INVITED REVIEWS

Numerical simulations for MHD coronal seismology

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Abstract Magnetohydrodynamic (MHD) processes are important for the transfer of energy over large scales in plasmas and so are essential to understanding most forms of dynamical activity in the solar atmosphere. The introduction of transverse structuring into models for the corona modifies the behavior of MHD waves through processes such as dispersion and mode coupling. Exploiting our understanding of MHD waves with the diagnostic tool of coronal seismology relies upon the development of sufficiently detailed models to account for all the features in observations. The development of realistic models appropriate for highly structured and dynamical plasmas is often beyond the domain of simple mathematical analysis and so numerical methods are employed. This paper reviews recent numerical results for seismology of the solar corona using MHD.

Key words: MHD — methods: numerical — Sun: corona — Sun: magnetic fields — Sun: oscillations — waves

1 INTRODUCTION

Magnetohydrodynamic (MHD) waves in the solar corona are widely studied as a seismological tool for the determination of fundamental plasma parameters (see, e.g., the recent review by De Moortel & Nakariakov 2012). As a means of transporting energy and momentum they also possibly have a significant role in the origins of coronal heating and solar wind acceleration (e.g. recent review by Parnell & De Moortel 2012). The modification of MHD wave behavior by the presence of transverse structuring of the plasma parameters was derived for slab and cylinder models (Zaitsev & Stepanov 1975, 1982; Edwin & Roberts 1982, 1983). Coronal loops are commonly modeled as field-aligned density enhancements ($\rho_0 > \rho_e$), having an internal Alfvén speed $C_{A0}$ less than the Alfvén speed $C_{Ae}$ of the (external) surrounding plasma and so behaving as a waveguide. The introduction of a characteristic spatial scale, $a$, by the transverse structuring makes the modes of oscillation dispersive, i.e the phase speed depends upon the longitudinal wavenumber $k$. For fast modes in a low $\beta$ plasma, the phase speed tends to the internal Alfvén speed $C_{A0}$ in the short wavelength limit ($ka \to \infty$). The phase speed increases with increasing wavelength $\lambda = 2\pi/k$ until it reaches either the external Alfvén speed $C_{Ae}$ in the case of sausage modes ($m = 0$) or the kink speed $C_k$ in the case of kink ($m = 1$) or fluting ($m > 1$) modes.

One of the first attempts at numerically modeling MHD seismology was the study of the time signatures of impulsively generated waves in a coronal plasma by Murawski & Roberts (1994) which reproduced the quasi-periodic behavior anticipated by Roberts et al. (1984). Numerical simulations
have also been used to study the behavior of waves in the corona for models such as resonant absorption (e.g. Steinolfson & Davila 1993; Ofman et al. 1995; Arregui et al. 2007; Terradas et al. 2008b; Russell & Wright 2010), phase mixing (e.g. Hood et al. 2005; McLaughlin et al. 2011), vertical kink modes (e.g. Gruszeczki et al. 2006; Wasiljew & Murawski 2009; Selwa et al. 2010), oscillations in active regions (e.g. McLaughlin & Ofman 2008; Selwa et al. 2011; Pascoe et al. 2009; De Moortel & Pascoe 2009; Pascoe & De Moortel 2014), fine structuring (e.g. Murawski et al. 2001; Pascoe et al. 2007b), magnetoacoustic-gravity waves (e.g. Murawski et al. 2013; Jelinek & Murawski 2013), instabilities (e.g. Terradas et al. 2008a; Möstl et al. 2013), magnetic reconnection (e.g. McLaughlin et al. 2009), and other wave behaviors associated with null points (e.g. Murawski et al. 2011; Thurgood & McLaughlin 2013). Comprehensive reviews of numerical simulations of coronal MHD waves and oscillations were produced by Ofman (2009) and Terradas (2009). This paper will focus on more recent results relevant to seismology of the transverse structuring of the solar corona using MHD waves, and in particular those studies which employ numerical modeling in their approach.

Section 2 gives an overview of recent developments in our understanding of leaky sausage modes in coronal waveguides. Section 3 considers the damping of kink modes by the process of mode coupling. The generation of quasi-periodic wave trains by the dispersive evolution of propagating sausage modes is described in Section 4. Concluding remarks are presented in Section 5.

2 LEAKY SAUSAGE MODES OF CORONAL PLASMA WAVEGUIDES

Recent studies have considered the behavior of the sausage mode \((m = 0)\) for wavenumbers less than its cut-off, for which the mode is leaky and wave energy propagates away from the structure. Figure 1 shows the behavior of the period of oscillation, in units of \(a/C_{Ae}\), of a sausage mode in a cylindrical waveguide as a function of the wavelength, in units of \(a\) (Nakariakov et al. 2012). The different curves represent different steepnesses of the density profile in the radial direction, with the thick solid line being a step-function profile (e.g. Edwin & Roberts 1983). The dependence of the period on the

![Fig. 1](image-url)
profile steepness is potentially useful for the seismological determination of this parameter, typically being smaller than modern instruments can accurately resolve. In all cases, the period increases with wavelength up to the cut-off \( \lambda = \lambda_0 C \) (solid line furthest to the right in Fig. 1) beyond which it tends to a constant value (independent of \( \lambda \)). This numerical parametric study therefore resolves the long-standing discrepancy of the sausage mode period being described in various studies as being dependent on (e.g. Nakariakov et al. 2003; Aschwanden et al. 2004; Pascoe et al. 2007a; Inglis et al. 2009) or independent of (e.g. Zaitsev & Stepanov 1982; Cally 1986; Kopylova et al. 2007) the wavelength. The leaky regime of sausage modes was recently considered in detail by Vasheghani Farahani et al. (2014) who derived analytical expressions for the frequency and damping behavior near the cut-off and in the long wavelength limit. These approximate analytical dependences are shown in Figure 2 (solid lines) and were supported by a series of numerical simulations (dashed lines). The signal quality (ratio of damping time to period \( \tau/P \)) for leaky modes was also shown to be approximately proportional to the density contrast ratio in the long wavelength limit, with \( \rho_0/\rho_e > 20 \) being the regime for reliably detectable observations.

In contrast with sausage modes that can have high quality for transverse profiles of any steepness, all other azimuthal harmonics (\( |m| > 0 \)) are subject to efficient coupling with small-scale torsional (Alfvénic) motions. For the most general case of the transverse structuring being smooth, i.e. the Alfvén speed varies continuously from its value inside \( (C_{A0}) \) to its value far from the structure \( (C_{Ae}) \), the behavior of kink modes is modified by the introduction of this mode coupling which will be discussed in detail in the following section.

