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Cosmology of a Chaplygin Gas Model Under $f(T)$ Gravity and Evolution of Primordial Perturbations

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Abstract

This paper reports a detailed study of generalized Chaplygin gas (GCG) with power law form of scale factor and truncated form of the scale factor using binomial expansion in both interacting and non-interacting scenarios along with its cosmological consequences, studied in terms of equation of state (EoS) parameter. In the non-interacting scenario, the EoS parameter behaves as quintessence in both forms of the scale factor. In the interacting scenario, the EoS parameter behaves as phantom and for the truncated form of the scale factor, it violates the constraints of the positive parameter α. The cosmological implementation of GCG interacting with pressureless dark matter is investigated in the framework of $f(T)$ modified gravity, where T is the torsion scalar in teleparallelism. The interaction term is directly proportional to the GCG density with positive coupling constant. In $f(T)$ gravity, the EoS is behaving like phantom. The stability of the reconstructed model is investigated and it is found to be stable against small gravitational perturbations, i.e., the squared speed of sound is non-negative and an increasing function of cosmic time t. We have observed that our reconstructed $f(T)$ model satisfies one of the sufficient conditions of a realistic reconstructed model and it is consistent with the CMB constraints and primordial nucleosynthesis. Cosmology of primordial perturbations has also been analyzed and the self-interacting potential has been found to be an increasing function of cosmic time t.

Key words: (cosmology:) dark energy – cosmology: theory – (cosmology:) cosmological parameters

1. Introduction

Our universe is mainly filled by an exotic component with negative pressure known as Dark Energy (DE henceforth), leading to the accelerated expansion and, by the latest observations of cosmic microwave background (CMB) anisotropies (Spergel et al. 2003), it is immensely justified. The most evident candidate for such component is vacuum energy, and a reasonable alternative is dynamical vacuum energy (Özer & Taha 1987; Ratra & Peebles 1988; Bertolami 2003), or quintessence. A single field (Ratra & Peebles 1988; Wetterich 1988; Caldwell et al. 1998; Ferreira & Joyce 1998; Amendola 1999; Bento & Bertolami 1999; Binetruy 1999; Chiba 1999; Kim 1999; Uzan 1999; Zlatev et al. 1999; Albrecht & Skordis 2000; Banerjee & Pavón 2001, 2001; Bertolami & Martins 2000; Bento et al. 2001; Sen & Sen 2001; Sen et al. 2001) or two coupled fields (Bento et al. 2002a; Kim & Nilles 2003) are mostly involved in these models. However, fine-tuning problems are usually faced by these models, in particular the cosmic coincidence problem, i.e., interrogation of the fact why the scalar field or vacuum energy dominates the universe only recently.

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Recently, it has been put forward that rather than the form of the potential, the change in the equation of state (EoS) of the background fluid regulated the change of behavior of the missing energy density, which means the aforesaid fine-tuning problems are avoided. Within the framework of FRW cosmology, it has been achieved through the introduction of a distant background fluid, the Chaplygin gas (Sultana et al. 2022), illustrated by the EoS

$$p = -\frac{A}{\rho^{\alpha}}, \tag{1}$$

where $0 < \alpha < 1$ and A is a positive constant. By putting this EoS into the relativistic energy conservation equation, a density is evolved as

$$p = \sqrt{A + \frac{B}{a^6}}, \tag{2}$$

where B is an integration constant and a is the scale factor of the universe. This plain and effective model easily interpolates between a De Sitter phase where $p \simeq -\rho$ and a dust dominated phase where $\rho \simeq \sqrt{B}\, a^{-3}$, though the in-between scheme is explained by the EoS for stiff matter, $p = \rho$ (Kamenshchik et al. 2001). Notably, this set up accepts a well demonstrated brane elucidation as Equation (1); for $\alpha = 1$, it is the EoS connected with the parameterization invariant Nambu–Goto d-brane action in a (d+1,1) spacetime. This action leads to the Galileo-invariant Chaplygin gas in a (d,1) spacetime, in the light-cone parameterization and to the Poincaré invariant Born–Infeld action in a (d,1) spacetime (Jackiw 2000). Furthermore, a super symmetric generalization (Jackiw 2000) is accepted only by the Chaplygin gas. Different forms of EoS parameter corresponding to generalized Chaplygin gas (GCG) have been discussed in Nojiri et al. (2005), Nojiri (2005), Capozziello et al. (2006a, 2006b), Bamba et al. (2012a), Nojiri & Odintsov (2004), Debnath et al. (2012a, 2012b).

The title of the paper itself reflects that the torsion contribution is considered due to the density ρ_T to evolve in the form of some Chaplygin gas (Saha & Chattopadhyay 2022). We would now like to discuss some features of the modified theory of gravity (Nojiri & Odintsov 2007, 2011b; Nojiri et al. 2017, 2018) which is an alternative explanation for late time acceleration of the universe, with more emphasis on $f(T)$ gravity. The General Theory of Relativity (GR) is extended to modified gravity and for this approach, an advantage is that a transition from deceleration to acceleration is led by the modified gravity in the evolution of the universe, and without depending upon any particular form of the exotic matter it describes the phase transition from non-phantom to phantom. $f(T)$, $f(R)$, $f(G)$, Brans–Dicke, Chern–Simons, Horava–Lifshitz, etc., are the candidates of modified gravity. By reconstructing the function $f(R)$ with the use of a scalar field, the $f(R)$ gravity can be marked in terms of a scalar field (like quintessence or phantom), and a conformal transformation is executed afterwards. $f(T)$-gravity (where T is torsion scalar) is another interesting choice of modified theories of gravity. As the order of field equations is four, the equations of motion for GR are different from $f(T)$-gravity which are of 2nd order. Without making use of DE (Bamba et al. 2008; Bengochea & Ferraro 2009; Linder 2010), a theoretical explanation for the late-time acceleration of the expanding universe can be provided by the $f(T)$-model. It can be accommodated easily with the regular thermal expanding history as well as the radiation and cold dark matter (DM) dominated phases.

The formation of large scale structure in our universe is eventually led by the nature of the primordial fluctuations which is a key challenge to understanding modern cosmology. There is one chance that the fluctuations were created in the inflation era before the radiation dominated period of the hot Big Bang. Far beyond the Hubble radius, the fluctuations were imprinted as primary conditions for the radiation as the inflation period ended. Another chance is that within the matter and standard Big Bang radiation eras, the structure was created through some causal mechanical operation. Nowadays, the main focus is on the chance that the primordial perturbations were adiabatic. If from the Lagrangian which is describing local physics, the relative abundances of different particle species were found directly; then irrespective of the long wavelength perturbations, one would anticipate those abundance ratios to be spatially constant as a similar early history is shared by all the regions of the universe. On large scales, the stress-energy present in the universe is then characterized by a uniform EoS. These fluctuations are called adiabatic and they are responsible for perturbation in the geometry and the matter content of the universe. The simplest inflationary models (Hawking 1982; Linde 2015) naturally predict those fluctuations, and other types of perturbations (Kofman & Linde 1987; Kofman et al. 1987; Vishniac et al. 1987; Salopek 1992) are described by more complex models. A recent discussion can be seen in Peebles (1999).

