Estimations of cosmological parameters from the observational variation of the fine structure constant *

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Received 2013 March 8; accepted 2013 August 14

Abstract We present constraints on the quintessence scalar field model from observational data of the variation of the fine structure constant obtained from the Keck telescope and VLT. Within the theoretical frame proposed by Bekenstein, the constraints on the parameters of the quintessence scalar field model are obtained. Considering the prior of $\Omega_{m0}$ as WMAP 7 suggests, we obtain various results from different samples. Based on these results, we also calculate the probability density function of the coupling constant $\zeta$. The best-fit values show a consistent relationship between $\zeta$ and the different experimental results. In our work, we test two different potential models, namely, the inverse power law potential and the exponential potential. The results show that both the large value of the parameters in the potential and the strong coupling can cause a variation in the fine structure constant.

Key words: quasars: general — scalar field — cosmological constraint

1 INTRODUCTION

Fundamental constants play important roles in physics and its mathematical laws. Through the information they contain, one can describe the phenomena of nature and obtain a better understanding of the real world (Uzan 2011). However, one may suspect whether the constants are real "constants," i.e. do the constants vary with time or space? This question was probably first asked by Dirac with his famous “Large Numbers Hypothesis” (LNH) (Dirac 1937, 1938). Thereafter, several works have investigated the underlying principle, including studies of the variations of the constants and measurements of their precise values. We refer readers to the reviews (Barrow 2005, 2010; Chiba 2011; Damour 2009; Flambaum 2008; García-Berro et al. 2007; Karshenboim 2006; Uzan 2003) for more comprehensive discussions.

* Supported by the National Natural Science Foundation of China.
In order to theoretically unify the fundamental interactions, different theories have been proposed including field theories derived from strings, brane-world theories and Kaluza-Klein theories, which are based on the introduction of extra dimensions. Among the constants, the fine structure constant $\alpha$ which measures the strength of the electromagnetic interaction has attracted a lot of attention. In 1982, Bekenstein proposed a different theoretical framework to study variability in $\alpha$ where a linear coupling between a scalar field and the electromagnetic field was introduced (Bekenstein 1982). This theory satisfies the general conditions: covariance, gauge invariance, causality and time-reversal invariance of electromagnetism. Later on, this proposal was generalized and improved by Sandvik et al. (2002); Barrow et al. (2002b,c,d); Barrow (2002). So far we have several different theories which can describe the time evolution of the gauge coupling constants. However, whether the theoretical predictions can provide consistent results with the experimental ones should be asked. In this paper, we limit ourselves to study the Bekenstein model and the behavior of the fine structure constant as it evolves with time. The experiments which imply $\alpha$ could be related to time include observation of the Oklo natural nuclear reactor (Damour & Dyson 1996; Olive et al. 2002), big bang nucleosynthesis (BBN) (Avelino et al. 2001; Martins et al. 2004; Nollett & Lopez 2002), cosmic microwave background (CMB) measurements (Avelino et al. 2001; Martins et al. 2004; Landau & Sócolola 2010; Menegoni et al. 2009; Nakashima et al. 2008), absorption spectra of distant quasars (QSOs) (Chand et al. 2004; Murphy et al. 2001a,b,c, 2003, 2004; Srianand et al. 2004; Webb et al. 1999, 2001, 2003), and so on. These observations give different measurements of $\alpha$ at different periods of cosmological evolution. Among these observations, QSO absorption lines provide a powerful probe of the variation of $\alpha$ with a large sample of data. The methods used to study these observational results include the alkali doublet (AD) method, the many-multiplet (MM) method, the revised many-multiplet (RMM) method and the single ion differential alpha measurement (SIDAM) method (Uzan 2011). Because the observation by the MM gives the widest range of redshift ($0.22 < z < 4.2$) (Murphy et al. 2001a,b,c, 2003, 2004; Landau & Simeone 2008), it may contain more information about cosmological evolution than the others. Thus we will mainly focus on these measurements in the present work. More details about the observational data will be presented in Section 3.