3 MODE COUPLING IN KINK WAVES

In physics, the term “resonant absorption” is usually given to the processes of effective energy transfer from macroscopic, collective oscillations to oscillations of small-scale microscopic resonators. The process of resonant absorption was first discussed by Sedláček (1971) in the context of electrostatic oscillations in a cold plasma. It was later proposed as a plasma heating mechanism by Chen & Hasegawa (1974) and applied to coronal loops by Ionson (1978). Although the coupling is itself ideal, the process leads to the spatial redistribution of energy (e.g., Tataronis 1975), which may initiate or enhance dissipative processes such as resistivity (e.g., Poedts et al. 1989, 1990). Resonant absorption was used to account for the strong damping of standing kink modes of coronal loops by Ruderman & Roberts (2002) and Goossens et al. (2002).
Tomczyk et al. (2007) used the ground-based coronagraph Coronal Multi-channel Polarimeter (CoMP) to observe spatially and temporally ubiquitous propagating transverse velocity oscillations with periods of about 5 minutes. They were initially interpreted as Alfvén waves, which was disputed by several authors (e.g., Van Doorsselaere et al. 2008a,b). Tomczyk & McIntosh (2009) determined that the propagating waves were being strongly damped, i.e. within just a few periods of oscillation. This is similar to the damping rate observed for standing kink modes and was interpreted by Pascoe et al. (2010) in terms of a coupled kink (collective) mode and small-scale Alfvénic oscillations of individual magnetic surfaces (that are the surfaces of the constant Alfvén speed). According to this model, the observed oscillation corresponds to the kink component, i.e. the bulk transverse motion of the a loop structure, whereas the azimuthal Alfvén oscillations remain unresolved by current observations. The observed damping reported by Tomczyk & McIntosh (2009) therefore only corresponds to the timescale in which energy is transferred from the kink mode to the Alfvén oscillations, which is an ideal process, rather than the actual dissipation of the energy. For a propagating kink wavepacket in an inhomogeneous loop, the mode coupling condition is satisfied at the magnetic surfaces where $C_k = V_A(r)$, i.e. where the phase speed of the kink mode wavepacket matches the Alfvén wave phase speed (e.g. Allan & Wright 2000; Pascoe et al. 2011).

Numerical simulations that consider the effect of line-of-sight integration on mode identification and energy budget calculations were performed by De Moortel & Pascoe (2012). The oscillations of multiple randomly-distributed structures were modeled with 3D simulations performed with LARE3D (Arber et al. 2001). This code solves the MHD equations with an adiabatic equation of state by taking a Lagrangian predictor-corrector time step, after which variables are conservatively remapped back onto the original Eulerian grid using van Leer gradient limiters. This method solves the MHD equations in their non-conservative form and so is particularly well suited to calculating accurate temperatures in low $\beta$ plasmas such as the solar corona.

Figure 3 shows velocity vectors in the horizontal plane taken at two different times and locations. The left panel shows a cut at an early time and low down and so reveals the form of the driver at
the lower boundary; each loop oscillates independently with a transverse (kink) displacement in a random direction. The right panel (not with the same scale) shows the velocity vectors at a later time and higher up, by which stage mode coupling within inhomogeneous regions (denoted by line contours) has converted a large fraction of the wave energy from the initially transverse motions to azimuthal motions corresponding to $m = 1$ Alfvén waves. Since the corona is optically thin, in the scenario that such a configuration is observed from the side then multiple structures can exist along the line-of-sight (e.g. the red line in Fig. 3, color online) and contribute to the measured (integrated) signal. De Moortel & Pascoe (2012) demonstrated that this has the effects of complicating mode identification and potentially producing significant underestimations of the energy budget provided by footpoint motions.

3.1 Spatial Damping Profile

Pascoe et al. (2012) performed numerical simulations to investigate the damping profile as a function of height for propagating waves driven by harmonic footpoint motions. They demonstrated that the commonly assumed exponential damping profile that had often been assumed in seismological estimations (e.g. Ruderman & Roberts 2002; Goossens et al. 2002; Terradas et al. 2010) provides a poor account of the decay at low heights (see Fig. 4) and so may be unsuitable for application to observations of strongly damped oscillations such as in Tomczyk & McIntosh (2009).

Pascoe et al. (2012) proposed instead that a Gaussian spatial damping profile best describes the behavior at low heights and hence, for strongly damped oscillations, this would be the dominant damping behavior as shown in Figure 4. Hood et al. (2013) followed up this result with a full analytical treatment of the problem. They obtained analytical expressions that describe a nonlinear damping rate that can be approximated by a Gaussian form at low heights. At larger heights the exponential damping profile similar to that produced by thin tube, thin boundary (TTTB) analysis (e.g. Ruderman & Roberts 2002; Goossens et al. 2002) is recovered.

The full expression for the spatial damping profile derived by Hood et al. (2013) is given by an integro-differential equation which must be solved numerically and so is not well-suited for seismological application to observations. Accordingly, Pascoe et al. (2013b) demonstrated a spatial damping profile combining two approximations which produces an accurate description of oscillations at all heights.

![Fig. 4](image_url) Transverse velocity component $v_x$ as a function of height at the loop axis. The dashed and dot-dashed lines represent exponential and Gaussian envelopes, respectively. Reproduced from Pascoe et al. (2012) with permission ©ESO.
The spatial damping profile applied to the first stage of the oscillation (i.e. for small heights) is based on the analysis of Hood et al. (2013) which can be approximated by a Gaussian profile of the form
\[ A(z) = \frac{A_0}{2} \left[ 1 + \exp \left( -\frac{z^2}{L_g^2} \right) \right], \tag{1} \]
where \( A_0 \) is the initial amplitude of the oscillation and \( L_g \) is the Gaussian damping length scale which depends upon the loop parameters as
\[ L_g^2 = \frac{16}{\epsilon k^2 \kappa^2}. \tag{2} \]
Here \( \epsilon = l/R \) is the (normalized) inhomogeneous layer width, \( \kappa = (\rho_0 - \rho_e)/(\rho_0 + \rho_e) \) is a ratio of densities inside and outside the loop, and the wavenumber is \( k = 2\pi/\lambda \) for a wavelength \( \lambda \). The constant of proportionality depends upon the chosen density profile in the inhomogeneous layer and here corresponds to a profile that varies linearly from \( \rho_0 \) at \( r \leq (R - l/2) \) to \( \rho_e \) at \( r > (R + l/2) \) (see Fig. 5).

