



# Supermassive Primordial Black Holes for Nano-Hertz Gravitational Waves and High-redshift JWST Galaxies

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## Abstract

Recently, observational hints for supermassive black holes have been accumulating, prompting the question: Can primordial black holes (PBHs) be supermassive, particularly with masses  $M \gtrsim 10^9 M_\odot$ ? A supercritical bubble, containing an inflating baby universe, that nucleated during inflation can evolve into a PBH in our observable universe. We find that when the inflaton slowly transitions past a neighboring vacuum, the nucleation rate of supercritical bubbles inevitably peaks, leading to a mass distribution of multiverse PBHs with a peak mass up to  $M \gtrsim 10^{11} M_\odot$ . Thus, our mechanism naturally provides a primordial origin for supermassive black holes.

**Key words:** (cosmology:) early universe – (cosmology:) inflation – (galaxies:) quasars: supermassive black holes

## 1. Introduction

In past years, the cosmological implications of primordial black holes (PBHs) (Zel'dovich & Novikov 1967; Hawking 1971; Carr & Hawking 1974), which might be responsible for dark matter and LIGO-Virgo gravitational wave (GW) events (Bird et al. 2016; Sasaki et al. 2016; Clesse & García-Bellido 2017), have been intensively studied, e.g., in Sasaki et al. (2018); Carr et al. (2021a, 2024). However, it has still been interesting to ask: Can PBHs be supermassive? In particular can the mass of PBHs reach  $M \gtrsim 10^9 M_\odot$  (Carr et al. 2021b)?

The origin of supermassive PBHs might have to be related with a period of inflation, since only inflation can stretch sub-horizon inhomogeneities to the scale that supermassive PBHs need. It has been widely thought that massive PBHs can be sourced by very large inflationary perturbations (Carr & Lidsey 1993; Ivanov et al. 1994; García-Bellido et al. 1996; Kawasaki et al. 1998; Yokoyama 1998),  $\delta\rho/\rho \gtrsim 0.1$ . However, current cosmic microwave background (CMB) spectral distortion observations have ruled out such a significant enhancement of the amplitudes of primordial perturbations (if the perturbations are Gaussian)<sup>7</sup> on the  $k \lesssim 10^4 \text{ Mpc}^{-1}$  scale, and hence PBHs with the mass  $M > 10^4 M_\odot$  (Nakama et al. 2018, see also De Luca et al. 2021, 2023; Franciolini et al. 2023).

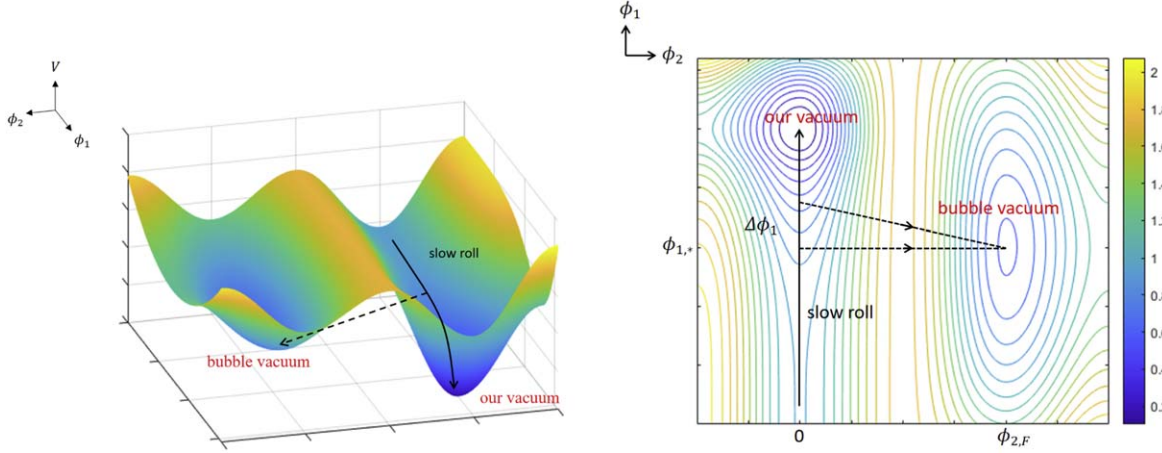
It has also been well-known that a supercritical bubble (with an inflating baby universe inside it) that nucleated during inflation can develop into a PBH in our observable universe (Garriga et al. 2016). In the corresponding multiverse scenario,<sup>8</sup> after inflation ended, the supercritical bubble will connect to our universe through a wormhole, and eventually we would see a PBH after the wormhole pinched off, see also Deng et al. (2017), Deng & Vilenkin (2017), Deng (2020), Wang et al. (2019), He et al. (2024) for further investigations. However, the mass distribution of such multiverse PBHs is  $\propto \frac{1}{M^{1/2}}$  (Garriga et al. 2016; Deng & Vilenkin 2017), and thus is negligible at supermassive band  $M \gtrsim 10^9 M_\odot$ , see also Kusenkov et al. (2020).

