Primordial non-Gaussianity from LAMOST surveys *

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\textbf{Abstract} The primordial non-Gaussianity (PNG) in the matter density perturbation is a very powerful probe of the physics of the very early Universe. The local PNG can induce a distinct scale-dependent bias on the large scale structure distribution of galaxies and quasars, which could be used for constraining it. We study the detection limits of PNG from the surveys of the LAMOST telescope. The cases of the main galaxy survey, the luminous red galaxy (LRG) survey, and the quasar survey of different magnitude limits are considered. We find that the Main1 sample (i.e. the main galaxy survey which is one magnitude deeper than the SDSS main galaxy survey, or \( r < 18.8 \)) could only provide a very weak constraint on PNG. For the Main2 sample (\( r < 19.8 \)) and the LRG survey, the 2\( \sigma \) (95.5\%) limits on the PNG parameter \( f_{\text{NL}} \) are \(|f_{\text{NL}}| < 145\) and \(|f_{\text{NL}}| < 114\) respectively, which are comparable to the current limit from cosmic microwave background (CMB) data. The quasar survey could provide a much more stringent constraint, and we find that the 2\( \sigma \) limit for \(|f_{\text{NL}}|\) is between 50 and 103, depending on the magnitude limit of the survey. With Planck-like priors on cosmological parameters, the quasar survey with \( g < 21.65 \) would improve the constraint to \(|f_{\text{NL}}| < 43\) (2\( \sigma \)). We also discuss the possibility of further tightening the constraint by using the relative bias method proposed by Seljak.

\textbf{Keywords}: cosmology; large scale structure

1 INTRODUCTION

The seed of large scale structure (LSS) is usually thought to be produced during a period of inflation in the early universe. Models of the standard scenario of single field slow roll inflation generally predict that these primordial perturbations are Gaussian distributed (Maldacena 2003; Acquaviva et al. 2003; Creminelli 2003; Bartolo, Komatsu & Matarrese 2004). However, significant primordial non-Gaussianity (PNG) may be produced by multi-field inflation (Linde & Mukhanov 1997) and reheating (Dvali, Gruzinov & Zaldarriaga 2004), if the inflaton potential has some sharp features

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(Chen, Easther & Lim 2007), or if the inflation is driven by a field with non-canonical kinetic terms (Chen et al. 2007). PNG is also predicted by alternative models to inflation, such as the ekpyrotic scenario (Lehners & Steinhardt 2008). Thus, non-Gaussianity in primordial density perturbations could be a very powerful discriminator for physical models of the very early Universe (Bartolo, Komatsu & Matarrese 2004; Komatsu et al. 2009).

The PNG in density perturbation is usually parametrized as

$$\Phi(x) = \phi(x) + f_{NL}(\phi^2(x) - \langle \phi^2 \rangle),$$

(1)

where $\Phi$ is Bardeen’s gauge-invariant potential and $\phi$ is a Gaussian random field. The parameter $f_{NL}$, which denotes the effect of the PNG, is typically of order $10^{-2}$ in standard inflation models (e.g., Gangui et al. 1994). The non-linear transformation from the primordial field fluctuation to observables generically gives rise to $f_{NL} \sim O(1)$. With $\phi \sim 10^{-5}$, this produces an extremely small non-Gaussianity in observables, such as the cosmic microwave background (CMB) temperature anisotropy or LSS bispectrum.

