**RAA 2021** Vol. **21** No. **3**, 69(9pp) doi: 10.1088/1674-4527/21/3/69

*http://www.raa-journal.org* *http://iopscience.iop.org/raa*

*Research in Astronomy and Astrophysics*

# The role of astronomical silicates during a cometary outburst *

Marcin Wesołowski

College of Natural Sciences, Institute of Physics, University of Rzeszów, Pigonia 1 Street, 35-310 Rzeszów, Poland; *marwes@ur.edu.pl*

Received 2020 May 14; accepted 2020 September 14

**Abstract** This paper presents a new approach to analyzing the change of cometary brightness. In our considerations, we assume that astronomical silicates (dust agglomerates) and gas are present in the coma. This assumption is a consequence of the analysis of the result observed during the *Rosetta* mission to comet 67P/Churyumov-Gerasimenko (abbreviated 67P/Ch-G). The dimensions of these agglomerates can be up to several centimeters. However, the large ones are few compared to particles with dimensions of several micrometers. This paper presents the results of calculations on the change in hypothetical comet brightness as a result of its outburst. The calculations take into account the percentage of carbonaceous particles and silicates rich in magnesium.

**Key words:** comets general — cometary outburst

## 1 INTRODUCTION

The source of comet outbursts is still one of the big unsolved problems in cometary science. The nuclei sometimes suddenly release a large amount of dust and gas during a relatively short time, often from a well-defined region but not always. This additional input of material to the coma leads to a jump in brightness which slowly decays as the dust (gas) cloud expands away from the nucleus. However, the explanation of what are the circumstances favorable for cometary outbursts to appear remains a challenge. Several hypotheses have been put forward so far in scientific literature, and many of them are trying to establish the ultimate cause of this phenomenon. However, careful analysis leads to the conclusion that most likely there are several different causes of cometary outbursts or even a combination of them, which in favorable conditions may initiate the outbursts of brightness.

When studying the physical evolution of comets, we focus on determining the amplitude of the change in brightness of these celestial bodies. This phenomenon is manifested by a sharp increase in the brightness of the comet in a short time, on the order of several hours, with an amplitude of about 5 mag. Talking about the real cases of cometary outbursts, comet 29P/Schwassmann-Wachmann should be mentioned, and in terms of outbursts, it is a model object. This is due to its unique outburst frequency (about seven times a year) as well as a specific location in the solar system on the border of water ice stability.

Observational materials also reveal cases of comet outburst with greater brightness jumps. One example is the famous comet 17P/Holmes, whose outburst amplitude was equal to 14 mag in 2007 (Montalto et al. 2008, Moreno et al. 2008). This is the most spectacular cometary outburst ever recorded.

The outburst of comet brightness is a complex process that occurs within the cometary nucleus. According to modern research, it seems highly likely that various mechanisms, or even combinations of them, may be responsible for comet outbursts (Richter 1954; Hughes 1991; Cabot et al. 1996; Enzian et al. 1997; Groussin et al. 2004; Trigo-Rodríguez et al. 2008, Trigo-Rodríguez et al. 2010; Ivanova et al. 2012; Gronkowski & Wesołowski 2012, 2015a; Merouane et al. 2016; Miles 2016; Wesołowski & Gronkowski 2018a; Wesołowski 2020a,b; Wesołowski et al. 2020a,b,c). It seems very probable that the diversity of comets in terms of their structure (chemical composition) and their location in the solar system also play an important role.

In this paper, we consider the effect of astronomical silicates found in a coma on changing the brightness of a comet. At this point, it should be explained that astronomical silicate is a general description concerning irregular particles of cometary dust. These irregular particles are called agglomerates, which are made up of individual

* University of Rzeszów, Poland.

monomers. Dust agglomerates are submicron and micron-sized model debris particles that approximately reproduce their highly irregular and fluffy morphology of dust in the Solar System (Kolokolova & Kimura 2010; Zubko 2013; Schulz et al. 2015; Zubko 2020). A detailed analysis of the formation of different types of agglomerates (for example: BA - Ballistic Aggregates, BAM1 - Ballistic Aggregates Type 1, BAM2 - Ballistic Aggregates Type 2 and BCCA - Ballistic Cluster-Cluster Aggregates) was presented in the paper Skorov et al. (2016).

It is also worth noting that dust agglomerates are characterized by complex chemical composition. Based on measurements made by mass spectrometers, we find that most of the particles emitted from the surface of the comet are a mixture of silicates with variable magnesium-iron composition, organic matter and even carbonaceous particles (Fomenkova et al. 1992; Dorschner et al. 1995; Zubko et al. 2020; Zubko 2020).

The results of the *Rosetta* mission to comet 67P/Churyumov-Gerasimenko (hereafter 67P/Ch-G) confirmed the existence of gas and dust (agglomerates) in a coma. Additionally, we could observe the development of a gas and dust coma. The obtained results contributed to a better understanding of the degassing effect and coma formation (Marschall et al. 2016). Our goal in this paper is to apply the *Rosetta* mission results and to attempt to develop a preliminary cometary outburst model that would be based on the idea of scattering sunlight on agglomerates. Due to different dimensions of silicates, the average value of the distribution function over time was used in the calculations.

## 2 THERMODYNAMIC MODEL OF COMETARY OUTBURST

### 2.1 Basic Assumptions of the Nucleus Structure

The chemical composition of comets is extremely impressive. We assume that water ice is the dominant chemical found in comet nuclei. Based on spectroscopic studies, we can determine the percentage of other chemical compounds in relation to water ice - the main component of the cometary nucleus. In the general case, we can distinguish the following components: carbon monoxide, carbon dioxide, astronomical silicates and organic compounds. These studies mainly focus on analyzing the coma. The dimensions of the coma depend on various factors. The intensive sublimation of given cometary ice plays a key role here. Analyzing the observational material, we see that sublimation does not occur from the entire surface of the nucleus, but only from a small fragment of it (on the order of a few percent).

In further discussion, we assume that:

- The nucleus of the comet has a layered structure.
- The outer layer of the nucleus is a mixture of various types of dust with different thermal conductivities, which depends on the position of the comet in the solar system.
- The dust mantle has a constant thickness which is regarded as a free parameter.
- Under the outer layer of the comet's surface, there is a region of dust and cometary ice.
- Ice is crystalline, or crystalline and amorphous.
- At a depth of several meters, there are cavities, which can play a key role in the outburst of a comet (Ipatov & A'Hearn 2011; Ipatov 2012; Gronkowski & Wesołowski 2015a; Wesołowski & Gronkowski 2018b).