Recently, observational hints for supermassive black holes (BHs) have been accumulating. The evidences for a nano-Hertz stochastic GW background have been found with a pulsar timing array (PTA) (Agazie et al. 2023b; EPTA Collaboration et al. 2023; Reardon et al. 2023; Xu et al. 2023), which might be interpreted with a population of  $M \gtrsim 10^9 M_\odot$  supermassive BH binaries (Agazie et al. 2023a,<sup>9</sup> see also Ellis et al. 2024). Supermassive BHs with  $M \sim 10^8 - 10^{10} M_\odot$  are believed to sit at the centers of galaxies observed at redshifts  $z \gtrsim 6$ , an aspect

<sup>7</sup> It seems that one needs to consider a scenario with highly non-Gaussian primordial perturbations to create supermassive PBHs, e.g., Nakama et al. (2016), Hasegawa & Kawasaki (2018), Kawasaki & Murai (2019), Kitajima & Takahashi (2020), Atal et al. (2021), however, see also Shinohara et al. (2021, 2023).

<sup>8</sup> This multiverse PBH scenario is reminiscent of the well-known eternally inflating multiverse (Vilenkin 1983; Linde 1986).

<sup>9</sup> An alternative might be an inflationary primordial GW (Vagnozzi 2021, 2023; Benetti et al. 2022; Afzal et al. 2023; Jiang et al. 2024a), which can be implemented by null energy condition violation, e.g., Piao & Zhang (2004), Liu et al. (2011), Cai & Piao (2021, 2022), see also Papanikolaou & Gourgoullos (2023), Ashoorioon et al. (2022), Sakharov et al. (2021), Bian et al. (2022).



**Figure 1.** A 2D potential and its contour diagram. Initially, the inflaton  $\phi_1$  rolls along its potential at  $\phi_2 = 0$ , and the nucleating rate of vacuum bubbles with a neighboring vacuum at  $\phi_2 = \phi_{2,F}$  and  $\phi_1 = \phi_{1,*}$  is highly suppressed. Only when  $\phi_1 \simeq \phi_{1,*}$  will the bubbles nucleate with the largest rate.

which still represents a challenge to the standard astrophysical accretion models (Volonteri 2010; Volonteri et al. 2021). The observations with the James Webb Space Telescope (JWST) have also discovered lots of early supermassive galaxies ( $M \gtrsim 10^{10} M_\odot$ ) at higher redshift  $z \sim 10$ , which is discordant with the  $\Lambda$ CDM model (Boylan-Kolchin 2023), but might be explained with  $\Lambda$ CDM+supermassive BHs (Liu & Bromm 2022; Hütsi et al. 2023). Currently, there has been a consensus that supermassive BHs can be either sourced by seed-like PBHs as massive as  $\sim 10^3 M_\odot$  (Düchting 2004; Serpico et al. 2020), which however must acquire sufficient accretion, or supermassive PBHs by birth, which might be most natural.