In the present work, $f(T)$ gravity is reconstructed by considering background evolution as interacting and non-interacting GCG with power law form of scale factor and along with that its cosmological consequences are studied. The paper is organized as follows: In Section 2 we consider GCG with power law form of scale factor and truncated form of the scale factor using binomial expansion in the non-interacting scenario and investigate its cosmology on EoS parameter. In Section 3 we study the cosmology of GCG with power law form of scale factor and truncated form of the scale factor using binomial expansion in the interacting scenario and its consequences are demonstrated. In Section 4 we demonstrate interacting GCG in the cosmological settings of $f(T)$ gravity with power law form of the scale factor, and investigate its stability against small gravitational perturbations, its consistency with the CMB constraints and primordial nucleosynthesis and also provide some discussions on primordial perturbations. One of the sufficient conditions needed for a reconstructed realistic model is also satisfied in this section. In Section 5 we conclude.

2. GCG with Power Law Form of Scale Factor in Non-interacting Scenario

A GCG is a perfect fluid with a polytropic EoS (Bento et al. 2002b). It is a phenomenological extension of the Chaplygin gas, which ends all the chances for a polytropic perfect fluid DE candidate, and on small scales its gravitational perturbations are stable (Fabris et al. 2002b; Carturan & Finelli 2003). The conduction of time of a GCG interpolates between a cosmological constant and dust, with an in-between behavior of $p = \alpha\rho$. A unified DM (UDM) or DE model is a model in which the GCG is referred to as the only dark component. In the context of supernovae (Fabris et al. 2002a; Makler et al. 2003) and other supportive data (Dev et al. 2003), the first analyzation was the validity of GCG as a DE and UDM model. The next step is the comparison with CMB data and much stronger constraints are led by this (Bean & Dore 2003). In this section we examine the cosmological consequences of GCG with a power law form of scale factor and truncated form of the scale factor using binomial expansion in non-interacting scenario. The scale factor chosen is

$$a = \left(1 + \frac{h_0 t}{\beta}(\alpha - 1)\right)^{\frac{\beta}{\alpha-1}};\ (0 < \alpha < 1), \tag{3}$$

that leads to Hubble parameter H (which is defined as $H = \frac{\dot{a}}{a}$) as

$$H = \frac{h_0}{1 + \frac{h_0 t(-1+\alpha)}{\beta}}. \tag{4}$$

The EoS of GCG is given by

$$p = -\frac{B}{\rho^{\alpha}}, \tag{5}$$

where p, ρ and B are the thermodynamical pressure, DE density and a constant parameter, respectively. Hence we get the EoS parameter as

$$w = -\frac{B}{\rho^{\alpha+1}}. \tag{6}$$

The conservation equation in non-interacting scenario is given by

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{7}$$

From Equation (7), we get the form of conservation equation as

$$\dot{\rho} + 3H\rho(1 + w) = 0. \tag{8}$$

Using Equations (6) and (4) in Equation (8), we write the reconstructed DE density as

$$\rho_{\rm GCG} = (B + e^{\frac{(1+\alpha)((-1+\alpha)C_1 - 3\beta \mathrm{Log}[h_0 t(-1+\alpha)+\beta])}{-1+\alpha}})^{\frac{1}{1+\alpha}}. \tag{9}$$

By putting the expression of DE density in Equation (5), we find reconstructed pressure as

$$p_{\rm GCG} = -B(B + e^{(1+\alpha)\left(C_1 - \frac{3\beta \mathrm{Log}[h_0 t(-1+\alpha)+\beta]}{-1+\alpha}\right)})^{\frac{-\alpha}{1+\alpha}}. \tag{10}$$

Hence we get the reconstructed EoS parameter in non-interacting scenario as

$$w_{\rm GCG} = -B(B + e^{(1+\alpha)\left(C_1 - \frac{3\beta \mathrm{Log}[h_0 t(-1+\alpha)+\beta]}{-1+\alpha}\right)})^{-1}. \tag{11}$$

We have plotted the reconstructed EoS parameter $w_{\rm GCG}$ Equation (11) with respect to the cosmic time t for GCG in non-interacting scenario in Figure 1. From Figure 1, we understand that $w_{\rm GCG}$ displays a quintessence behavior and it is decreasing with the evolution of time. Finally it converges to -1 and becomes asymptotic in its neighborhood of -1, i.e., the phantom boundary in the near future.

In the next phase we expand Equation (3) for binomial expansion and truncate the expanded form up to the term involving t^2 and neglect the higher powers because of small coefficients; we have a new form of scale factor as

$$a = 1 + h_0 t + \frac{(\beta - \alpha + 1)}{\beta} h_0^2 t^2, \tag{12}$$

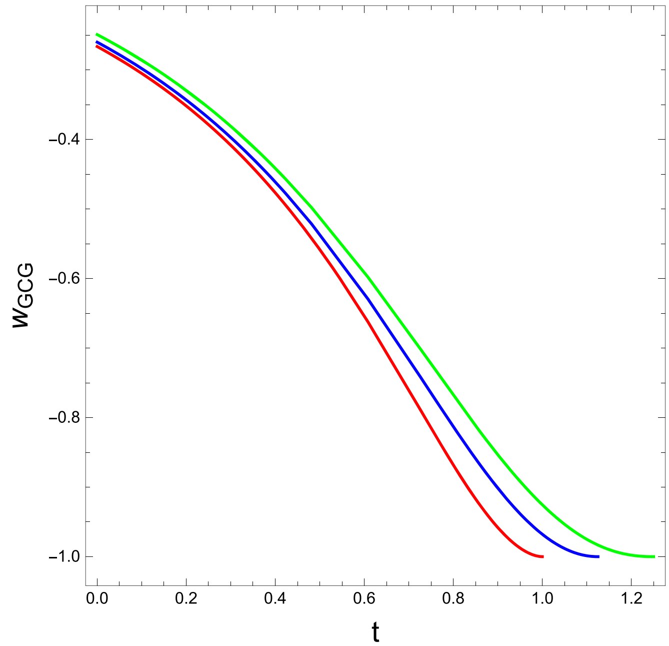

Figure 1. Evolution of EoS parameter $w_{\rm GCG}$ with respect to cosmic time t for GCG in non-interacting scenario. We have chosen $\alpha = 0.2$, $B = 0.1$, $C_1 = 0.3$, $h_0 = 0.5$. The red, blue and green lines correspond to $\beta = 0.4$, 0.45 and 0.5 respectively.