On the other hand, since its discovery more than ten years ago, cosmic accelerated expansion has been demonstrated by the observations of type Ia supernovae (SNe Ia) and this phenomenon has been widely accepted (Eisenstein et al. 2005; Hicken et al. 2009; Komatsu et al. 2011; Percival et al. 2010; Riess et al. 1998; Spergel et al. 2007). In order to explain this amazing discovery, a great variety of attempts have been made, including the introduction of dark energy and theories of modified gravity (Tsujikawa 2010; Copeland et al. 2006). Among these proposals, the scalar field as a dynamical dark energy model was seriously investigated (Chen & Ratra 2011; Li et al. 2011; Samushia 2009; Samushia & Ratra 2006, 2009). Therefore, the cosmological variation of $\alpha$ induced by coupling with quintessence, which is a typical scalar field dark energy, is worth studying in order to find if the QSO observations contain information about the cosmic accelerated expansion. In other words, whether the QSO observations can give a consistent result with other cosmological probes, such as SNe Ia, CMB, baryon acoustic oscillation (BAO), observational Hubble parameter data (OHD) (Ma & Zhang 2011; Moresco et al. 2012; Zhang et al. 2010) and so forth, should be tested. Moreover, if the observation of QSO absorption lines provides results that are consistent with the ones listed above, can it be thought of as an indirect proof of the existence of the scalar field (quintessence)? This will be a very interesting question. In addition, we should notice that there are some differences between the QSO observations in Murphy et al. (2001a,b,c, 2003, 2004); Webb et al. (1999, 2001, 2003) and Chand et al. (2004); Srianand et al. (2004). The results analyzed by these two MM methods show an inconsistency in the time evolution of $\alpha$. Thus what information about cosmological evolution is contained in these data should be studied.

Following this direction, we constrain the cosmological parameters of the quintessence dark energy model with data on the variation of $\alpha$ from observations of the QSO absorption lines. One should note that researchers have freedom to choose several different forms of the scalar field poten-
tial, which plays an important role in the scalar field evolution. In our paper, we firstly focus on the inverse power law potential \( V(\phi) \propto \phi^{-n} \), where \( n \) is a nonnegative constant (Peebles & Ratra 1988; Ratra & Peebles 1988). This assumption has several advantages such as it can reduce to the standard ΛCDM case when \( n = 0 \) and contains solutions which can alleviate the fine-tuning problem (Watson & Scherrer 2003). Recent studies on the mass scale of the inverse power law potential show that the field value at present is of the order of the Planck mass (\( \phi_0 \sim M_p \)) (Tsujikawa 2010; Copeland et al. 2006; Steinhardt et al. 1999; Zlatev et al. 1999). For comparison, we also consider another potential model \( V(\phi) \propto e^{-\lambda \phi} \), where \( \lambda \) is a positive constant (Ratra & Peebles 1988). This model was first motivated by the anomaly of the dilatation symmetry in particle physics and has the tracker solution at late time (Wetterich 1988; Doran & Wetterich 2003). In this paper we just consider a spatially-flat quintessence model.

Many previous works that constrain the parameters of the quintessence dark energy model show that the universe is composed of about 30% nonrelativistic matter while dark energy contributes nearly 70%. The parameter \( n \) (of the inverse power law potential) and \( \lambda \) (of the exponential potential) which directly affects the evolutionary behavior of the scalar field both favor small values (Samushia 2009; Samushia & Ratra 2006, 2009 and references therein; Bozek et al. 2008; Wang et al. 2012). We should ask to what extent are the constraints from QSO observations consistent with these results. The possibility of studying the fine structure constant under the dark energy models has been proposed from various aspects, including the reconstruction of the dark energy equation of state (Avelino et al. 2006b; Nunes & Lidsey 2004; Parkinson et al. 2004) or combined with other cosmological observations (Amendola et al. 2012). In this paper we will discuss the possibility of constraining the quintessence dark energy model with the direct measurements of the variation of the fine structure constant.