However, it is still unclear how supermassive PBHs can form. In this work, we present such a mechanism. It is found that in a slow-roll inflation model with multiple neighboring metastable vacua, the mass distribution of multiverse PBHs formed by supercritical bubbles that nucleated during inflation would inevitably have a multi-peak spectrum, and the peak mass can reach  $M \gtrsim 10^{11} M_\odot$ . Thus, our multiverse PBHs can naturally serve as supermassive BHs needed to explain nano-Hertz GW and supermassive JWST galaxies.

## 2. Our Multiverse PBH Model

In our phenomenological model, see Figure 1 for  $V(\phi_1, \phi_2)$ , the inflaton  $\phi_1$  slowly rolls along its potential  $V_{\text{inf}}(\phi_1)$  at which  $\phi_2 = 0$ , and a neighboring vacuum with  $V_b < V_{\text{inf}}$  is at  $\phi_2 = \phi_{2,F}$  and  $\phi_1 = \phi_{1,*}$ . It is expected that only when  $\phi_1$  rolls to the vicinity of  $\phi_{1,*}$  the nucleating rate of the bubble with  $V = V_b$  is maximized.

In the thin-wall regime, the bubble nucleating rate per Hubble spacetime volume  $1/H_i^4$  in the inflating background (the Hubble parameter is  $H_i^2 = \frac{8\pi V_{\text{inf}}(\phi_1)}{3M_P^2}$ ) is

(Garriga et al. 2016)

$$\lambda \sim e^{-B}, \quad \text{with} \quad B = \frac{2\pi^2 \sigma}{H_i^3}, \quad (1)$$

where

$$\sigma = \int_{\text{path}} \sqrt{2(V(\phi_1, \phi_2) - V_b)} \left( \sum_{i=1,2} d\phi_i^2 \right)^{1/2}, \quad (2)$$

is the wall tension with the “path” representing the “least- $\sigma$ ” path (Ahlqvist et al. 2011).<sup>10</sup> Here, we will not calculate  $\sigma$  exactly, but consider such an approximation (which helps to highlight the essential aspect of how  $\lambda$  is affected by the roll of inflaton), i.e., when the inflaton passes through  $\phi_{1,*}$ , we have  $d\phi_1 \simeq \Delta\phi_1 = \phi_1 - \phi_{1,*}$  and  $d\phi_2 \simeq \phi_{2,F}$ , thus

$$\sigma \approx \left( 1 + \frac{(\Delta\phi_1)^2}{\phi_{2,F}^2} \right)^{1/2} \int_0^{\phi_{2,F}} \sqrt{2(V(\phi_{1,*}, \phi_2) - V_b)} d\phi_2. \quad (3)$$

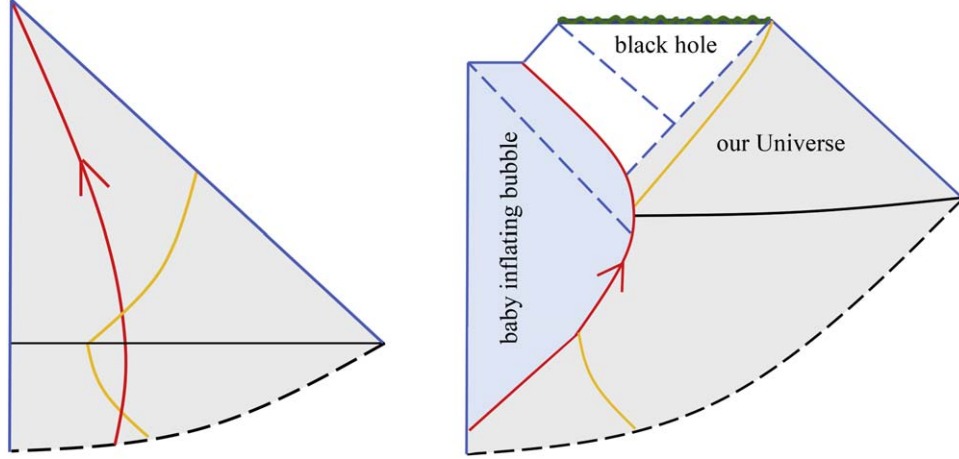
According to (1), we have

$$B \approx B_* \left( 1 + \frac{(\Delta\phi_1)^2}{\phi_{2,F}^2} \right)^{1/2}, \quad (4)$$

with

$$B_* = \frac{2\pi^2}{H_i^3} \int_0^{\phi_{2,F}} \sqrt{2(V(\phi_{1,*}, \phi_2) - V_b)} d\phi_2, \quad (5)$$

