



# The Metallicity Dimension of the Super Earth-cold Jupiter Correlation

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## Abstract

The correlation between close-in super Earths and distant cold Jupiters in planetary systems has important implications for their formation and evolution. Contrary to some earlier findings, a recent study conducted by Bonomo et al. suggests that the occurrence of cold Jupiter companions is not excessive in super-Earth systems. Here we show that this discrepancy can be seen as a Simpson's paradox and is resolved once the metallicity dependence of the super-Earth–cold Jupiter relation is taken into account. A common feature is noticed that almost all the cold Jupiter detections with inner super-Earth companions are found around metal-rich stars. Focusing on the Sun-like hosts with super-solar metallicities, we show that the frequency of cold Jupiters conditioned on the presence of inner super Earths is  $39^{+12}_{-11}\%$ , whereas the frequency of cold Jupiters in the same metallicity range is no more than 20%. Therefore, the occurrences of close-in super Earths and distant cold Jupiters appear correlated around metal-rich hosts. The relation between the two types of planets remains unclear for stars with metal-poor hosts due to the limited sample size and the much lower occurrence rate of cold Jupiters, but a correlation between the two cannot be ruled out.

**Key words:** Planetary Systems – stars: abundances – methods: statistical

## 1. Introduction

Planets with masses/radii between Earth and Neptune are abundant in the Galaxy, with each system expected to have several of these so-called super Earths within the inner  $\sim 1$  au (e.g., Mayor et al. 2011; Fressin et al. 2013; Petigura et al. 2013; Mulders et al. 2018; Zhu et al. 2018; Hsu et al. 2019). Consequently, these systems contain significantly more solid material compared to the solar system. If the planets in these systems were formed in situ, then the planet-forming disks might have been (or near) gravitationally unstable (Chiang & Laughlin 2013; Schlichting 2014). As a result, theories involving disk-driven migration of (proto-) planets and/or planet-forming material have been popular.

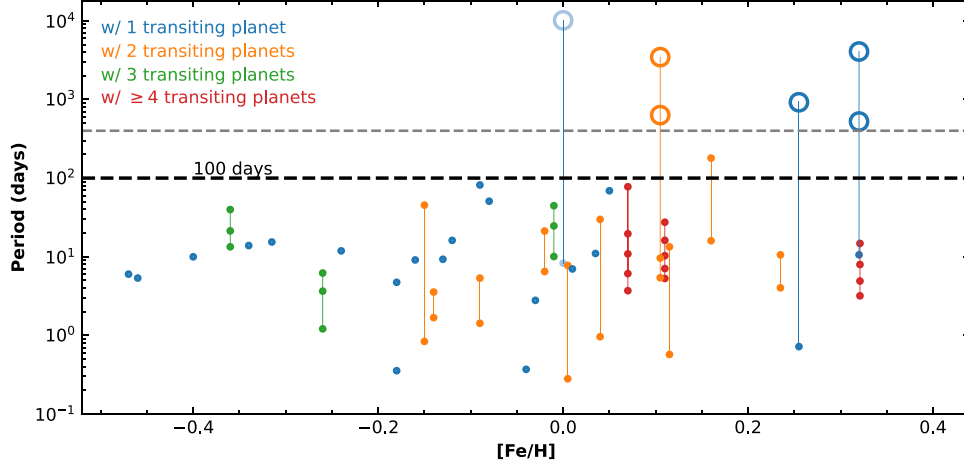
To further constrain the formation models, the presence/absence of cold Jupiter companions (with semimajor axis  $\gtrsim 1$  au and mass or minimum mass  $> 0.3 M_J$ ) in systems with inner super Earths has been considered a useful test (e.g., Ida & Lin 2010; Izidoro et al. 2015). Based on carefully constructed samples of radial velocity (RV) and transiting systems, Zhu & Wu (2018) showed that cold Jupiters were found in roughly one-third of super-Earth systems, outnumbering the frequency of cold Jupiters around field stars with similar properties by a factor of  $\sim 3$ . This excess of cold Jupiters was subsequently confirmed by several other studies (Bryan et al. 2019; Rosenthal et al. 2022). It is also in alignment with the excess of long-period transiting events in Kepler transiting systems, if these transiting events are indeed due to genuinely cold Jupiters (Foreman-Mackey et al. 2016; Herman et al.