The later stage of the spatial damping profile is an exponential profile (e.g. Terradas et al. 2010 and references therein) with the form
\[ A(z) = A_0 \exp \left( -z/L_d \right), \tag{3} \]
where \( L_d \) is the exponential damping length scale given by
\[ L_d = \frac{8}{\pi \epsilon k \kappa}. \tag{4} \]
Fig. 6 Number of wavelengths after which the switch to the exponential damping profile occurs \( h = 1/\kappa \) as a function of the density contrast ratio \( \rho_0/\rho_e \). For \( \rho_0/\rho_e < 2 \) the Gaussian damping profile is expected to dominate the observed signal, whereas for \( \rho_0/\rho_e > 3 \) only the first 1–2 wavelengths exhibit the Gaussian damping behavior. Reproduced from Pascoe et al. (2013b) with permission ©ESO.

The switch from one profile to the other occurs at a height \( z = h \) and so the full general spatial damping profile is given by

\[
A(z) = \begin{cases} 
\frac{4A_0}{\lambda^2} \left[ 1 + \exp \left( \frac{-z^2}{\lambda^2} \right) \right] & z \leq h, \\
A_h \exp \left( \frac{-z-h}{L_d} \right) & z > h,
\end{cases}
\]

(5)

where \( A_h = A(z = h) \) and the height of the switch is

\[
h = \frac{L_g^2}{L_d},
\]

(6)

which can be expressed in terms of the number of wavelengths \( \lambda \) as a function of the density contrast ratio (see Fig. 6)

\[
\frac{h}{\lambda} = \frac{1}{\kappa} = \frac{\rho_0/\rho_e + 1}{\rho_0/\rho_e - 1}.
\]

(7)

This is a key result for the seismological application of kink waves damped by mode coupling as it may inform the observer about which analytical formula is the most appropriate to use for an inversion. For larger density contrasts (e.g. \( \rho_0/\rho_e > 3 \)) the switch occurs at \( h < 2\lambda \). On the other hand, for fainter loops with low density contrast ratios \( \rho_0/\rho_e < 2 \), the Gaussian profile applies to at least the first three wavelengths. Since this is the number of wavelengths typically measured by actual observations, the Gaussian damping profile is applicable to the entire signal, whereas the exponential stage is likely to not be observed or is too weak for accurate inversions.

To demonstrate the applicability of this general damping profile, Pascoe et al. (2013b) performed a series of numerical simulations using a Lax-Wendroff code to solve the linear MHD equations in cylindrical coordinates. The mode coupling process corresponds to a kink wave with azimuthal symmetry \( m = 1 \) transferring energy to an Alfén wave. The symmetry of the mode is preserved i.e. the excited Alfén wave also has an \( m = 1 \) azimuthal dependence (e.g. Pascoe et al. 2010). Assuming this \( m = 1 \) symmetry in the equations solved by the code therefore allows efficient 2D simulations calculating the \( r \) and \( z \) dependences only. The lower boundary is driven harmonically
with velocity perturbations that correspond to moving the loop footpoint back and forth about its equilibrium position. The driver efficiently excites kink oscillations that propagate along the loop, although the kink normal mode structure is not assumed in the radial direction.

The numerical domain has a radial extent of \( r = [0, 6R] \) while in \( z \) it scales according to the driving period such that 10 periods of oscillation are accommodated by \( z = [0, 10C_A \epsilon P] \). The typical resolution is \( 2400 \times 3000 \) grid points and simulation parameters \( R = 1, \rho_e = 1 \) and \( C_A = 1 \) were kept constant while \( P, l \) and \( \rho_0 \) were varied. The simulations run for a time \( t = 10P \) after which point the spatial damping profile can be investigated by considering \( v_r \) as a function of \( z \) at the center of the loop \( r = 0 \).

Figure 7 shows snapshots of the transverse (left) and azimuthal velocities (middle) as functions of \( r \) and \( z \) for a simulation with \( P = 48 \) s, \( \epsilon = 0.2 \) and \( \rho_0/\rho_e = 5 \). Inside the loop both \( v_r \) and \( v_\theta \) are damped oscillations, while within the inhomogeneous layer \( v_\theta \) increases with \( z \) (see also Fig. 9). The right panel of Figure 7 shows \( \sqrt{E_w} \), where \( E_w \) is the wave energy density. At low \( z \) the energy is concentrated in the high density core of the tube, and at larger \( z \) it is focused in the inhomogeneous layer. Cally & Hansen (2011) consider the case of mode conversion at a reflection level in a stratified atmosphere, in which only a fraction of the energy is exchanged, though here the process continues until saturation.

Figure 8 shows an example of the general spatial damping profile (Eq. (5)) applied to a numerical simulation with \( P = 24 \) s, \( \epsilon = 0.2 \) and \( \rho_0/\rho_e = 2 \). The left panel shows the transverse velocity \( v_r \) at the center of the loop as a function of propagation distance \( z \). The amplitude decreases with \( z \) as energy is transferred by mode coupling to azimuthal perturbations \( v_\theta \) in the inhomogeneous layer \( r \sim R \) (see Fig. 9). The envelope of the oscillation is the general spatial damping profile given by Equation (5), with \( h = 3\lambda \) (Eq. (7)) denoted by the vertical dashed line. The right panel shows the same oscillation using a logarithmic scale to highlight the switch between Gaussian and exponential damping behavior i.e. from a quadratic envelope to a linear one, respectively. The dotted line shows the extrapolation of the Gaussian damping profile to \( z > h \), and similarly the dot-dashed line shows the exponential profile for \( z \rightarrow 0 \). Evidently, the general spatial damping profile (Eq. (5)) combines the best aspects of the Gaussian and exponential profiles to produce an envelope which is accurate for all \( z \) (the first period of oscillation after the driver is turned on is ignored due to the known result of it having a different damping rate as described by Hood et al. 2005).

Figure 9 (left panel) shows the Alfvén wave relation \( -B_\theta(z) / \sqrt{\mu_0 \rho} \) superposed on \( v_\theta \) as a dotted line. Since the two lines are nearly indistinguishable their difference is plotted as a dashed curve. The mode coupling process itself is ideal and does not dissipate energy but the result, i.e. wave energy eventually being entirely converted to Alfvén waves propagating in the inhomogeneous layer, inevitably leads to phase-mixing (e.g., Heyvaerts & Priest 1983). Figure 9 also shows the ra-
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Fig. 8 General spatial damping profile for $P = 24$ s, $\epsilon = 0.2$ and $\rho_0/\rho_e = 2$. The solid lines show the numerical simulation of the damped kink mode and the analytical damping envelope given by Eq. (5). The dashed vertical line shows the change from Gaussian to exponential damping regimes, located at $h = 3\lambda$ as given by Eq. (7) (see also Fig. 6). Reproduced from Pascoe et al. (2013b) with permission ©ESO.