<sup>10</sup> The nucleating rate of bubbles in multiple-field space has been also investigated in, e.g., Kusenko (1995), Konstandin & Huber (2006), Masoumi et al. (2017), Espinosa & Konstandin (2019), Ashoorioon et al. (2021).



**Figure 2.** Right larger panel: Penrose diagram of inflation followed by a radiation era with multiverse PBH. The red curve is the comoving bubble wall, the yellow curves are the comoving Hubble horizon, and the black dashed and solid curves represent when the inflation started and ended, respectively. The bubble that nucleated during slow-roll inflation will evolve to be supercritical, i.e.,  $r \gtrsim 1/H_b$ , and its interior contains a baby inflating universe (blue region). The bubble entered into the horizon of our observable universe at  $t = t_H$ , after which it will be hidden behind the horizon of a PBH. Left panel (A contrast): Penrose diagram of inflation followed by a radiation era. The red curve is the comoving primordial perturbation.

where the effect of the rolling velocity of inflaton on  $\lambda$  has been imprinted in  $\Delta\phi_1 = \int \frac{\dot{\phi}_1}{H} d\mathcal{N}$ , with  $\mathcal{N} = \int H dt = \int \frac{H d\phi}{\dot{\phi}}$  being the e-folding number before inflation ended.

After its nucleating occurs, the bubble will rapidly expand. Inflation ends at  $t = t_i$ , at which time the number density of bubbles is

$$dn(t_i) = \lambda \frac{dr_i}{(r_i + H_i^{-1})^4}, \quad (6)$$

( $r_i = \frac{e^{\mathcal{N}}}{H_i} \gg 1/H_i$  is the radius of the bubble at  $t_i$ ), and the energy of the inflaton is rapidly converted to that of radiation, so  $V \sim 0$  outside the bubble and is completely negligible.

In Figure 2, we present the full Penrose diagram for the supercritical bubble<sup>11</sup> evolving into a PBH. In the corresponding spacetime, the interior of the supercritical bubble contains a baby inflating universe, which is connected to the exterior through a wormhole being closed. Eventually a PBH will come into being in our observable universe.<sup>12</sup>

In light of the causality, the region affected by the Schwarzschild radius of the PBH cannot exceed the Hubble radius of the parent universe  $1/H \simeq t_H$ . During the radiation era, the bubble will expand comovingly with  $a(t) = \left(\frac{t}{t_i}\right)^{1/2}$ , until its physical radius  $r = a(t_H)r_i \sim t_H$  (so  $t_H = H_i r_i^2$ ), after

which the bubble will be hidden behind the horizon of a PBH with mass  $M \sim M_p^2 t_H$ , so

$$M \sim H_i r_i^2 M_p^2 = \frac{M_p^2}{H_i} e^{2\mathcal{N}}. \quad (7)$$

The dark matter density is  $\rho_{\text{DM}}(t) \sim \frac{M_p^3}{t^{3/2} \mathcal{M}_{\text{eq}}^{1/2}}$ , where  $\mathcal{M}_{\text{eq}} \sim 10^{17} M_\odot$ . Thus the mass distribution of such PBHs is (Garriga et al. 2016; Deng & Vilenkin 2017)

$$f(M) = \frac{1}{\rho_{\text{DM}}(t)} \left( M^2 \frac{dn}{dM} \right) \sim \lambda \left( \frac{\mathcal{M}_{\text{eq}}}{M} \right)^{1/2}. \quad (8)$$

According to (4),  $f(M) \propto \frac{e^{-B_*}}{M_*^{1/2}}$  is maximized<sup>13</sup> at  $\phi_1 = \phi_{1,*}$

(equivalently  $M_* = \frac{M_p^2}{H_i} e^{2\mathcal{N}_*}$ ).