Our paper is organized as follows. In Section 2, we present the basic formulas of the quintessence -α model. The data used and the corresponding constraints are shown in Section 3. The conclusion is presented in Section 4.

2 QUINTESSENCE AND THE ELECTROMAGNETIC COUPLINGS

We consider a spatially-flat FRW cosmology where the metric can be written as

\[
ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) ,
\]

where \( a \) is the scale factor. Under this geometrical background, the evolution of the quintessence scalar field \( \phi \) is determined by the Friedmann equation and the Klein-Gordon equation

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3M_p^2} \sum \rho_i ,
\]

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{dV}{d\phi} = 0,
\]

where \( M_p \) is the Planck mass, the overdot is the derivative with respect to the cosmic time \( t \), \( \rho \) stands for the density and \( i = m \) runs over the matter (including dark matter) and \( \phi \) runs over the scalar components. The relevant equations of state are \( \omega_m = 0 \) for matter and \( \omega_\phi = p_\phi/\rho_\phi \) for the scalar field where

\[
p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) , \quad \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi).
\]

In our calculation, the functional forms of the potential are

Model I \( V(\phi) = \kappa M_p^2 \phi^{-n} \),

Model II \( V(\phi) = V_0 e^{-\lambda \phi} \),

(5)
where $\kappa$ and $V_0$ are non-negative constants; $n$ and $\lambda$ are parameters that will be constrained by the data. These kinds of scalar field models were first studied by Peebles and Ratra in 1988 and further explored, especially in explaining the dark energy problem (Chen & Ratra 2011; Samushia & Ratra 2006, 2009; Russo 2004; Binétruy 2000, 1999; Ferreira & Joyce 1998 and references therein). By the definitions of the dimensionless parameters

$$
\Omega_m = \frac{8\pi \rho_m}{3M_p^2 H^2} = \frac{\rho_m}{\rho_m + \rho_\phi}, \quad \Omega_\phi = \frac{8\pi \rho_\phi}{3M_p^2 H^2} = \frac{\rho_\phi}{\rho_m + \rho_\phi},
$$

the Friedmann Equation (2) can be rewritten in a simple form

$$
\Omega_m + \Omega_\phi = 1.
$$

So far our model is determined by only two parameters ($\Omega_{m0}, n$) for Model I and ($\Omega_{m0}, \lambda$) for Model II, where the subscript 0 stands for the present value. This parameter set is the key point that will be constrained by the observational data.

Considering an interaction between a quintessence field $\phi$ and an electromagnetic field $F_{\mu\nu}$, we can write its Lagrangian density as

$$
\mathcal{L}_F(\phi) = -\frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu},
$$

where $B_F(\phi)$ is the function that describes the coupling behavior. One should note that the addition of this interaction term does not affect the evolution of the quintessence scalar field. This is due to the fact that the statistical average of $F_{\mu\nu} F^{\mu\nu}$ over a current state of the universe is zero (Copeland et al. 2004; Marra & Rosati 2005). Thus Equation (3) is still applicable. The Lagrangian form of Equation (8) allows us to define a new “effective” fine structure constant

$$
\alpha(\phi) = \frac{\alpha_0}{B_F(\phi)},
$$

where $\alpha_0$ is the current value. By the use of this equation we can obtain a relative variation of $\alpha$

$$
\frac{\Delta \alpha}{\alpha} = \frac{\alpha(\phi) - \alpha_0}{\alpha_0} = 1 - B_F(\phi) B_F(\phi).
$$

Apparenty, the evolution of $\alpha$ is directly affected by $\phi$ and the functional form $B_F(\phi)$. From a theoretical point of view, there are many choices in defining $B_F$ which lead to different behaviors for $\alpha$. The authors of Marra & Rosati (2005) give a detailed discussion about $B_F(\phi)$ which contains many different cases. In our paper, we will consider the simplest case which is a linear form and corresponds to the original Bekenstein proposal (Bekenstein 1982),

$$
B_F(\phi) = 1 - \zeta (\phi - \phi_0),
$$

where the constant $\zeta$ describes the strength of the coupling between the scalar field and the electromagnetic field. We will see that the parameter sets $(\Omega_{m0}, n, \zeta)$ and $(\Omega_{m0}, \lambda, \zeta)$ completely describe the evolutionary behavior of quintessence-$\alpha$ Model I and Model II respectively.