2019; Masuda et al. 2020). The inversion of this conditional rate seems to suggest that most, if not all, of cold Jupiter systems should have inner super Earths (Zhu & Wu 2018; Bryan et al. 2019; but see also Barbato et al. 2018 and Rosenthal et al. 2022 for some lower estimates). The observed strong correlation between super Earths and cold Jupiters has motivated further development of various theoretical models (e.g., Bitsch et al. 2020; Chen et al. 2020; Schlecker et al. 2021; Best et al. 2024; Bitsch & Izidoro 2023; Chachan & Lee 2023).

A recent study by Bonomo et al. (2023) arrived at an opposite conclusion. Based on RV observations from the HARPS-N survey of 37 Kepler and K2 systems with close-in ( $P < 100$  days) super Earths,<sup>1</sup> Bonomo et al. (2023) reported the detections of five cold Jupiters and two potential cold Jupiter candidates in four systems. Statistical analysis led to an occurrence rate of  $9.3^{+7.7}_{-2.9}\%$  for cold Jupiters between 1 and 10 au around systems with inner super Earths. According to Bonomo et al. (2023), this fraction is lower than the fraction of similar cold Jupiters around field stars reported in Wittenmyer et al. (2020). Although their sample remains too small to draw any firm conclusion, it hints at an anti-correlation between small planets and cold Jupiters.

Putting the discrepancy on the overall conditional rate aside, it is interesting to note that all the cold Jupiter detections are from host stars that are relatively metal-rich. The average

<sup>1</sup> We follow Bonomo et al. (2023) and also exclude Kepler-22 from the analysis. With a host metallicity of  $[\text{Fe}/\text{H}] = -0.26$ , this has no impact on the conditional rate that is derived in Section 2.



**Figure 1.** Illustration of the planetary systems observed by HARPS-N. The  $x$ -axis shows the bulk metallicity of the host star, and the  $y$ -axis displays the orbital period of the detected planets. Systems with different observed transit multiplicities have been differentiated with different colors, and the detected cold Jupiters (including the tentative detection in K2-12 at  $[\text{Fe}/\text{H}] = 0$ ) are shown with open circles. For the purpose of illustrations, systems with nearly the same host metallicities have been offset by  $\pm 0.005$  in  $[\text{Fe}/\text{H}]$ .

metallicity of the HARPS-N sample is  $[\text{Fe}/\text{H}] \approx -0.05$ , but for simplicity we will set the threshold at  $[\text{Fe}/\text{H}] = 0$ , i.e., (nearly) solar metallicity. As illustrated in Figure 1, the bulk metallicities of the three cold Jupiter hosts are  $[\text{Fe}/\text{H}] = 0.11 \pm 0.06$ ,  $0.255 \pm 0.065$ , and  $0.32 \pm 0.08$  for Kepler-68, K2-312/HD 80653, and Kepler-454, respectively, and another system (K2-12) with a candidate companion that remains consistent with a cold Jupiter within 10 au has  $[\text{Fe}/\text{H}] = 0.00 \pm 0.07$ . This is a striking feature given that only 35% (13/37) of the selected systems have  $[\text{Fe}/\text{H}] > 0$ . In other words, the derived conditional rate could be nearly three times higher if only super-solar metallicity stars are considered.

The feature that cold Jupiters with inner small planet companions are preferentially (or exclusively) found around metal-rich stars is already seen in the original samples used by Zhu & Wu (2018) and Bryan et al. (2019) and also seen in the recent samples of Rosenthal et al. (2021), Van Zandt et al. (2023), and Weiss et al. (2024). In particular, the pure-RV sample used in Zhu & Wu (2018) and tabulated in their table 2 shows that cold Jupiters were not detected in super-Earth systems with  $[\text{Fe}/\text{H}] \lesssim 0.1$ . As for the transiting systems, all those with cold Jupiter detections in resolved orbits have stellar  $[\text{Fe}/\text{H}] > 0$  (Zhu & Wu 2018; Bryan et al. 2019; Van Zandt et al. 2023; Weiss et al. 2024). See Figure 8 of Zhu & Dong (2021) for a similar illustration to Figure 1. There were two metal-poor systems (Kepler-93 and Kepler-97) which showed long-term RV trends and could have been explained by distant cold Jupiters, but the more recent RV observations have ruled out the cold Jupiter explanation (Bonomo et al. 2023; Weiss et al. 2024).

