranmer & Kohl 2001; Habbal et al. 2001). The temperature of O⁺⁵ is much greater than that of H, \( T_{O^+5}/T_H \approx 10 – 100 \). The Mg⁺⁹ ions are also heated significantly (Esser et al. 1999). The

* Supported by the National Natural Science Foundation of China.
heating of Mg$^{+9}$ is comparable with the heating of O$^{+5}$ at $\sim 1.75R_\odot$ and is even more efficient than the heating of O$^{+5}$ in a region $\lesssim 1.6R_\odot$. In a region $\gtrsim 1.9R_\odot$, however, the heating of O$^{+5}$ is generally more efficient. The heating of H is weak relative to O$^{+5}$ and Mg$^{+9}$. The temperature of H increases by only a factor of $\sim 2\sim 3$.

Plasma waves in the ion-cyclotron frequency range are generally believed to play an essential role in the preferential heating of particles in solar corona, solar winds, and/or solar coronal holes (Parker 1991; Zirker 1993; Narain & Ulmschneider 1996; Tu & Marsch 1997; Marsch & Tu 1997; Axford et al. 1999; Hu et al. 1999; Isenberg 2001; Hollweg 2002; Cranmer 2002; Zhang 2003a; Li & Habbal 2003; Chen & Li 2004). High-frequency Alfvén wave and electrostatic ion-cyclotron wave are two important candidates. The former can be cascaded from a lower-frequency Alfvén wave (Isenberg & Hollweg 1982; Isenberg 1990), excited by electron beams (Temenin & Roth 1992; Miller & Viñas 1993), or/and generated by magnetic reconnections at a coronal base (Axford & McKenzie 1992; McKenzie et al. 1997). The latter can be generated by global MHD modes, currents, or/and electron beams (Kindel & Kennel 1971; Markovskii 2001; Zhang 2003a).

Heating of H, O$^{+5}$ and Mg$^{+9}$ by H-cyclotron waves generated by a global MHD mode in solar coronal holes (Markovskii 2001) has been quantitatively studied by Zhang (2003a) in terms of the heating mechanism developed previously for solar $^3$He-rich events (Palmadesso et al. 1974; Fisk 1978; Zhang 1995, 1999; Zhang 2003b; Zhang & Wang 2003, 2004). It is shown that H-cyclotron waves can be excited at (1.15–1.2) times the H-cyclotron frequency $\Omega_H$ and so can heat O$^{+5}$ through the fourth harmonic resonance and Mg$^{+9}$ through the third harmonic resonance. The H-cyclotron waves are efficient at heating O$^{+5}$ if $\omega \simeq 1.2\Omega_H$, and Mg$^{+9}$ if $\omega \simeq 1.15\Omega_H$. Protons can only be weakly heated by H-cyclotron waves through non-resonant dissipation. The amount of heating for H, O$^{+5}$ and Mg$^{+9}$ can be of the same order as the measurements.

Heating of ions by Alfvén-waves in solar coronal holes has been extensively studied for more than two decades (Hollweg & Turner 1978; Dusenbery & Hollweg 1981; Isenberg & Hollweg 1982; Emslie & Sturrock 1982; Heyvaerts & Priest 1983; Davila 1987; Wingée et al. 1987; Grossmann & Smith 1988; Poedts et al. 1989; Moore et al. 1991; Ofman et al. 1994; Tu & Marsch 1997; Marsch & Tu 1997; Cranmer, Field & Kohl 1999; Hollweg 1999; Hu et al. 2000; Isenberg et al. 2001). One of the most interesting coronal hole heating models is the fluid/MHD model based on fluid mechanics or magnetohydrodynamics that contains in the energy equation a heat transfer/dissipation rate ($Q_\sigma$ for the ion species $\sigma$) due to Alfvén-wave turbulences. Dusenbery & Hollweg (1981) first derived a heating rate with a quasilinear theory and parametrically studied the heating rate of minor ions with a parallel propagating Alfvén wave. Later on, the quasilinear heating rate was used as the heat transfer/dissipation rate of the energy equation in the fluid/MHD coronal hole heating model to determine the radial dependences of ion temperatures in solar coronal holes (Marsch et al. 1982; Isenberg & Hollweg 1983; Isenberg 1984; Hu & Habbal 1999; Cranmer et al. 1999).