3 THE OBSERVATIONAL QSO DATA AND CONSTRAINTS

3.1 The Observational QSO Data

The MM method as a generalization of the AD method was first proposed in Dzuba et al. (1999). It was first applied in Webb et al. (1999, 2001, 2003) to analyze distant QSO absorption lines observed by using the Keck telescope, which is located in Hawaii. Their result shows a variation of $\alpha$ in...
the redshift range of 0.6 < z < 1.6. Later on, more QSO systems were observed and the data sample was enlarged. The updated results, which are based on a statistical analysis including 143 absorption systems, show that $\Delta \alpha / \alpha = (-0.57 \pm 0.11) \times 10^{-5}$ in the redshift range of 0.2 < z < 4.2 (Murphy et al. 2001a,b,c, 2003, 2004). We will use this data sample to test the quintessence-$\alpha$ model. For convenience, we use “KWM143” as an abbreviation for this sample. Although there are some differences in analyzing the low-z and high-z absorption systems, we will combine a total of 143 data to do the calculation and neglect the tiny discrepancies.

On the other hand, a further independent statistical study was completed in Chand et al. (2004); Srianand et al. (2004) based on the observations with VLT. Their calculation favors a different result of $\Delta \alpha / \alpha = (-0.06 \pm 0.06) \times 10^{-5}$ which shows a nearly unchanged $\alpha$ in the redshift range of 0.4 < z < 2.3. However, this analysis was challenged by Murphy et al. (2007, 2008) who used the same reduced data and got a result of $\Delta \alpha / \alpha = (-0.44 \pm 0.16) \times 10^{-5}$ in the same redshift range. However, this result is not reliable because of its larger value of the reduced $\chi^2$ in Murphy et al. (2007, 2008). Therefore, it is necessary to consider the additional scatter and the systematic error which can be used to derive the most conservative weighted mean result $\Delta \alpha / \alpha = (-0.64 \pm 0.36) \times 10^{-5}$. This result also prefers a non-zero variation of the fine structure constant. Since then, these contradictory results have been discussed several times by different research groups and the principles behind the observations have been explored from many different aspects (King et al. 2012; Barrow & Li 2008; Bento & González Felipe 2009; Bisabr 2010; Calabrese et al. 2011; Fujii 2009; Gutiérrez & López-Corredoira 2010; Lee et al. 2004; Mosquera et al. 2008; Tedesco 2011; Toms 2008; Avelino et al. 2012; Farajollahi & Salehi 2012; Martinelli et al. 2012; Thompson 2012). Recently, an intensive debate in the literature was proposed by Berengut et al. (2011, 2012); Webb et al. (2011). They proposed that the observed spatial variation of $\alpha$ is not really an artificial effect, resulting from the fact that Keck and VLT are located in different hemispheres. Therefore, perhaps it is worth studying the possibility of the spatial variation of $\alpha$ in the quintessence model, where new physics may be taken into account. This will be the future focus of our research. In the present calculation, we will use these two different data samples independently and constrain the various quintessence-$\alpha$ models.

One of our goals is to explore the factors that cause the above two different results, i.e., aiming at finding out whether the coupling strength or the cosmological evolution of quintessence leads to the discrepancy between them. In the following, we use ”VCS23” and ”VWM23” as abbreviations for these two samples. In Figure 1, we plot the direct measurements of $\Delta \alpha / \alpha$ for the three data samples listed above.

