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Investigating the Hubble Tension Through Hubble Parameter Data

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Abstract

The Hubble constant (H_0), which represents the expansion rate of the universe, is one of the most important cosmological parameters. The recent measurements of H_0 using the distance ladder methods such as type Ia supernovae are significantly greater than the CMB measurements by Planck. The difference points to a crisis in the standard model of cosmology termed Hubble tension. In this work we compare different cosmological models, determine the Hubble constant and comment on the Hubble tension using the data from differential ages of galaxies. The data we use are free from systematic effects as the absolute age estimation of the galaxies is not needed. We apply the Bayesian approach along with the commonly used maximum likelihood method to estimate H_0 and calculate the AIC scores to compare the different cosmological models. The non-flat cosmological model provides a higher value for matter density as well as the Hubble constant compared to the flat ΛCDM model. The AIC score is smaller for the flat ΛCDM cosmology compared to the non-flat model indicating the flat model a better choice. The best-fit values of H_0 for both these models are 68.7 ± 3.1 km s^{-1} Mpc^{-1} and 72.2 ± 4 km s^{-1} Mpc^{-1}, respectively. Our results are consistent with the CCHP measurements. However, the flat model result does not agree with the SH0ES result, while the non-flat result is inconsistent with the Planck value.

Key words: (cosmology:) cosmological parameters – galaxies: high-redshift – (stars:) supernovae: general

1. Introduction

The linear relation between the distance to various galaxies from us and their recessional velocities was the first evidence for the state of expansion of the universe (Hubble 1929). The slope of the graph, also known as the Hubble constant, measures the expansion rate of the universe. Additionally, observations of type Ia supernovae (SNe) show that the expansion is accelerating (Perlmutter et al. 1998; Riess et al. 1998). The ΛCDM model (Astier & Pain 2012) is the simplest cosmological model which provides a good fit for available cosmological data.

The Hubble constant (H_0) is one of the most important parameters in modern cosmology; and along with other cosmological parameters it sets the age, size and shape of the universe. Determining an accurate value of H_0 has been a challenging task for cosmologists during the last few decades. Measuring the value within 10% accuracy has been one of the key projects of the Hubble Space Telescope. The current estimate of the key project is $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1} (Freedman et al. 2001). Lately some excellent progress has been made toward measuring the Hubble constant as a number of different methods of measuring distances have been developed and refined. Supernovae, H_0, Equation of State of Dark energy (SH0ES) is among the most precise measurements of type Ia SN distances for the above purpose. From the SH0ES program (Riess et al. 2016) a value of $H_0 = 73.24 \pm 1.74$ km s^{-1} Mpc^{-1} was obtained. On the other hand, observations of Cosmic Microwave Background (CMB) anisotropies can also provide a global value of H_0 when ΛCDM cosmology is applied to it. Coincidentally, these two measurements of the Hubble constant disagree at more than the 3σ level. The discrepancy is termed "Hubble tension" (Dainotti et al. 2021). The conflict is alarming and it possibly indicates new physics beyond the standard ΛCDM cosmological model (Freedman 2017; Feeney et al. 2018; De Felice et al. 2020; Vagnozzi 2020, 2021). Recently Freedman et al. (2019) calibrated type Ia SNe using tip of the red giant branch (TRGB) stars. Their value of $H_0 = 69.8 \pm 0.8 \pm 1.1$ km s^{-1} Mpc^{-1} is smaller than that of Riess et al. (2016), leading to a reduction in the discrepancy level. However, the tension between the local and global value of H_0 has not disappeared and requires attention of researchers (Haslbauer et al. 2020; Di Valentino et al. 2021; Freedman 2021; Thakur et al. 2021a; Łukasz Lenart et al. 2023; Adhikari 2022; Cai et al. 2022; Dainotti et al. 2022; Rezazadeh et al. 2022; Dainotti et al. 2023; Thakur et al. 2023).