leading to the Hubble parameter

$$H = \frac{h_0(\beta + 2h_0 t(1 - \alpha + \beta))}{\beta + h_0 t(\beta + h_0 t(1 - \alpha + \beta))}. \tag{13}$$

Now using the conservation Equation (8) for this choice of truncated scale factor, we get the reconstructed DE density as

$$\rho = (B + e^{C_1(1+\alpha)}(h_0 t(h_0 t(-1 + \alpha - \beta) - \beta) - \beta)^{-3(1+\alpha)})^{\frac{1}{1+\alpha}}, \tag{14}$$

and from Equation (14), we write the reconstructed EoS parameter as

$$w = -B(B + e^{C_1(1+\alpha)}(h_0 t(h_0 t(-1 + \alpha - \beta) - \beta) - \beta^{-3(1+\alpha)})^{-1}. \tag{15}$$

In Figure 2 we plot reconstructed EoS parameter w with respect to cosmic time t for the choice of truncated scale factor in non-interacting scenario. From Figure 2, we understand that it is behaving the same as in Figure 1, i.e., it is showing quintessence behavior and is asymptotic to -1.

In the next section we examine cosmological evolution of GCG with power law form of scale factor and truncated form of the scale factor using binomial expansion in interacting scenario.

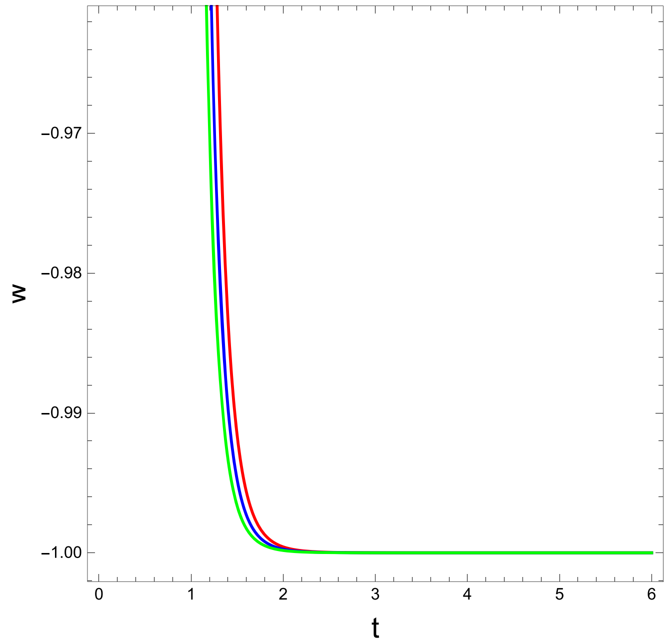

Figure 2. Evolution of reconstructed EoS parameter w with respect to cosmic time t for the choice of truncated scale factor in non-interacting scenario. We have chosen $\alpha = 1.0$, $B = 0.9$, $C_1 = 0.005$, $h_0 = 2.5$. The red, blue and green lines correspond to $\beta = 0.12$, 0.13 and 0.14 respectively.

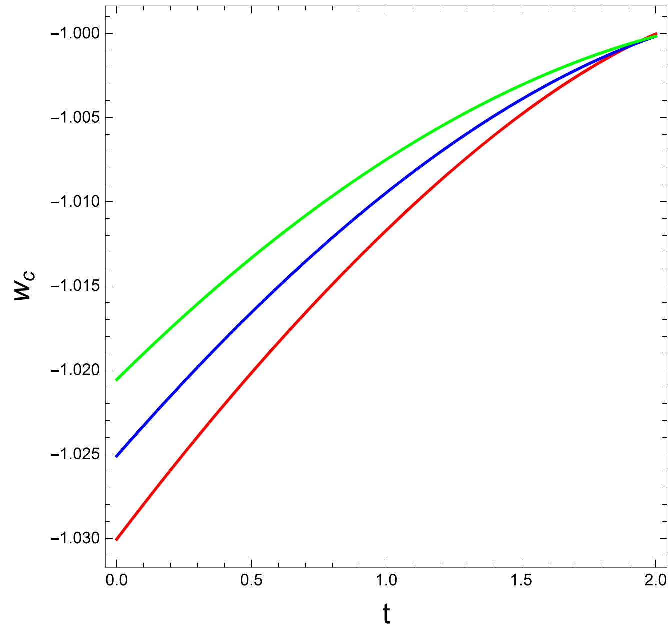

Figure 3. Evolution of EoS parameter w_c with respect to cosmic time t for GCG in interacting scenario. We have chosen $\beta = 0.4$, $B = 5$, $C_1 = 0.5$, $h_0 = 0.2$, $\delta = 0.001$. The red, blue and green lines correspond to $\alpha = 0.1$, 0.15 and 0.2 respectively.

3. GCG with Power Law Form of Scale Factor in Interacting Scenario

In this section we study the cosmology of GCG with power law form of scale factor and truncated form of scale factor using a binomial expansion in interacting scenario. The conservation equation in interacting scenario is given by

$$\dot{\rho_c} + 3H(\rho_c + p_c) = 3H\delta\rho_c, \tag{16}$$

where δ is the interacting term. From Equation (16), we get the form of conservation equation in interacting scenario as

$$\dot{\rho_c} + 3H\rho_c(1 + w_c) = 3H\delta\rho_c. \tag{17}$$

Using Equations (6) and (4) in Equation (17), we arrive at the reconstructed DE density for GCG in interacting scenario as

$$\rho_c = \left(\frac{-B + e^{\frac{(1+\alpha)(-1+\delta)((-1+\alpha)C_1+3\beta\mathrm{Log}[h_0 t(-1+\alpha)+\beta])}{-1+\alpha}}}{-1+\delta}\right)^{\frac{1}{1+\alpha}}. \tag{18}$$

By putting the expression of DE density in Equation (5), we get reconstructed pressure for GCG in interacting scenario as

$$p_c = -B\left(\frac{-B + e^{\frac{(1+\alpha)(-1+\delta)(C_1(-1+\alpha)+3\beta\mathrm{Log}[h_0 t(-1+\alpha)+\beta])}{-1+\alpha}}}{-1+\delta}\right)^{\frac{-\alpha}{1+\alpha}}. \tag{19}$$

Hence we express the reconstructed EoS parameter for GCG in interacting scenario as

$$w_c = -B\left(\frac{-B + e^{\frac{(1+\alpha)(-1+\delta)(C_1(-1+\alpha)+3\beta\mathrm{Log}[h_0 t(-1+\alpha)+\beta])}{-1+\alpha}}}{-1+\delta}\right)^{-1}. \tag{20}$$

We plot the reconstructed EoS parameter w_c Equation (20) with respect to the cosmic time t for GCG in interacting scenario in Figure 3. From Figure 3, we understand that w_c manifests a phantom behavior and it is increasing with the evolution of time. It can also be seen that it is possible to go beyond the phantom boundary under the reconstruction made and the EoS parameter is approaching -1 at $t = 2$, i.e., $w_c \approx -1$, which is consistent with our observation.