The CMB anisotropy so far offers the cleanest observational probe of PNG (Wang & Kamionkowski 2000; Verde et al. 2001). Yadav & Wandelt (2008) recently claimed the detection of a positive $f_{NL}$ at 99.5% significance ($27 < f_{NL} < 147$ at 2$\sigma$) from the Wilkinson Microwave Anisotropy Probe (WMAP) 3 year data. Analysis of the WMAP 5-year data using the bispectrum also favors a slightly positive $f_{NL}$, $-9 < f_{NL} < 111$ at the 95.5% confidence level (C.L.) (Komatsu et al., 2008), while the Minkowski functional method yields $-178 < f_{NL} < 64$ (Komatsu et al. 2008). In the future, the Planck satellite should significantly improve the constraint on $f_{NL}$, which may eventually push the uncertainty in $f_{NL}$ to be around 5 (Komatsu & Spergel 2001). Alternatively, one may use the LSS observations to search for the PNG. However, even if the initial fluctuations are Gaussian, during the gravitational growth of the LSS, a non-linear effect also generates non-Gaussianity, which may swamp the primordial one. Typically, the LSS constraints on PNG using the bispectrum technique are weak (Verde et al. 2001; Scoccimarro, Sefusatti & Zaldarriaga 2004; Sefusatti & Komatsu 2007). One consequence of PNG in structure formation is to significantly enhance the number of rare peaks in fluctuation, boosting the formation of massive objects (Lucchin, Matarrese & Vittorio 1986; Matarrese, Verde & Jimenez 2000). One could observe this effect from the abundance of galaxy clusters (Benson, Reichardt & Kamionkowski 2002) or dark matter halos harboring the first generation of stars (Chen et al. 2003). However, the required observational task is difficult due to the associated low-number statistics.

Recently, Dalal et al. (2008) proposed a powerful and practical method for probing the local PNG with LSS observations. The PNG-caused enhancement in the formation of massive dark matter halos induces a distinctive scale-dependent bias on the largest scales. Subsequent works confirmed and extended this result (Matarrese & Verde 2008; Slosar et al. 2008; Afshordi & Tolley 2008; McDonald 2008; Carbone, Verde & Matarrese 2008; Kamionkowski, Verde & Jimenez 2008), showing that useful limits on PNG could be obtained from LSS surveys.

In the present work, we investigate the sensitivity to PNG from the extragalactic surveys of the Large sky Area Multi-Object Spectroscopic Telescope (LAMOST). The LAMOST is a recently built Chinese 4-meter Schmidt telescope with a field of view of 20$^2$. Equipped with 4000 optical fibers that are individually positioned by a computer during observation, it can take spectra of 4000 targets simultaneously (Chu 1998; Stone 2008). The LAMOST is currently undergoing engineering tests and calibrations, and it is expected to begin conducting surveys by 2010/2011. In a recent paper, we have forecasted the constraining power of the LAMOST extragalactic surveys on dark energy measurement (Wang et al. 2008) by considering different samples of galaxies and quasars. Here, we make a similar forecast on the constraining power of PNG.

1 http://www.lamost.org/
2 METHOD

On large scales, the PNG induces a distinct scale-dependent bias in the distribution of tracer objects, which can be measured from the correlation function or power spectrum of galaxies and quasars. Extending the work of Dalal et al. (2008), Slosar et al. (2008) found that this bias departs from the Gaussian case as

$$\Delta b(z, M, k) = 2 f_{NL} (b - p) \delta_c \frac{3 \Omega_{m0}}{2k^2 T(k) D(z)} \left( \frac{H_0}{c} \right)^2,$$

where \( b \) is the Eulerian bias and can be calculated as usual with the halo model, \( \delta_c = 1.686, D(z) \) is the linear growth factor normalized to be the scale factor in the matter dominated epoch, \( \Omega_{m0} \) is the present matter energy density parameter, \( T(k) \) is the transfer function, \( H_0 \) is the current Hubble parameter and \( c \) is the speed of light.

The parameter \( p \) is introduced to account for the effect of merger bias. If the effect of the merger on the formation of tracer objects is negligible, its value should be unity. However, the formation of some objects may be triggered by mergers. For example, quasars are believed to be accreting massive black holes in the centers of galaxies, and the activity of the quasar may be triggered by galaxy mergers, which send gas to the center of the galaxy, providing the materials needed for accretion. If the merger effect is dominant, \( p = 1.6 \). In general, the value of \( p \) is between 1 and 1.6.