We plan to analyze the Hubble parameter data sets using the flat and non-flat ΛCDM models by applying Bayesian analysis to test if the Hubble tension is real. This paper is organized as follows: The data and method of our analysis are described in Section 2. The results and conclusions are presented in Sections 3 and 4 respectively.

2. Methodology and Data

We begin with the maximum likelihood method which is a common approach to estimate best-fit parameters for a model. One can define the likelihood in terms of χ^2 as follows

$$P(D|M) \propto \exp(-\chi^2/2). \tag{1}$$

Here likelihood, $P(D|M)$, is the probability of obtaining the data assuming that the given cosmological model M is correct. In the present analysis we have considered the flat and non-flat ΛCDM cosmological models. One can maximize the likelihood or minimize χ^2 with respect to the model parameters to obtain the best-fit. χ^2 is defined for the above cosmological models as

$$\chi^2(a_j) = \sum_i \left[\frac{H^{\rm th}(z_i, a_j) - H_i^{\rm obs}}{\sigma_{H_i}}\right]^2, \tag{2}$$

where the free parameters of our model, a_j, are Ω_M and H_0 for flat while Ω_k, Ω_M and H_0 for non-flat cosmology. $H^{\rm th}$ and $H^{\rm obs}$ denote the theoretical and observed value of the Hubble parameter respectively while σ_{H_i} stands for the standard error in $H^{\rm obs}$. The Hubble parameter $H^{\rm th}$ for spatially flat ΛCDM model is

$$H^{\rm th} = H_0\sqrt{\Omega_M(1+z)^3 + 1 - \Omega_M}, \tag{3}$$

where Ω_M is the present value of the density parameter. In the non-flat ΛCDM model the expansion rate function is given by

$$H^{\rm th} = H_0\sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + 1 - \Omega_M - \Omega_k}, \tag{4}$$

where Ω_k is the current value of the spatial curvature energy density parameter.

2.1. Akaike Information Criterion

Akaike Information Criterion, popularly known as AIC, is a technique for assessing how well the data fit a specific model. It is used to compare different models to determine which one fits the data better. AIC can be computed from the likelihood, L, and the number of independent variables, K, in the following manner (Akaike 1974)

$$\text{AIC} = -2\log L + 2K. \tag{5}$$

A smaller value of AIC indicates a better fit. A difference of more than 2 AIC units between the AIC scores of different models is considered significant. The default value of K is 2 with no independent parameters. Here we shall compare the flat ΛCDM model with Ω_M and H_0 as independent variables with the non-flat ΛCDM model in which Ω_k is an additional independent variable. Our cosmological models have two and three parameters, respectively, hence the values of K are 4 and 5 for each model.

2.2. The Bayesian Approach

We use both the maximum likelihood method as well as the Bayesian approach to estimate the best-fit values of cosmological parameters. The posterior probability of the parameters can be calculated using Bayes theorem

$$P(M|D) \propto P(D|M) \times P(M). \tag{6}$$

The main criticism of the Bayesian approach arises from the prior probability which represents our state of knowledge about the model itself since it could be subjective. One should be careful while selecting the prior probability, and stringent priors should be avoided. However the Bayesian approach is useful as it allows one to calculate the direct probability of model parameters. The other advantage of this approach is the marginalization over the undesired model parameters. For instance, Ω_M and H_0 are often used as the essential parameters in most of the cosmological models. Since we are only interested in the expansion rate, we prefer marginalizing over Ω_M using the following equation

$$P(H_0) = \int P(\Omega_M, H_0)P(\Omega_M, H_0)d\Omega_M. \tag{7}$$

Two different types of priors have been considered in our analysis: i) uniform prior ($0 \leqslant \Omega_M \leqslant 1$) and ii) Gaussian priors centered around the best-fit value. We have carefully chosen the prior probability of Ω_M within a reasonable range.