### 2.2 Thermodynamic Parameters and Amplitude of the Outburst

In considering the outburst of a comet, the key issue is to adopt the appropriate thermodynamic model. The proposed approach is to solve the energy balance taking into account the sublimation occurring from the surface of the comet nucleus. This equation can be written in the following form

$$\frac{S_{\odot}(1-A_{\rm N})\cos\varphi}{r^2} = \epsilon\,\sigma\,T^4 + \frac{\dot{Z}\,L(T)}{N_{\rm A}}\,. \qquad (1)$$

The left part of this equation describes the input of solar energy into the comet. The right side of the equation represents the energy consumed by the nucleus for irradiation and sublimation of given comet ice. The following notation is adopted: $S_{\odot}$ - the solar constant at heliocentric distance, $A_{\rm N}$ - the albedo, $\varphi$ - the angle between the normal to the surface of the nucleus and the direction to the Sun, $r$ - the heliocentric distance of the comet, $\epsilon$ - the infrared emissivity of the nucleus, $\sigma$ - the Stefan-Boltzmann constant, $\dot{Z}$ - the rate of sublimation of cometary ice (expressed in molecules $\rm m^{-2}s^{-1}$), $L(T)$ - the latent heat of sublimation of cometary material and $N_{\rm A}$ - Avogadro's number. Because we are developing a simple comet outburst model, in Equation (1) we omit the factor related to heat conductivity to its interior. This approach was followed, among others, by Huebner et al. (2006). It has also been demonstrated that the thermal conductivity of the comet nucleus does not significantly affect the temperature value on the nucleus' surface (Groussin et al. 2013; Schloerb et al. 2015). Therefore, this approach seems correct at least for a simple model. Note that advanced thermodynamic models should take into account thermal conductivity, as this factor has a direct impact on the rate of gas sublimation from inside the nucleus. A measure of the rate of sublimation of the gas is its velocity that depends on the temperature determined based on the energy balance. The rate of sublimation was calculated via the equation

$$\dot{Z} = Ae^{-B/T}\sqrt{\frac{\pi}{2m_{\mathrm{g}}k_{\mathrm{B}}T}}\,. \tag{2}$$

In Equation (2), the individual symbols mean: $A$ and $B$ are constants associated with sublimation of cometary gas. The values of these parameters are expressed in units of pressure and temperature, respectively. Also, $m_{\mathrm{g}}$ is the mass of sublimating cometary gas molecules, $k_{\mathrm{B}}$ is the Boltzmann constant and $T$ is the temperature on the surface of the nucleus, which is calculated utilizing Equation (1).

It is worth noting here that in the last several years significant progress has been made in modeling gas dynamics near a cometary nucleus. *Rosetta*'s mission provided us with a lot of interesting information related to the sublimation activity of comet 67P/Ch-G (Gicquel et al. 2016; Marschall et al. 2016; Rinaldi et al. 2017). The discussed sublimation activity of the comet nucleus is responsible for the scale of the following processes: emission of matter from the surface of the nucleus, movement of particles both on the surface (migration), and in a coma, as well as on the amplitude of the brightness jump.

When considering the dynamics of particles, we need to make some division, if only because of their size. In the simplest approach we can talk about two types of behavior of particles (Wesołowski et al. 2019; Wesołowski 2020a):

– the smallest particles due to the sublimation of a given ice are emitted into the cometary coma,
– slightly larger crumbs can move on the surface causing, for example, initiation of avalanches.

The dimensions of individual particles depend on many factors. The most important of them include: the heliocentric distance which translates into the sublimation rate of a given comet ice, the density of particles, the shape of the nucleus and its radius (Gronkowski & Wesołowski 2015b, 2017).

These particles are made up of ice water, organic matter, astronomical silicates or their mutual combinations. To estimate the maximum size $r_{\mathrm{ej}}$ of particles emitted from the surface of the nucleus, we utilize the following equation

$$m_{\mathrm{gr}}g_{\mathrm{c}} \leq \frac{1}{2}C_{\mathrm{D}}\pi r_{\mathrm{ej}}^2(v_{\mathrm{g}}-v_{\mathrm{gr}})^2\rho_{\mathrm{g}} + m_{\mathrm{gr}}\omega^2 R_{\mathrm{N}}\cos^2\varphi\,. \tag{3}$$

Note that in Equation (3) we considered only three forces (gravitation of the nucleus, drag force coming from the molecules of sublimating gases and centrifugal force related to the rotation of the nucleus), which have the greatest impact on the size of emitted particles (Gronkowski & Wesołowski 2015b). In this equation, the individual symbols mean: $m_{\mathrm{gr}}$ the mass of cometary particles, $g_{\mathrm{c}}$ the gravitational acceleration from the cometary nucleus, $C_{\mathrm{D}}$ the modified free-molecular drag coefficient for a spherical cometary particle with radius $r_{\mathrm{ej}}$, $v_{\mathrm{g}}$ the gas flow velocity, $v_{\mathrm{gr}}$ the velocity of dust particles, $\rho_{\mathrm{g}}$ the gas density, $\omega$ the angular velocity of the nucleus, $R_{\mathrm{N}}$ is the radius of cometary nucleus and $\varphi$ the cometocentric latitude.

We note that the modified-molecular drag coefficient for spherical particles $C_{\mathrm{D}}$ which occurs in Equation (3) is a complicated function of factor $s$ (Crifo et al. 2005 and literature therein). It depends on the factor $s$ according to the following formula

$$\begin{aligned} C_{\mathrm{D}} = &\frac{2s^2+1}{s^3\sqrt{\pi}}\exp(-s^2) \\ &+ \frac{4s^4+4s^2-1}{2s^4}\cdot\frac{2}{\pi}\int_0^s \exp(-t^2)dt \\ &+ \frac{2\sqrt{\pi}}{3s}\sqrt{\frac{T_{\mathrm{gr}}}{T_{\mathrm{g}}}}\,, \end{aligned} \tag{4}$$

where $T_{\mathrm{gr}}$ and $T_{\mathrm{g}}$ stand for the grains and gas temperature, respectively. The parameter $s$ appearing in Equation (4) can be defined as

$$s = |v_{\mathrm{g}} - v_{\mathrm{gr}}|/\sqrt{2\mathrm{R}T_{\mathrm{g}}/(M\mu)}\,. \tag{5}$$

Here R stands for the universal gas constant, $M$ denotes the unit of atomic mass and $\mu$ is the molar mass of sublimating cometary ices.

