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A New Consistency Test for ΛCDM Cosmology Using Galaxy Surveys

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Abstract

We propose a new consistency test for the ΛCDM cosmology using baryonic acoustic oscillations (BAO) and redshift space distortion (RSD) measurements from galaxy redshift surveys. Specifically, we determine the peak position of $f\sigma_8(z)$ in redshift z offered by an RSD measurement, and compare it to the one predicted by the BAO observables assuming a flat ΛCDM cosmology. We demonstrate this new test using the simulated data for the DESI galaxy survey, and argue that this test complements those using the background observables alone, and is less subject to systematics in the RSD analysis, compared to traditional methods using values of $f\sigma_8(z)$ directly.

Key words: Cosmology – (cosmology:) dark energy – galaxies: distances and redshifts

1. Introduction

The ΛCDM model, in which the cold dark matter (CDM) and a cosmological constant, Λ, contribute to roughly 1/3 and 2/3 energy budget of the current universe respectively, has become the standard cosmological paradigm (Riess et al. 1998; Perlmutter et al. 1999). Although this "vanilla" model is favored by most observations available so far in terms of model selection, it is being challenged, especially by the "Hubble crisis" (see Di Valentino et al. 2021 for a recent review).

Performing consistency tests for the ΛCDM model is one of the most efficient ways to discover new physics, if any, beyond the standard cosmological scenario, and efforts have been made along these lines. For example, the quantity $Om(z)$ (Sahni et al. 2008), derived from $H(z)$ using cosmic chronometers measuring the age of passive galaxies at various redshifts (Moresco et al. 2012), complemented with the local H_0 measurement (Riess et al. 2021) can be used for a consistency test. $Om(z)$ is a constant and coincides with $\Omega_{\rm M,0}$ only if the underlying cosmology is ΛCDM, while it evolves with redshift otherwise. This quick test relies on measuring both $H(z)$ and H_0 precisely, which is challenging. Further, this test only accounts for the expansion history of the universe.

In this paper, we propose a new consistency test for the Λ CDM model only using observables delivered by spectroscopic galaxy surveys; namely, we use baryonic acoustic oscillations (BAO) (Eisenstein et al. 2005) and redshift space distortion (RSD) (Kaiser 1987) measurements at multiple redshifts to hunt for deviations from the ΛCDM model at both the background and perturbation levels, and demonstrate our method using simulated Dark Energy Spectroscopic Instrument (DESI) measurements (DESI Collaboration et al. 2016).

The new method is presented in Section 2, including the relevant formalism and procedure; we then show the main result in Section 3, before concluding in Section 4.

2. Methodology

2.1. The Idea and Procedure

We start from the well-known relations for the evolution of the matter density parameter and growth of structure (Dodelson 2003),

$$\Omega_{\rm M}(a) = \frac{\Omega_{\rm M,0}}{a^3}\left[\frac{H_0}{H(a)}\right]^2 \tag{1}$$

$$f(a) \equiv \frac{d\log\delta(a)}{d\log a} = \Omega_{\rm M}^{\gamma}(a) \tag{2}$$

$$\sigma_8(a) \propto \delta(a) \tag{3}$$

where H denotes the expansion rate of the universe, $\Omega_{\rm M}$ is the fractional matter density, δ is the overdensity of matter and σ_8 is the root-mean-square (rms) matter fluctuation on a scale of $8\,h^{-1}$Mpc. Symbols with a subscript 0 mean quantities at redshift 0, and Equation (2) is a reasonable approximation relating the expansion history with structure growth, through the growth index γ (Linder 2005).

Combining Equations (1)–(3), we obtain

$$\frac{(f\sigma_8)'}{f\sigma_8} = \frac{1}{a}\{\Omega_{\rm M}^{\gamma}(a) - \gamma[1 - 2q(a)]\} \tag{4}$$

in which the prime denotes the derivative with respect to the scale factor, and q is the deceleration parameter defined as $q \equiv -\ddot{a}a/\dot{a}^2$, where the dot is the derivative with respect to time.