It is significant to see the resulting  $f(M)$  in different slow-roll inflation models. As a simple example, we consider  $V_{\text{inf}}(\phi_1) \sim \phi_1^p$ . In large- $\mathcal{N}$  limit,

$$\mathcal{N} \simeq \frac{4\pi}{p M_p^2} \phi_1^2. \quad (9)$$

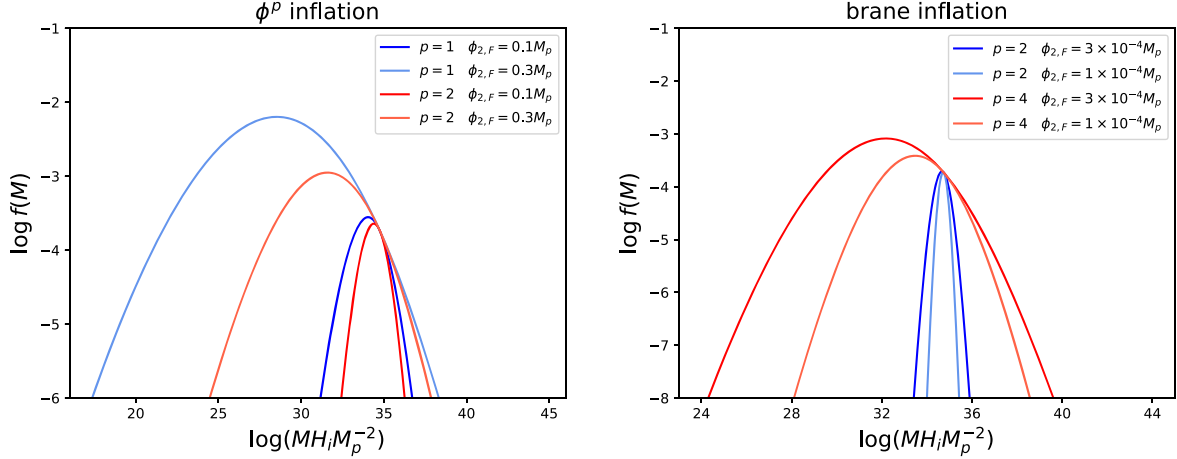
Here,  $p=2$  corresponds to chaotic inflation (Linde 1983; Kaloper & Sorbo 2009),<sup>14</sup> and  $p=2/3, 1, 4/3$  correspond to

<sup>13</sup> In Garriga et al. (2016),  $\lambda \sim \text{const.}$ , so  $f(M) \propto \frac{1}{M^{1/2}}$ .

<sup>14</sup> Recently, based on the  $\Lambda$ CDM model, the Planck collaboration has obtained  $n_s \approx 0.965$  (Planck Collaboration et al. 2020). However,  $n_s = 1$  ( $n_s - 1 \sim -\mathcal{O}(0.001)$ ) is also observationally allowed (Ye & Piao 2020; Ye et al. 2021; Jiang & Piao 2022; Jiang et al. 2024b) in light of resolution of recent Hubble tension. Thus chaotic inflation might be still consistent in its hybrid extension (Kallosh & Linde 2022; Ye et al. 2022).

<sup>11</sup> A supercritical bubble refers to a bubble with its radius larger than the Hubble length of spacetime inside the bubble, i.e.,  $r \gtrsim \frac{1}{H_b}$ , where  $H_b^2 = \frac{8\pi}{3M_p^2} V_b$ .

<sup>12</sup> In earlier works Maeda et al. (1982), Kodama et al. (1981), Farhi & Guth (1987), Blau et al. (1987), the scenarios with baby inflating universes inside BHs in de Sitter or asymptotically flat spacetime have been explored.