Recently, the time evolution of ion temperatures has been studied according to the quasilinear heating rate in a laboratory plasma with parallel propagating ion-cyclotron Alfvén waves (Zhang & Li 2004). It was shown that the interaction process between particles and Alfvén waves, similar to the electrostatic ion-cyclotron waves, is transient or microscopic with a timescale of the ion gyro-period $\Omega_H^{-1}$. This implies that simply replacing the heat transfer/dissipation rate of the fluid/MHD energy equation by the quasilinear heating rate to obtain the radial dependences or long time evolutions of ion temperatures will lead to numerical and physical difficulties because the timescale of the quasilinear heating rate does not match the fluid/MHD equations. To guarantee the numerical solution to be accurate, the radial step $\Delta r$ must be chosen to be $\Delta r \sim V_0/\Omega_H$, or about 0 to 1 km in the coronal hole at $1-5R_\odot$. To
acquire the whole ion temperature profile through the inner coronal hole to the outer coronal hole will take an unreasonably long time. For the radial dependences of ion temperatures to be obtained in a reasonable calculation time, the radial step should be about its usual size, $\sim 0.01 R_\odot$, say. The corresponding time step will be as large as seconds or minutes and the calculated ion temperatures will have a sudden increase in a single time step because $Q_\sigma$ is very large (see figures 1, 2, 5, 6, and 7 of Isenberg & Hollweg 1983 and figures 1, 2, and 5 of Isenberg 1984). In some other simulation studies (Chen & Hu 2002; Chen et al. 2003), the heat transfer/dissipation rate was usually chosen according to some rate models instead of the quasilinear heating rate such as Kolmogorov rate model (Hollweg 1986) and Kraichnan rate model (Hollweg & Johnson 1988). These rate models, however, lack details on the microscopic particle-wave interactions.

In this paper, resonant heating of H, O$^{+5}$, and Mg$^{+9}$ by parallel propagating ion-cyclotron waves in solar coronal holes are studied. According to the quasilinear heating rate, the time evolution and species dependence of the ion temperatures due to the resonant heating of ions by the Alfvén waves in solar coronal holes are determined. The results obtained by this study indicate that the resonant heating of ions by the Alfvén waves is significant. Within a short timescale, the O$^{+5}$ and Mg$^{+9}$ ions can be preferentially heated relative to protons by a factor of $\sim 10 - 100$, the order of the UVCS measurements of solar coronal holes at some given heliocentric distance.

2 HEATING OF H, O$^{+5}$, AND Mg$^{+9}$ BY ALFVÉN WAVES

In a hot (collisionless), magnetized, and multi-ion plasma with the composition of coronal hole, the heating rate of any ion species by a parallel propagating ion-cyclotron Alfvén wave can be derived kinetically with the quasilinear theory as (Dusenbery & Hollweg 1981; Marsch et al. 1982; Zhang & Li 2004; see details in Appendix A)

$$Q_\sigma = \frac{dT_\sigma}{dt} = \frac{2\pi \sqrt{2} \pi \Omega_\sigma c^2}{3 B_0^2 v_{T,\sigma}^2} \sum_{k \neq 0} W_k(t) \frac{\Omega_\sigma}{k_{\|} v_{T,\sigma}} (6 + R_\sigma^2) \exp \left( -\frac{R_\sigma^2}{2} \right),$$  

(1)

where the subscript $\sigma$ refers to the ion species; $B_0$ is the magnetic field of the background plasma; $v_{T,\sigma} = (k_B T_\sigma/m_\sigma)^{1/2}$ is the ion thermal velocity with $k_B$ the Boltzmann constant, $T_\sigma$ the ion temperature, and $m_\sigma$ the ion mass; $\Omega_\sigma = Z_\sigma^2 e B_0/(m_\sigma c)$ the ion-cyclotron frequency with $Z_\sigma^2$ the ion charge (or ionization) state, $e$ the proton electric charge, and $c$ the light speed; $W_k(t) = |E_k(t)|/(8\pi)$ the Alfvén-wave-energy density with $E_k$ the wavenumber vector, $k_{\|}$ the parallel wavenumber, and $E_k(t)$ the wave electric field; and $R_\sigma = (\omega - \Omega_\sigma)/(k_{\|} v_{T,\sigma})$ is the resonance factor with $\omega$ the wave frequency. For a left-hand polarized Alfvén wave with a frequency less than the H-cyclotron frequency, we have $R_\| < 0$ for proton and $R_\sigma < V_A/v_{T,\sigma}$ for heavy ions, $V_A$ being the Alfvén wave speed defined by $v_A = B_0/\sqrt{4\pi \rho}$ and $\rho$, the plasma density defined by $\rho = \sum n_{\sigma,0} m_\sigma$ with $n_{\sigma,0}$ the initial ion-number density.