2.3. H(z) Data and the Differential Ages of Galaxies

The data set consists of 31 H(z) values, recently compiled by Cao & Bharat (2022). The redshift range covered by the measurements is $z \leqslant 0.07 \leqslant 2.42$. Earlier attempts at estimating H_0 from Hubble parameter data can be found in Cao & Bharat (2022). This technique uses passively evolving early-type galaxies and does not depend on the cosmological model (Dhawan et al. 2021; Vagnozzi et al. 2021). This method can provide constraints on the cosmological parameters as it does not rely on the nature of the metric between the observer and the chronometers. The differential approach instead of the real ages of the galaxies is the reason for the above advantage. Additionally, this technique is immune to systematic effects as the absolute age estimation of the galaxies is not required. Luminous red galaxies (LRGs) are regarded as a good candidate for this method as their photometric properties are consistent with an old passively evolving stellar population.

3. Results and Discussion

We first calculate the best-fit parameters from the $H(z)$ data set by minimizing χ^2 defined in Equation (2). The minimum value of χ^2 and the best-fit cosmological parameters for both the flat and non-flat ΛCDM models are presented in Tables 1 and 2. It is clear that χ^2_ν is smaller than 1, indicating that the error bars probably have been overestimated. The large error

Table 1
Best-fit Value of Parameters for a Flat ΛCDM Cosmology from $H(z)$ Data by Minimizing χ^2

Ω_M	H_0	χ^2_ν	AIC
0.28	68.8	0.972	36.21

Table 2
Best-fit values of Cosmological Parameters by Minimizing χ^2 for non-flat ΛCDM Model

Ω_M	H_0	Ω_k	χ^2_ν	AIC
0.45	73.1	−0.53	0.973	37.25

Table 3
H_0 Best fit Values after Marginalization from the Hubble Parameter Data

Probe	Model	H_0	σ
Diff. Ages	Flat ΛCDM	68.7	3.1
Diff. Ages	Non-flat ΛCDM	72.2	4
Planck Ade et al. (2014)	⋯	67.8	0.90
SH0ES Riess et al. (2016)	⋯	73.24	1.74
CCHP Freedman et al. (2019)	⋯	69.8	0.8

Note. Both the Gaussian and uniform priors in a reasonable range provide the same values of H_0. Other measurements from the literature are shown for comparison.

bars in the data also indicate the same. A comparison of the tables affirms that flat ΛCDM favors lower matter density and Hubble constant compared to the non-flat ΛCDM model. We further calculate the likelihood and AIC score for both the models using Equations (1) and (5). The AIC score for flat cosmology is smaller and hence this model should be favored. Now we apply the Bayesian analysis and calculate the posterior probability using Equation (6). Finally, marginalization over the matter density, Ω_M, is performed and the corresponding best-fit value of Hubble constant is calculated which is presented in Table 3. Both Gaussian as well as uniform priors have been used for the marginalization. The best-fit values of H_0 are almost the same in the two cases of marginalization. For non-flat ΛCDM cosmology, marginalization over Ω_k and Ω_M have been performed. The final value of H_0 is shown in Table 3 which is again higher than the value obtained for flat cosmology.

Finally, we compare the numerical value of H_0 for both flat and non-flat ΛCDM models obtained from the Hubble parameter data with the latest measurements of H_0. The posterior probability of H_0 for flat ΛCDM cosmology from the $H(z)$ data is plotted in Figure 1. The best-fit value is 68.7 and the area between the vertical dashed lines corresponds to the 1σ confidence level. For comparison, the H_0 values from Planck, SH0ES collaboration and Carnegie-Chicago Hubble Program (CCHP) (Freedman et al. 2019) have also been shown in the same graph. Planck and CCHP values are within the 1σ region of our result. However, SH0ES value is higher than all other values and lies outside the 1σ region. Figure 2 displays the distribution of posterior probability of H_0 for non-flat ΛCDM model. As noted earlier, the best-fit in this case is slightly higher. Thus, the CCHP and SH0ES values are within the 1σ region in this case, but the Planck value is just outside the 1σ region. It should be noted that in both cases the CCHP value is consistent with the Hubble parameter data.