**Figure 3.** The mass spectra  $f(M)$  for different models of slow-roll inflation. We set  $\mathcal{N}_* = 40$ , and specially for KKLT brane inflation,  $\mu = \phi_{2,F} \ll M_p$ .

monodromy inflation (Silverstein & Westphal 2008; McAllister et al. 2010). Thus with (4), we have

$$B \approx B_* \sqrt{1 + \frac{pM_p^2}{4\pi\phi_{2,F}^2}(\mathcal{N}^{1/2} - \mathcal{N}_*^{1/2})^2}, \quad (10)$$

which suggests

$$f(M) \sim e^{-B(\mathcal{N})} \left(\frac{\mathcal{M}_{\text{eq}}}{M}\right)^{1/2} = \left(\frac{\mathcal{M}_{\text{eq}H_i}}{M_p}\right)^{1/2} \cdot \exp \left[ -B_* \sqrt{1 + \frac{pM_p^2}{4\pi\phi_{2,F}^2}(\mathcal{N}^{1/2} - \mathcal{N}_*^{1/2})^2} - \mathcal{N} \right], \quad (11)$$

where  $\mathcal{N} = \ln \sqrt{\frac{MH_i}{M_p}}$ . The results with  $\mathcal{N}_* = 40$  are shown in Figure 3. Different  $p$  and  $\phi_{2,F}$  (“shortest paths” to a neighboring vacuum) result in different profiles of  $f(M)$ .

In our multiverse PBH model, we can have  $\epsilon = -\dot{H}/H^2 \sim 0.01$  on all scales (outside the bubbles) for  $\phi^p$  inflation. However, it must be mentioned that in primordial-perturbation-sourced PBH models, a large amplitude  $\delta\rho/\rho \gtrsim 0.1$  of primordial scalar perturbations requires that the standard slow-roll evolution,  $\epsilon \sim 0.01$ , of inflaton must be broke on corresponding PBH scales, e.g., so-called ultra-slow roll.<sup>15</sup>

It is also interesting to consider Kachru-Kalosh-Linde-Trivedi (KKLT) brane inflation,  $V_{\text{inf}}(\phi_1) \sim 1 - \left(\frac{\mu}{\phi_1}\right)^p$  (Kachru et al. 2003a; Kalosh et al. 2019), where  $p = 1, 2, 3, 4$ . In such

models, for  $\mu \ll M_p$ , we have

$$\mathcal{N} = \frac{8\pi\mu^2}{p(p+2)M_p^2} \left(\frac{\phi}{\mu}\right)^{p+2}. \quad (12)$$

Thus with (4), we have

$$B \approx B_* \sqrt{1 + \mathcal{A} \frac{\mu^{2p/(p+2)} M_p^{4/(p+2)}}{\phi_{2,F}^2} (\mathcal{N}^{1/(p+2)} - \mathcal{N}_*^{1/(p+2)})^2}, \quad (13)$$

where  $\mathcal{A} = \left(\frac{p(p+2)}{8\pi}\right)^{2/(p+2)}$ . The corresponding results are also displayed in Figure 3.

### 3. Discussion

In this work, we found that if the inflaton is passed by a neighboring vacuum, the mass spectrum of multiverse PBHs formed by the supercritical bubbles that nucleated during inflation would attain a peak. It is usually expected that inflation happened at  $V_{\text{inf}} \sim (10^{15} \text{ GeV})^4$  and lasts for  $\mathcal{N} \approx 60$  e-folds, thus if the bubbles nucleated at  $10 \lesssim \mathcal{N}_* \lesssim 50$ , the mass peak will be at

$$10^{-22} M_\odot \lesssim M_* = \frac{M_p^2}{H_i} e^{2\mathcal{N}_*} \lesssim 10^{11} M_\odot. \quad (14)$$

In Liu & Bromm (2022), it has been shown that early supermassive galaxies observed by JWST can be explained with supermassive ( $M \gtrsim 10^9 M_\odot$ ) BHs that make up a small fraction ( $\sim 10^{-6} - 10^{-3}$ ) of dark matter. According to (14), our multiverse PBHs can be supermassive, up to  $10^{11} M_\odot$ , which thus can naturally serve as such supermassive BHs.