It is seen that the Alfvén-wave heating depends explicitly on $Z_\sigma^2$, $m_\sigma$, $T_\sigma$, $B_0$, $\omega$, $k_{\|}$, and $W_k(t)$. The frequency $\omega$ and the parallel wavenumber $k_{\|}$ are related to each other through the Alfvén-wave dispersion relation, which includes the Alfvén speed. Therefore, the Alfvén-wave heating also depends on the plasma density or the initial ion-number density $n_{\sigma,0}$. Zhang & Li (2004) have shown that the resonant heating of ions in a multi-ion laboratory plasma including H, $^2$H, $^3$H, $^3$He and $^4$He by parallel propagating Alfvén waves increases with increasing wave-energy density but decreases with increasing plasma density, increasing temperature and decreasing magnetic field. An Alfvén wave with a higher energy density is more efficient at heating ions. A plasma with a higher density has more ions to be heated per unit volume and thus needs more wave energies. As the ions are heated, the heating rate of the ions decreases.
This shows that the heating of ions by the Alfvén waves will saturate, i.e., an Alfvén wave of finite power cannot heat up the ions infinitely. The magnetic field also tends to decrease the heating rate.

The most important question for resonant heating of ions by Alfvén waves is whether or not the cyclotron resonance condition, \( \omega - \Omega_{\sigma} - k ||= v_\parallel = 0 \), is satisfied for most particles. If most particles of species \( \sigma \) satisfy the cyclotron resonance condition, \( |v_\parallel| \) should be just below the thermal velocity, \( |v_\parallel| \lesssim v_\sigma, |R_\sigma| \lesssim 1 \). In this case, the cyclotron resonance is significant and the heating of ions can be efficient. For the best resonance (\( R_\sigma = 0 \)), the heating of heavy ions by the Alfvén waves with a power law spectrum (i.e., \( W_k(t) \propto k_{\parallel}^{-\gamma_w} \)) depends on the ion species and is approximately proportional to \((Z^*_\sigma)^{3/2}(A_\sigma/Z^*_\sigma)^{\gamma_w+1/2}\), with \( A_\sigma \) the ion mass number and \( \gamma_w \) the spectral power index of the Alfvén waves, usually chosen to be \( \gamma_w \approx 1.5 - 2 \). In solar coronal holes the higher the charge state, the greater the mass-to-charge ratio of the ion will be if the ion is heavier, the Alfvén waves will heat a heavier ion more than a lighter ion. Therefore, the heating of \( \text{Mg}^{+9} \) should be generally more efficient than the heating of \( \text{O}^{+5} \), as was measured by UVCS below 1.6\( R_\odot \).