From Equations (10) and (11), we can see that the value of $\zeta$ directly effects the evolution of $\Delta \alpha / \alpha$. From the tests of the equivalence principle, the coupling is constrained to be $|\zeta| < 10^{-3}$ (Copeland et al. 2006; Olive & Pospelov 2002). Furthermore, Copeland et al. (2004) used a simple estimation to obtain an approximate value of $\zeta \approx 10^{-5}$ which is under the assumption of an inverse

Fig. 1 The direct measurements of $\Delta \alpha / \alpha$ with respect to the redshift z: VCS23 (left); VWM23 (middle); KWM143 (right). The dashed curve is the horizontal line indicating there is no variation.
power law potential and the QSO observations. In our paper, we consider $\zeta$ as a free parameter to be constrained by the data and compare the results with the previous works such as the equivalence principle test (Avelino et al. 2004, 2006a; Damour 2003; Will 2001), which shows that $|\zeta| < 5 \times 10^{-4}$.

3.2 Constraints with a Prior of $\Omega_{m0}$

In order to get the best-fit results for the parameters of the quintessence-$\alpha$ model, we apply $\chi^2$ statistics to the observational QSO data

$$\chi^2(z; \Omega_{m0}, n; \zeta) = \sum_i \left( \frac{(\Delta \alpha/\alpha)_{th,i} - (\Delta \alpha/\alpha)_{obs,i}}{\sigma_i} \right)^2,$$

where the subscripts “th” and “obs” stand for the theoretically predicted value and observed ones respectively. In order to obtain the pure results, we do not take into account other experimental bounds for our $\chi^2$ calculation as mentioned in Section 1, but the comparisons of the single QSO constraints with other experiments are worth studying and are carried out in later sections. For the purpose of reducing the unnecessary distractions arising from the intrinsic complexity of this scalar field model, it is convenient to set reasonable priors on some of the parameters.

As mentioned in the previous sections, one important discovery in the field of cosmology is the present accelerated expansion. This discovery indicates that the universe contains much more of the so-called “dark energy” component than ordinary matter, i.e. in our quintessence-$\alpha$ model, the parameter $\Omega_{m0}$ should occupy a relatively smaller proportion. Therefore, we adopt a Gaussian distribution of $\Omega_{m0} = 0.275 \pm 0.016$ as WMAP 7 suggests (Komatsu et al. 2011) to be a prior to constrain the quintessence-$\alpha$ model. Thus the parameters which will be constrained are $(n, \zeta)$ for Model I and $(\lambda, \zeta)$ for Model II.

Our results for the constraints are shown in Figures 2–5. The corresponding $1\sigma$ errors of the parameters are summarized in Table 1. Generally speaking, for Model I, the best-fit values of the quintessence-$\alpha$ model obtained from three data samples all favor a small value of $n$. This feature is consistent with most other cosmic probes which show that $n < 1.5$ (Samushia 2009; Samushia & Ratra 2006, 2009). This phenomenon shows that the scalar field slowly evolves in the universe. When the value of $n$ is smaller, the scalar field model is closer to the standard $\Lambda$CDM, except for the two contradictory samples VCS23 and VWM23 which both indicate $n = 0.1$. The main differences between these constraint results are the big discrepancies in the value of $\zeta$. The value obtained by VWM23 is much larger than the other two samples, while VCS23 gives the smallest one.

An exception is that, from Table 1, one can see that the constraint of $\zeta$ in Model II is much worse than in Model I. Both the best-fit values of $\zeta$ and the $1\sigma$ upper bound are larger in Model II. This can be seen as a signal of the different choices of the potential of the scalar field in Equation (5),

<table>
<thead>
<tr>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta(10^{-5})$</td>
<td>$\zeta(10^{-5})$</td>
</tr>
<tr>
<td>VCS23</td>
<td>$(0, 0.10)$</td>
</tr>
<tr>
<td>VWM23</td>
<td>$(0.08, 0.85)$</td>
</tr>
<tr>
<td>KWM143</td>
<td>$(0.33, 0.77)$</td>
</tr>
<tr>
<td>Total</td>
<td>$(0.06, 0.22)$</td>
</tr>
</tbody>
</table>
Fig. 2 Left (Model I): The confidence regions of \((n, \zeta)\) obtained from VCS23 with a prior of \(\Omega_m^0 = 0.275 \pm 0.016\). The best-fit result which is indicated by the star is \((n, \zeta) = (0.1, 0.18 \times 10^{-5})\) with \(\chi^2_{\text{min}} = 27.7919\). Right (Model II): The confidence regions of \((\lambda, \zeta)\) obtained from VCS23 with a prior of \(\Omega_m\). The best-fit result which is indicated by the star is \((\lambda, \zeta) = (0.12, 0.80 \times 10^{-5})\) with \(\chi^2_{\text{min}} = 27.6141\).