By using Equation (3), we can determine the dependence on the maximum radius of a particle that is emitted into the coma

$$r_{\mathrm{max,ej}} = \frac{3C_{\mathrm{D}}v_{\mathrm{g}}^2\rho_{\mathrm{g}}}{8\rho_{\mathrm{gr}}\left(g_{\mathrm{c}} - \frac{4\pi^2}{P^2}R_{\mathrm{N}}\cos^2\varphi\right)}, \tag{6}$$

where $\rho_{\mathrm{gr}}$ is the density of particles, and $P$ is its rotational period.

When considering the dynamics of particles in a coma, we must first consider the size and shape distribution of emitted agglomerates. Our calculations were carried out for the average size distribution of agglomerates over time.

$$dN = CN_0h(r) = CN_0r^{-q}dr\,, \tag{7}$$

where $N_0$ is the number of particles in a coma, $r$ and $C$ are the radius of the effective cross-section of fluffy aggregate and the normalization constant respectively, and $q$ is the size distribution power-law indexes. In the calculations, we adopted a constant power-law index of $q$ = 2.1, (Zubko et al. 2016). It should be clearly emphasized that the value of the power factor in the size distribution is a function of the radius of the dust particle (Virrki et al. 2019). Furthermore, such a power index is consistent with the range detected in situ in the 1P/Halley coma in submicron and micron-sized particles (Mazets et al. 1986). In our considerations, we assumed that the agglomerates consist of silicate particles (refractive index $m = 1.6 + 0.01i$) and carbonaceous particles ($m = 1.855 + 0.45i$). Note that the refractive index above was applied for the decaying comet C/2019 Y4 (ATLAS) (Zubko et al. 2020). Based on Equation (7), we can determine the normalization constant $C$

$$C = \frac{1}{\int_{r_{\min}}^{r_{\max}} r^{-q} dr} \,. \tag{8}$$

Using Equations (7)–(8), we can determine the average particle radius $r_{\rm av}$ in a coma

$$r_{\rm av} = C \int_{r_{\min}}^{r_{\max}} h(r) r dr = \frac{\int_{r_{\min}}^{r_{\max}} r^{-q+1} dr}{\int_{r_{\min}}^{r_{\max}} r^{-q} dr} \,. \tag{9}$$

In the calculations, we assume that the values of $r_{\min}$ = $10^{-7}$ and $r_{\max}$ = $10^{-2}$ (Gronkowski & Wesołowski 2015a). The above Equstions (1)–(9) define the basic parameters of the thermodynamics involved. They allow also determining the maximum particle size and the average size of an agglomerate which is located in a coma. Using them, one can determine the amplitude of the cometary brightness change utilizing Pogson's law (Gronkowski et al. 2018; Wesołowski 2019; Wesołowski et al. 2020b; Wesołowski 2020c)

$$\Delta m \approx -2.5 \times \log \frac{p(\theta_2) C_2 + p(\theta_2)_{\rm Si} a C_{\rm (Si)_{ej}} + p(\theta_2)_{\rm C} b C_{\rm (C)_{ej}}}{p(\theta_1) C_1} , \tag{10}$$

where $p(\theta_2)$ is the value of the phase function during quiet sublimation and $p(\theta_1)$ is the value of the phase function during outburst. Note that $a$ means fractional content (percentage) of silicate particles in the total amount of particles contained in the comet's coma and $b$ stands for the percentage of carbonaceous particles in the total amount of grains in the cometary coma (of course, $a$ + $b$ = 1). The individual scattering cross-sections that occur in Equation (10) can be defined as:

$$C_1 = \frac{3\pi \eta(t_2) \kappa R_{\rm N}^2 \dot{Z} R_{\rm h}(t_1) m_{\rm g} \int_{\rm r_{min}}^{\rm r_{max}} Q_{\rm ice} r^2 h(r) dr}{v_{\rm g} \rho_{\rm N} \int_{\rm r_{min}}^{\rm r_{max}} r^3 h(r) dr} , \tag{11}$$

$$C_2 = \frac{9\pi \eta(t_1) \kappa R_{\rm N}^2 \dot{Z} R_{\rm h}(t_2) m_{\rm g} \int_{\rm r_{min}}^{\rm r_{max}} Q_{\rm ice} r^2 h(r) dr}{v_{\rm g} \rho_{\rm N} \int_{\rm r_{min}}^{\rm r_{max}} r^3 h(r) dr} , \tag{12}$$

$$C_{\rm (Si)_{ej}} = \frac{3 M_{\rm ej} \int_{\rm r_{min}}^{\rm r_{max}} Q_{\rm Si} r^2 h(r) dr}{4 \rho_{\rm Si} \int_{\rm r_{min}}^{\rm r_{max}} r^3 h(r) dr} , \tag{13}$$

and

$$C_{\rm (C)_{ej}} = \frac{3 M_{\rm ej} \int_{\rm r_{min}}^{\rm r_{max}} Q_{\rm C} r^2 h(r) dr}{4 \rho_{\rm C} \int_{\rm r_{min}}^{\rm r_{max}} r^3 h(r) dr} \,. \tag{14}$$

A detailed discussion and analysis of individual symbols appearing in Equations (11)–(14) are presented in Wesołowski et al. (2020b). The $Q_{\rm dust}$ parameter depends on: the wavelength of electromagnetic solar radiation ($\lambda$), the effective diffraction parameter ($x_{\rm eff}$) and the complex refractive index ($m$). The parameter $\eta(t_1)$ signifies the percentage of the comet nucleus' active sublimation surface in a calm sublimation phase before the outburst. Based on the analysis of many observational data obtained during space missions (*Deep Impact*, *EPOXI*, *Rosetta*), we were able to estimate the real value of the parameter $\eta(t_1)$. The parameter $\eta(t_2)$ signifies the percentage of active surface during the outburst.