For \( \text{O}^{+5} \) to be heated over \( \text{Mg}^{+9} \) as measured by UVCS above 1.9\( R_\odot \), the energy density of the Alfvén wave at the \( \text{O}^{+5} \)-cyclotron frequency must be at least double that at the \( \text{Mg}^{+9} \)-cyclotron frequency. This requires, assuming a power law spectrum for the wave, that the spectral power index must be \( \gamma_w > 4.5 \). As the spectral power index is usually valued in the range of \( \gamma_w \approx 1.5 - 2 \), a power law spectral Alfvén wave with a normal spectral power index cannot cause \( \text{O}^{+5} \) to be heated more than \( \text{Mg}^{+9} \) in a solar coronal hole. Here we show that a power law spectral Alfvén wave with a normal spectral power index (\( \gamma_w = 1.5 - 2 \)) cannot cause \( \text{O}^{+5} \) to be heated more than \( \text{Mg}^{+9} \) as measured in the solar coronal hole above \( \sim 1.9 \) solar radii. In fact, a power law spectral Alfvén wave excited at the coronal base will deform its spectrum due to uneven heating of ions with different ion-cyclotron frequencies as it propagates outward along the magnetic field lines. This uneven heating causes the temperature ratio between \( \text{O}^{+5} \) and \( \text{Mg}^{+9} \) to be different for holes at different distances. In the coronal hole below \( \sim 1.6 \) solar radii, \( \text{Mg}^{+9} \) is heated more than \( \text{O}^{+5} \), which leads to the wave energy at the \( \text{Mg}^{+9} \)-cyclotron frequency being absorbed more than that at the \( \text{O}^{+5} \)-cyclotron frequency. In the region above \( \sim 1.9 \) solar radius, the heating of \( \text{O}^{+5} \) becomes the more efficient because the energy-density difference between the waves with \( \omega \sim \Omega_{\text{O}^{+5}} \) and \( \Omega_{\text{Mg}^{+9}} \) is significantly greater than the difference that a power law spectral Alfvén wave gives.

To quantitatively study the resonant heating of ions by high-frequency Alfvén waves in solar coronal holes, we numerically calculate the increases of the ion temperatures according to the heating-rate expression (1). The typical ions \( \text{H}, \text{O}^{+5} \) and \( \text{Mg}^{+9} \) are considered. The initial plasma parameters are chosen in terms of the coronal hole plasma properties at a given radial distance/height such as \( n_{e,0} = 10^6 \text{cm}^{-3}, n_{\text{H}^+0}/n_{\text{H}} = 0.1, T_{e,0} = T_{\sigma,0} = 10^9 \text{K}, \) and \( B_0 = 10 \text{ Gauss} \). The high-frequency Alfvén waves are assumed to exist in the plasma and propagate along the magnetic field lines. The dispersion relation of the Alfvén waves in the ion-cyclotron frequency range is given by \( \omega = k_\parallel v_A(1 - \omega/\Omega_{\text{H}})^{1/2} \) (Hollweg 1999; Cranmer et al. 1999).

Figure 1 plots the temperatures of \( \text{H}, \text{O}^{+5} \), and \( \text{Mg}^{+9} \) (normalized by the initial values) as functions of the heating time (normalized by the H-cyclotron frequency). For the resonance of these ions to be significant or best, the resonant factor should be small. Here, as an example, the resonant factor of each species is chosen to be \( R_\sigma = -1 \). In Figure 1a, the wave-energy density is chosen to be a constant (i.e., a flat spectrum) at \( 8 \times 10^{-10} \text{ erg cm}^{-3} \) in the frequency range \( \omega/\Omega_{\text{H}} = 0.3 - 1 \); while in Figure 1b, the wave-energy density at frequency close to the \( \text{O}^{+5} \)-cyclotron frequency is higher at \( 3 \times 10^{-9} \text{ erg cm}^{-3} \) (an uneven spectrum). It is seen that the high-frequency Alfvén waves with sufficient energies can significantly heat the particles through the cyclotron resonance if the resonant factor is in the range of \( |R_\sigma| \lesssim 1 \). The heating of the
Coronal Hole Heating by Alfvén Waves 289

Fig. 1 Temperatures of H, O$^{+5}$, and Mg$^{+9}$ vs. time of heating by Alfvén waves due to cyclotron resonances in two cases. (a) Same energy density at $\omega \simeq \Omega_H$, $\Omega_{O^{+5}}$, and $\Omega_{Mg^{+9}}$; (b) higher energy density at $\omega \simeq \Omega_{O^{+5}}$.

Fig. 2 Resonant factors of H, O$^{+5}$ and Mg$^{+9}$ in terms of a wave with a frequency $\omega/\Omega_H = 0.1 - 1$.

Particles is ordered along the particle mass if the wave-energy density is independent of the wave frequency. If the wave-energy density at frequency close to the O$^{+5}$-cyclotron frequency is significantly higher, the heating of O$^{+5}$ can be more efficient than the heating of Mg$^{+9}$. The heating of H is general weak due to its smaller mass and the larger $k_\parallel$ close to the H-cyclotron frequency. Therefore, for an appropriate wave-energy spectrum, we can fit the model results with the measurements of UVCS in solar coronal holes at some given heliocentric distance.