Fig. 3 Left (Model I): The confidence regions of \((n, \zeta)\) obtained from VWM23 with a prior of \(\Omega_m^0 = 0.275 \pm 0.016\). The best-fit result which is indicated by the star is \((n, \zeta) = (0.1, 2.0 \times 10^{-5})\) with \(\chi^2_{\text{min}} = 29.2648\). Right (Model II): The confidence regions of \((\lambda, \zeta)\) obtained from VWM23 with a prior of \(\Omega_m\). The best-fit result which is indicated by the star is \((\lambda, \zeta) = (0.16, 6.63 \times 10^{-5})\) with \(\chi^2_{\text{min}} = 28.8861\).

Because the coupling strength between the electromagnetic field and the scalar field seems to be not as sensitive to the cosmological evolution in Model II as in Model I.

For Model II, the constraints show similar trends in the parameters. The best-fit value of \(\lambda\) obtained by KWM143 is apparently larger than the other two data samples, but the coupling constant \(\zeta\) of VCS23 is larger than KWM143 which is different from the case of Model I.

Additionally, these results for the two models are also consistent with a test of the equivalence principle. However, we should note that observations of the variations in the fine structure constant cannot give an efficient constraint on the cosmological parameters \(n\) and \(\lambda\). Therefore it is difficult to identify the current evolutionary state of the universe, i.e. the evolution of \(\Delta \alpha/\alpha\) is not as sensitive to the cosmological parameters as to the coupling constant \(\zeta\). One more point worth noticing is
that from our results, the variation of $\alpha$ can be caused by a large value of $n$ or $\lambda$, or the strong coupling constant $\zeta$. This can be obtained from comparisons of the constraints, and it is shown that the different weighted values of VWM23 and VCS23 are attributed to the variance of $\zeta$ instead of cosmological parameters. However, similar results for KWM143 and VWM23 do not give consistent results for the constraint, because VWM23 gives a smaller $n$ and $\lambda$ but a larger $\zeta$. Therefore, the goal of finding the reasons causing the variation of $\alpha$ is still vague. We should emphasize that the above conclusions are not certain enough because of the insufficient constraints on $n$ and $\lambda$, even when the 1$\sigma$ confidence regions are not perfectly obtained.
Fig. 6  Left (Model I): The PDF of $\zeta$ obtained from VCS23. The most probable point is located at $\zeta = 0.04 \times 10^{-5}$.  Right (Model II): The PDF of $\zeta$ obtained from VCS23. The most probable point is located at $\zeta = 0.10 \times 10^{-5}$.

Fig. 7  Left (Model I): The PDF of $\zeta$ obtained from VWM23. The most probable point is located at $\zeta = 0.46 \times 10^{-5}$.  Right (Model II): The PDF of $\zeta$ obtained from VWM23. The most probable point is located at $\zeta = 1.06 \times 10^{-5}$.

3.3 The Probability Density Function of $\zeta$

The previous constraints show a two dimensional distribution of the parameters $(n, \zeta)$ and $(\lambda, \zeta)$. In order to compare the value of $\zeta$ with other tests, it is necessary to calculate the probability density function (PDF) of $\zeta$ by marginalizing the parameter $n$ or $\lambda$. Our results are presented in Figure 6 to Figure 9.