Now using the truncated form of scale factor for GCG in the interacting scenario, we get the reconstructed DE density in the interacting scenario as

$$\rho_{\text{interacting}} = \left(\frac{\psi}{-1+\delta}\right)^{\frac{1}{1+\alpha}}, \tag{21}$$

where

$$\begin{aligned}\psi = {} & e^{-(1+\alpha)C_1}(h_0^2 t^2(-1+\alpha-\beta) - \beta - h_0 t\beta)^{-3(1+\alpha)} \\ & \times(-Be^{(1+\alpha)C_1}(h_0^2 t^2(-1+\alpha-\beta) - \beta - h_0 t\beta)^{3(1+\alpha)}, \\ & + e^{(1+\alpha)\delta C_1}(h_0^2 t^2(-1+\alpha-\beta) - \beta - h_0 t\beta)^{3(1+\alpha)\delta})\end{aligned} \tag{22}$$

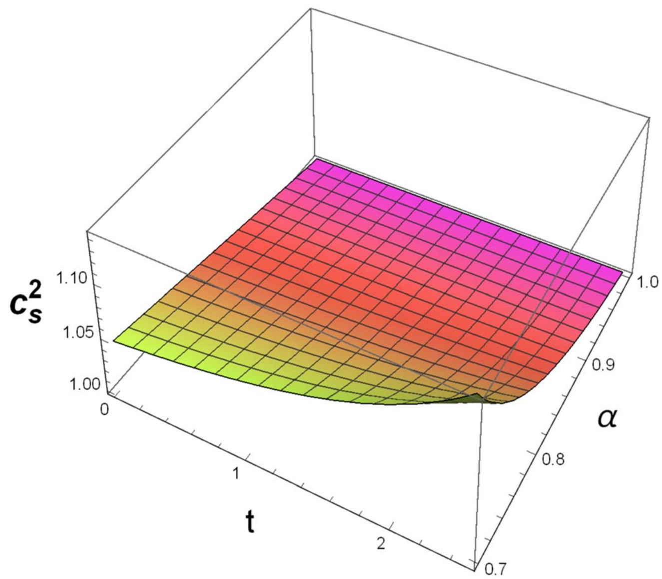

Figure 4. Evolution of squared speed of sound c_s^2 with respect to cosmic time t and positive parameter α for GCG in the framework of $f(T)$ gravity. We have chosen $\beta = 0.32$, $B = 5$, $C_1 = 1.5$, $h_0 = 0.2$, $C_2 = 0.1$, $\delta = 0.04$.

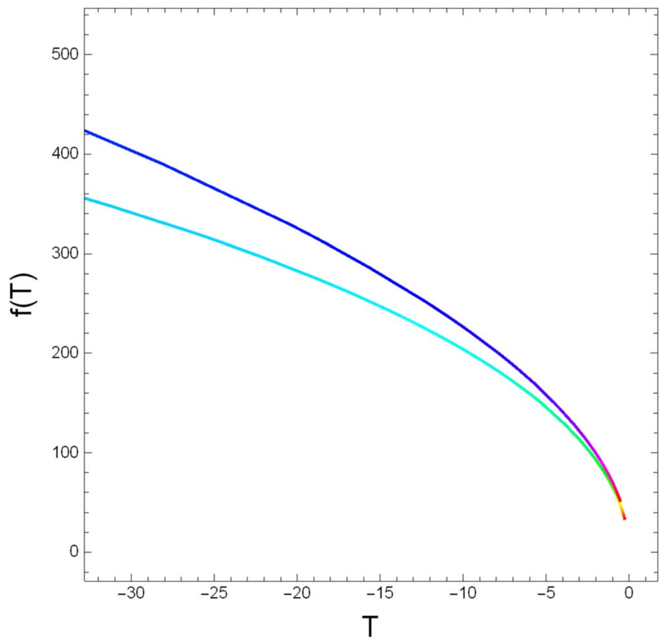

Figure 6. Reconstructed $f(T)$ against torsion scalar T. We have chosen $h_0 = 0.06$, $\alpha = 0.001$, $\beta = 0.05$, $C_2 = 0.5$, $B = 0.09$, $\delta = 0.002$, $C_1 = 0.003$.

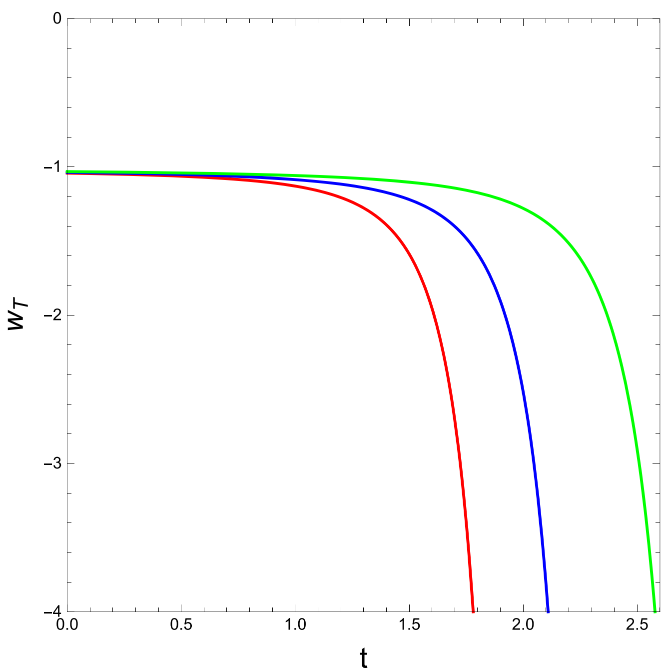

Figure 5. Evolution of EoS parameter w_T with respect to cosmic time t for GCG in the framework of $f(T)$ gravity. We have chosen $\beta = 0.32$, $B = 5$, $C_1 = 1.5$, $C_2 = 0.1$, $h_0 = 0.2$, $\delta = 0.04$. The red, blue and green lines correspond to $\alpha = 0.3$, 0.4 and 0.5 respectively.