Using Equation (9), we can define the diffraction parameter (size parameter) as

$$x_{\rm eff} = \frac{2\pi}{\lambda} r_{\rm av} = \frac{2\pi}{\lambda} \frac{\int_{r_{\min}}^{r_{\max}} r^{-q+1} dr}{\int_{r_{\min}}^{r_{\max}} r^{-q} dr} \,. \tag{15}$$

A measure of a comet's brightness jump is the number of particles located in the coma which scatter the incident sunlight. This parameter is defined by the mass ejected at the moment of the outburst. It should be noted that the size of the dust particles and their chemical composition play important roles. Based on many years of research on comets, we conclude that a coma contains particles with dimensions of the order of micrometers or centimeters. However, the largest ones are few in comparison to micron-sized particles. In light of what has been said above, the mass ejected ($M_{\rm ej}$) can be defined as

$$M_{\rm ej} = \frac{4}{3} \pi \rho_{\rm dust} \int_{r_{\min}}^{r_{\max}} h(r) r_{\rm dust}^3 dr \,. \tag{16}$$

In the above equation, the same integration limits were left as in Equation (9). We note that in Equation (10), a phase function occurs whose value depends on the scattering angle $\theta$ (angle between Sun-comet-observer, see Fig. 1). To specify a value for the phase function, we need to determine the asymmetry coefficient $g$ based on the equation

$$g = \frac{2 \sum_{n=1}^{\infty} \left( \frac{n(n+2)}{n+1} {\rm Re}(a_n a_{n+1}^* + b_n b_{n+1}^*) + \frac{2n+1}{n(n+1)} {\rm Re}(a_n b_n^*) \right)}{\sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2)} , \tag{17}$$

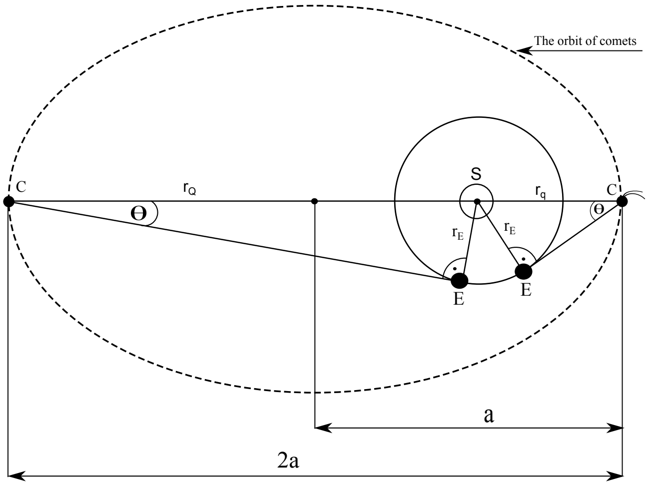


**Fig. 1** Graphical interpretation used to determine the phase angle. The individual symbols mean: S - Sun, E - Earth, C - comet, $\theta$ - the phase angle, $r_q$ - the distance of a comet being at the perihelion from the Sun, $r_{\rm Q}$ - the distance of a comet being at the aphelion from the Sun, $r_{\rm E}$ - distance of the Earth from the Sun ($r_{\rm E}$ = 1 au), a - semi-major axis.

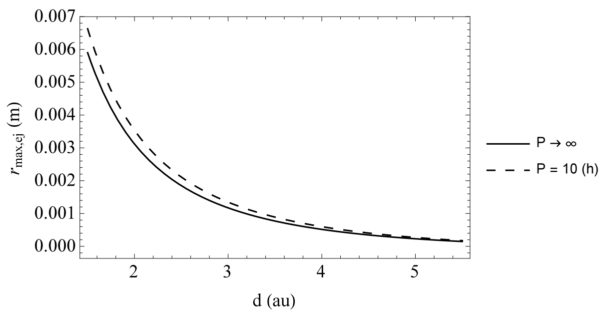


**Fig. 2** The maximum radius of the cometary particles, $r_{\rm max,ej}$ in meters, that can be emitted from the equator of the cometary nucleus by gentle sublimation controlled by water ice as a function of the heliocentric distance. We also assume that the density of the average silicate particles (agglomerates Mg-rich) is in the range (1.5 – 3.5 g cm$^{-3}$, Zubko 2020). The calculations show two models: rotating ($P$=10 h) and non-rotating ($P = \infty$) which are based on Eqs. (1)–(6).

and the asterisks * denote complex conjugate. The symbols $a_n$ and $b_n$ appearing in Equation (16) are scattering coefficients that we calculate based on Mie theory. The phase function $p(\theta)$ can be defined as

$$p(\theta) = \frac{(1-g^2)}{4\pi\left(1+g^2-2g\cos(\theta_{\rm i})\right)^{3/2}}, \tag{18}$$

where $\theta_{i=1}$ signifies the value of the phase angle at perihelion and $\theta_{i=2}$ that at aphelion of the cometary orbit.

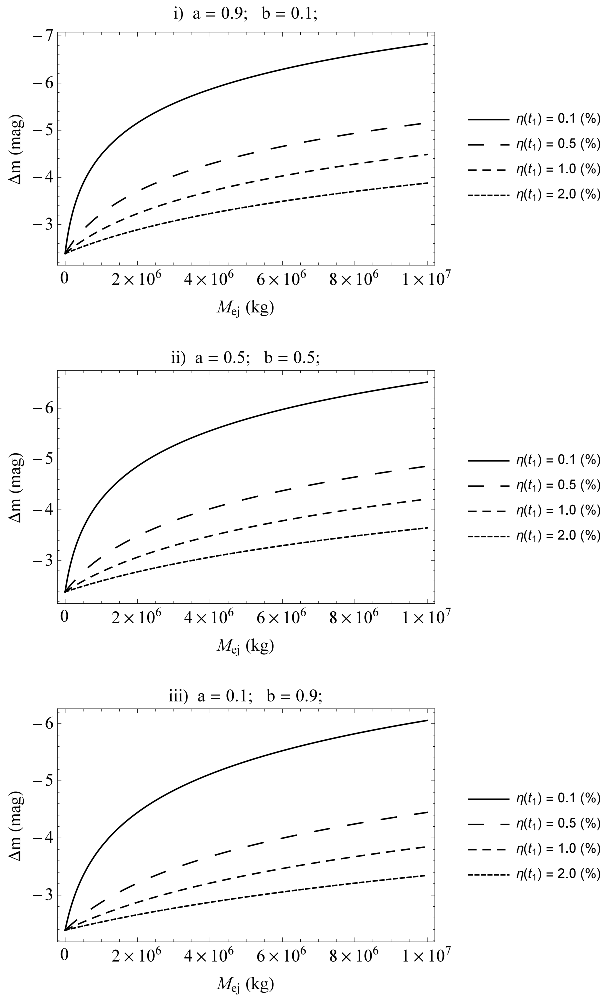


**Fig. 3** The jump in comet X/PC brightness $\Delta$m as a function of ejected mass $M_{\rm ej}$ for different values of the parameter $\eta$(t$_1$). It is assumed that the comet is at the perihelion of its orbit ($r_q$ = 1.5 au), and its activity is controlled by sublimation of water ice. The calculations were based on Eqs. (1)–(18).