Generally speaking, the results obtained are compatible with a test of the equivalence principle which is $|\zeta| < 5 \times 10^{-4}$. However, the differences between these calculations are also significant. Figure 6 shows an apparent result that the best-fit value of $\zeta$ is nearly zero for both models. The small coupling constant indicates a case that the coupling between the electromagnetic field and the scalar field is so weak that the fine structure constant is nearly unchanged. Figure 7 and Figure 8
show different results. Both VVM23 and KWM143 give a similar value of $\zeta$ in each model. Their results are consistent with each other and favor a variation of $\alpha$. It is noteworthy that the constraint from VVM23 is not as strict as KWM143 which may be attributed to the smaller size of this sample. Comparing the two models, the best-fit values of $\zeta$ are larger in Model II than in Model I, which implies that the coupling between the scalar field and electromagnetic field is stronger in the exponential potential than in the inverse power law potential. Besides that, we also notice that on comparison with the results obtained in Section 3.2, an apparent discrepancy between the constraints of $\zeta$ from VVM23 (Fig. 3 and Fig. 7) emerge. This result is understandable because the two dimensional constraints or the corresponding likelihood function is non-Gaussian. This comes from the quality of the data or the non-linearity of the theoretical function.
Fig. 10  **Left Top:** The evolution of scalar field $\phi$ under potential I with respect to redshift $z$; the red and blue curves are obtained by the best-fit values of KWM143 and VCS23 (VWM23) respectively.  
**Left Middle:** the evolution of $\Delta \alpha / \alpha$ with respect to $z$. The red, blue and green curves are obtained by the use of the best-fit values of KWM143, VWM23 and VCS23 respectively, while the boxes are the corresponding weighted values of QSO observations.  
**Left Bottom:** A comparison of the QSO results with the Oklo bound (dashed lines) (left panel) and meteorite bound (the solid and dotted lines correspond to $1\sigma$ and $2\sigma$, respectively) (right panel).  
**Right:** The same as the **Left** but for Model II.
3.4 Comparison with Other Experiments

In this section, we consider the comparison of results on the QSO constraint with other experiments. In order to obtain a clear impression, we plot the evolutions of the scalar field $\phi$ and $\Delta \alpha / \alpha$ with respect to redshift $z$ in Figure 10 (the top and middle panels). Firstly, we consider the Oklo natural reactor which provides a bound at the 95% confident level,

$$-0.9 \times 10^{-7} < \frac{\Delta \alpha}{\alpha} < 1.2 \times 10^{-7}$$

for $z = 0.14$ (Damour & Dyson 1996; Fujii et al. 2000; Fujii 2003). Besides that, estimates on the age of iron meteorites at $z = 0.45$ combined with a measurement of the Os/Re ratio resulting from the radioactive decay $^{187}\text{Re} \rightarrow ^{187}\text{Os}$ gives (Olive et al. 2002, 2004; Fujii & Iwamoto 2003)

$$\frac{\Delta \alpha}{\alpha} = (-8 \pm 8) \times 10^{-7}$$

at 1$\sigma$ and

$$-24 \times 10^{-7} < \frac{\Delta \alpha}{\alpha} < 8 \times 10^{-7}$$

at 2$\sigma$ (Bento et al. 2004).

The results are presented in the bottom panels of Figure 10. Compared with the QSO results, the Oklo measurements and meteorite estimates both favor an unchanged value of the fine structure constant, because they are consistent with the VCS23 constraint except for a tiny deviation in Model II bounded by the Oklo measurement. Compared with Oklo, the meteorite observations give a wider range of uncertainty, however, VWM23 and KWM143 both violate this bound. Furthermore, the larger value of the slope of Model II at low redshift shows a more drastic deflection from the meteorite constraint. However, we should note that we discuss these results only with the best-fit values of the parameters. Once the uncertainties of the parameters are taken into account, the above tendencies would be weakened. Visualizations of the corresponding uncertainties are expressed in the constraints shown in Figure 2 to Figure 5.