Fig. 9 The induced Alfvénic motions at the center of the inhomogeneous layer for the simulation in Fig. 7 with the Alfvén wave relation $-B_\theta(z)/\sqrt{\mu_0 \rho}$ overplotted as a dotted line. The two lines are nearly indistinguishable and so their difference is plotted as a dashed curve. $v_\theta$ as a function of $r$ at the values of $z$ is denoted by vertical dashed lines in the left panel. Reproduced from Pascoe et al. (2013b) with permission ©ESO.

dial structure of $v_\theta$ at large $z$ (at the two locations denoted by vertical dashed lines in the left panel). The radial profile corresponding to the larger value of $z$ has more extrema within the inhomogeneous layer (outlined by vertical dotted lines) and so demonstrates increased (radial) spatial gradients compared to the profile at smaller $z$. The gradients will continue to increase as the wave propagates and so can lead to the wave energy being converted to heat by some non-ideal process whose efficiency is enhanced by the large spatial gradients.

Figure 10 summarizes the results of a parametric study performed by Pascoe et al. (2013b) to demonstrate the dependence of the spatial damping profile upon the period of oscillation $P$, and the density structure parameters $\epsilon$ and $\rho_0/\rho_e$. The expected period dependences (assuming the long
The numerical length scales were calculated using a least squares fit of a general spatial damping profile as given by Equations (5) and (6) to the maxima and minima of the oscillation. Equations (2) and (4) give the dependence of the damping length scales on the inhomogeneous layer width as $L_g \propto 1/\sqrt{\epsilon}$, $L_d \propto 1/\epsilon$ and $h$ is independent of $\epsilon$.

wavelength limit $\lambda = C_k P$ are

$$L_g = \frac{2C_k}{\pi \epsilon \kappa^{1/2}} P, \quad L_d = \frac{4C_k}{\pi^2 \epsilon \kappa} P, \quad h = \frac{C_k P}{\kappa}.$$  

(8)

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Fig. 10 Damping length scale $L_g$ (triangles) and the height $h$ (pluses) as a function of the period of oscillation $P$, the inhomogeneous layer width $\epsilon$, and the density contrast $\rho_0/\rho_e$. The dotted lines show the analytical dependences given by Eqs. (6) and (8). Reproduced from Pascoe et al. (2013b) with permission ©ESO.
Equations (2) and (4) give the dependence of the damping length scales on the density contrast as $L_{g,d} \propto 1/\kappa$. Figure 10 shows $L_g$, $L_d$ and $h$ as a function of the density contrast $\rho_0/\rho_e$ for numerical simulations with $P = 120$ s and $\epsilon = 0.15$. The damping length scales have an asymptotic dependence that is well reproduced by the numerical fits.

3.2 Application of Spatial Damping Profiles to Coronal Seismology

Initial studies using damping of kink oscillations for seismology (e.g. Nakariakov et al. 1999; Ruderman & Roberts 2002; Goossens et al. 2002) were based on the use of an exponential damping profile alone. For that profile, the single fitted parameter $L_d$ is a function of the inhomogeneous layer width and the density contrast. A seismological inversion therefore has infinite possible solutions, though reasonable bounding values may be estimated (e.g. Arregui et al. 2007; Goossens et al. 2008, 2012). For the general spatial damping profile proposed by Pascoe et al. (2013b) there are three fitted parameters ($L_g$, $h$ and $L_d$) and so for data of sufficient quality a unique solution for both the layer width and density contrast may be calculated, for example the density contrast can be calculated from $h$ using Equation (7) and then Equations (2) and/or (4) can be used to find $\epsilon$. Arregui et al. (2013) also performed seismological inversions based on the general spatial damping profile (Eqs. (5) and (6)) using a Bayesian inference technique which properly returns the physical parameters and also gives estimates of error bars based on the propagation of uncertainty.

Numerical simulations (e.g. Fig. 8) can provide data with a large number of oscillations and negligibly small errors (in addition to excluding other potential complications such as line-of-sight effects). The Gaussian and exponential damping regimes are readily recognized and the most suitable spatial damping profile(s) can be applied accordingly. Observational data usually do not have sufficient quality and in this case a simpler method based on fitting a single Gaussian or exponential profile may be more appropriate. Note also that here we consider different damping behaviors arising only due to the process of mode coupling and so additional effects may also need to be included. For example, Nakariakov et al. (2009) demonstrate the departure of a kink oscillation from exponential damping due to the process of vortex shedding.

In the case of observational data not taking a form that suggests either the Gaussian or exponential damping profile is more suitable, the dependence of the height $h$ upon the density contrast (Eq. (6)) may be exploited to determine the best approximation. Pascoe et al. (2013b) propose that the Gaussian damping profile is generally more appropriate for seismological inversions except when the density contrast is known or expected to be very high.

Figure 11 summarizes this procedure in which an estimate of the density contrast is compared with the number of observed wavelengths or periods of oscillation $N_{obs}$ (also known as the oscillation quality) to determine the most suitable approximation.

Figure 12 demonstrates the scenario of fitting to data with a low signal quality typical of actual observations. Here just the first three wavelengths are considered to be “observed” and used for fitting Gaussian and exponential damping profiles. The left panel shows the error in the seismologically inferred density $\rho_S$ plotted as a function of the actual density contrast. The triangles correspond to values calculated using the Gaussian damping profile, while the squares are calculated from the exponential profile. The inhomogeneous layer width $\epsilon = 0.3$ is assumed to be known for the purposes of these calculations. The vertical dotted line denotes the density contrast at which $h = 3\lambda$ (Eq. (6)). Since all three wavelengths will be in the Gaussian damping regime for $\rho_0/\rho_e \leq 2$, the inversion based on a Gaussian damping profile (triangles) produces a very low error in this range. For larger densities, part of the decaying oscillation will be influenced by the exponential damping regime and so the Gaussian estimate becomes increasingly inaccurate. The inversion using an exponential damping profile (squares) gives a poor estimate, except in the limit of very weak damping ($\rho_0/\rho_e \rightarrow 1$), although the error does not increase much more for density contrast $\rho_0/\rho_e > 2$. Note that inversions based on an exponential damping profile tend to underestimate the density contrast, since the initial
Fig. 11 Flowchart of a simple method to determine the appropriate spatial damping profile for seismology based on the number of observed wavelengths $N_{\text{obs}}$ and an estimate of the density contrast $\rho_0/\rho_e$. Reproduced from Pascoe et al. (2013b) with permission ©ESO.

stage of the damping starts from a gradient of zero, rather than the maximum gradient assumed by the exponential spatial damping profile. In contrast, the inversion based on the Gaussian damping profile tends to overestimate the density contrast when applied outside of its known validity.