Observational bias is unlikely the reason for the observed feature, because cold Jupiters around metal-poor stars can be and have been detected. According to a query made on 2023

June 26, to the NASA Exoplanet Archive (Akeson et al. 2013), there have been  $\sim 130$  cold Jupiters around Sun-like stars, and  $> 30$  have host metallicity  $[\text{Fe}/\text{H}] < 0$ . Therefore, it seems to be the case that the occurrence of cold Jupiters in systems with inner small planets must correlate with the host star metallicity, a point that has been made in Zhu & Wu (2018). According to that study, the rate of cold Jupiters conditional on the presence of inner super Earths,  $P(\text{CJ}|\text{SE})$ , increases to  $\sim 50\%$  for systems with bulk metallicities above the solar value, whereas it averages  $\sim 30\%$  if the whole dynamical range of stellar metallicities is considered. This is not much of a surprise, given that the occurrence of giant planets in general correlates with the host star metallicity (e.g., Santos et al. 2001, 2004; Fischer & Valenti 2005) and that if indeed the majority of the cold Jupiters have inner small companions (Zhu & Wu 2018; Bryan et al. 2019). However, whether or not the metallicity dependence of the super-Earth–cold Jupiter correlation is entirely due to the giant planet–metallicity correlation requires further investigations with much larger planet samples.

In this work, we show that the discrepancy between Bonomo et al. (2023) and previous works can be resolved by adding the metallicity dimension of the super-Earth–cold Jupiter correlation. Specifically, the super-Earth systems with metal-rich ( $[\text{Fe}/\text{H}] > 0$ ) hosts have an excess of cold Jupiters, whereas it is unclear for systems with metal-poor hosts, but a correlation between the two planet populations cannot be ruled out.

## 2. Excess of Cold Jupiters in Metal-rich Systems

### 2.1. The Conditional Rate $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] > 0)$

We first estimate the conditional rate  $P(\text{CJ}|\text{SE})$  in the super-solar metallicity regime, which is denoted  $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] > 0)$ .

**Table 1**  
The Conditional Rate  $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] > 0)$  Derived from Different Samples

Sample	$P(\text{CJ} \text{SE}, [\text{Fe}/\text{H}] > 0)$	Label	Note
Kepler & K2 sample ( $R_{\text{SE}} < 4 R_{\oplus}$ )	3/13 ( $29^{+13}_{-11}\%$ )	B23	(1)
Kepler & K2 sample ( $R_{\text{SE}} < 4 R_{\oplus}$ , w/o transit compact multi)	3/10 ( $36^{+15}_{-14}\%$ )	B23, clean	(2)
RV sample ( $m_{\text{SE}} < 20 M_{\oplus}$ )	9/17 ( $53 \pm 11\%$ )	RV, high	(3)
RV sample ( $m_{\text{SE}} < 10 M_{\oplus}$ )	3/7 ( $44^{+17}_{-16}\%$ )	RV, low	(4)
RV+Transit	$39^{+12}_{-11}\%$	Joint	(5)

**Notes.** The ratios give the numbers of systems with both cold Jupiter(s) and super Earth(s) out of the numbers of systems with SEs. The fractions in the parentheses are the estimated rates with uncertainties.

<sup>1</sup>From the sample of Bonomo et al. (2023). The rate is calculated by adopting a mean efficiency of 87.9%.

<sup>2</sup>Similar to (1), but now with the transit compact multis (i.e., systems with at least four transiting planets) removed.

<sup>3</sup>From the RV sample of Zhu & Wu (2018). Systems with at least one inner planet whose mass is below  $20 M_{\oplus}$  are considered. The rate is calculated by adopting 100% detection efficiency.

<sup>4</sup>From the RV sample of Bryan et al. (2019) after the removal of systems with M-dwarf hosts. Systems with at least one inner planet whose mass is below  $10 M_{\oplus}$  are considered. The rate is calculated by adopting 100% detection efficiency.

<sup>5</sup>The joint constraint on the conditional rate, derived from combination of the “B23, clean” and “RV, low” samples.