The nano-Hertz stochastic GW background recently detected might be interpreted with a population of  $M \gtrsim 10^9 M_\odot$  supermassive BH binaries, but such an explanation seems to have a mild tension with North American Nanohertz Observatory for

<sup>15</sup> In such a single-field inflation model, a recent dispute is whether the enhanced small-scale perturbations might lead to large one-loop correction for perturbations on CMB scales or not, e.g., Kristiano & Yokoyama (2024a), Riotto (2023), Kristiano & Yokoyama (2024b), Fumagalli (2023), see also Cai et al. (2024) for PBHs from null energy condition violation during inflation (Cai & Piao 2021, 2022).





- Deng, H., & Vilenkin, A. 2017, *JCAP*, 2017, 044
- Düchting, N. 2004, *PhRvD*, 70, 064015
- Ellis, J., Fairbairn, M., Hütsi, G., et al. 2024, *PhRvD*, 109, L021302
- EPTA Collaboration, InPTA Collaboration, Antoniadis, J., et al. 2023, *A&A*, 678, A50
- Espinosa, J. R., & Konstandin, T. 2019, *JCAP*, 2019, 051
- Farhi, E., & Guth, A. H. 1987, *PhLB*, 183, 149
- Franciolini, G., Iovino, A. J., Vaskonen, V., & Veermäe, H. 2023, *PhRvL*, 131, 201401
- Fumagalli, J. 2023, arXiv:2305.19263
- García-Bellido, J., Linde, A., & Wands, D. 1996, *PhRvD*, 54, 6040
- Garriga, J., Vilenkin, A., & Zhang, J. 2016, *JCAP*, 2016, 064
- Guo, S.-Y., Khlopov, M., Liu, X., et al. 2023, arXiv:2306.17022
- Hasegawa, F., & Kawasaki, M. 2018, *PhRvD*, 98, 043514
- Hawking, S. 1971, *MNRAS*, 152, 75
- He, J., Deng, H., Piao, Y.-S., & Zhang, J. 2024, *PhRvD*, 109, 044035
- Huang, H.-L., Jiang, J.-Q., & Piao, Y.-S. 2024, *PhRvD*, 109, 063515
- Huang, H.-L., & Piao, Y.-S. 2023, *Phys. Rev. D*, 110, 023501
- Hütsi, G., Raidal, M., Urrutia, J., Vaskonen, V., & Veermäe, H. 2023, *PhRvD*, 107, 043502
- Ivanov, P., Naselsky, P., & Novikov, I. 1994, *PhRvD*, 50, 7173
- Jiang, J.-Q., Cai, Y., Ye, G., & Piao, Y.-S. 2024a, *JCAP*, 2024, 004
- Jiang, J.-Q., & Piao, Y.-S. 2022, *PhRvD*, 105, 103514
- Jiang, J.-Q., Ye, G., & Piao, Y.-S. 2024b, *MNRAS*, 527, L54
- Kachru, S., Kallosh, R., Linde, A., et al. 2003a, *JCAP*, 2003, 013
- Kachru, S., Kallosh, R., Linde, A., & Trivedi, S. P. 2003b, *PhRvD*, 68, 046005
- Kallosh, R., & Linde, A. 2022, *PhRvD*, 106, 023522
- Kallosh, R., Linde, A., & Yamada, Y. 2019, *JHEP*, 2019, 8
- Kaloper, N., & Sorbo, L. 2009, *PhRvL*, 102, 121301
- Kawasaki, M., & Murai, K. 2019, *PhRvD*, 100, 103521
- Kawasaki, M., Sugiyama, N., & Yanagida, T. 1998, *PhRvD*, 57, 6050
- Khlopov, M. Y., Rubin, S. G., & Sakharov, A. S. 2005, *Aph*, 23, 265
- Kitajima, N., & Takahashi, F. 2020, *JCAP*, 2020, 060