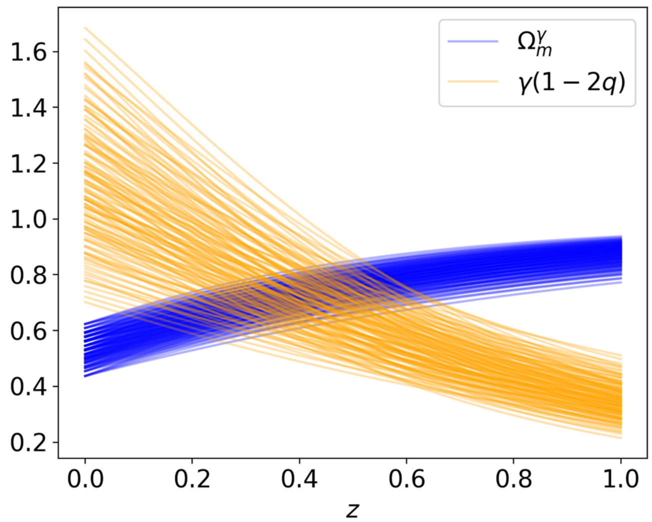

Figure 1. The orange and blue curves show quantities $\gamma(1-2q)$ and $\Omega_{\rm M}^{\gamma}$ as a function of a, respectively. The collections of curves represent models with five values of w, $\Omega_{\rm M,0}$ and γ each, evenly distributed in ranges of $w \in [-1.2, -0.8]$, $\Omega_{\rm M,0} \in [0.28, 0.35]$ and $\gamma \in [0.45, 0.65]$, so that there are 125 curves in total for both $\gamma(1-2q)$ and $\Omega_{\rm M}^{\gamma}$.

According to Equation (4), $f\sigma_8(a)$ has a peak in a (and in z) at a specific redshift, namely, $a = a_{\rm p}$, if

$$\Omega_{\rm M}^{\gamma}(a_{\rm p}) = \gamma[1 - 2q(a_{\rm p})] \\ = 3\gamma[1 - \Omega_{\rm M}(a_{\rm p})] \quad (5)$$

where the last equation holds in the ΛCDM scenario. To investigate whether the peak of $f\sigma_8$ exists for a range of cosmologies, we allow the growth index γ, the equation of state of dark energy w (assumed to be a constant), and the fractional matter density at current epoch $\Omega_{\rm M,0}$, to vary within a wide range. Figure 1 shows two groups of curves of $\Omega_{\rm M}^{\gamma}$ and $\gamma(1-2q)$ as functions of the scale factor a, calculated using Equations (1) and (2) with $H_0 = 67.4\,{\rm km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{\rm M} = 0.31$, $\gamma = 6/11$, which are the values favored by the Planck 2018 observations in the ΛCDM model (Planck Collaboration et al. 2020).

The two groups of curves intersect in all cases, which means that the peak exists for all these cosmologies. This is confirmed by Figure 2, where $f\sigma_8$ for various cosmologies is shown.

Given $\Omega_{\rm M,0}$, w and γ, the position of the peak in $f\sigma_8(a)$ can be found by solving Equation (5) explicitly. In ΛCDM, where $w = -1$ and $\gamma = 6/11$ (Linder 2005), the peak position of $f\sigma_8(a)$ in a is determined once $\Omega_{\rm M,0}$ is known. On the other hand, $\Omega_{\rm M,0}$ in a flat ΛCDM universe can be found from $D_{\rm A}(z)H(z)$, the product of the radial and transverse distances at redshift z, which is provided by a BAO measurement. Note that in a flat ΛCDM universe, $D_{\rm A}(z)H(z)$ only depends on $\Omega_{\rm M,0}$.

This motivates a new consistency test for ΛCDM model using BAO and RSD measurements, which are provided by galaxy surveys. The procedure for this new test is as follows:

1. Given a pair of measured $D_{\rm A}/r_{\rm d}$ and $Hr_{\rm d}$ from BAO in galaxy surveys at a specific effective redshift, compute $D_{\rm A}H$, from which $\Omega_{\rm M,0}$ is computed assuming a ΛCDM model, denoted as $\Omega_{\rm M,0}^{\rm BAO}$;
2. Take measurements of $D_{\rm A}H$ available from BAO at other redshifts; repeat step 1 to extract $\Omega_{\rm M,0}^{\rm BAO}$ for those redshifts;
3. Quantify the agreement of $\Omega_{\rm M,0}$ extracted from observables at various redshifts, by fitting a constant, denoted as $\bar{\Omega}_{\rm M,0}^{\rm BAO}$, to all the data points of $\Omega_{\rm M,0}^{\rm BAO}$;
4. Given $\bar{\Omega}_{\rm M,0}^{\rm BAO}$, compute the expected peak position of $f\sigma_8$ in ΛCDM, denoted as $z_{\rm p}^{\rm BAO}$, using Equation (5);
5. Using the measured $f\sigma_8$ data points, determine the actual peak position using a Taylor expansion approach (see next subsection) and denote this as $z_{\rm p}^{\rm RSD}$;
6. Compare $z_{\rm p}^{\rm BAO}$ with $z_{\rm p}^{\rm RSD}$.