Alfvén waves of a single frequency cannot significantly resonate H, O$^{+5}$ and Mg$^{+9}$ simultaneously, because the ion-cyclotron frequencies of H, O$^{+5}$ and Mg$^{+9}$ are different. To illustrate this point, we show, in Fig. 2, the absolute resonant factors of H, O$^{+5}$ and Mg$^{+9}$ as functions of the wave frequency in the range $\omega/\Omega_H = 0.1 - 1$. It can be seen that the resonant factor of each species is small when the wave frequency is close to its ion-cyclotron frequency. For Alfvén waves with a unique frequency, $|R_\sigma| < 1$ cannot hold for all species, except for $\omega \sim \Omega_H$. The Alfvén waves with $\omega \sim \Omega_H$, however, cannot significantly heat O$^{+5}$ and Mg$^{+9}$ together with H itself because the parallel wavenumber $k_\parallel$ at $\omega \sim \Omega_H$ is very large.

To heat different species of ions simultaneously, the Alfvén waves must have a wide range of frequency, e.g., a continuous spectrum with a constant energy density (i.e., independent of the frequency or a flat spectrum). Figure 3 plots the heating of H, O$^{+5}$ and Mg$^{+9}$ as a function
Fig. 3 Temperatures of H, O$^{+5}$ and Mg$^{+9}$ vs. frequency of Alfvén waves in two cases. (a) Same energy density at $\omega \simeq \Omega_H$, $\Omega_{O^{+5}}$, and $\Omega_{Mg^{+9}}$; (b) higher energy density at $\omega \simeq \Omega_{O^{+5}}$. 

of the wave frequency, where the heating time is fixed at 100/$\Omega_H$ ($\sim 10^{-3}$ seconds) and the wave-energy density is fixed at $8 \times 10^{-10}$ erg cm$^{-3}$. Figure 3b shows the case where the wave-energy density at the frequency close to the O$^{+5}$-cyclotron frequency is higher, as shown in Figure 1b. It is seen that significant or best heating occurs when the wave frequency is close to the ion-cyclotron frequency. For appropriate and reasonable wave and plasma parameters, the numerical results for the heating of H, O$^{+5}$ and Mg$^{+9}$ agree with the measurements.

3 DISCUSSION AND CONCLUSIONS

This study mainly focuses on the time evolution of local ion temperature due to Alfvén wave turbulence at a given radial distance or height in the solar coronal hole. The timescale of the heating is microscopic and resonant heating is significant. In the inner coronal holes, transient heating of ions at one place does not instantly affect the heating at another place since the waves propagate much faster than the wind. The heating of ions at different radial distances may be separately calculated to find the temperature radial profiles. In addition, we have ignored the influence of ionization on the heating of O$^{+5}$ and Mg$^{+9}$ in this short time, because the ionization state of the ions depends on the electron temperature. Furthermore, in comparison with the particle-wave interaction process, effects due to column collisions and variations of plasma properties including density, temperature, magnetic field, gravity, and flow speed are negligible. To predict the heating in the whole heating and acceleration region, we need use the empirical relations for the spatial/radial dependence of the initial plasma and wave properties $n_e(r)$, $B_0(r)$, $W_k(r)$, $T_{e,0}(r)$ and $V_\sigma(r)$ to determine $T_{\sigma}(r)$. We leave this as a topic for future study.

The heating model is isotropic: the parallel temperature ($T_\parallel$) is not separated from the perpendicular one ($T_\perp$). Using an isotropic temperature may increase the number of resonant particles but will not significantly increase the resonant heating because the Alfvén waves resonate with the thermal particles rather than with the particles at the distribution tail. We leave the development of a fully anisotropic heating model on the basis of the isotropic one to a future study. In addition, the plasma streaming effect has not been included in this study. When a streaming velocity $V_\sigma$ is considered, we can still have the same heating rate expression (1): only the resonant factor will now involve $V_\sigma$: $R_\sigma = (\omega - \Omega_\sigma - k_\parallel V_\sigma)/(k_\parallel v_{T,\sigma})$. Plasma streaming decreases the heating of H but not the heating of heavy ions because the resonant factor of H cannot be less than $V_\sigma/v_{T,\sigma}$, while that of heavy ions can always tend to zero if $V_\sigma < V_A$.