- Kodama, H., Sasaki, M., Sato, K., & Maeda, K. 1981, *PhThPh*, 66, 2052
- Konstandin, T., & Huber, S. J. 2006, *JCAP*, 2006, 021
- Kristiano, J., & Yokoyama, J. 2024a, *PhRvL*, 132, 221003
- Kristiano, J., & Yokoyama, J. 2024b, *PhRvD*, 109, 103541
- Kusenko, A. 1995, *PhLB*, 358, 51
- Kusenko, A., Sasaki, M., Sugiyama, S., et al. 2020, *PhRvL*, 125, 181304
- Linde, A. D. 1983, *PhLB*, 129, 177
- Linde, A. D. 1986, *PhLB*, 175, 395
- Liu, B., & Bromm, V. 2022, *ApJL*, 937, L30
- Liu, Z.-G., Zhang, J., & Piao, Y.-S. 2011, *PhRvD*, 84, 063508
- Maeda, K.-I., Sato, K., Sasaki, M., & Kodama, H. 1982, *PhLB*, 108, 98
- Masoumi, A., Olum, K. D., & Shlaer, B. 2017, *JCAP*, 2017, 051
- McAllister, L., Silverstein, E., & Westphal, A. 2010, *PhRvD*, 82, 046003
- Nakama, T., Carr, B., & Silk, J. 2018, *PhRvD*, 97, 043525
- Nakama, T., Suyama, T., & Yokoyama, J. 2016, *PhRvD*, 94, 103522
- Papanikolaou, T., & Gourgouliaos, K. N. 2023, *PhRvD*, 108, 063532
- Piao, Y.-S. 2023, *PhRvD*, 107, 123509
- Piao, Y.-S., & Zhang, Y.-Z. 2004, *PhRvD*, 70, 063513
- Planck Collaboration, Akrami, Y., Arroja, F., et al. 2020, *A&A*, 641, A10
- Reardon, D. J., Zic, A., Shannon, R. M., et al. 2023, *ApJL*, 951, L6
- Riotto, A. 2023, arXiv:2301.00599
- Rubin, S. G., Sakharov, A. S., & Khlopov, M. Y. 2001, *JETP*, 92, 921
- Sakharov, A. S., Eroshenko, Y. N., & Rubin, S. G. 2021, *PhRvD*, 104, 043005
- Sasaki, M., Suyama, T., Tanaka, T., & Yokoyama, S. 2016, *PhRvL*, 117, 061101
- Sasaki, M., Suyama, T., Tanaka, T., & Yokoyama, S. 2018, *CQGrA*, 35, 063001
- Serpico, P. D., Poulin, V., Inman, D., & Kohri, K. 2020, *PhRvR*, 2, 023204
- Shinohara, T., He, W., Matsuoka, Y., et al. 2023, *PhRvD*, 108, 063510
- Shinohara, T., Suyama, T., & Takahashi, T. 2021, *PhRvD*, 104, 023526
- Silverstein, E., & Westphal, A. 2008, *PhRvD*, 78, 106003
- Susskind, L. 2003, *The Anthropic Landscape of String Theory* (Davis, CA: The Davis Meeting On Cosmic Inflation), 26
- Vagnozzi, S. 2021, *MNRAS*, 502, L11
- Vagnozzi, S. 2023, *JHEAp*, 39, 81
- Vilenkin, A. 1983, *PhRvD*, 27, 2848
- Volonteri, M. 2010, *A&ARv*, 18, 279
- Volonteri, M., Habouzit, M., & Colpi, M. 2021, *NatRP*, 3, 732
- Wang, Y.-T., Zhang, J., & Piao, Y.-S. 2019, *PhLB*, 795, 314
- Xu, H., Chen, S., Guo, Y., et al. 2023, *RAA*, 23, 075024
- Ye, G., Hu, B., & Piao, Y.-S. 2021, *PhRvD*, 104, 063510
- Ye, G., Jiang, J.-Q., & Piao, Y.-S. 2022, *PhRvD*, 106, 103528
- Ye, G., & Piao, Y.-S. 2020, *PhRvD*, 101, 083507
- Yokoyama, J. 1998, *PhRvD*, 58, 083510
- Zel'dovich, Y. B., & Novikov, I. D. 1967, *SvA*, 10, 602