## 3 RESULTS OF THE NUMERICAL CALCULATIONS

The numerical values of individual parameters are presented in Table 1. In this paper, we calculate sunlight scattering on dust particles that are present in the coma. Figure 1 illustrates the orbit of a hypothetical comet (X/PC) for which numerical calculations were performed. Based on this, the phase angle can be determined for perihelion and aphelion. The obtained results of numerical simulations are presented in the following Figures 2–7. Figure 2 plots the distribution of the maximum particle sizes that can be emitted in a coma as a result of the local sublimation of water ice. Based on the average size distribution of coma dust, its value was estimated to be $r_{\rm av}$ = 0.752×10$^{-6}$ m. Figures 3–4 display the change

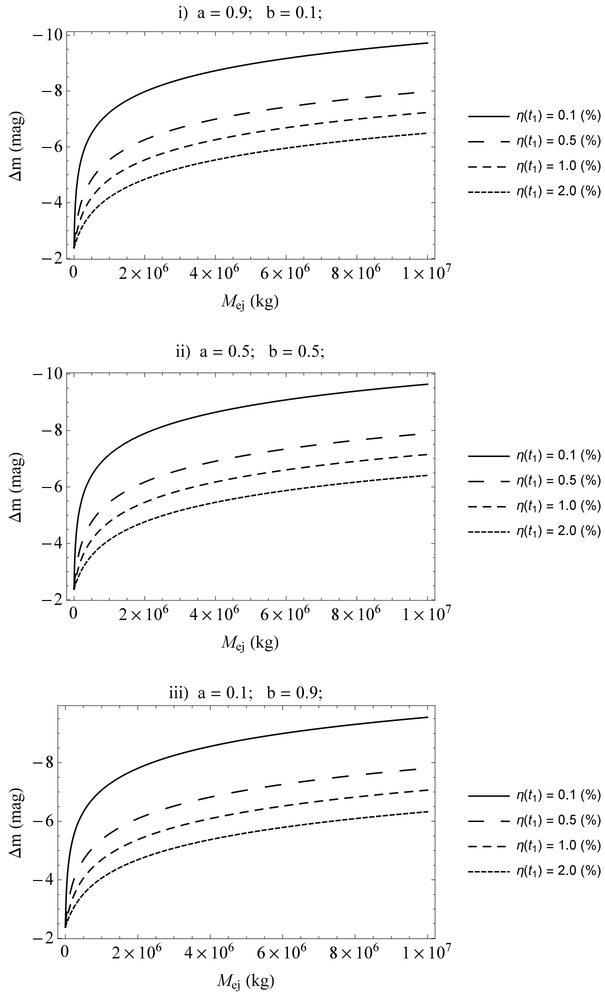


**Fig. 4** Similar to Fig. 3, but these calculations were made for aphelion ($r_Q$ = 5.5 au) of comet X/PC.

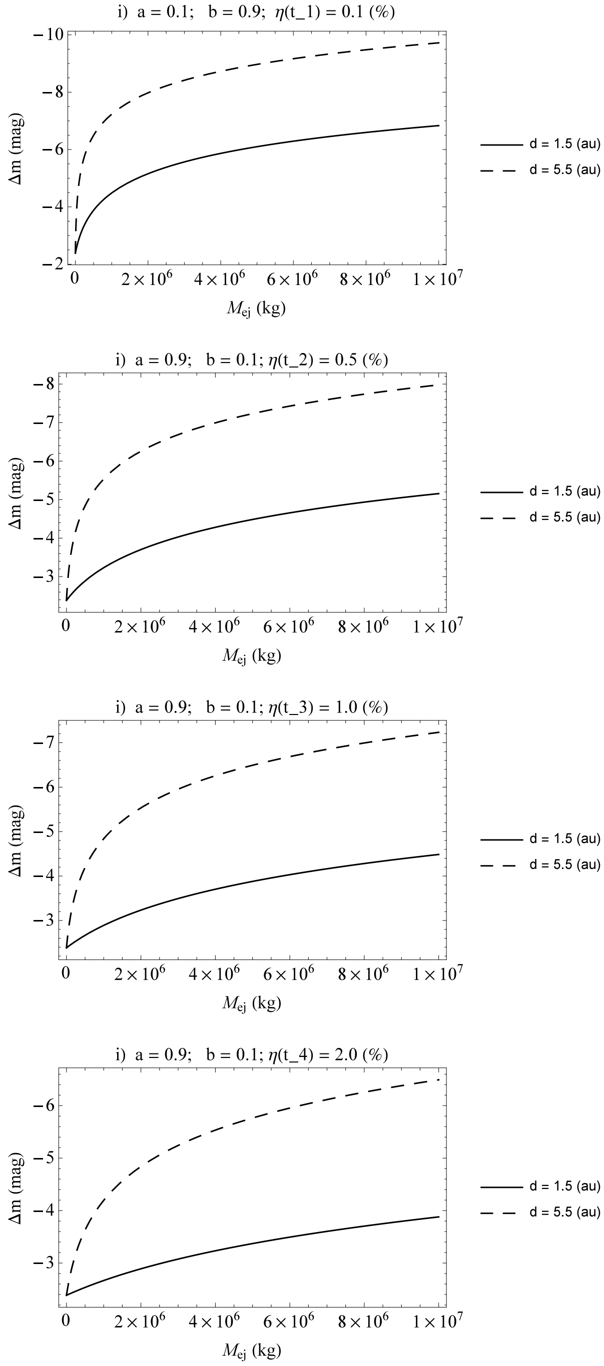


**Fig. 5** The jump in comet X/PC brightness Δm as a function of ejected mass $M_{\rm ej}$. The calculation compares the brightness jump of the comet at perihelion and aphelion. The calculations assume that parameters $a$ and $b$ are equal to 0.9 and 0.1, respectively.