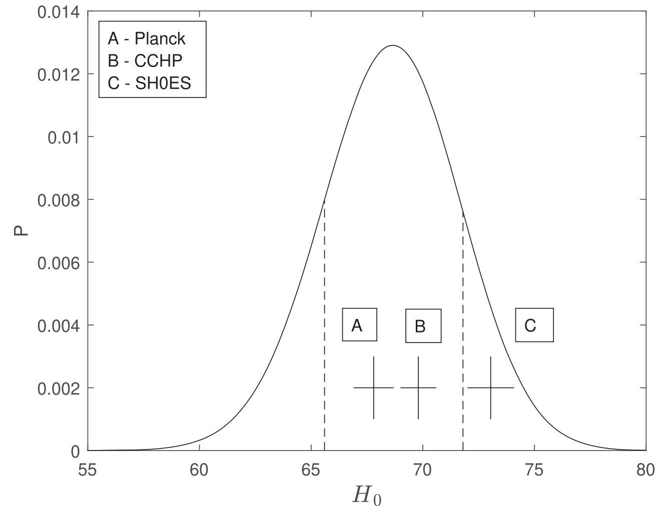

Figure 1. Probability Distribution of H_0 values for flat ΛCDM model after marginalization over Ω_M for Hubble parameter data. Planck and CCHP values are within 1σ however SH0ES value is outside of the 1σ level.

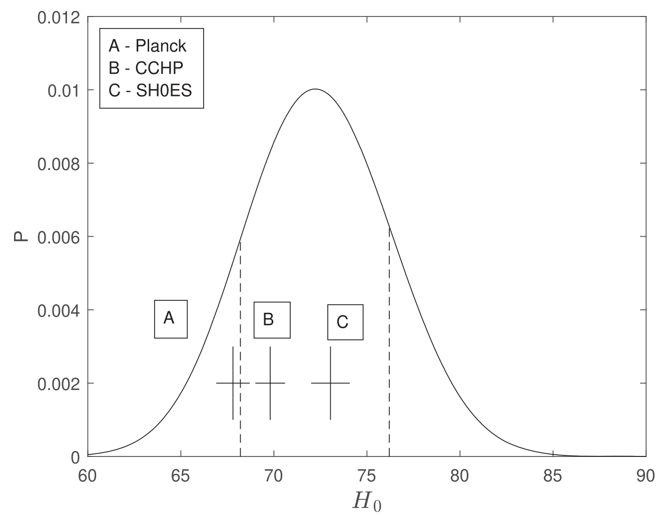

Figure 2. Probability Distribution of H_0 values for non-flat ΛCDM model after marginalization over Ω_k and Ω_M for Hubble parameter data. The Planck value lies outside of 1σ however CCHP and SH0ES values are within the 1σ level.

4. Conclusion

We compare flat and non-flat ΛCDM cosmologies in the current work and calculate the expansion rate using Hubble parameter data. We applied a variety of statistical techniques to assess the data from differential galaxy ages for this reason. The following are our main conclusions: (i) Non-flat ΛCDM cosmology favors a higher value of density as well as expansion rate, in comparison to flat ΛCDM cosmology. (ii) AIC score is smaller for the flat ΛCDM model which also has less number of parameters. Both these facts make it a better choice. (iii) For the value of Hubble constant, Planck (Ade et al. 2014) and CCHP are consistent with the ΛCDM results. However SH0ES results are quite high and are not consistent at the 1σ confidence level. (iv) SH0ES (Riess et al. 2016) and CCHP values (Freedman et al. 2019) of H_0 are consistent with our results using non-flat ΛCDM model as it provides a higher value. (v) CCHP value is consistent with Hubble parameter data in both cases as well as with other SNe Ia data (Thakur et al. 2021b). (vi) Since the number of data points is only 31 and the error bars in the data are large, the posterior probability curve is wide. A concrete statement about the Hubble tension can be made once we have sufficient Hubble parameter data.

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