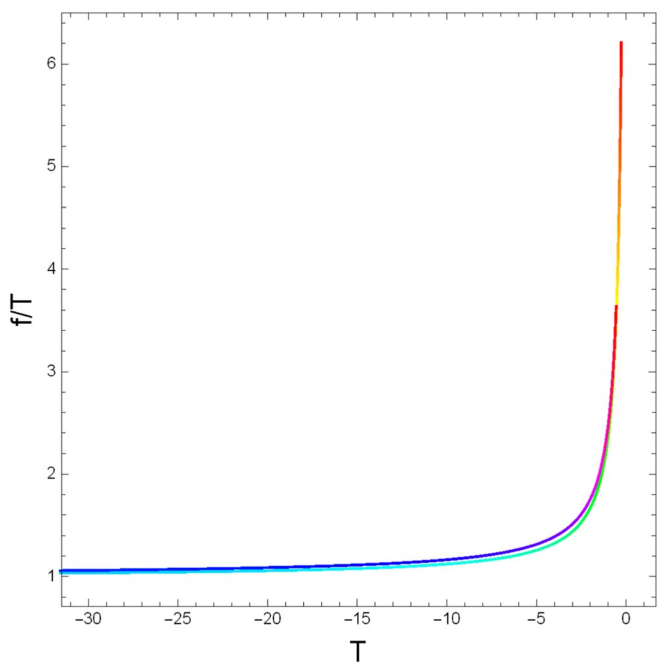

Figure 7. The given figure shows that $\frac{f}{T} \to 1$ as $|T| \to \infty$. We have chosen $h_0 = 0.06$, $\alpha = 0.002$, $\beta = 0.05$, $C_2 = 0.0005$, $B = 0.09$, $\delta = 0.05$, $C_1 = 0.003$.

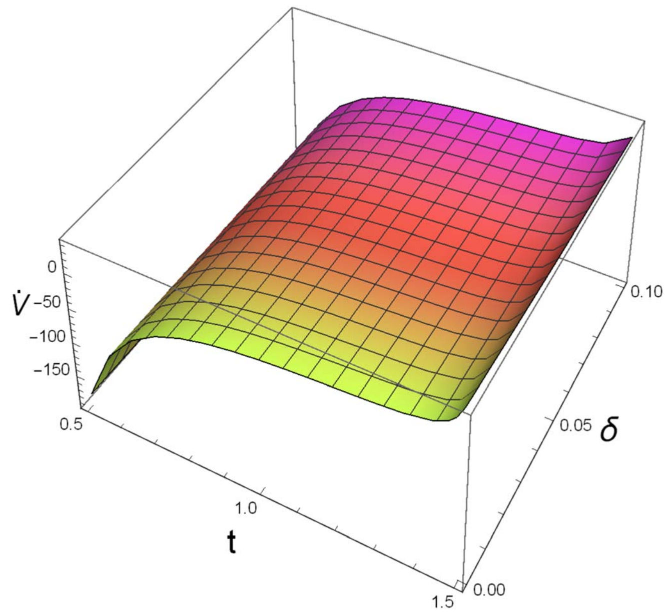

Figure 8. Evolution of the self interacting potential $\dot{V}$ with respect to cosmic time t and positive parameter α for GCG in the framework of $f(T)$ gravity. We have chosen $\beta = 0.32$, $B = 5$, $C_1 = 0.001$, $C_2 = 2$, $h_0 = 5$, $\delta = 0.04$.

and from Equation (21), we write the reconstructed DE pressure in the interacting scenario as

$$p_{\text{interacting}} = -B\left(\frac{-B + e^{C_1(1+\alpha)(-1+\delta)}(h_0 t(h_0 t(-1 + \alpha - \beta) - \beta) - \beta)^{3(1+\alpha)(-1+\delta)}}{-1 + \delta}\right)^{\frac{-\alpha}{1+\alpha}}. \tag{23}$$

Hence, we get the reconstructed EoS parameter in the interacting scenario as

$$w_{\text{interacting}} = -B\left(\frac{-B + e^{C_1(1+\alpha)(-1+\delta)}(h_0 t(h_0 t(-1 + \alpha - \beta) - \beta) - \beta)^{3(1+\alpha)(-1+\delta)}}{-1 + \delta}\right)^{-1}. \tag{24}$$

In Equation (24), $h_0t(h_0t(-1 + \alpha - \beta) - \beta > 0$ is possible if and only if $\alpha > 1 + \beta + \frac{\beta}{h_0 t} + \frac{\beta}{h_0^2 t^2}$, which contradicts the constraints given for α, i.e., $0 < \alpha < 1$. Hence, it violates the theory reported in the literature.

In the subsequent section, we examine GCG in the cosmological settings of $f(T)$ modified gravity.

4. Interacting GCG in the Framework of $f(T)$ Gravity

A current discussion on $f(T)$ gravity is available in Cai et al. (2016). Capability of $f(T)$ gravity to realize early inflation and late time acceleration of the universe (Ferraro & Fiorini 2007; Bamba et al. 2011, 2010, 2012b, 2013b; Dent et al. 2011) has been shown in the literature. A reconstruction approach for $f(T)$ based on GCG interacting with pressureless DM has been adopted in this work. In Aghamohammadi (2014), Chattopadhyay & Pasqua (2013), Chattopadhyay et al. (2015), it is shown that from various variants of holographic DE, reconstruction of $f(T)$ gravity has been attempted. The action of modified teleparallel gravity in the framework of $f(T)$ theory is written as (Bengochea & Ferraro 2009; Karami & Abdolmaleki 2013)

$$I = \frac{1}{2\kappa^2}\int d^4xe[f(T) + \mathcal{L}_m], \tag{25}$$

where $\kappa^2 = M_p^{-2} = 8\pi G$ and $e = \det(e_\mu^i) = \sqrt{-g}$. The torsion scalar and Lagrangian density of the matter inside the universe are T and $\mathcal{L}_m$ respectively. In the present work, we have assumed $8\pi G = 1$. In Karami & Abdolmaleki (2013), densities of $f(T)$ gravity are mentioned. The modified Friedmann's equations in the framework of $f(T)$ gravity are

$$3H^2 = \rho_m + \rho_T, \tag{26}$$

$$2\dot{H} + 3H^2 = -(p_m + p_T), \tag{27}$$

where

$$\rho_T = \frac{1}{2}(2Tf_T - f - T), \tag{28}$$

and

$$p_T = -\frac{1}{2}(-8\dot{H}Tf_{TT} + (2T - 4\dot{H})f_T - f + 4\dot{H} - T). \tag{29}$$