in brightness $\Delta m$ of the comet as a function of mass ejected $M_{\rm ej}$ for perihelion and aphelion, respectively. At intermediate distances, numerical values of the parameter $\Delta m$ oscillate between those shown in Figures 3–4. In the next step, a comparison was made between the change in brightness of the comet located at perihelion and aphelion respectively (Figs. 5–7). These calculations take into account the same surface ($\eta(t_1)$), which is active sublimation. It is worth noting that the calculations presented in Figures 3–7 take into account the variable percentage of silicate and carbonaceous particles. The approach utilized in the simulations allows us to determine the change in brightness of the comet depending on the mass ejected during the outburst and the heliocentric distance.

## 4 SUMMARY AND CONCLUSIONS

The paper presents an analysis of cometary matter emission. For this purpose, two mechanisms were discussed that are associated with the activity of the comet. The first

**Table 1** Values of the Physical Cometary Parameters for Object X/PC Used in the Numerical Calculations and Simulations

| *Parameters* | *Value*(s) | Reference |
|---|---|---|
| The semimajor axis of the cometary orbit (au) | $a = 3.5$ | Value adopted |
| The eccentricity of the cometary orbit | $e$=0.571 | Value adopted |
| The radius of the comet's nucleus (m) | $R_{\mathrm{N}}$ = 1000 | Value adopted |
| The density of the comet nucleus ($\mathrm{kg\,m^{-3}}$) | $\rho_{\mathrm{N}}$=500 | Value adopted |
| The albedo | $A_{\mathrm{N}} = 0.04$ | Richardson et al. (2007) |
| Constant $A_{H_2O}$ for water ice (Pa) | $A_{H_2O}$=3.56$\times 10^{12}$ | Fanale & Salvail (1984) |
| Constant $B_{H_2O}$ for water ice (K) | $B_{H_2O}$=6141.667 | Fanale & Salvail (1984) |
| Latent heat of water ice sublimation ($\mathrm{J\,kg^{-1}}$) | L(T)=2.83$\times 10^{6}$ | Prialnik (2006) |
| The porosity | $\psi$=0.7 | Kossacki & Szutowicz (2013) |
| Dust - gas mass ratio | $\kappa$ = 1 | Gronkowski & Wesołowski (2015a) |
| Emissivity | $\epsilon$ = 0.9 | Wesołowski (2019) |
| Solar constant (for $d$=1 au) ($\mathrm{W\,m^{-2}}$) | $S_{\odot}$ = 1360.8$\pm$0.5 | Kopp & Lean (2011) |
| The radius of the cometary coma during the outburst (m) | $R'_{\mathrm{h}} = 3\times 10^{8}$ | Hughes (1991) |
| The radius of the cometary coma during gentle sublimation (m) | $R_{\mathrm{h}} = 1\times 10^{8}$ | Hughes (1991) |
| The refractive index for silicate particles | $m_{Si}$=1.6+0.01$i$ | Zubko et al. (2020) |
| The refractive index for carbonaceous particles | $m_{C}$=1.855+0.45$i$ | Zubko et al. (2020) |
| Mean value of solar radiation wavelength (m) | $\lambda$=0.5$\times 10^{-6}$ | Value adopted |
| The average particle radius (m) | $r_{av}$ = 0.752$\times 10^{-6}$ | Calculated value |
| The scattering efficiencies of silicate particles | $Q_{Si}$=2.557 | Calculated value |
| The scattering efficiencies of carbonaceous particles | $Q_{C}$=1.252 | Calculated value |
| Asymmetry coefficient for silicate particles | $g_{Si}$=0.785 | Calculated value |
| Asymmetry coefficient for carbonaceous particles | $g_{C}$=0.873 | Calculated value |
| The phase angle in the comet perihelion (deg) | $\Theta_{q_i}$=41.81 | Calculated value |
| The phase angle in the comet aphelion (deg) | $\Theta_{Q_i}$=10.48 | Calculated value |
| The effective diffraction parameter | $x_{eff}$ = 9.449 | Calculated value |
| The value of phase function at the comet's perihelion for silicate particles | $p(\Theta_{q_i})_{Si}$ = 0.103 | Calculated value |
| The value of phase function at the comet's perihelion for carbonaceous particles | $p(\Theta_{q_i})_{C}$ = 0.061 | Calculated value |
| The value of phase function at the comet's aphelion or silicate particles | $p(\Theta_{Q_i})_{Si}$ = 1.568 | Calculated value |
| The value of phase function at the comet's aphelion or carbonaceous particles | $p(\Theta_{Q_i})_{C}$ = 1.968 | Calculated value |

is the emission of dust from the surface of the nucleus to a coma, and the second is the change in the brightness of the comet as a result of an outburst. The sublimation of comet ice as well as the particle size plays an important role in both the first and second processes. Therefore, this paper relies on a function that describes the particle size belonging to a range from micrometers to millimeters. Knowledge of this function allows us to determine the approximate value of the dust particle radius.

In our calculations, we assumed that the coma contains astronomical silicates and cometary gas. We note that in the initial phase of the outburst of the comet, there are also grains of ice in the coma, which also sublimate. We omit their participation for two main reasons. Firstly, their lifetime in a coma is short, and secondly the *Rosetta* mission did not register the presence of ice in the form of grains in the coma.

In the context of searching for the cause of the comet's outburst, its mantle (the exterior of the nucleus) performs an important function. More specifically, we mean its tensile strength (cracking). It is generally accepted that the subsurface layers located at a depth of up to several meters have numerous cavities that can be filled with gas and dust. If the pressure of gas trapped in the cavity exceeds the mechanical strength of the mantle, then local destruction of the comet nucleus surface occurs. As a result, significant amounts of gases and dust are released into its coma. Additionally, the rate of sublimation increases with newly discovered layers that contain more volatile cometary material. Then the incident sunlight is more effectively dispersed on the dust agglomerates. As a result of this process, we observe an increase in the brightness of the comet, i.e. its outburst.