The right panel of Figure 12 shows the errors in seismological estimates of $\epsilon$ based upon fitting the Gaussian and exponential damping profiles to oscillations with low signal quality (three wavelengths). Using mode coupling in this way as a seismological tool to determine the transverse scale...
of the density structure assumes that an estimate of the density contrast is already known. In this case, the use of either a Gaussian or exponential damping profile to fit the data is more readily informed by the method proposed in Figure 11. The data considered here are for the case of a density contrast \( \rho_0/\rho_e = 2 \), for which we expect the change from Gaussian to exponential damping regimes to occur at \( h = 3\lambda \). The seismological inversions based on a Gaussian damping profile (triangles) should therefore be more accurate than those based on an exponential damping profile (squares), as is the case. For larger density contrasts the exponential damping profiles would be more suitable for seismological inversions, in accordance with Equation (6).

### 3.3 Standing Kink Modes

The mode coupling process is a robust physical mechanism applying to standing kink modes as well as propagating ones. Rapidly decaying standing kink modes are readily observed in coronal loops using EUV imaging satellites such as SOHO, TRACE (e.g. Aschwanden et al. 1999; Nakariakov et al. 1999) or more recently the Solar Dynamics Observatory (SDO) (e.g. White & Verwichte 2012; Nisticò et al. 2013). Their strong damping has been interpreted as mode coupling or resonant absorption using an exponential damping profile. Since oscillating loops are identified by their EUV intensity, it is typically assumed in interpretations (e.g. Nakariakov et al. 1999; Ruderman & Roberts 2002; Goossens et al. 2002) that the loops have a high density contrast \( \rho_0/\rho_e \approx 10 \). The more general damping behavior discussed by Pascoe et al. (2013b) is consistent with these previous studies since in the limit of large density contrasts the initial Gaussian damping behavior is only expected to apply for the first wavelength or period of oscillation (Eq. (6) and Fig. 11). However, as observations improve it is likely that standing kink waves will increasingly be detectable in fainter loops which have a lower density contrast for which mode coupling would occur with a Gaussian damping profile.

Changing our attention from propagating to standing kink modes requires us to consider temporal damping profiles rather than spatial ones. Pascoe et al. (2010) estimated the damping timescale for a finite wave train propagating at a group speed \( V_g \) (see Fig. 5) using the change of variable \( t = z/V_g \), where \( V_g = V_p = C_k \) in the long wavelength limit. For a standing mode decaying by resonant absorption with an exponential damping envelope, Ruderman & Roberts (2002) give

\[
\frac{\tau_d}{P} = L_d/\lambda = \frac{4}{\pi^2 \epsilon \kappa}.
\]

Similarly, for a standing mode damping with a Gaussian envelope (see also Ruderman & Terradas 2013) and a characteristic damping time \( \tau_g \) we have (Pascoe et al. 2013b)

\[
\frac{\tau_g}{P} = L_g/\lambda = \frac{2}{\pi \kappa \epsilon^{3/2}}.
\]

So far one potential candidate for a standing kink mode in a low density contrast loop is the observation by Van Doorsselaere et al. (2008c) using Hinode/EIS. They estimate the density contrast of their oscillating loop as \( \rho_0/\rho_e \approx 1.2 \). Additionally, they report a damping rate much lower than typically seen in higher density loops. Pascoe et al. (2013b) estimated the Gaussian damping timescale \( \tau_g \) for this particular observation and found it to be consistent with the actual reported damping rate, whereas the exponential damping timescale \( \tau_d \) was much shorter.

The initial damping behavior of standing kink modes in coronal loops has also been studied in detail by Ruderman & Terradas (2013). Following Hood et al. (2013), their analytical and numerical modeling predicts and confirms that standing waves also undergo damping behavior that initially has a Gaussian envelope and is followed by an exponential damping stage. Inspection of their results (e.g. Fig. 13) demonstrates that the Gaussian behavior in their simulations is consistent with the timescale given by Equation (10).
Hood et al. (2013) find that the classical theory of resonant absorption (e.g. Ruderman & Roberts 2002) underestimates the damping length scale during the exponential damping stage (in addition to excluding the initial Gaussian stage entirely). Similarly, Ruderman & Terradas (2013) find that for standing kink modes the damping timescale is underestimated by the classical resonant absorption formula (Eq. (9)). They report typical differences of the order of approximately 10%–20%. On the other hand, the general spatial damping profile used by Pascoe et al. (2013b) and Arregui et al. (2013) for seismological inversions does employ the damping rate given by classical resonant absorption for the exponential stage of the profile but with an accuracy typically better than the results of Hood et al. (2013) and Ruderman & Terradas (2013) might suggest. This may be understood by considering the effect of boundary conditions upon the oscillation. Analytical studies can assume that the radial boundary is located at $r \rightarrow \infty$, whereas for numerical simulations it must be located at some finite distance for calculations to complete in a reasonable runtime. In Hood et al. (2013) the numerical simulations used to confirm the analytical result produce excellent agreement for the radial boundary located at 20 times the loop radius. In the parametric study performed by Pascoe et al. (2013b) the radial boundary is located at six times the loop radius. Evidently this closer radial boundary modifies the oscillation such that the classical resonant absorption condition becomes a more accurate approximation. In the actual corona such a boundary does not exist, but on the other
hand neither does an isolated loop. Instead there exist multiple threads or loops (e.g. De Moortel & Pascoe 2012 and Fig. 3) which have been demonstrated to interact such that the classical exponential damping formula remains a reasonable estimate (e.g. Terradas et al. 2008b, Pascoe et al. 2011) and so the different rates for the exponential damping regime calculated by the full analytical solutions of Hood et al. (2013) and Ruderman & Terradas (2013) are likely to not be significant from the point of view of seismological applications.

An additional challenge in understanding and applying damping profiles comes from the recently discovered regime of undamped low-amplitude transverse oscillations (Nisticò et al. 2013; Anfinogentov et al. 2013). These observations can be understood in terms of a balance between damping, such as that provided by mode coupling, and a continuous driver, however the lack of a readily identifiable damping length scale makes their seismological application complicated. In the case of an additional high-amplitude impulsive driver that produces a decaying signal, the superposition of a low-amplitude harmonic signal may also modify the measured oscillation parameters (Nisticò et al. 2013) and possibly affect any subsequent seismological inversion.

4 DISPERSE EVOLUTION OF SAUSAGE WAVES

The dispersive evolution of sausage waves was described by Roberts et al. (1984) as a mechanism for producing quasi-periodic wave trains from a broadband initial excitation. Initial observations of propagating fast wave trains were made with the Solar Eclipse Coronal Imaging System (SECIS) (Katsiyannis et al. 2003; Cooper et al. 2003). More recently, quasi-periodic wave trains have been observed using the Atmospheric Imaging Assembly (AIA) onboard the SDO (e.g. Shen & Liu 2012; Liu et al. 2012; Yuan et al. 2013).