$[\text{H}] > 0$ ). There are 13 planetary systems with  $[\text{Fe}/\text{H}] > 0$  in the 37 HARPS-N systems of Bonomo et al. (2023).<sup>2</sup> As shown in Figure 1, this subset of the sample contains all of the confirmed cold Jupiter detections. According to Bonomo et al. (2023), the HARPS-N survey has an average completeness of 87.9% for cold Jupiters within 10 au. With these numbers, we can estimate the conditional rate  $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] > 0)$ . We assume binomial statistics and adopt a flat prior on the conditional rate. With  $n_s$  detections and  $n_f$  non-detections, the posterior distribution of the conditional rate is given by a beta distribution with shape parameters  $a = n_s + 1$  and  $b = n_f + 1$

$$f(p; a, b) = \frac{1}{B(a, b)} p^{a-1} (1-p)^{b-1}. \quad (1)$$

Here  $B(a, b)$  is the beta function, and we report the median value and the 68% confidence interval centered on the median as the uncertainties, which can be efficiently computed via the `scipy.stats.beta` function (Virtanen et al. 2020). In the present case, we have  $n_s = 3$ ,  $n_f = 13 \times 87.9\% - 3$ , and a conditional rate  $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] > 0) = 29^{+13}_{-11}\%$ . This is a factor of  $\sim 3$  higher compared to the conditional rate of Bonomo et al. (2023) over the full metallicity range. We list this new conditional rate in Table 1.

The HARPS-N sample has an over-representation of transiting systems with dynamically compact configurations (i.e., transiting compact multis), which also has implications to a proper derivation of the conditional rate. If we define a transiting compact multi as a system with at least four transiting planets within 100 days, then the HARPS-N sample contains three such systems, all with  $[\text{Fe}/\text{H}] > 0$ . See Figure 1 for an

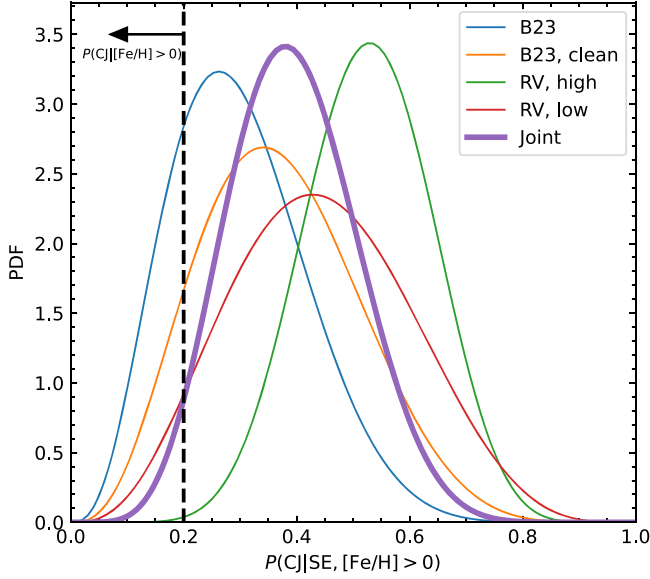
illustration. The fraction of transiting compact multis in the super-solar regime, 3/9 (excluding K2 systems, which have a much shorter time baseline, Howell et al. 2014), is much higher than the fraction of transiting compact multis in the overall Kepler systems ( $< 3\%$ , Zhu et al. 2018; Zink et al. 2019; He et al. 2020; Zhu & Dong 2021). This latter fraction would be further reduced if one only selects Kepler systems with super-solar metallicities, because the transiting compact multis are preferentially found around metal-poor stars (e.g., Brewer et al. 2018; Anderson et al. 2021). The selection of the HARPS-N sample is therefore biased toward transiting compact multis.

Theoretical studies have shown that the transiting compact multis can be easily dynamically perturbed by one or more distant massive companions, unless these companions are in very special configurations (e.g., Carrera et al. 2016; Becker & Adams 2017; Hansen 2017; Huang et al. 2017; Lai & Pu 2017; Pu & Lai 2021). It remains unclear whether these transiting compact multis follow the same correlation with cold Jupiters as the general super-Earth population, and thus the bias toward the selection of transiting compact multis may imply a bias against the detection of cold Jupiters in such systems.<sup>3</sup> Given this and the fact that these transiting compact multis are over-represented, it is reasonable to remove the three dynamically compact systems in the derivation of the conditional rate. By doing so, we arrived at a slightly higher, but yet statistically consistent, rate of  $36^{+15}_{-14}\%$ , as also listed in Table 1.