In this paper, numerous simulations and numerical tests were carried out and on this basis, the following conclusions can be inferred.

- A measure of a comet brightness jump is the amount of cometary dust (agglomerates) in a coma that causes more effective scattering of sunlight.
- The following parameters play a key role: heliocentric distance, the rate of sublimation, the dimensions of the cavity, the type of cometary ice, particle size and the fraction of the surface that is active in terms of sublimation.
- In particular, attention should be paid to the close relationship between the heliocentric distance and the sublimation rate which translates into a change in brightness. As the heliocentric distance increases, the sublimation rate decreases, and the brightness amplitude of the comet increases regardless of the surface of the active sublimation.

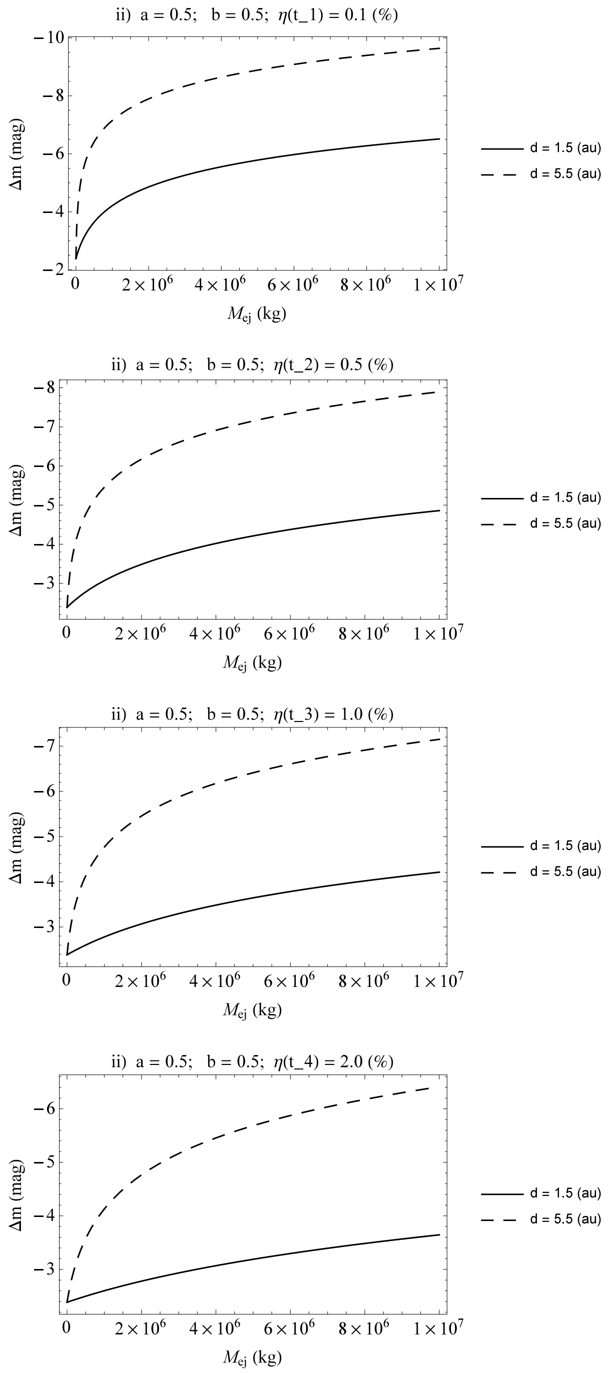


**Fig. 6** Similar to Fig. 5, but calculations assume that parameters $a$ and $b$ are equal to 0.5 and 0.5, respectively.

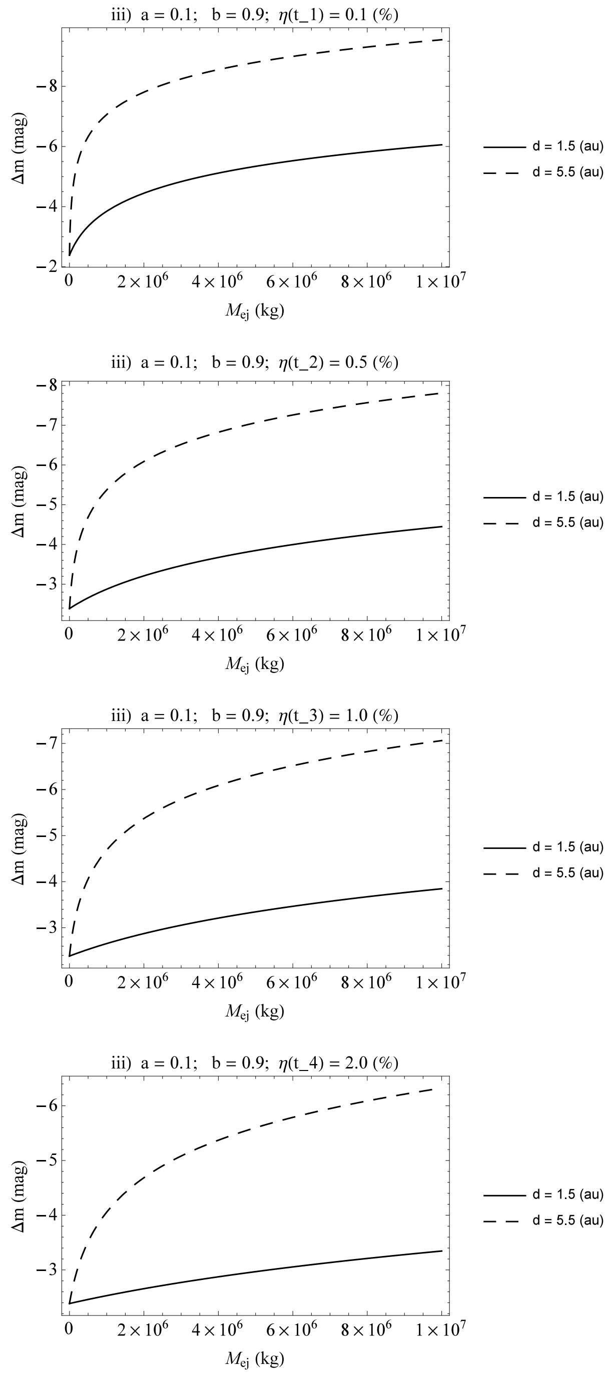


**Fig. 7** Similar to Fig. 5, but calculations assume that parameters $a$ and $b$ are equal to 0.1 and 0.9, respectively.