Details on this topic will be given in a future study.
Theoretically, both the H-cyclotron waves and the parallel propagating high-frequency Alfvén waves can preferentially heat O\(^{+5}\) and Mg\(^{+9}\) to the measured extents if these two kinds of waves are excited in the coronal hole with sufficient wave energy densities. Since the measurements on the energy spectra of the H-cyclotron waves and the Alfvén waves are not available, we cannot conclude, at present, which mechanism is the more effective one for the heating of O\(^{+5}\) and Mg\(^{+9}\) in solar coronal holes. In general, the Alfvén waves with a wide frequency range can heat ions with various charge states, while the H-cyclotron waves with a narrow frequency range can only heat ions with appropriate charge states. For instance, an H-cyclotron wave that heats Mg\(^{+9}\) does not heat Mg\(^{+8}\) (Zhang 2003b), while an Alfvén wave that heats Mg\(^{+9}\) can surely heat Mg\(^{+8}\). Therefore, to measure the heating of various ionization states of O and Mg in solar coronal hole will be important to discriminate these two mechanisms.

To summarize, we have theoretically and numerically investigated the preferential heating of H, O\(^{+5}\) and Mg\(^{+9}\) due to cyclotron resonance with parallel propagating Alfvén waves in the ion-cyclotron frequency range. In terms of the heating-rate expression for any species of particles that is heated by the Alfvén waves, we have numerically studied the heating of particles including H, O\(^{+5}\) and Mg\(^{+9}\) by Alfvén waves in solar coronal holes at a given radial distance or height. The results show that the Alfvén waves in the ion-cyclotron frequency range can be efficient at heating particles through the cyclotron resonance.

The rate of heating depends on the wave-energy density, the magnetic field, the plasma density, the temperature, and the particle species. Alfvén waves with a higher energy density are more efficient at heating ions. A plasma with a higher density has more ions to be heated per unit volume and thus needs more wave energies. As the ions are heated, the heating rates of ions decrease. This shows that the heating will be saturated; this means that a finite power Alfvén wave cannot heat ions infinitely. The magnetic field also acts in the direction of decreasing the heating rate.

Resonant heating of ions by Alfvén waves in solar coronal holes is approximately proportional to the mass. Such waves with a continuous energy spectrum (either a constant or a power law spectra with index below 4.5) are capable of heating H, O\(^{+5}\) and Mg\(^{+9}\) in the mass order. If the wave energy density at \(\sim \Omega_{\text{Mg}^{+9}}\) is about half or less than that at \(\sim \Omega_{\text{O}^{+5}}\), the heating of O\(^{+5}\) can be more efficient than the heating of Mg\(^{+9}\). If the wave spectrum is power law one, then the spectral power index must be \(\gamma_w > 4.5\). As the spectral power index is usually in the range of \(\gamma_w \approx 1.5 - 2\), a power law spectral Alfvén wave with a normal spectral power index cannot heat O\(^{+5}\) more than Mg\(^{+9}\) in solar coronal holes. Therefore, it is in the existence of Alfvén waves with appropriate energy spectra rather than the power law form in the ion-cyclotron frequency range, that the amount and profile of heating of H, O\(^{+5}\) and Mg\(^{+9}\) can be made consistent with the measurements of UVCS in solar coronal holes at the given heliocentric distance.

**Acknowledgements** The work was supported by National Natural Science Foundation of China (Grant No. 10233050), National Science Foundation (ATM 00-70385), and AFOSR (F49620-00-0-0204).
Appendix A: HEATING RATES OF IONS

In a collisionless, magnetized and multi-ion plasma with Alfvén-waves, the velocity-distribution function \( f_\sigma(\mathbf{r}, \mathbf{v}, t) \) of species \( \sigma \) is determined by the Vlasov equation,

\[
\frac{\partial f_\sigma(\mathbf{r}, \mathbf{v}, t)}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\sigma(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{r}} + \frac{q_\sigma}{m_\sigma} \left[ E(\mathbf{r}, t) + \frac{1}{c} (\mathbf{v} \times \mathbf{B}(\mathbf{r}, t)) \right] \frac{\partial f_\sigma(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{v}} = 0, \tag{A1}
\]

where \( E(\mathbf{r}, t) \) is the electric field, \( c \) is the light speed, and \( \mathbf{B}(\mathbf{r}, t) \) is the magnetic field. The coordinate system is a frame with the origin on the Sun’s surface and the \( z \)-axis along the magnetic field line through the solar coronal hole.