For comparisons, we also derive the same conditional rate from the pure-RV sample, in which all planets are detected via the RV method. This sample has not been expanded so much as the transit sample since the studies of Zhu & Wu (2018) and Bryan et al. (2019). For example, there is only one new system

<sup>2</sup> Two systems have reported  $[\text{Fe}/\text{H}] = 0$ , including one (K2-12) which shows a linear trend consistent with a cold Jupiter within 10 au. Including or not these two systems makes no statistically significant difference in the final result. We therefore decide to exclude them to avoid discussing the ambiguity on the nature of the linear trend in K2-12.

<sup>3</sup> In fact, the absence of Kepler compact multis around metal-rich stars has been used by Zhu & Wu (2018) as a supporting evidence for the super-Earth-cold Jupiter correlation.



**Figure 2.** Illustration of the conditional rate,  $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] > 0)$ , derived from different samples, in comparison to the estimated upper limit of the rate  $P(\text{CJ}|\text{[Fe/H]} > 0)$ . See Table 1 for the meanings of the labels.

(HD 168009) with Sun-like hosts and low-mass ( $< 10 M_{\oplus}$ ) close-in planets in the recently released California Legacy Survey (CLS, Rosenthal et al. 2021). We therefore rely on the pure-RV samples of Zhu & Wu (2018) and Bryan et al. (2019). If we define a super-Earth by its mass  $m_{\text{SE}} < 20 M_{\oplus}$ , then the sample of Zhu & Wu (2018) is more suitable. According to the Table 2 of that study, there are nine systems with cold Jupiter companions out of a total of 17 super-Earth systems with  $[\text{Fe}/\text{H}] > 0$ , resulting in a conditional rate of  $53\% \pm 11\%$ . If we adopt a lower mass limit (i.e.,  $10 M_{\oplus}$ ) for a super-Earth and use the sample of Bryan et al. (2019), the numbers are three and seven, and the resulting conditional rate is  $44^{+17}_{-16}\%$ . In the above discussion we have limited ourselves to planetary systems with Sun-like hosts, defined by stellar mass in the range  $0.7\text{--}1.3 M_{\odot}$ , as the planet distribution seems to vary with host star types (see Zhu & Dong 2021 and references therein). This excludes 23 of the 65 systems in Bryan et al. (2019), making their sample comparable in size with that of Zhu & Wu (2018). We have also adopted a 100% detection efficiency for the detection of cold Jupiters in such pure-RV systems. This is shown to be reasonable according to the efficiency estimation of Bryan et al. (2019, see their Figure 5).

As listed in Table 1 and illustrated in Figure 2, the conditional rates of cold Jupiters around Sun-like stars with super-solar metallicities, derived from both transit and pure-RV samples under different criteria, appear broadly consistent with each other. When the two independent samples, namely the HARPS-N sample without the transiting compact multis and the pure-RV sample with the lower mass limit of super Earths,

are combined, we find a conditional rate of  $39^{+12}_{-11}\%$ . In other words,  $\sim 40\%$  of Sun-like stars with inner super Earths and super-solar metallicities should have cold Jupiters in the distance range of 1–10 au.

## 2.2. On the Frequency of Cold Jupiters

Is the conditional rate derived above higher than the rate of cold Jupiters in the same metallicity range, namely  $P(\text{CJ}|\text{[Fe/H]} > 0)$ ? A detailed derivation of this rate is beyond the scope of the current paper. Instead, we provide a reasonable estimate based on a simple approach. According to Cumming et al. (2008) and as further explained in Zhu & Wu (2018), the cold Jupiter rate, integrated over the full metallicity range, is  $\sim 10\%$ . The average metallicity of stars in the RV survey is usually around or slightly below the solar value (e.g., Cumming et al. 2008; Johnson et al. 2010; Mayor et al. 2011). Therefore, even in the most extreme case that the more metal-rich half of the stars contribute all the cold Jupiter detections, the frequency of such planets around Sun-like stars with super-solar metallicities is 20% at most. More realistic giant planet–metallicity correlations would give lower values. For example, the giant planet–metallicity correlation of Johnson et al. (2010) indicates that the frequency of giant planets with  $[\text{Fe}/\text{H}] > 0$  is only  $\sim 1.4$  times the frequency of giant planets across the whole metallicity range. Therefore, the rate of cold Jupiters around metal-rich Sun-like stars is statistically lower than the conditional rate of cold Jupiters in the same metallicity range.