Recent studies have used numerical simulations to investigate more realistic models for magnetohydrodynamic waveguides. Specifically, more complex magnetic field geometries have been considered to better represent the range of structures present in the solar corona. Current sheets attract interest as a basic component of the standard model of a solar flare. Jelínek & Karlický (2012) performed numerical simulations of impulsively-generated magnetoacoustic waves in a 2D Harris current sheet. The equations for ideal magnetohydrodynamics were solved using a two-step Lax-Wendroff algorithm stabilized with artificial smoothing (Kliem et al. 2000). For comparison with the Harris current sheet, Jelínek & Karlický (2012) also numerically modeled a basic density slab in which the initial equilibrium is based on a uniform magnetic field and the generalized symmetric Epstein profile (with steepness parameter \( p = 8 \)) for the density structuring in the transverse direction. For the Harris current sheet the magnetic field was defined as

\[
B = B_{\text{out}} \tanh \frac{y - H/2}{w_{\text{cs}}} \hat{e}_x ,
\]

where

\[
B_{\text{out}} = \sqrt{\frac{2\mu_0 p_{\text{cs}}}{1 + \beta}}.
\]

The plasma beta was taken to be 0.1 outside the current sheet and then the gas pressure at the center of the current sheet \( p_{\text{cs}} \) was calculated from the condition of total pressure balance.

Fast sausage wave trains are readily generated by applying an anti-symmetric perturbation to the transverse velocity component, with many studies (e.g. Nakariakov et al. 2004, 2005) using the form

\[
v_y = A_0 y \exp \left[ -\frac{(x - x_0)^2}{\sigma_x^2} \right] \exp \left[ -\frac{(y - y_0)^2}{\sigma_y^2} \right].
\]

Here, the transverse direction in which the density is structured is taken to be \( y \) and a compressive, anti-symmetric driver is obtained by multiplying by this coordinate. \( A_0 \) is a constant which determines the initial amplitude of the perturbation, and the remaining terms in Equation (13) describe
Fig. 14 Tadpole wavelet signatures for a standard density slab (left column) and a Harris current sheet (right column). The top, middle and bottom rows are for signal detection points located at positions $L/4$, $L/2$ and $3L/4$, with the initial perturbation being located close to $L/4$. Reproduced from Jelíněk & Karlický (2012) with permission ©ESO.

the spatial localization of the pulse i.e. centered at $(x_0, y_0)$ which have transverse and longitudinal spatial scales $\sigma_x$ and $\sigma_y$, respectively.

In order to describe the range of frequencies present in the quasi-periodic wave trains generated by the dispersive evolution of fast waves it is convenient to use wavelet analysis (Torrence & Compo 1998). For dispersive wave trains which have propagated a sufficient distance from their source, a tadpole signature is typically produced by the wavelet analysis (Nakariakov et al. 2004).

Figure 14 shows the tadpole signatures generated by Jelíněk & Karlický (2012) for a standard density slab (left column) and a Harris current sheet (right column). The top, middle and bottom rows are for signal detection points located at positions $L/4$, $L/2$ and $3L/4$, with the initial perturbation being located close to $L/4$, where $L = 100$ Mm is the loop length. The signatures for the density slab and current sheet are very similar in all cases, and the tadpoles demonstrate the expected behavior of becoming more developed in time. In addition to the tadpoles highlighted by the authors, additional weaker tadpole signatures can be seen arriving afterwards, particularly in the top and middle panels. These are probably the result of reflections from the loop footpoints and so the fainter tadpoles are more developed in accordance with their longer propagation paths to the detection point. (The authors apply open boundary conditions though these rarely work perfectly and some energy is usually reflected back into the numerical domain.)

The simulations of Jelíněk & Karlický (2012) demonstrate that it would be difficult to distinguish between a density slab or a current sheet on the basis of the dispersive evolution of fast wave trains alone. On the other hand, the lack of sensitivity to the details of the particular model demonstrates that such wave trains could be a robust means of inferring the characteristic transverse length scale of observed loops. In the case of the slab the transverse scale corresponds to the density enhancement, whereas for the current sheet it corresponds to the magnetic field.

Recently, Karlický et al. (2013) applied this model to interpret decimetric band radio fiber bursts in terms of propagating fast sausage wave trains. Figure 15 shows examples of the dynamic spectra
Fig. 15 Examples of the dynamic spectra for fiber bursts from 1998 (top left) and 2003 (top right). The bottom panels show the corresponding wavelet power spectra. Reproduced from Karlický et al. (2013) with permission ©ESO.

Fig. 16 Calculated radio spectra based upon a semi-empirical model (left) and a full numerical simulation (right) of sausage wave trains in a waveguide. Reproduced from Karlický et al. (2013) with permission ©ESO.

for fiber bursts from 1998 (top left) and 2003 (top right). The bottom panels show the corresponding wavelet power spectra, showing characteristic tadpole signatures. Karlický et al. (2013) calculated radio spectra based upon the model of fast sausage modes in a waveguide, as shown in Figure 16. The left panel is for a semi-empirical model in which an analytical wave train is superimposed on chosen profiles for the density and magnetic field and the radio emission is calculated as having a constant intensity and a frequency of twice that of upper hybrid oscillations. The wave train generates a series of fiber bursts with a drifting frequency, consistent with observations. The right panel in Figure 16 shows the radio spectrum calculated from a 2D numerical simulation, allowing the effects
of dispersive evolution to be included, which produces wavy fiber bursts. However, Karlický et al. (2013) suggest that their chosen waveguide of a vertical current sheet is not ideal for modeling fiber bursts due to its high levels of energy leakage.

### 4.1 Dispersive Evolution in Stratified Atmospheres

Observations by Mészárosová et al. (2013) using the Giant Metrewave Radio Telescope (GMRT) have also revealed tadpole wavelet signatures in a fan structure above a coronal magnetic null point, with periods in the range of 10–83 s. Wang et al. (2013) recently performed 3D numerical simulations of propagating disturbances (PDs) in fan-like coronal loops, suggesting that the PDs may be produced by numerous small-scale impulsive perturbations and are consistent with the widely accepted interpretation in terms of slow magnetoacoustic waves (see also Kiddie et al. 2012; Threlfall et al. 2013 for recent discussions of PDs).

Jelínek et al. (2012) extend the work of Jelínek & Karlický (2012) to consider the effect of a current sheet in a gravitationally stratified atmosphere (Fig. 17).