In summary, this new consistency test contains two key ingredients: testing the agreement among $\Omega_{\rm M,0}^{\rm BAO}$ derived at multiple redshifts, and the agreement between $z_{\rm p}^{\rm BAO}$ and $z_{\rm p}^{\rm RSD}$. To quantify the (dis)agreement, we compute two quantities as follows:

1. $({\rm S/N})_{\boldsymbol{B}}$: This is to quantify the significance of $\Omega_{\rm M,0}^{\rm BAO}$ derived at various redshifts not being a constant. Practically, we fit a constant, which is $\bar{\Omega}_{\rm M,0}^{\rm BAO}$, to all $\Omega_{\rm M,0}^{\rm BAO}$ data points and record the χ^2 for the best-fit value of $\bar{\Omega}_{\rm M,0}^{\rm BAO}$. The covariance between D_A and H is properly taken into account using a Fisher matrix analysis. Then $({\rm S/N})_{\boldsymbol{B}}$ is defined as $\sqrt{\chi^2}$.
2. $({\rm S/N})_{\boldsymbol{P}}$: This is to quantify the agreement between $z_{\rm p}^{\rm BAO}$ and $z_{\rm p}^{\rm RSD}$, and is calculated as $|z_{\rm p}^{\rm BAO} - z_{\rm p}^{\rm RSD}|/\sigma$, where σ is the uncertainty of $(z_{\rm p}^{\rm BAO} - z_{\rm p}^{\rm RSD})$; we discuss how this is evaluated in Section 2.2. We assume that measurements at the different redshifts have negligible covariance.

2.2. Determine $\bar{\Omega}_{\rm M,0}^{\rm BAO}$, $z_{\rm p}^{\rm BAO}$ and $z_{\rm p}^{\rm RSD}$

The new consistency test requires measuring $\bar{\Omega}_{\rm M,0}^{\rm BAO}$, $z_{\rm p}^{\rm BAO}$ and $z_{\rm p}^{\rm RSD}$ from galaxy surveys. Here we describe how to measure these quantities from the simulated data, assuming a DESI sensitivity for a demonstration.

Given a cosmological model, which in this work is considered to be $w\gamma$ CDM (a CDM model with a constant w for dark energy and a growth index γ for matter in a flat universe), we first perform a Fisher matrix forecast for $D_{\rm A}H$ and $f\sigma_8$ (with all relevant correlation coefficients) at 18