The recent CLS provided another estimate of the cold Jupiter rate (Fulton et al. 2021; Rosenthal et al. 2021). By focusing on the Sun-like sample and applying a proper method that takes into account the impact of planet multiplicity, Zhu (2022) reported a rate of 17% for the frequency of planetary systems containing at least one cold Jupiter around all metallicities. However, one should be cautious in directly applying this value to the current context. Some fraction of the cold Jupiters have inner giant companions, as inferred from the RV follow-up observations of close-in giant planets (namely hot and warm Jupiters, Knutson et al. 2014; Bryan et al. 2016), whereas none of the planetary systems in the Bonomo et al. (2023) sample contain such close-in giants. This difference usually has a negligible impact on the correlation study, as the frequency of close-in giants is typically small compared to the frequency of cold Jupiters. This is not the case for the CLS Sun-like sample, whose frequency of close-in giants is surprisingly high at 7.2% (Zhu 2022). In particular, the hot Jupiter rate of 2.8% is almost three times the canonical value ( $\sim 1\%$ ) from previous studies (e.g., Cumming et al. 2008; Mayor et al. 2011). In order to have a fair comparison, one should exclude the cold Jupiters with close-in giant companions from the overall cold Jupiter population. By doing so, we get  $P(\text{CJ})$  between  $\sim 10\%$  (if all close-in giants have cold Jupiter companions) and  $\sim 13\%$  (if



half of the close-in giants have cold Jupiter companions). These values are in closer agreement with the adopted value of 10%.

Out of all 474 stars in the CLS Sun-like sample, 59% have  $[\text{Fe}/\text{H}] > 0$  and contribute 41 out of 49 systems with at least one cold Jupiter detection. These values imply that the cold Jupiter frequency around stars with super-solar metallicities is 1.4 times the overall cold Jupiter frequency. This factor is consistent with our estimation based on the giant planet–metallicity correlation of Johnson et al. (2010). Therefore, the value of  $P(\text{CJ}|\text{[Fe/H]} > 0)$  is probably in the range 14%–19%, which is below the upper limit of 20% that we have adopted.

Finally, we comment on the unconditional rate,  $P(\text{CJ})$ , used in Bonomo et al. (2023). Bonomo et al. (2023) quoted 20.2% by summing up the occurrence rates of cold Jupiters from 300 to 10,000 days, using the occurrence rates of gas giants from Wittenmyer et al. (2020). The above orbital period range does not match exactly the semimajor axis range of 1–10 au, and the integrated occurrence rate does not account for the impact of giant planet multiplicity. A closer examination of the planet sample from Wittenmyer et al. (2020) reveals that half of the cold Jupiter detections in the period range of 300–1000 days have  $P < 400$  days, including five with period below one year (although only two have derived semimajor axis below 1 au). If we adopt the lower limit in orbital period at 1 yr, then the integrated occurrence rate in the period range  $1 \text{ yr} < P < 10,000$  days ( $\sim 9$  au) becomes 16.2%. The revision of the upper limit to 10 au does not further revise this value. Furthermore, in order to obtain the frequency of cold Jupiter systems, it is necessary to correct for planet multiplicity, as a significant fraction of cold Jupiters reside in systems with multiple cold Jupiters. Based on the estimated average multiplicity of 1.27 for cold Jupiters in the range of 1–10 au (Zhu 2022),<sup>4</sup> we obtain an unconditional rate of 12.8%. Given the statistical uncertainty and probably systematic uncertainty arising from the use of the inverse detection efficiency method (see Section 1.2 of Zhu & Dong 2021 for further discussions), it is not significantly different from the value used in this study for the full metallicity range (i.e., 10%).

### 3. Discussion

The coexistence (or not) of inner super Earths and outer cold Jupiters in planetary systems has significant theoretical implications for their formation and evolution. Several studies have shown that Sun-like stars with super Earths are more likely to have cold Jupiter companions compared to random field stars, suggesting that these two types of planets indeed tend to coexist.