We consider three types of extragalactic surveys for LAMOST. (1) The main galaxy survey, which is a magnitude-limited general survey of all types of galaxies. By Main1 and Main2, we denote respectively the samples which are 1 and 2 magnitudes deeper than the SDSS surveys, i.e. Main1 for \( r < 18.8 \), and Main2 for \( r < 19.8 \). (2) The LRG survey, a survey of color-selected luminous red galaxies (LRGs). Here for illustration, we have used the MegaZ LRG sample (Collister et al. 2006) for calculation, though in reality the LAMOST LRG will be selected independently. (3) The quasar survey. Depending on the magnitude limit, we define three samples, QSO1 (\( g < 20.5 \)), QSO2 (\( g < 21 \)), and QSO3 (\( g < 21.65 \)). As in Wang et al. (2008), we assume that the surveyed area is 8000 deg\(^2\), for which we have the SDSS imaging data, even though the actual survey may cover a larger area if the input catalog for the south Galactic cap is available. For more details of these samples, including their number densities, redshift distributions, and the values of bias (estimated in the Gaussian models), see Wang et al. (2008).

We model the galaxy power spectrum as

$$P_g(k) = (b_g + \Delta b)^2 P_{lin}(k) \left( 1 + \frac{Qk^2}{1 + Ak} \right),$$

where \( b_g \) is the bias of the galaxies calculated for the Gaussian case, \( P_{lin} \) is the linear matter power spectrum (Eisenstein & Hu 1998), \( k \) is the wavenumber \( (10^{-2} \text{ hMpc}^{-1} < k < 0.2 \text{ hMpc}^{-1}) \), and \( Q \) and \( A \) are free parameters which describe the non-linear effect. For fiducial models, we fix \( Q = 4.6 \) for the main samples, \( Q = 30 \) for the LRG sample, and adopt \( A = 1.4 \) (Cole et al. 2005; Tegmark et al. 2006). For the quasar power spectrum, we adopt the simpler form

$$P_q(k) = (b_q + \Delta b)^2 P_{lin}(k),$$

where \( b_q \) is the bias of quasars. The value of bias for each of the samples has been given in Wang et al. (2008). We assume that the PNG is small, so that the value of bias does not change much.

The deviation of bias could be discovered with the precise measurement of the power spectrum of the galaxy or quasar distribution. The measurement error of the power spectrum is given by (Feldman, Kaiser & Peacock 1994; Tegmark 1997)

$$\frac{\sigma_P}{P} = 2\pi \sqrt{\frac{1}{V_s k^2 \Delta k} \left( 1 + n P \right) / n P} = 2\pi \sqrt{\frac{1}{V_{eff} k^2 \Delta k}},$$
where \( V_s \) is the survey volume and \( V_{\text{eff}} \) is the effective volume, defined by

\[
V_{\text{eff}} \equiv \int \left[ \frac{n(r)P}{n(r)P + 1} \right]^2 dr = \left[ \frac{nP}{nP + 1} \right]^2 V_s.
\]

The second equality holds if the comoving number density \( n(r) \) is constant. If we divide the samples into different redshift bins as in Wang et al. (2008), then the total likelihood function is the product of the likelihood functions for all the different bins.

In order to estimate the error in the measurement of \( f_{NL} \), we use a Markov Chain Monte Carlo (MCMC) technique to explore the parameter space, then marginalize over (i.e. integrate out) the other parameters to obtain a constraint on the PNG. For details of the MCMC technique, see, e.g., our earlier paper (Gong & Chen 2007). The fiducial cosmology we adopt is the WMAP 5-year best fit \( \Lambda \)CDM model without PNG, with \( \Omega_{\Lambda 0} = 0.05, \Omega_{m0} = 0.24, \Omega_{A0} = 0.76, \sigma_8 = 0.82, n_s = 0.96, f_{NL} = 0.0 \) and \( h_0 = 0.7 \). The allowed ranges of parameters in the Monte Carlo are \( \Omega_{\Lambda 0} \in (0, 0.1), \Omega_{m0} \in (0, 1), \sigma_8 \in (0.6, 0.9), n_s \in (0.9, 1.1), f_{NL} \in (-400, 400), Q \in (0, 40) \) and \( h_0 \in (0.5, 0.9) \).