– The outbursts of comet brightness last from several to several dozens of days. This is a very short time in comparison with the comet's orbital period, and hence its phase angle changes very little. Therefore, it can be safely assumed that just before and during the outburst, the phase angle is constant for the comet under consideration.

– The mass ejected is a key parameter determining the amplitude of the change in brightness of comets. This result was not a surprise but it confirmed many years of observations of comets during their outbursts.

At the end of our considerations, it is worth asking an open question: what mechanism is responsible for the outburst of comets? Another issue is how the results of numerical calculations reflect the actual state? Yet another important issue is how we can generalize the results obtained for one comet to others. We can no doubt say that future space missions and more accurate numerical models can answer these questions.

**Acknowledgements** This work has been done due to the support the author received from the Centre for Innovation and Transfer of Natural Sciences and Engineering Knowledge, University of Rzeszów, Poland.

## References

Cabot, H., et al. 1996, Planet. Space Sci., 44, 1015
Crifo, J. F., Loukianov, G. A., Rodionov, A. V., & Zakharov, V. V. 2005, Icarus, 176, 192
Dorschner, J., et al. 1995, A&A, 300, 503
Enzian, A., et al. 1997, A&A, 319, 995
Fanale, F. P., & Salvail, J. R. 1984, Icarus, 60, 476
Fomenkova, M. N., Kerridge, J. F., Marti, K., & McFadden, L. A. 1992, Science, 258, 266
Gicquel, A., Vincent, J. B., Agarwal, J., et al. 2016, MNRAS, 462, S57
Gronkowski, P., Tralle, I., & Wesołowski, M. 2018, Astronomische Nachrichten, 339, 37
Gronkowski, P., & Wesołowski M. 2012, Astronomische Nachrichten, 333, 721
Gronkowski, P., & Wesołowski, M. 2015a, MNRAS, 451, 3068
Gronkowski, P., & Wesołowski, M. 2015b, Astronomische Nachrichten, 336, 362
Gronkowski P., Wesołowski M., 2017, Astronomische Nachrichten, 338, 385
Groussin, O., Lamy, P., & Jorda, L. 2004, A&A, 413, 1163
Groussin, O., Sunshine, J. M., Feaga, L. M., et al. 2013, Icarus, 222, 580
Huebner W. F., et al. 2006, in Heat and Gas Diffusion in Comet Nuclei (Noordwijk: ESA Publications Division)
Hughes, D. W. 1991, Possible Mechanisms for Cometary Outbursts, eds. J. Newburn, R. L., M. Neugebauer, & J. Rahe, 167, IAU Colloq. 116: Comets in the Post-Halley Era, eds. J. Newburn, R. L., M. Neugebauer, & J. Rahe, 167, 825
Ipatov, S. I. 2012, MNRAS, 423, 3474
Ipatov, S. I., & A'Hearn, M. F. 2011, MNRAS, 414, 76
Ivanova, A. V., et al. 2012, Solar System Research, 46, 313
Kolokolova, L., & Kimura, H. 2010, Earth, Planets, and Space, 62, 17
Kopp, G., & Lean, J. L. 2011, Geophys. Res. Lett., 38, L01706
Kossacki, K. J., & Szutowicz, S. 2013, Icarus, 225, 111
Marschall, R., Su, C. C., Liao, Y., et al. 2016, A&A, 589, A90
Mazets, E. P., Aptekar, R. L., Golenetskii, S. V., et al. 1986, Nature, 321, 276
Merouane, S., Zaprudin, B., Stenzel, O., et al. 2016, A&A, 596, A87
Miles, R. 2016, Icarus, 272, 356
Montalto, M., Riffeser, A., Hopp, U., Wilke, S., & Carraro, G. 2008, A&A, 479, L45
Moreno, F., Ortiz, J. L., Santos-Sanz, P., et al. 2008, ApJL, 677, L63
Prialnik, D. 2006, in Asteroids, Comets, Meteors, ed. D. Lazzaro, S. Ferraz-Mello, & J. A. Fernández, Vol. 229, 153
Richardson, J. E., Melosh, H. J., Lisse, C. M., & Carcich, B. 2007, Icarus, 190, 357
Richter, N. 1954, Astronomische Nachrichten, 281, 241
Rinaldi, G., Della Corte, V., Fulle, M., et al. 2017, MNRAS, 469, S598
Schloerb, F. P., Keihm, S., von Allmen, P., et al. 2015, A&A, 583, A29
Schulz, R., Hilchenbach, M., Langevin, Y., et al. 2015, Nature, 518, 216
Skorov, Y., Reshetnyk, V., Lacerda, P., Hartogh, P., & Blum, J. 2016, MNRAS, 461, 3410
Trigo-Rodríguez, J. M., García-Hernández, D. A., Sánchez, A., et al. 2010, MNRAS, 409, 1682
Trigo-Rodríguez, J. M., García-Melendo, E., Davidsson, B. J. R., et al. 2008, A&A, 485, 599
Virrki, A., et al. 2019, Icarus, 325, 94
Wesołowski, M., & Gronkowski, P. 2018a, New Astron., 62, 55
Wesołowski, M., & Gronkowski, P. 2018b, Earth Moon and Planets, 121, 105
Wesołowski, M. 2019, Journal of Astrophysics and Astronomy, 40, 20
Wesołowski, M., et al. 2019, MNRAS, 484, 2309
Wesołowski, M., et al. 2020a, Icarurs, 338, 113546
Wesołowski, M., et al. 2020b, Planet. Space Sci., 184, 104867
Wesołowski M., et al. 2020c, Icarus, 352, 114005
Wesołowski, M. 2020a, Journal of Astrophysics and Astronomy, 41, 1
Wesołowski, M. 2020b, Icarus, 351, 113950
Wesołowski, M. 2020c, RAA (Research in Astronomy and Astrophysics), 8, 132
Zubko, E. 2013, Earth, Planets, and Space, 65, 139
Zubko, E. 2020, MNRAS, 492, 810
Zubko, E., Videen, G., Hines, D. C., & Shkuratov, Y. 2016, Planet. Space Sci., 123, 63
Zubko, E., Zheltobryukhov, M., Chornaya, E., et al. 2020, MNRAS, 497, 1536