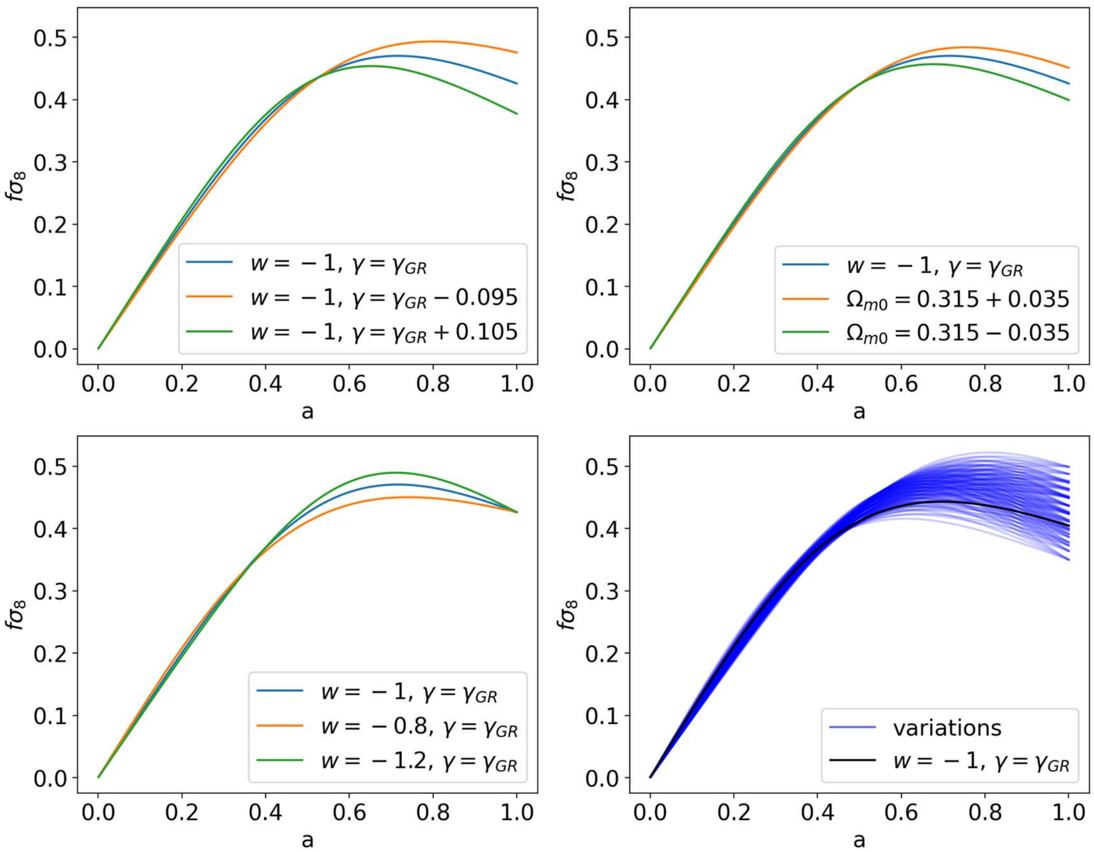

Figure 2. The quantity $f\sigma_8$ as a function of a for various cosmologies. In all panels, the fiducial ΛCDM model is shown in the middle for reference. "Variations" in the bottom right panel means $f\sigma_8$ for models (125 in total) with all parameters varied.

effective redshifts (uniform in z from $z = 0.05$ to 1.75) jointly covered by the LRGs and ELGs to be observed by a 14,000 deg^2 DESI survey, with dN/dz specified in Table 2.3 in the official DESI forecast paper (DESI Collaboration et al. 2016).

Determine $\bar{\Omega}_{\rm M,0}^{\rm BAO}$: Given the uncertainties on $D_{\rm A}H$ at multiple redshifts (18 for the DESI example) obtained in the Fisher forecast, generate a large sample of $D_{\rm A}H$ at each redshift following a Gaussian distribution, with the mean and variance given by the fiducial model assumed and the Fisher forecast, respectively. This allows for a derivation of $\Omega_{\rm M,0}^{\rm BAO}$ from $D_{\rm A}H$ at each redshift assuming a flat ΛCDM cosmology. Then fit a constant, which is $\bar{\Omega}_{\rm M,0}^{\rm BAO}$, to the derived collection of $\Omega_{\rm M,0}^{\rm BAO}$ at all redshifts. The fitting is performed using `zeus` (Karamanis et al. 2021), the Python package for slice sampling.

Determine $z_{\rm p}^{\rm BAO}$: Given the posterior of $\bar{\Omega}_{\rm M,0}^{\rm BAO}$ obtained from the previous step, draw a large sample of $\bar{\Omega}_{\rm M,0}^{\rm BAO}$, and solve for the peak location using the peak equation, Equation (5) for each $\bar{\Omega}_{\rm M,0}^{\rm BAO}$ in the sample using $\gamma = 6/11$ (i.e., assuming GR); then compute the mean and uncertainty of $z_{\rm p}^{\rm BAO}$ accordingly.

Determine $z_{\rm p}^{\rm RSD}$: To determine $z_{\rm p}^{\rm RSD}$ from the simulated RSD measurements, we choose to use a cubic function to fit to a collection of simulated $f\sigma_8$ data points. The fitting function takes the form

$$f\sigma_8(a) = A + B(a - a_{\rm p})^2 + C(a - a_{\rm p})^3 \qquad (6)$$

where $A = f\sigma_8(a_{\rm p})$, $B = f\sigma_8''(a_{\rm p})/2$, $C = f\sigma_8'''(a_{\rm p})/6$ and $z_{\rm p}^{\rm RSD} = 1/a_{\rm p} - 1$. This is a Taylor expansion of $f\sigma_8(a)$ around $a_{\rm p}$, so the linear term vanishes by definition. We have performed tests on simulated data to confirm that this peak finder is sufficiently accurate given the DESI sensitivity (the bias is less than 4% in estimating the peak position in all cases).

3. Results

In this section, we demonstrate the proposed new consistency test using the simulated BAO and RSD data assuming a