The value of bias for the sample is an important parameter which is partially degenerate with \( f_{NL} \). We model the value of bias in different redshift bins using two possible forms:

\[
b(z) = b_0(1 + z)\gamma,
\]

or

\[
b(z) = c_1 + c_2/D_0(z),
\]

where \( D_0(z) \) is the growth factor normalized such that \( D_0(z = 0) = 1 \). This latter form is inspired by \( b(z) = 1 + (b_p - 1)/D_0(z) \), which can be derived in the linear regime for a passively evolved tracer population without any mergers by using \( \delta_g = \delta_m \) and \( \delta_g(z) = b(z)\delta_m(z) \) (Fry 1996). Here, \( b_p \) is the present day bias factor of the tracer; \( \delta_g \) and \( \delta_m \) are the tracer and matter density contrast, respectively. We then try to use these two parametrization forms to fit the bias for our samples which were given in Wang et al. (2008). Our test shows that for the second form (Eq. (8)) to yield a good fit, we must have \( c_1 < 0 \), which is physically un PLAUSIBLE given the original motivation for this form of bias evolution. Thus, we choose to use the first form (Eq. (7)) to fit biases at different redshifts. The parameters \( b_0 \) and \( \gamma \) are then free parameters in the MCMC with the priors given by \( b_0 \in (0, 10) \) and \( \gamma \in (-5, 5) \). The predicted values of bias given by the best fit parameter for several samples (Main1, LRG, QSO3) and those given by Wang et al. (2008) are shown in Figure 1. We see that the functional form we adopt (Eq. (7)) is adequate to describe the value of bias as a function of redshift for each sample.

Note that to see how the LSS probes the PNG, here we do not consider joint constraints on \( f_{NL} \) from CMB observations (e.g., bispectrum). We also do not use the currently available CMB angular power spectrum data in our MCMC unless otherwise stated (see next section about Planck prior).

\section{3 RESULTS AND CONCLUSIONS}

The results of our analysis, i.e., the \( 1\sigma (68.3\%) \), \( 2\sigma (95.5\%) \), and \( 3\sigma (99.7\%) \) limits of \( f_{NL} \) inferred from the power spectrum of the galaxy and quasar samples, are given in Tables 1 and 2, and are also shown in Figure 2. We see that for the Main1 sample, the \( 3\sigma (99.7\%) \) limit on \( |f_{NL}| \) reaches 379, so it provides only a very weak constraint on PNG. The Main2 and LRGs surveys have almost the same effective volume on large scales (Wang et al. 2008), so they provide a comparable constraint on \( f_{NL} \). Compared with the current SDSS spectroscopic LRG survey (Slosar et al. 2008), the constraint on \( |f_{NL}| \) could be improved by a factor of about 1.5, to reach a \( 2\sigma \) limit of about 114, which is comparable to current limits from WMAP data (Komatsu et al. 2008).

Quasars are luminous and can be observed at high redshifts, so one could obtain a large survey volume with quasars. For the large scales considered here, the quasars provide the largest effective
Fig. 1  Fiducial values of bias and the fits with the functional form $b(z) = b_0 (1 + z)^{\gamma}$ for the Main1, LRG, and QSO3 samples. For each sample, the fiducial bias values (symbols) are from Wang et al. (2008), the thick curves are the best fit with the thin curves denoting the central 68% distribution marginalized over the MCMC results.

Table 1  The 1σ, 2σ, and 3σ ranges of $f_{NL}$ constrained with the LAMOST Galaxy surveys. The last row ($LRG-\alpha$) is from the relative bias method proposed by Seljak (2008).