The torsion contribution to the energy density and pressure are Equations (28) and (29) respectively. Conservation equation for $\rho = \rho_T + \rho_m$ and $p = p_T + p_m$ can be represented as

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{30}$$

The EoS parameter for the torsion contribution is expressed as

$$w_T = \frac{p_T}{\rho_T} = -1 + \frac{4\dot{H}(2Tf_{TT} + f_T - 1)}{2Tf_T - f - T}. \tag{31}$$

In the subsequent subsection below, we have chosen the scale factor $a(t)$ in power law form and the cosmological consequences are studied accordingly. Earlier, a power law solution for scale factor was attempted in $f(T)$ theory by Bamba et al. (2013a), Setare & Darabi (2012). The squared speed of sound has the following form

$$c_s^2 = \frac{1 + f_T}{1 + f_T - 12H^2 f_{TT}}. \tag{32}$$

The model is found to be stable against small gravitational perturbations if the squared speed of sound c_s^2 is positive.

4.1. Scale Factor $a(t) = \left(1 + \frac{h_0 t}{\beta}(\alpha - 1)\right)^{\frac{\beta}{\alpha - 1}}$

In this section we consider the scale factor of the form Equation (3) and not the truncated form of the scale factor Equation (12), since it is not introducing any additional cosmological feature. We have

$$\dot{H} = -\frac{h_0^2(-1 + \alpha)\beta}{(h_0 t(-1 + \alpha) + \beta)^2};\ h_0 > 0, \tag{33}$$

and

$$T = -\frac{6h_0^2\beta^2}{(h_0 t(-1 + \alpha) + \beta)^2};\ \dot{T} = \frac{12h_0^3(-1 + \alpha)\beta^2}{(h_0 t(-1 + \alpha) + \beta)^3}. \tag{34}$$

Considering the correspondence between ρ_c Equation (18) and ρ_T Equation (28) for GCG, we have the following differential equation

$$\frac{144h_0^5(-1 + \alpha)\beta^4}{(h_0 t(-1 + \alpha) + \beta)^5}\frac{df(t)}{dt} + f(t) = \frac{6h_0^2\beta^2}{(h_0 t(-1 + \alpha) + \beta)^2} - 2\left(\frac{-B + e^{\frac{(1+\alpha)(-1+\delta)((-1+\alpha)C_1 + 3\beta \mathrm{Log}[h_0 t(-1+\alpha)+\beta])}{-1+\alpha}}}{-1 + \delta}\right)^{\frac{1}{1+\alpha}}, \tag{35}$$

which on solving, we get the reconstructed $f(T)$ with respect to cosmic time t for GCG as

$$f(T) = -\frac{6h_0^2\beta^2}{(h_0 t(-1 + \alpha) + \beta)^2} + \frac{C_2}{h_0 t(-1 + \alpha) + \beta} - \frac{\left(-\frac{B}{-1+\delta}\right)^{1+\frac{1}{1+\alpha}}(-2B(-1 + \alpha^2)(-1 + \delta) + e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(-2 + 2\alpha + 3\beta + 3\alpha\beta + 3h_0 t(-1 + \alpha^2)(-1 + \delta) - 3(1 + \alpha)\beta\delta))}{B^2(-1 + \alpha^2)}. \tag{36}$$

The time derivative of $f(T)$ is

$$\dot{f}(T) = h_0\left(\frac{12h_0^2(-1+\alpha)\beta^2}{(h_0t(-1+\alpha)+\beta)^3} + \frac{C_2 - C_2\alpha}{(h_0t(-1+\alpha)+\beta)^2} + \frac{3e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B}\right). \tag{37}$$

Now, the derivative with respect to the torsion scalar T is

$$f_T(t) = \frac{12h_0^2\beta^2 - C_2(h_0t(-1+\alpha)+\beta) + \frac{3e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0t(-1+\alpha)+\beta)^3\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B(-1+\alpha)}}{12h_0^2\beta^2}, \tag{38}$$

and the second derivative is

$$f_{TT}(t) = \frac{C_2(h_0 - h_0\alpha) + \frac{9e_1^C(1+\alpha)(-1+\delta)}{h}{}_0\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0t(-1+\alpha)+\beta)^2\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B} 12h_0^2\beta^2. \tag{39}$$

We get ρ_T, p_T and ρ_m in the framework of reconstructed $f(T)$ gravity from Equations (28), (29) and (26) respectively as

$$\rho_T = -\frac{(e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(-1 - 3h_0t(1+\alpha)(-1+\delta)) + B(1+\alpha)(-1+\delta))\left(-\frac{B}{-1+\delta}\right)^{1+\frac{1}{1+\alpha}}}{B^2(1+\alpha)}, \tag{40}$$

$$\begin{aligned} p_T = \frac{1}{2}\Bigg(&\frac{4h_0^2(-1+\alpha)\beta}{(h_0t(-1+\alpha)+\beta)^2} - \frac{12h_0^2\beta^2}{(h_0t(-1+\alpha)+\beta)^2} + \frac{C_2}{h_0t(-1+\alpha)+\beta} \\ &+ \frac{4h_0^2(-1+\alpha)\beta\left(C_2(h_0 - h_0\alpha) + \frac{9e^{C_1(1+\alpha)(-1+\delta)}h_0\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0t(-1+\alpha)+\beta)^2\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B}\right)}{(h_0t(-1+\alpha)+\beta)^4} \\ &- \frac{(-1+\alpha-3\beta)\left(12h_0^2\beta^2 - C_2(h_0t(-1+\alpha)+\beta) + \frac{3e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0t(-1+\alpha)+\beta)^3\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B(-1+\alpha)}\right)}{3\beta(h_0t(-1+\alpha)+\beta)^2} \\ &- \frac{\left(-\frac{B}{-1+\delta}\right)^{1+\frac{1}{1+\alpha}}(-2B(-1+\alpha^2)(-1+\delta) + e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(-2+2\alpha+3\beta+3\alpha\beta+3h_0t(-1+\alpha^2)(-1+\delta) - 3(1+\alpha)\beta\delta))}{B^2(-1+\alpha^2)}\Bigg), \end{aligned} \tag{41}$$

and

$$\begin{aligned} \rho_m = &\frac{3h_0^2\beta^2}{(h_0t(-1+\alpha)+\beta)^2} \\ &+ \frac{(e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(-1 - 3h_0t(1+\alpha)(-1+\delta)) + B(1+\alpha)(-1+\delta))\left(-\frac{B}{-1+\delta}\right)^{1+\frac{1}{1+\alpha}}}{B^2(1+\alpha)}. \end{aligned} \tag{42}$$