With the quasi-linear approximation, Equation (A1) can be divided into the following two quasi-linear equations (Kennel & Engelfried 1966),

\[
\frac{\partial G_{\sigma,0}(\mathbf{v}, t)}{\partial t} + \frac{q_\sigma}{m_\sigma} \sum_{k \neq 0} \delta E_{-k}(t) \frac{\partial \delta f_{\sigma,k}(\mathbf{v}, t)}{\partial \mathbf{v}} = 0, \tag{A2}
\]

\[
\frac{\partial \delta f_{\sigma,k}(\mathbf{v}, t)}{\partial t} + i \mathbf{k} \cdot \mathbf{v} \delta f_{\sigma,k}(\mathbf{v}, t) + \frac{q_\sigma}{m_\sigma} \delta E_{k}(t) \frac{\partial f_{\sigma,0}(\mathbf{v})}{\partial \mathbf{v}} + \frac{q_\sigma}{m_\sigma c} (\mathbf{v} \times \mathbf{B}_0) \cdot \frac{\partial \delta f_{\sigma,k}(\mathbf{v}, t)}{\partial \mathbf{v}} = 0, \tag{A3}
\]

where \( G_{\sigma,0}(\mathbf{v}, t) \) is defined by \( G_{\sigma,0}(\mathbf{v}, t) = f_{\sigma,0}(\mathbf{v}) + \delta f_{\sigma,0}(\mathbf{v}, t); \delta f_{\sigma,0}(\mathbf{v}, t) \) and \( \delta f_{\sigma,k}(\mathbf{v}, t) \) are the small derivations; and \( f_{\sigma,0}(\mathbf{v}) \) is the initial unperturbed distribution function, which is usually assumed to be bi-Maxwellian at a given radial distance as

\[
f_{\sigma,0}(\mathbf{v}) = \frac{1}{(2\pi v_{T,\sigma})^{3/2}} \exp\left(\frac{-v^2}{2v_{T,\sigma}^2}\right), \tag{A4}\]

where \( v_{T,\sigma} = \sqrt{k_BT_0/m_\sigma} \) is the thermal velocity with \( k_B \) the Boltzmann constant and \( T_\sigma \) the temperatures. In the derivation of Equations (A2) and (A3), we have assumed that \( f_{\sigma,0}(\mathbf{v}) = f_{\sigma,0}(v_\parallel, v_\perp), \delta f_{\sigma,0}(\mathbf{v}, t) = \delta f_{\sigma,0}(v_\parallel, v_\perp, t), \delta f_{\sigma,k}(\mathbf{v}, -\infty) = 0, \delta E_{k=0}(t) = 0, \) and \( \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_\perp \) along the \( \mathbf{z} \)-axis.

Equation (A2) includes a nonlinear term for the interactions between particles and wave turbulences up to the second order of perturbations; while Equation (A3) is linear and can be analytically solved (Ichimaru 1973; Stix 1992). For the Alfvén waves that propagates along the magnetic field lines (\( \mathbf{k} \parallel \mathbf{B}_0 \) and \( \mathbf{E} \perp \mathbf{B}_0 \)), the linear solution of Equation (A3) for the distribution derivation due to the Alfvén-wave perturbation is obtained by

\[
\delta f_{\sigma,k}(\mathbf{v}, t) = -\frac{q_\sigma}{m_\sigma} \delta E_{k}(t) \frac{\partial f_{\sigma,0}(v_\parallel, v_\perp)}{\partial v_\perp} \times 
\quad \left[ \frac{\exp(-i\phi)}{i(\omega - \Omega_\sigma - k_\parallel v_\parallel + i\eta)} + \frac{\exp(i\phi)}{i(\omega + \Omega_\sigma - k_\parallel v_\parallel + i\eta)} \right], \tag{A5}\]