Figure 17 shows the tadpole wavelet signatures for a current sheet in a gravity-free (left column) and gravitationally stratified (right column) medium. The top, middle and bottom wavelets are points where signals were detected at distances of 50, 60, and 70 Mm, respectively. The panels on the far left and right show the corresponding models for the magnetic field lines (solid lines), while the contour represents the vertical component of the magnetic field. The circles show the location of the initial perturbation and the crosses show the three signal detection points.

The numerical simulations were performed using the FLASH code to solve the 2D ideal MHD equations (Fryxell et al. 2000; Lee & Deane 2009). An adaptive mesh refinement module was used to determine the appropriate resolution for reducing the error in the gradient of mass density and hence limit the numerical diffusion within the 100 Mm by 20 Mm numerical domain. For the case of a gravitationally stratified atmosphere, the current sheet expands with height and the typical period of the propagating wave trains is found to be slightly higher. The wavelet signature is also found to be more irregular than the typical tadpole shape demonstrated for the gravity-free case.

Pascoe et al. (2013a) also consider the propagation of fast magnetoacoustic waves in a model with expanding flux tubes, but with a much larger range of magnetic stratification than that considered by Jelínek et al. (2012). A potential magnetic field is used with the form

\[
B = B_0 \sin \left( \frac{x + \pi}{l} \right) \exp \left( -\frac{z}{l} \right) \hat{e}_x + B_0 \cos \left( \frac{x + \pi}{l} \right) \exp \left( -\frac{z}{l} \right) \hat{e}_z ,
\]

where \( B_0 \) and \( l \) are constants determining the magnitude of the field and the characteristic spatial scale, respectively. The horizontal and vertical coordinates are taken to be \( x \) and \( y \), respectively. The absolute value of the field is \( B = B_0 \exp(-z/l) \), which is constant in the horizontal direction (see contour plot of Fig. 18).

The equilibrium plasma density is based on a symmetric Epstein profile with steepness parameter \( p = 8 \) (as in Jelínek & Karlický 2012) but with an increase in the width of the profile with height so that it follows the expansion of the magnetic field lines. At each horizontal level, the density in the funnel is given by an Epstein profile (see, e.g. Nakariakov & Roberts 1995; Pascoe et al. 2007a). Thus, the equilibrium density is given by the expression

\[
\rho_0 = \exp \left( -\frac{z + \pi}{\Lambda_{\text{eff}}} \right) \times \left[ (\rho_F - \rho_\infty) \text{sech}^2 \left( \frac{x/l}{\arccos \exp(z/l) - \pi/2} \right)^p + \rho_\infty \right] ,
\]

where \( \rho_\infty \) is the (constant) density far from the funnel and \( \rho_F \) is the density enhancement at the center of the funnel (at the bottom of the numerical domain). Pascoe et al. (2013a) also considered the role of hydrostatic stratification by modifying the effective scale height parameter \( \Lambda_{\text{eff}} \), though this was found to have a smaller influence than the magnetic stratification (left panel of Fig. 18) defined by
Fig. 17  Tadpole wavelet signatures for a current sheet in a gravity-free (left column) and gravitationally stratified (right column) medium. The panels on the far left and right show the corresponding models for the magnetic field lines (solid lines). The top, middle and bottom wavelets are points where signals were detected at distances of 50, 60, and 70 Mm (crosses), respectively, for an initial perturbation at 30 Mm (circles). Reproduced from Jelínek et al. (2012) with permission ©ESO.

Fig. 18  Equilibrium magnetic field (left) and density profile (right) used by Pascoe et al. (2013a). The magnetic field describes an expanding funnel at the center of the numerical domain. The colored contour represents the magnitude of the magnetic field, which varies exponentially with height. The density profile corresponds to a field-aligned funnel with density contrast $\rho_F/\rho_\infty = 3$, $p = 8$, and no gravitational stratification $\Lambda_{\text{eff}} \to \infty$. Reproduced with permission ©ESO.

the magnetic field geometry. The right panel of Figure 18 shows the density profile for a funnel with $\rho_F/\rho_\infty = 3$, $p = 8$, $l = 1$ and no density stratification $\Lambda_{\text{eff}} \to \infty$. Figure 19 shows the dispersive nature of the phase and group speeds for trapped sausage modes in this funnel structure. Note that the funnel geometry means the structure width $a$ increases with height, and the wavenumber $k$ may also vary with height due to the effect of vertical stratification on the Alfvén speed.

The funnel density structure was initially placed in a state of equilibrium by defining the gas pressure to be a constant, and the plasma beta to be small, $\beta(z) < 6 \times 10^{-4}$. A spatially localized
transverse perturbation was then applied as
\[ V_x = A \frac{x}{l} \exp \left[ -\left( \frac{x}{\Delta_{x}} \right)^2 \right] \exp \left[ -\left( \frac{z/l + 3\pi/4}{\Delta_z} \right)^2 \right], \]  
(16)
where the initial amplitude \( A \) was chosen to be small enough to avoid nonlinear effects, and the parameters \( \Delta_{x} = 0.05 \) and \( \Delta_{z} = 0.05 \) are the width of the initial pulse in the horizontal and vertical directions, respectively. Note the different coordinates used compared with Equation (13), i.e. here the transverse direction is taken to be \( x \) (and \( z \) is the longitudinal direction).

Simulations were preformed using the 2.5D MHD code \textsc{Lare2D} (Arber et al. 2001) with a typical resolution of \( 4000 \times 2000 \) grid points for a numerical domain of size \( 2\pi \times \pi \). Line-tied boundary conditions were used in conjunction with damping layers near the edges of the numerical domain to avoid perturbations reflecting back inwards. The \textsc{Lare2D} code solves the MHD equations with an adiabatic equation of state and the 2.5D approximation here applies to the \( y \)-direction, i.e. \( \partial / \partial y = 0 \).

Figure 20 shows snapshots of the absolute velocity (left) and density perturbations (right) generated by applying Equation (16) to the model shown in Figure 18, with the equilibrium density profile indicated by the line contours. The initial perturbation was applied below the plotted region and by this time the waves have propagated far enough to separate into two identifiable components. The first is the wave train which can be seen inside the funnel density structure at \( z \approx -1 \). This component is formed from the trapped \( (k > k_c) \) sausage perturbations which remain within and propagate along the waveguide (note a downwards propagating wave train also exists but is not shown). This wave train undergoes dispersive evolution as it propagates, although the expansion of the loop with height means the extent of the evolution is less than in straight waveguides and so the wavelet signature (Fig. 21) is less developed in terms of having a “tadpole” shape (compared with Fig. 14).