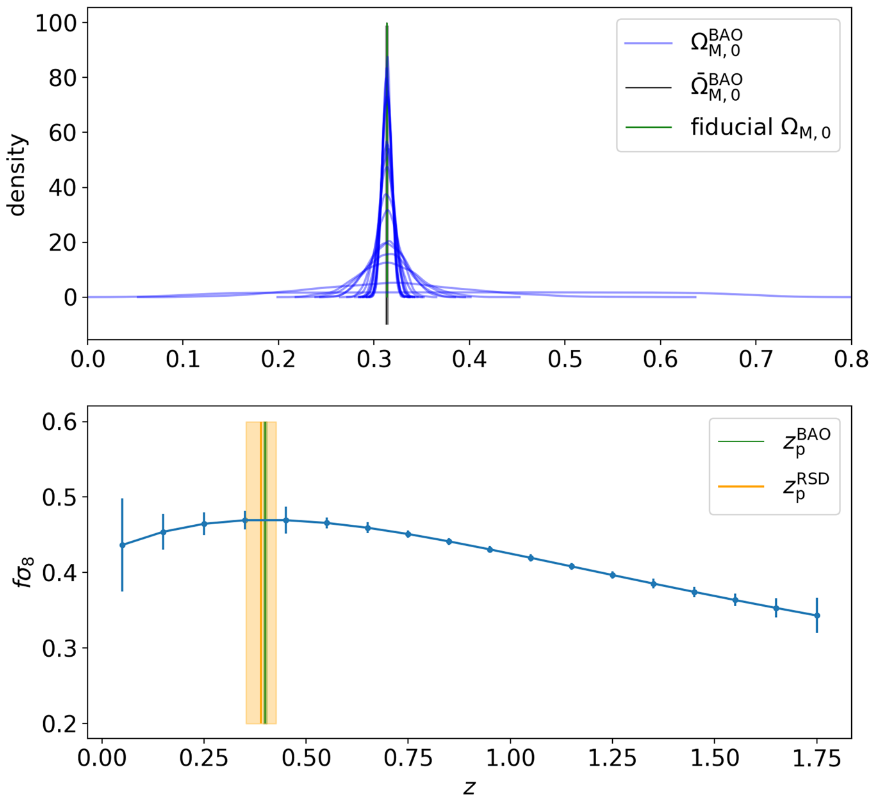

Figure 3. The derived $\bar{\Omega}_{M,0}^{BAO}$, z_p^{BAO} and z_p^{RSD} for the ΛCDM model. Top: The 1D distribution of $\Omega_{M,0}^{BAO}$ at 18 redshifts (blue) and $\bar{\Omega}_{M,0}^{BAO}$ (black). The green vertical line denotes the fiducial $\Omega_{M,0}$, which is consistent with the Planck 2018 cosmology used for the forecast. Bottom: The forecast $f\sigma_8$ (blue data points with error bars), z_p^{BAO} (thin green band) and z_p^{RSD} (thick orange).

DESI sensitivity. We perform the new consistency test on four different models:

1. Model I: $w=-1$, $\gamma=\gamma_{GR}=6/11$ (ΛCDM)
2. Model II:$w=-1$, $\gamma=\gamma_{GR}-0.095=0.45$
3. Model III: $w=-1$, $\gamma=\gamma_{GR}+0.105=0.65$
4. Model IV: $w=-0.8$, $\gamma=\gamma_{GR}+0.105=0.65$

We carry out all of the procedures described in Section 2 for these models; the results are summarized in Figures 3–6 and Table 1.

Figure 3 shows the test result for the ΛCDM model. The upper panel shows the derived $\Omega_{M,0}^{BAO}$ from the simulated DESI D_AH measurement at 18 effective redshifts, together with $\bar{\Omega}_{M,0}^{BAO}$, which is the compressed quantity from all the $\Omega_{M,0}^{BAO}$ data points. All $\Omega_{M,0}^{BAO}$ and $\bar{\Omega}_{M,0}^{BAO}$ agree with each other as expected, confirming that the model being tested is consistent with ΛCDM at the background level. On the other hand, the lower panel compares z_p^{BAO} with z_p^{RSD}, and an excellent agreement is reached, which further confirms that the model being tested is also consistent with ΛCDM at the perturbation level.

In Figures 4 and 5, we show results for the cases of $w=-1$, $\gamma=0.45$ and $w=-1$, $\gamma=0.65$, respectively. In both cases, we see a discrepancy (around 2σ and 4σ) between z_p^{BAO} and z_p^{RSD} in the lower panel. We also tried testing on models in which both expansion and growth history deviate from those in ΛCDM, e.g., $w=-0.8$, $\gamma=0.65$. As shown in Figure 6, $\Omega_{M,0}^{BAO}$ at various redshifts do not agree (with a discrepancy around $\sim7.4\sigma$), and z_p^{BAO} and z_p^{RSD} differ at $\sim6.3\sigma$.