<table>
<thead>
<tr>
<th>Survey</th>
<th>1σ</th>
<th>2σ</th>
<th>3σ</th>
</tr>
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<tbody>
<tr>
<td>Main1</td>
<td>±127</td>
<td>±252</td>
<td>±379</td>
</tr>
<tr>
<td>Main2</td>
<td>±71</td>
<td>±145</td>
<td>±206</td>
</tr>
<tr>
<td>LRG</td>
<td>±53</td>
<td>±114</td>
<td>±153</td>
</tr>
<tr>
<td>LRG–α</td>
<td>±13</td>
<td>±28</td>
<td>±37</td>
</tr>
</tbody>
</table>

Table 2  The 1σ, 2σ, and 3σ Ranges of $f_{NL}$ Constrained with the LAMOST Quasar Surveys

<table>
<thead>
<tr>
<th>Survey</th>
<th>Merger (p = 1.6)</th>
<th>Non-merger (p = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QSO1</td>
<td>1σ</td>
<td>2σ</td>
</tr>
<tr>
<td>QSO2</td>
<td>±39</td>
<td>±82</td>
</tr>
<tr>
<td>QSO3</td>
<td>±33</td>
<td>±69</td>
</tr>
</tbody>
</table>
volume (c.f. fig. 3 of Wang et al. 2008), so we expect the LAMOST quasar surveys to provide a better constraint on \( f_{\text{NL}} \). To account for the possible effect of mergers, we did calculations for the case of \( p = 1 \) (non-merger) and \( p = 1.6 \) (merger). We see that even for the QSO1 sample \((g < 20.5)\), the 95.5\%\(2\sigma\) limit on \(|f_{\text{NL}}|\) reaches 103 for the merger case and 82 for the non-merger case. For the QSO2 sample \((g < 21)\), the corresponding limits are 82 (merger) and 63 (non-merger). For the QSO3 sample \((g < 21.65)\), the limits shrink to 69 (merger) and 50 (non-merger). Compared with the SDSS quasar surveys (Slosar et al. 2008), the LAMOST could improve the constraint of \( f_{\text{NL}} \) by a factor of 1.3. We note that we assumed a lower QSO bias than Slosar et al. (2008), so the limits obtained here are conservative.

The posterior probability distribution function of \( f_{\text{NL}} \) appears to be nearly Gaussian here, so the \( 1\sigma \), \( 2\sigma \), and \( 3\sigma \) errors are almost uniformly spaced. We have also investigated the degeneracy between \( f_{\text{NL}} \), \( \Omega_m \) and \( n_s \). We find that \( f_{\text{NL}} \) is positively correlated with \( \Omega_m \), but it is almost independent of \( n_s \), confirming the results of Slosar et al. (2008).

By the time that the LAMOST surveys are completed, the Planck satellite should have been observing the CMB for a few years. With these CMB data, the uncertainties in cosmological parameters should shrink further. Here, we simulate this effect by adding priors to the other cosmological parameters based on the forecast of the Planck surveys. We set the standard deviation and associated values to be \( \Omega_{b0} = 0.05 \pm 4.0 \times 10^{-4} \), \( \Omega_{m0} = 0.24 \pm 5.0 \times 10^{-3} \), \( \sigma_8 = 0.82 \pm 6.7 \times 10^{-3} \), \( n_s = 0.96 \pm 4.0 \times 10^{-3} \) and \( h_0 = 0.7 \pm 6.9 \times 10^{-3} \) (Colombo et al., 2008). We present the results for the LRG survey and QSO3 survey, and the impacts on other cases are similar. With the stronger priors, the constraints are improved slightly. With the QSO3 sample, we obtain \(|f_{\text{NL}}| < 65\) (merger case) and \(|f_{\text{NL}}| < 43\) (non-merger case) at 95.5\% C.L., in comparison with \(|f_{\text{NL}}| < 69\) and \(|f_{\text{NL}}| < 50\) without the Planck priors. These results are given in Table 3 and are also shown in Figure 2.