The second component is formed by the leaky sausage perturbations \( (k < k_c) \). These initially propagate away from the funnel waveguide in the \( x \)-direction. However, refraction due to the decrease in magnetic field strength (and hence Alfvén speed) with height causes the leaky waves to propagate upwards and so form the “wing” wave trains located on each side of the density structure in Figure 20. Whereas the trapped wave train has the form of anti-symmetric perturbations with respect to the transverse velocity \( v_x \), the wing wave train mainly comprises vertical velocity perturbations \( v_z \), propagating in phase with density perturbations. On the other hand it retains the property of being quasi-periodic, reflecting its origin as an impulsive sausage perturbation.

The quasi-periodic nature of both wave trains is demonstrated by Figure 21 which shows the density perturbation signals as a function of time and their corresponding wavelets (Torrence & Compo 1998). The trapped wave train is measured along the funnel axis \( (x = 0) \) while the wing wave train is taken at \( x = -\pi/2 \). (The signals are taken at the same height of \( z = -\pi/2 \).) The magnetic stratification that causes refraction also causes the periods of oscillation of the wave trains to decrease as they propagate to higher \( z \). This means the wing wave train in Figure 21 is composed of slightly shorter periods than the trapped wave train, despite initially being formed from the (longer period) leaky spectral components of the initial perturbation.

The condition for efficient generation of quasi-periodic wave trains by dispersive evolution also depends upon the spatial localization of the driver, particularly the spatial scale in the longitudinal direction. Initial perturbations with larger longitudinal sizes \( \Delta_z \) excite a narrower range of wave numbers (and hence a narrower range of phase/group speeds). Larger perturbations therefore produce wave trains that are more monochromatic as demonstrated by Nakariakov et al. (2005) for a straight slab. Pascoe et al. (2013a) show that the wing wave trains similarly depend upon the longitudinal spatial size \( \Delta_z \), albeit the wing wave trains form from the leaky components rather than the trapped ones. In Figure 22 the wing density signal becomes more monochromatic for higher values of \( \Delta_z \), reflecting the more monochromatic spatial spectra of the driver.
Fig. 19  The dependence of the phase (left) and group (right) speeds on the normalized wavenumber $ka$ for an symmetric Epstein density profile with steepness parameter $p = 8$. The upper and lower dashed lines represent the external and internal Alfvén speeds, respectively. Reproduced from Pascoe et al. (2013a) with permission ©ESO.

Fig. 20  Snapshot of velocity $|v|$ (left) and density (right) perturbations for the model shown in Fig. 18. The line contours outline the equilibrium density profile. Reproduced from Pascoe et al. (2013a) with permission ©ESO.

Fig. 21  Density signals (left) and the corresponding wavelets (right) measured inside (top) and outside (bottom) the funnel density structure. Reproduced from Pascoe et al. (2013a) with permission ©ESO.
Pascoe et al. (2013a) therefore demonstrate that quasi-periodic fast magnetoacoustic wave trains naturally arise for an expanding magnetic field geometry such as Figure 18. The “wing” wave trains propagate outside of any particular density structure, though they are dependent on transverse structuring existing at the location of the initial driving perturbation to provide the dispersive evolution. Such wave trains therefore have potential seismological applications since their properties depend upon the details of transverse structuring and the driver. Since the driver is required to be spatially and temporally localized, a natural candidate is flaring energy releases. Recently, Yuan et al. (2013) used SDO/AIA to observe a series of quasi-periodic propagating fast wave trains (Fig. 23) associated with flaring radio bursts. The reported expansion of the intensity perturbation wave front, increase in amplitude and deceleration are all consistent with the numerical model of Pascoe et al. (2013a).
5 DISCUSSION AND CONCLUSIONS

The behavior of fast MHD waves in low $\beta$ plasmas such as the solar corona is heavily modified by the presence of transverse structuring. The geometrical dispersion arising from the introduction of a characteristic spatial scale applies to the commonly observed kink and sausage modes of oscillation. On the other hand studies also reveal that the properties of MHD waves are often insensitive to the details of fine structuring (e.g. Pascoe et al. 2007b; Terradas et al. 2008b; Pascoe et al. 2011). The identification of the most important parameter(s) for a particular process is key to the development and implementation of simple and robust seismological techniques for probing the coronal plasma.

The understanding of the dispersive dependence of the oscillation frequency on plasma parameters is the basis of seismological methods. A simple and robust method is the determination of the kink speed $C_k$ from observations of the global standing kink mode (e.g. Nakariakov et al. 1999; Nakariakov & Ofman 2001). Further information about the dispersive structuring may be obtained by the observation of more than one frequency of oscillation (e.g. review by Andries et al. 2009 and references therein; Macnamara & Roberts 2011).

This paper has focused on reviewing recent numerical investigations of the effects of transverse structuring on MHD wave behavior, and in particular on those effects which may produce quasi-periodic oscillations. In the case of kink oscillations (Sect. 3) the role of mode coupling was discussed, which is an intrinsic property of kink modes in smoothly varying structures. While the coupling is itself an ideal process, it can play a role in enhancing dissipation by transferring wave energy from bulk transverse motions to smaller scale phase-mixed Alfvénic oscillations. It is therefore potentially very important for coronal heating, particularly in light of observations of ubiquitous propagating transverse waves (e.g. Tomczyk et al. 2007). The rate at which the coupling occurs is most strongly dependent upon the density contrast ratio and the steepness of the density profile. Observations with sufficient quality could allow estimates for both parameters to be seismologically inferred from the oscillation damping profile, or in the case of one parameter being known the other may be determined.

The behavior of sausage modes also depends upon the density contrast ratio and density profile steepness, as discussed in Section 2. The dependence of the dispersive evolution of impulsively-generated quasi-periodic wave trains upon transverse structuring was considered in Section 4. This area of research is particularly suited to numerical investigations due to the time-dependent nature of the dispersive evolution. Investigations also benefit from the fact that the behavior is weakly dependent on the choice of slab or cylindrical geometry and so the computational expense of full 3D simulations is not necessary. The dependence of the wave train properties on the transverse structuring is another potential source of seismological information. To what extent the density profile and its steepness varies between different types and instances of structures in the corona is not yet known but potentially the transverse structuring revealed by quasi-periodic sausage oscillations could also be applied to the mode coupling of kink oscillations.

As models for wave behavior in a structured corona become more sophisticated, numerical simulations offer the opportunity to investigate scenarios which are not readily tractable with purely analytical methods. Of course any results or discoveries must be understood in terms of some theoretical framework to be reliably applied to observational results. However even analytical studies can benefit from development and testing in conjunction with numerical simulations (e.g. Hood et al. 2013; Vasheghani Farahani et al. 2014).

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