More test results are shown in Table 1. Generally speaking, z_p^{BAO} and z_p^{RSD} are different if either $w\neq-1$, or $\gamma\neq\gamma_{GR}$, making the new peak statistic highly complementary to that for the background, e.g., the $\bar{\Omega}_{M,0}^{BAO}$ test or the *Om* test (Sahni et al. 2008). For models where $w\neq-1$ and $\gamma\neq\gamma_{GR}$, z_p^{BAO} and z_p^{RSD} are also distinct if $w>-1$, $\gamma>\gamma_{GR}$ or $w<-1$, $\gamma<\gamma_{GR}$. For example, $(S/N)_{\boldsymbol{P}}$ can reach 4.71 and 6.3σ level for $w=-1.2$, $\gamma=0.45$ and $w=-0.8$, $\gamma=0.65$, respectively. For models where both w and γ deviate from those in ΛCDM but in opposite directions, e.g., when $w>-1$, $\gamma<\gamma_{GR}$ or $w<-1$, $\gamma>\gamma_{GR}$, z_p^{BAO} and z_p^{RSD} can approach each other, because of the degeneracy between w and γ given the peak position. This could actually be used as a diagnosis for the model, i.e., a large

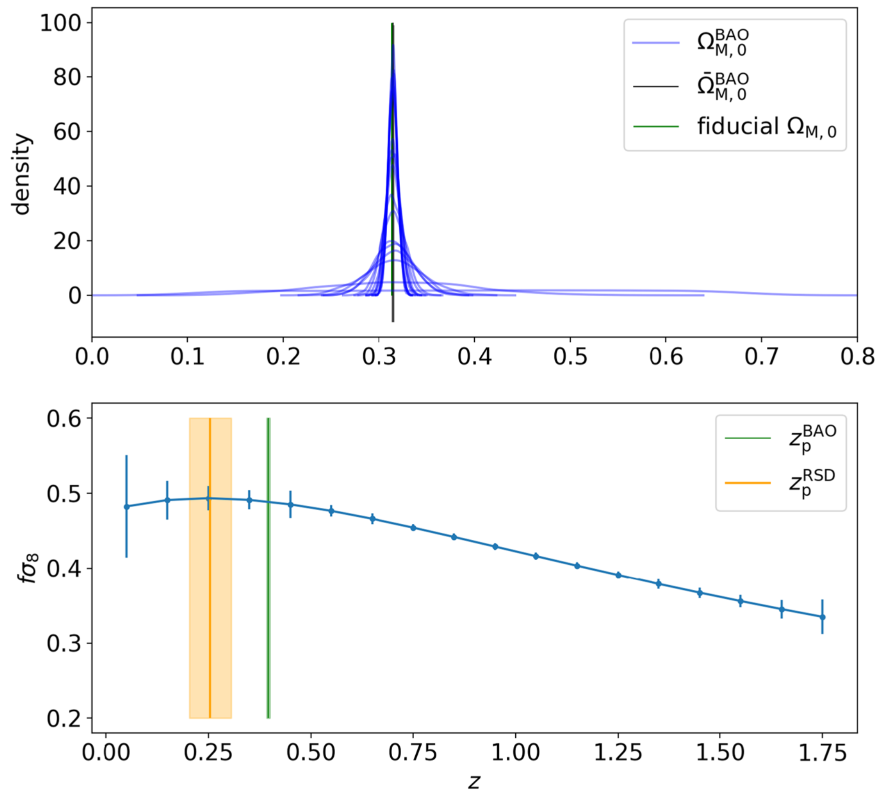

Figure 4. Same as Figure 3 but for model II ($w = -1$, $\gamma = 0.45$).