where \( \omega \) is the Alfvén wave frequency, \( k_\parallel \) is the parallel wave number, \( \Omega_\sigma = q_\sigma B_0/m_\sigma c \) is the particle-cyclotron frequency with \( m_\sigma \) the ion mass, \( q_\sigma \) the ion electric charge, and \( c \) the light speed, \( \phi \) is the angle between \( \mathbf{E} \) and \( v_\perp \), and \( \eta \) is a small positive constant. Substituting the linear solution (A5) into (A2), we have

\[
\frac{\partial G_{\sigma,0}(\mathbf{v}, t)}{\partial t} = \frac{q_\sigma^2}{m_\sigma^2} \sum_{k \neq 0} |\delta E_{k}(t)|^2 \cos(\phi) \frac{\partial^2 f_{\sigma,0}(v_\parallel, v_\perp)}{\partial v_\perp^2} \times 
\quad \left[ \frac{i \exp(-i\phi)}{k_\parallel v_\parallel - \omega + \Omega_\sigma - i\eta} + \frac{i \exp(i\phi)}{k_\parallel v_\parallel - \omega - \Omega_\sigma - i\eta} \right]. \tag{A6}\]
Equation (A6) shows that the time-dependent velocity distribution of each species is significantly varied by the Alfvén waves when the resonance condition, \( \omega \pm \Omega_\sigma = k_\parallel v_\parallel \), is satisfied.

Based on the time-dependent distribution evolution Equation (A5), the heating rates of particles by the Alfvén waves can be obtained via completing the following integration,

\[
\frac{3}{2} k_B \frac{dT_\sigma}{dt} = \int \frac{1}{2} m_\sigma v^2 \frac{\partial G_{\sigma,0}(v, t)}{\partial t} d^3v
\]

\[
= \frac{1}{2} m_\sigma \int_0^{2\pi} \int_0^\infty \int_{-\infty}^\infty d\phi dv_\perp dv_\parallel (v_\perp^2 + v_\parallel^2) \frac{\partial G_{\sigma,0}(v, t)}{\partial t},
\]

where \( k_B \) is the Boltzmann constant. Using Equation (A5) to replace \( \frac{\partial G_{\sigma,0}(v, t)}{\partial t} \) and integrating (A6) with respect to \( \phi, v_\parallel \) and \( v_\perp \), we can derive, for hot (or collisionless), magnetized and multi-ion plasma, the heating rate of ions by the parallel propagating ion-cyclotron Alfvén waves (Zhang & Li 2004) is

\[
\frac{1}{T_\sigma} \frac{dT_\sigma}{dt} = \frac{4\pi \sqrt{2} \Omega_\sigma c^2}{3B_0^2 v_\sigma T} \sum_{k\neq0} W_k(t) \frac{\Omega_\sigma}{k_\parallel v_\sigma T} \left[ 3 + \left( \frac{\omega - \Omega_\sigma}{\sqrt{2} k_\parallel v_\sigma T} \right)^2 \right] \exp \left[ -\left( \frac{\omega - \Omega_\sigma}{\sqrt{2} k_\parallel v_\sigma T} \right)^2 \right],
\]

where \( W_k(t) = |E_k^2(t)|/(8\pi) \) is the wave-energy density.

References

Hollweg J. V., 1999, JGR, 104, 24781
Hollweg J. V., Johnson W., 1988, JGR, 93, 9547
Ichimaru S., 1973, Basic Principles of Plasma Physics, Benjamin, Massachusetts
Isenberg P. A., Hollweg J. V., 1982, JGR, 87, 5023
Isenberg P. A., Hollweg J. V., 1983, JGR, 88, 3923
Isenberg P. A., 1984, JGR, 89, 6613
Isenberg P. A., 1990, JGR, 95, 6437
Kindel J. M., Kennel C. F., 1971, JGR, 76, 3055
Marsch E., Goerts C. K., Richter K., 1982, JGR, 87, 5030
McKean J. F., Axford W. I., Banaszkiewicz M., 1997, GRL, 24, 2877
Zhang T. X., Li B., 2004, PhPl, 11, 2172