A recent study by Bonomo et al. (2023) conducted RV follow-up observations of a sample of 37 planetary systems with inner super Earths (with periods less than 100 days) and found that this sample of systems did not show an excess of cold Jupiter detections. It is first noticed that all the cold Jupiter detections in their sample are found around metal-rich stars (see Figure 1). This feature is also seen in almost all the other known cold Jupiter systems with inner super-Earth companions (Zhu & Wu 2018; Bryan et al. 2019; Rosenthal et al. 2021; Van Zandt et al. 2023; Weiss et al. 2024). The study of the correlation (or not) between inner small planets and outer cold giants should take the stellar metallicity dependence into account.

This work derives  $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] > 0)$ , the frequency of cold Jupiters around Sun-like stars conditioned on the presence of inner super Earths and super-solar metallicities. For the HARPS-N sample of Bonomo et al. (2023), this quantity is estimated to be  $36^{+15}_{-14}\%$  after removing the transiting systems with dynamically compact configurations, on the basis that such systems are over-represented in the sample and potentially bias against the detection of cold Jupiters. The derived conditional rate is statistically consistent with the rate derived from the pure-RV sample. Combining the two samples, we estimate that  $39^{+12}_{-11}\%$  of Sun-like stars with inner super Earths and bulk metallicity  $[\text{Fe}/\text{H}] > 0$  should have cold Jupiter companions in the range of 1–10 au. For comparisons, the frequency of cold Jupiters in the same metallicity range is  $\lesssim 20\%$ . Therefore, the current results support that super-Earth systems around metal-rich hosts have an excess of cold Jupiters.

Almost no cold Jupiter has been detected in super-Earth systems with Sun-like stars and sub-solar metallicities (i.e.,  $[\text{Fe}/\text{H}] < 0$ ).<sup>5</sup> Does this mean that there is no correlation or even an anti-correlation between inner super Earths and outer cold Jupiters in the metal-poor environment? Unfortunately we cannot answer this question with the currently available data. The frequency of cold Jupiters around metal-poor stars is intrinsically low, likely at the few percent level, so a much larger planet sample is needed to address the super-Earth–cold Jupiter relation in this metallicity range. For example, using the HARPS-N sample of Bonomo et al. (2023), which contains 22 stars and no cold Jupiter detections with  $[\text{Fe}/\text{H}] < 0$ , one finds the 95% upper limit on the conditional rate to be  $P(\text{CJ}|\text{SE}, [\text{Fe}/\text{H}] < 0) < 14\%$  following the same procedure as in Section 2. Either correlation or anti-correlation remains possible.

<sup>4</sup> As has been discussed above, the estimated rates of cold Jupiters in Zhu (2022) might suffer from systematics in the sample selection. However, the average multiplicity, which is the ratio between the frequency of planets and the frequency of planetary systems, should be less affected.

<sup>5</sup> According to NASA Exoplanet Archive (Akeson et al. 2013), HD 137496 and HD 191939 are perhaps the only exceptions. The former contains one confirmed cold Jupiter and have slightly sub-solar metallicity ( $[\text{Fe}/\text{H}] = -0.027 \pm 0.040$ , Azevedo Silva et al. 2022). The latter has  $[\text{Fe}/\text{H}] = -0.15 \pm 0.06$  and a long-term RV trend that is consistent with a super-Jupiter beyond 3 au (Orell-Miquel et al. 2023), but more observations are needed to confirm the signal.

In both the metal-rich and metal-poor regimes, the frequency of cold Jupiters conditioned on the presence of super Earths appears consistent between the HARPS-N sample of Bonomo et al. (2023) and the original samples of Zhu & Wu (2018) and Bryan et al. (2019). Therefore, the discrepancy on the overall conditional rate can be explained by Simpson's paradox: the same trend appears in different subsets of the data, but disappears or even reverses in the joint data set.

The current planet sample remains too small to fully establish the correlation between inner small planets and outer giant planets and explore its dependence on properties of the host stars and the planetary systems (see some early attempts by Zhu 2019 and He & Weiss 2023), making it difficult to test predictions of formation models (e.g., Schlecker et al. 2021; Best et al. 2024; Bitsch & Izidoro 2023). The ongoing RV follow-up observations of transiting systems (e.g., Bonomo et al. 2023; Van Zandt et al. 2023; Weiss et al. 2024) and upcoming Gaia astrometric detections (e.g., Perryman et al. 2014; Espinoza-Retamal et al. 2023) aided by careful statistical analysis will be helpful in this regard. Further studies of this correlation can also be a task for future missions (e.g., Ge et al. 2022; Ji et al. 2022; Wang et al. 2023).

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