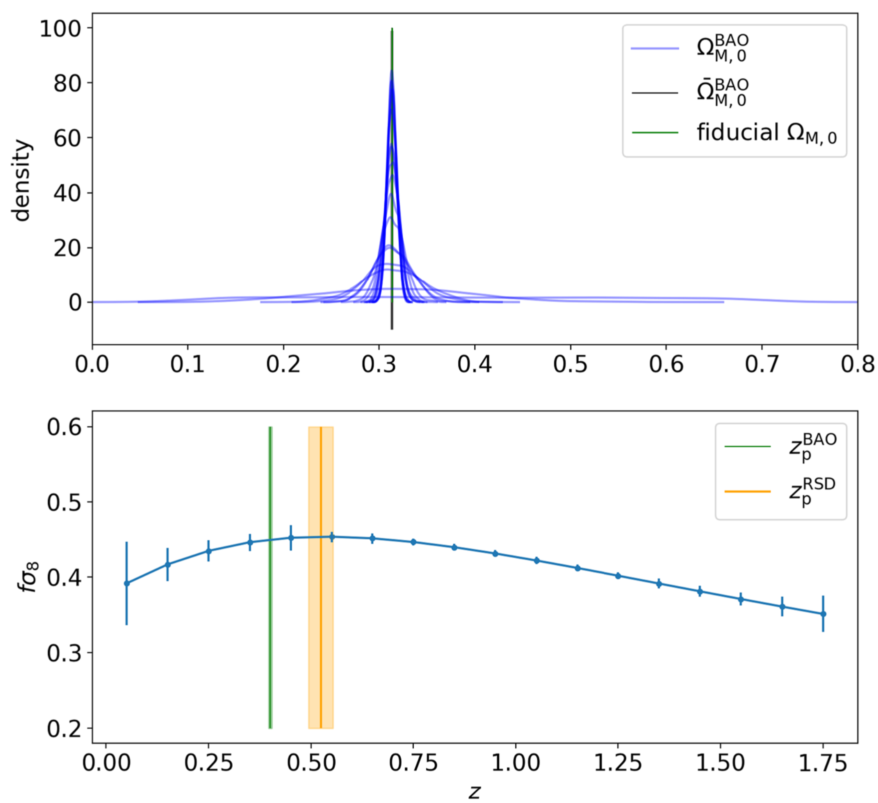

Figure 5. Same as Figure 3 but for model III ($w = -1$, $\gamma = 0.65$).

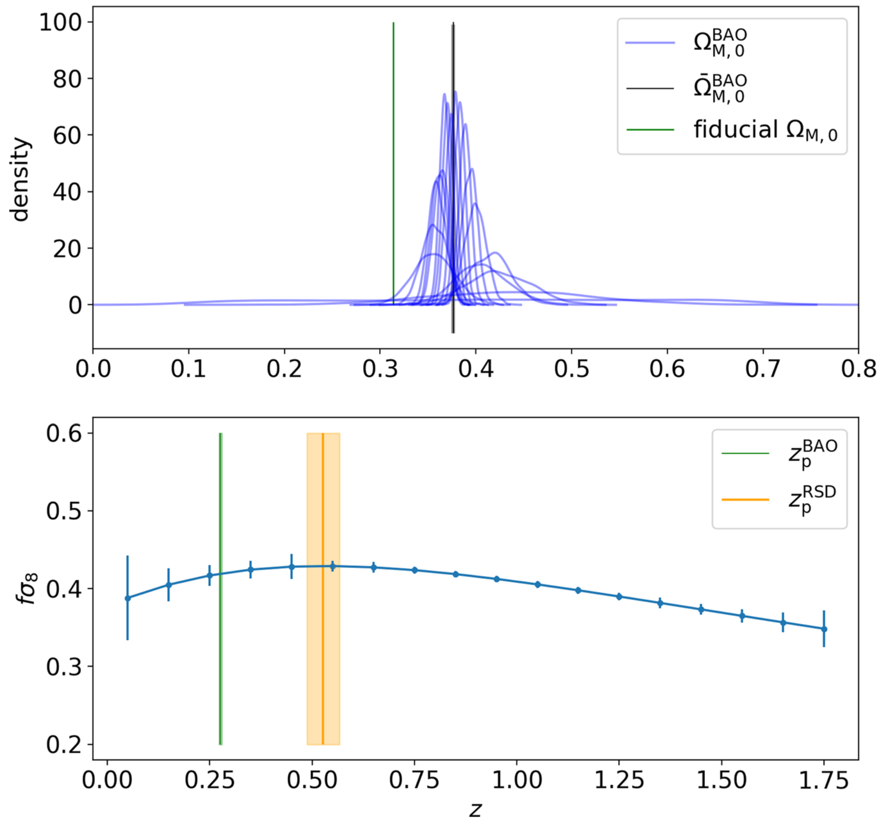

Figure 6. Same as Figure 3 but for model IV ($w = -0.8$, $\gamma = 0.65$).

Table 1
The Significance of $\Omega_{\rm M,0}^{\rm BAO}$ Derived from Different Redshifts not Being a Constant (Middle Column), and of $z_{\rm p}^{\rm BAO} \neq z_{\rm p}^{\rm RSD}$ (Right Column) for Various Models as Listed

Models	$(S/N)_B$	$(S/N)_P$
$w = -1$, $\gamma = \gamma_{\rm GR}$	0	0.41
$w = -1$, $\gamma = \gamma_{\rm GR} - 0.095$	0	2.43
$w = -1$, $\gamma = \gamma_{\rm GR} + 0.105$	0	4.14
$w = -1.2$, $\gamma = \gamma_{\rm GR}$	8.47	3.86
$w = -1.2$, $\gamma = \gamma_{\rm GR} - 0.095$	8.47	4.71
$w = -1.2$, $\gamma = \gamma_{\rm GR} + 0.105$	8.47	0.23
$w = -0.8$, $\gamma = \gamma_{\rm GR}$	7.36	1.47
$w = -0.8$, $\gamma = \gamma_{\rm GR} - 0.095$	7.36	1.96
$w = -0.8$, $\gamma = \gamma_{\rm GR} + 0.105$	7.36	6.30