Directly comparing the density fields of tracers with different biases in the same survey volume helps to eliminate the measurement error due to cosmic variance and break the degeneracy with other cosmological parameters (Seljak 2008; McDonald & Seljak 2008). The relative bias \( \alpha = b_1/b_2 \) is proposed to be used as the estimator of PNG (Seljak 2008; McDonald & Seljak 2008). Under the assumption that the biases of the two tracers are deterministic, and one tracer is unbiased \((b = 1)\), the constraint of \( f_{\text{NL}} \) would be improved by a factor of \( 2 \left[ (\hat{n}_1 P_1)^{-1} + (\hat{n}_2 P_2)^{-1} \right]^{-1/2} \). The feasibility of this method depends on the number density of the tracers. Here, we consider the LRG as the high bias tracer, and a sample of galaxies with photometric redshifts as the low bias tracer. The average redshift of the LRGs is \( z \approx 0.4 \). Using the halo model method described in Wang et al. (2008), we find that for a sample of galaxies with good photometry up to \( r < 21.5 \), the average comoving density is \( n = 8.11 \times 10^{-3} h^3 \text{Mpc}^{-3} \), and the average bias is \( b = 1.08 \). This sample of galaxies satisfies the requirement for the low bias tracer and has available SDSS photometry. With the relative bias method, we find that the limit on \( f_{\text{NL}} \) obtainable with LRGs could be improved by a factor of about 4.1, with the \( 1\sigma \), \( 2\sigma \), and \( 3\sigma \) limits on \(|f_{\text{NL}}|\) being 13, 28, and 37, respectively.

<table>
<thead>
<tr>
<th>Survey</th>
<th>1σ</th>
<th>2σ</th>
<th>3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRG</td>
<td>±34</td>
<td>±69</td>
<td>±102</td>
</tr>
<tr>
<td>QSO3 merger</td>
<td>±32</td>
<td>±65</td>
<td>±93</td>
</tr>
<tr>
<td>QSO3 non-merger</td>
<td>±21</td>
<td>±43</td>
<td>±60</td>
</tr>
</tbody>
</table>

Table 3: The 1\(\sigma\), 2\(\sigma\), and 3\(\sigma\) Constraints on \( f_{\text{NL}} \) from the LAMOST Surveys with Planck Priors.
In fact, since the evolution of the number density and clustering of the two galaxy samples could be different, the assumption of perfect correlation between the two tracer populations may not be valid, especially for the large volumes used for the non-Gaussianity test discussed here. This could introduce an additional error. The detailed study of this effect is beyond the scope of the present paper, and we plan to investigate it in subsequent studies. For the problem discussed here, crude estimates show that in the worst case, it may cause a degradation of the statistical power by as large as 50%. The LRGα-r in Figure 2 shows the degraded precision.

In summary, we forecast constraints on the primordial non-Gaussianity from the LAMOST surveys. Using the scale-dependent bias effect proposed by Dalal et al. (2008), and based on the estimations of the galaxy and quasar surveys for the LAMOST (Wang et al. 2008), we find that the galaxy surveys provide a relatively weak constraint on the $f_{NL}$ parameter, but the QSO surveys could provide strong constraints, with a $2\sigma$ limit on $|f_{NL}|$ somewhere between 50 to 103, depending on
the sample size of the LAMOST QSO survey. After adding the Plank priors for other cosmological parameters, the QSO3 survey could give a $2\sigma$ limit of 43 for $|f_{NL}|$. We also considered using the relative bias method (Seljak, 2008). Correlating the LAMOST LRG sample with a photometric sample of galaxies with $r < 21.5$, for which $b \approx 1$ at $z = 0.4$, we obtain a $2\sigma$ limit of $|f_{NL}| < 28$. These results show that LAMOST could set useful constraints on the primordial non-Gaussianity.

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