Using Equations (4), (38) and (39) in Equation (32), we obtain the reconstructed squared speed of sound for GCG in the framework of $f(T)$ gravity as

$$c_s^2 = \frac{\theta_1}{\theta_2}, \tag{43}$$

where

$$\theta_1 = 1 + \frac{12h_0^2\beta^2 - C_2(h_0t(-1+\alpha)+\beta) + \frac{3e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0t(-1+\alpha)+\beta)^3\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B(-1+\alpha)}}{12h_0^2\beta^2}, \tag{44}$$

and

$$\theta_2 = 1 - \frac{C_2(h_0 - h_0\alpha) + \frac{9e^{C_1(1+\alpha)(-1+\delta)}h_0\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0t(-1+\alpha)+\beta)^2\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B}}{(h_0t(-1+\alpha)+\beta)^2}$$
$$+ \frac{12h_0^2\beta^2 - C_2(h_0t(-1+\alpha)+\beta) + \frac{3e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0t(-1+\alpha)+\beta)^3\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B(-1+\alpha)}}{12h_0^2\beta^2}. \tag{45}$$

We plot squared speed of sound c_s^2 for GCG Equation (43) with respect to cosmic time t and α in Figure 4 for GCG in the framework of $f(T)$ gravity. Figure 4 affirms that squared speed of sound is positive and is increasing with the evolution of time. Hence we can infer that the reconstructed $f(T)$ gravity maintains its stability under small gravitational perturbations. Now by using Equations (33), (34) and reconstructed $f(T)$ Equation (36), $f_T(t)$ Equation (38) and $f_{TT}(t)$ Equation (39) in Equation (31), the reconstructed EoS parameter w_T due to torsion contribution is

$$w_T = -1 - \frac{\xi_1}{\xi_2}, \tag{46}$$

where

$$\xi_1 = (-1+\alpha)(1+\alpha)\beta\Bigg(C_2\left(-\frac{h_0t(-1+\alpha)}{\beta^2} - \frac{1}{\beta} + \frac{12h_0^3(-1+\alpha)}{(h_0t(-1+\alpha)+\beta)^2}\right)$$
$$+ \frac{3e^{C_1(1+\alpha)(-1+\delta)}\beta^{-2+\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0^3t^3(-1+\alpha)^3 + 3h_0^2t^2(-1+\alpha)^2\beta - 3h_0(12h_0^2 - t)(-1+\alpha)\beta^2 + \beta^3)\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B(-1+\alpha)}\Bigg)$$
$$\left(-\frac{B}{-1+\delta 6(h_0t(-1+\alpha)+\beta)^2(e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(1+3h_0t(1+\alpha)(-1+\delta)) - B(1+\alpha)(-1+\delta))}\right)^{\frac{\alpha}{1+\alpha}}(-1+\delta)^2, \tag{47}$$

and

$$\xi_2 = 6(h_0t(-1+\alpha)+\beta)^2(e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}$$
$$\times (1 + 3h_0t(1+\alpha)(-1+\delta)) - B(1+\alpha)(-1+\delta)). \tag{48}$$

We plot EoS parameter w_T Equation (46) with respect to cosmic time t in Figure 5 for GCG in the framework of $f(T)$ gravity. From Figure 5 we can understand that it is behaving like phantom and it is decreasing with the evolution of time. It can also be noted that it is not crossing the phantom boundary.

In Figure 6, reconstructed $f(T)$ is plotted against T. We can observe from the figure that $f(T) \to 0$ as $T \to 0$ which shows that one of the sufficient conditions of a realistic model (Bamba et al. 2013a) is satisfied. In Figure 7 we demonstrate that $\frac{f}{T} \to 1$ as $|T| \to \infty$ which is consistent with the CMB constraints and primordial nucleosynthesis (Linder 2010; Wu & Yu 2010; Karami & Abdolmaleki 2012).

4.2. Cosmology of Primordial Perturbations

Cosmological reconstruction, a method for constructing a model for an arbitrary evolution of the scale factor, was discussed for the evolution behavior of the matter density contrast in the work of Matsumoto et al. (2015). In a noteworthy work, Nojiri & Odintsov (2003) discussed a model consisting of tachyon and scalar phantom with conformal quantum matter and their model, perturbed by quantum effects and that realized two de Sitter phases where inflation in the early universe was produced by quantum effects and the late time accelerating universe was due to phantom/tachyon. In a recent study, a nonsingular bounce cosmology was discussed by Nojiri et al. (2019) in the context of $f(R)$ generalized by the Lagrange multiplier and the authors (Nojiri et al. 2019) showed explicitly that the perturbation modes exit the horizon at a large negative time during the pre-bounce contraction era. Studying the spectral index of the curvature perturbations and the tensor-to-scalar ratio of the density perturbations, Bamba & Odintsov (2016) investigated a fluid model involving the EoS for a fluid including bulk viscosity. In a very recent work, Odintsov et al. (2020) showed that the early-time bounce has a nearly scale-invariant power spectrum of primordial scalar curvature perturbations.

The current work demonstrates a density perturbation analysis of the reconstructed $f(T)$ gravity under the purview of GCG. Earlier studies on density perturbation of a universe dominated by other versions of Chaplygin gas include e Costa et al. (2008), Pourhassan (2016). Fabris et al. (2002b) demonstrated that it is possible to have the value for the density contrast observed in large scale structure of the universe by fixing a free parameter in the EoS of Chaplygin gas. In our work, we reconstructed the $f(T)$ in Equation (36) by considering the background evolution in the form of GCG and accordingly the derivatives of different orders have been obtained in Equations (37)–(39). This reconstructed $f(T)$ can now be explored for the study of primordial perturbation. This approach was earlier adopted in different contexts by Chattopadhyay (2017, 2018), Chattopadhyay et al. (2018). The details will be demonstrated in the subsequent paragraphs.