Table 2
The Cosmological Implication given $(S/N)_B$ and $(S/N)_P$

$(S/N)_B$	$(S/N)_P$	Implication
∼0	∼0	ΛCDM
∼0	Large	Growth history deviates from that in ΛCDM
Large	∼0 or large	Expansion history deviates from that in ΛCDM; growth may or may not deviate

$(S/N)_B$ with a small $(S/N)_P$ may suggest a deviation from the ΛCDM model at both the background and perturbation level. The cosmological implication given $(S/N)_B$ and $(S/N)_P$ is summarized in Table 2.

4. Discussion and Conclusions

In this work we propose a new consistency test for the standard ΛCDM paradigm using the BAO and RSD measurements directly accessible from galaxy redshift surveys.

This new test contains two essential ingredients: a test for the expansion history and a test for the structure growth. Consistency tests for the expansion history have been proposed in the literature, e.g., the *Om* statistic, which also checks the constancy of $\Omega_{\rm M,0}$ derived from observables at different redshifts. However, the key difference between *Om* and our test is that Om relies on measurements of $H(z)$ and H_0, while ours only requires the BAO measurement. Direct $H(z)$ measurements are performed using the age of passive galaxies, and may be subject to large statistical and systematical uncertainties. The local H_0 measurement, on the other hand, is in serious tension with the indirect inference from the CMB, which may suggest new physics beyond ΛCDM, or unknown systematics. This makes our new test for the background more robust—the BAO measurements are known to be less contaminated by systematics (Ross et al. 2012), and are easier to access from existing galaxy surveys, including 2dFGRS (Lahav et al. 2002), SDSS-III BOSS (Dawson et al. 2013), SDSS-IV eBOSS (Dawson et al. 2016), WiggleZ (Blake et al. 2011) and ongoing galaxy surveys such as DESI (DESI Collaboration et al. 2016), PFS (Takada et al. 2014) and the Euclid mission (Amendola et al. 2018).

Our new test for the structure growth essentially scrutinises the consistency relation between the background and perturbation in the flat ΛCDM universe. Since we only examine the peak position of $f\sigma_8$, we argue that this is less subject to possible systematics in the RSD measurement—the absolute value of $f\sigma_8$ is not directly used in this test. We note that the dominating error budget of $|z_{\rm p}^{\rm BAO} - z_{\rm p}^{\rm RSD}|$ is the uncertainty in $z_{\rm p}^{\rm RSD}$, which is primarily due to the low redshift resolution of $f\sigma_8$ measurements in traditional analyses. Recent developments, including the optimal redshift weighting method for RSD measurements (Ruggeri et al. 2019; Zhao et al. 2019), may improve the $z_{\rm p}^{\rm RSD}$ measurement, which will be presented in a follow-up work using existing observations.

As we have access to observational data from stage-IV galaxy surveys in the near future, the new tests proposed in this work allow for an imminent and robust consistency check of the flat ΛCDM model.

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Appendix
Derivation of Equation (4)

Let us start from

$$f\sigma_8 = Af\delta \tag{A1}$$

Then

$$(f\sigma_8)' = A(f\delta)' = Af\delta\left(\frac{f}{a} + \frac{f'}{f}\right) \tag{A2}$$

where $' \equiv \frac{d}{da}$. Thus

$$\frac{(f\sigma_8)'}{f\sigma_8} = \frac{f}{a} + \frac{f'}{f} \tag{A3}$$

Note that $f = \Omega_{\rm M}^{\gamma}$ and $\Omega_{\rm M}(a) = \Omega_{\rm M,0}a^{-3}(H_0/H)^2$, so we have

$$\frac{f'}{f} = \gamma\frac{\Omega_{\rm M}'}{\Omega_{\rm M}} \tag{A4}$$

$$\frac{\Omega'_{\rm M}}{\Omega_{\rm M}} = -\frac{3}{a} - 2\frac{H'}{H} = -\frac{3}{a} + 2\frac{1+q}{a} = \frac{2q-1}{a} \tag{A5}$$

where $q(a) \equiv -\frac{\ddot{a}a}{\dot{a}^2}$ is the deceleration parameter. This naturally leads to

$$f = a\frac{(f\sigma_8)'}{f\sigma_8} + \gamma(1-2q) \tag{A6}$$

which is exactly Equation (4).

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