4 DISCUSSION AND CONCLUSIONS

In the present paper, we show constraints of the cosmological parameters on the quintessence model by measurements in the variation of the fine structure constant $\alpha$ from distant QSOs. By the use of the Gaussian prior of $\Omega_m$, three data samples KWM143, VWM23 and VCS23 apply various constraints on the parameters. For both of the two potential models, VCS23 shows the smallest $\zeta$ which can be treated as an explanation of the results of Chand et al. (2004) because the weak coupling derives a weak interaction with the electromagnetic field. This leads to an unchanging $\alpha$. On the other hand, VWM23 and KWM143 present a result that the values of $\zeta$ are larger, especially the VWM23 one, but the constraints of $n$ for Model I and $\lambda$ for Model II are different. The strong coupling strength implies the possibility of a variation in $\alpha$. In order to further study this problem, we marginalize the cosmological parameter $n$ or $\lambda$ and obtain the PDF of $\zeta$. The results confirm our analysis that the VCS23 favors a nearly null result for $\zeta$. Except for that, the discrepancy between VWM23 and KWM143 about $\zeta$ disappears and $\rho$ provides consistent constraints. Combining the two models, we find that either a strong coupling or a large value of the cosmological parameters (here referred to as $n$ or $\lambda$) can lead to an apparent variation of $\alpha$. Our results show that the difference between VWM23 and VCS23 is caused by the different coupling constant, while the similar results of VWM23 and KWM143 have different reasons. The former is attributed to a stronger coupling and the latter is caused by a different evolution of the quintessence scalar field. In order to obtain a complete analysis, we also constrain the quintessence models with all the available data as shown in
Figure 5 and Figure 9. As a corollary, we find that the coupling between the electromagnetic field and the scalar field is stronger in Model II (the exponential potential) than in Model I. This means that it is relatively easier for the inverse power law potential to derive a change of the fine structure constant than the exponential potential.

Furthermore, we should point out that the observations of the variations of $\alpha$ by QSOs are not as efficient as other cosmic probes. This feature is reflected in their insensitivity to the cosmological parameters such as $n$ or $\lambda$, because the sufficient constraints of the confident regions are also necessary for the best-fit values. From the confidence regions and the constraint errors of the parameters, we see that the constraint on $n$ or $\Lambda$ is not as strict as other cosmic probes such as supernovae or CMB (Samushia 2009; Samushia & Ratra 2006, 2009). This is a relatively more obvious shortcoming of the observations of the variation in $\alpha$. On the other hand, both the observations and theoretical calculations do not directly give information about the coupling strength between the electromagnetic field and the scalar field. Therefore, the combination of the supernovae data and the $\Delta\alpha/\alpha$ data may provide us with a more complete description of the universe. One possible way may be to first use the supernovae data and find the constraints on parameters such as $n$ or $\lambda$, and then apply the results to constrain $\zeta$ and decide the coupling strength, because the mechanism associated with supernovae is relatively clearer than that associated with QSOs and thus the uncertainties in the cosmological parameters are bound to be smaller.

Further research on the distribution or the precise value of $\zeta$ is important. Comparisons between that research and research on QSOs are also imperative. To get a more accurate description of $\Delta\alpha/\alpha$, measuring $\zeta$ accurately or combining results with other observations will be necessary (Amendola et al. 2012). Moreover, the relationship between $\zeta$ and another quantity related to dark energy, such as the equation of state, is also meaningful (Amendola et al. 2012), since the connection between the fundamental constant and the dark energy models can be indicated. Therefore the power of using observations related to fundamental constants in studying cosmic evolution could be searched more deeply.

We recently noticed that the correlation between cosmic dipoles, the fine structure constant and supernovae are studied in Mariano & Perivolaropoulos (2012). From a theoretical view, exploring this correlation under the assumption of a scalar field and reconstructing the quintessence model is also worth studying. We will discuss this question in our future research.

Acknowledgements The authors would like to thank Liu, W., Wang, H., Chen, Y., Landau, S. J., Meng, X. and Ma, C. for their helpful discussions and valuable suggestions. This research is supported by the National Natural Science Foundation of China (Grant Nos. 11173006, 10773002, 10875012 and 11175019), the National Basic Research Program of China (973 project, Grant No. 2012CB821804), and the Fundamental Research Funds for Central Universities.

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