The matter component has been taken in the form of canonical scalar field ϕ with a Lagrangian and is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi). \tag{49}$$

The evolution of scalar-sector metric perturbations is understood by Cai et al. (2011) and hence for the gravitational potential Φ we use the perturbed equation of motion. As per the discussion in Cai et al. (2011), the final form of the equation of motion of one Fourier mode Φ_κ is given by

$$\ddot{\Phi}_\kappa + \alpha\dot{\Phi}_\kappa + \mu^2\Phi_\kappa + c_s^2\frac{\kappa^2}{a^2}\Phi_\kappa = 0, \tag{50}$$

where

$$\alpha = 7H + \frac{2V_\phi}{\dot{\phi}} - \frac{36H\dot{H}(f_{TT} - 4H^2 f_{TTT})}{1 + f_T - 12H^2 f_{TT}}, \tag{51}$$

and

$$\mu^2 = 6H^2 + 2\dot{H} + \frac{2HV_\phi}{\dot{\phi}} - \frac{36H^2\dot{H}(f_{TT} - 4H^2 f_{TTT})}{1 + f_T - 12H^2 f_{TT}}. \tag{52}$$

The effective mass and the frictional term are μ_2 and α respectively for the gravitational potential Φ. The background equation for scalar field is expressed as

$$\ddot{\phi} + 3H\phi + V_\phi = 0. \tag{53}$$

In the present case, the equation $\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT})}$ is read as

$$(1 + f_T - 12H^2 f_{TT})\dot{H} = -4\pi G\dot{\phi}^2, \tag{54}$$

where $4\pi G = \frac{1}{2}$. Using Equations (4), (33), (38) and (39), we get

$$\dot{\phi}^2 = 2h_0^2(-1 + \alpha)\beta\left(1 + \frac{C_2 h_0(-1 + \alpha)}{(h_0 t(-1 + \alpha) + \beta)^2} + \frac{12h_0^2\beta^2 - C_2(h_0 t(-1 + \alpha) + \beta) + \frac{3e^{C_1(1+\alpha)(-1+\delta)}\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}(h_0 t(-1+\alpha)+\beta)^3\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B(-1+\alpha)}}{12h_0^2\beta^2} - \frac{9e^{C_1(1+\alpha)(-1+\delta)}h_0\beta^{\frac{3(1+\alpha)\beta(-1+\delta)}{-1+\alpha}}\left(-\frac{B}{-1+\delta}\right)^{\frac{1}{1+\alpha}}}{B}\right)(h_0 t(-1 + \alpha) + \beta)^{-2}. \tag{55}$$

Now using Equation (55) in Equation (53), we get the time derivative of the self interacting potential $\dot{V}$. We have plotted self interacting potential $\dot{V}$ with respect to cosmic time t and coupling constant δ in Figure 8 for a set of values of the coupling constant δ. From Figure 8 we conclude that regardless of the value of the coupling constant δ, i.e., regardless of the strength of the interaction, the potential is a monotonically increasing function of cosmic time t as $\dot{V} > 0$.

5. Conclusions

In the current work, we report a detailed cosmology of GCG in the cosmological setting of $f(T)$ gravity. In the first phase of the study, we take the scale factor in power law form (see Equation (3)) and also derive a truncated form of the scale factor using binomial expansion (see Equation (12)). For both the cases we have reconstructed the EoS parameter for GCG in non-interacting scenario. We observe that in each case the EoS parameter behaves like quintessence and with the evolution of the universe it converges asymptotically to -1. It does not cross the -1 boundary in either of the cases (see Figures 1 and 2).

In the next phase we consider an interacting scenario with interaction term $Q = 3H\delta\rho_c$ where ρ_c stands for density of GCG. Considering the conservation equation reframed for interacting scenario, the EoS parameter was reconstructed for this case (see Equation (3)). Contrary to the non-interacting scenario, the EoS parameter behaves like phantom, i.e., $w_c \leqslant -1$ in this case for the power law form of scale factor. Here with the passage of cosmic time t, the EoS parameter is converging to -1 but not crossing the -1 boundary (see Figure 3). When we consider the truncated form of scale factor in the interacting scenario, we observe that $\alpha > 1$ which contradicts the constraints given for α, i.e., $0 < \alpha < 1$ (see Equation (24)). Hence, it violates the theory described in the literature.

In the next phase of the study we consider GCG in the framework of $f(T)$ gravity. As we have observed that expanding the scale factor through binomial expansion and truncating up to the third term do not introduce any additional cosmological feature, we consider the power law form of the scale factor in the original form (see Equation (3)). For this choice of scale factor we get the torsion scalar as a function of cosmic time t in Equation (34). Considering Equation (28) we carry out a correspondence between ρ_c Equation (18) and ρ_T Equation (28) in the sense that the density contribution due to torsion behaves like a fluid with the GCG density. This leads to an ordinary differential equation expressed in Equation (35). As we have expressed T as a function of cosmic time t, the solution to Equation (28) yields a reconstructed $f(T)$ as a function of cosmic time t and the solution is presented in Equation (36). To understand the stability of the model, we calculate the squared speed of sound for this reconstructed $f(T)$ using Equations (44) and (45). For different values of parameters associated with GCG, i.e., α, we plot squared speed of sound c_s^2 in Figure 4 and we observe that the c_s^2 is staying at the positive level with the evolution of the universe and for smaller values of α the increasing pattern of c_s^2 is more prominent than higher values of α. Therefore the reconstructed $f(T)$ appears to be stable against small perturbations. For this reconstructed $f(T)$ we have plotted the EoS parameter in Figure 5 and a phantom behavior is observed. In this case it may be noted that in the earliest stage of the universe $w_T \approx -1$, i.e., it behaves like a cosmological constant. In Figure 6 we plot $f(T)$ against T and observed that $f(T) \to 0$ as $T \to 0$. This satisfies the sufficient conditions for a realistic reconstructed model. In Figure 7, we found that $\frac{f}{T} \to 1$ as $|T| \to \infty$ which is consistent with the CMB constraints and primordial nucleosynthesis. Finally, we studied primordial perturbations for this reconstructed $f(T)$ in Figure 8 and concluded that the potential is monotonically increasing with the evolution of cosmic time t as $\dot{V} > 0$, independent of the value of the coupling constant δ, i.e., independent of the strength of the interaction.

While concluding, let us comment on our choice of modified gravity with respect to existing literatures relevant to the current work. Let us mention the work of Nojiri & Odintsov (2011a), where the authors reviewed realistic $F(R)$-models that unify inflation with DE epoch. Nojiri & Odintsov (2011a) demonstrated how the five-dimensional $F(R)$-gravity could have a non-perturbative stringy effective action leading to a universal relation for viscous bound ratio. We endeavor to extend our study in light of Nojiri & Odintsov (2011a) and Nojiri et al. (2022) by making an attempt to unify early inflation and late-time acceleration of the universe with perturbative as well as non-perturbative approaches. In this context it may be mentioned that the suitability of $f(T)$ gravity for studying the early universe was already reported in Awad et al. (2018), Cai et al. (2016), Bamba et al. (2013b, 2014), Oikonomou (2017).

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