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Following up the afterglow: strategy for X-ray observation triggered by gravitational wave events

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Abstract The multi-messenger observation of coalescing compact binary systems promises great scientific treasure. However, synthesising observations from both gravitational wave and electromagnetic channels remains challenging. In the context of the day-to-week long emission from a macronova, the binary neutron star merger GW170817 remains the only event with successful electromagnetic followup. In this manuscript, we explore the possibility of using the early stage X-ray afterglow to search for the electromagnetic counterpart of a gravitational wave event. Two algorithms, the simple and straightforward *sequential observation* (SO) and the step-wise optimizing *local optimization* are considered and applied to some simulated events. We consider the WXT from the proposed *Einstein Probe* as a candidate X-ray telescope, which has a very wide field of view of $3600 \deg^2$. Benefiting from the large field of view and high sensitivity, we find that the SO algorithm not only is easy to implement, but also promises a good chance of actual detection.

Key words: gravitational waves: X-ray afterglow: multi-messenger astronomy

1 INTRODUCTION

The successful operation of ground-based gravitational wave (GW) detectors like LIGO and Virgo has marked the beginning of a new era of GW astronomy (Abbott et al. 2016). During the first to the third observing runs (O3a) of LIGO and Virgo, a series of detections of compact binary coalescence has been made, including binary black holes, binary neutron stars and neutron star-black hole binaries (Abbott et al. 2021b), and a large variety of characteristics of the detected events, including their mass, mass ratio, spin, and distance, has been inferred (Abbott et al. 2019, 2021a). The operation of GW observatories promises to greatly deepen our understanding of the most dense objects in the Universe.

On 17 August, 2017, a GW signal from a binary neutron star merger later denoted as GW170817 was observed (Abbott et al. 2017b), and a gamma ray burst (GRB) event GRB170817A was observed simultaneously from the same location (Abbott et al. 2017a). Such synthetic observations of both GW and electromagnetic (EM) waves sparked huge interest among astronomers, leading to a series of scientific discoveries, from confirming the link between short GRBs and binary neutron star mergers, to revealing their role in producing heavy elements throughout the Universe (Abbott et al. 2017a).

However, it should be noted that due to the exceptional nature of GRB170817A, which is the closest short GRB detected to date, it is the only published event with multimessenger observations from both GW and EM channels. It is also by far the weakest short GRB in terms of intrinsic luminosity (Abbott et al. 2017a). Furthermore, no

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detection of an EM counterpart of the next confirmed neutron star binary merger event GW190425 or the neutron star-black hole mergers GW200105 and GW200115 was made (Abbott et al. 2020). The *status quo* of multimessenger observation triggered by GW detections reflects its intrinsic difficulty.

The EM counterpart of GW events can cover a wide range of spectrum, but all face its own challenges. The prompt emission of GRBs are powerful sources of EM emission, but they are highly beamed. Only those observers who are located in a small solid angle close to their collimated jet could easily detect them (e.g., Sari et al. 1999). Furthermore, detections of gamma rays sources are often accompanied with large uncertainties (\sim degrees) in their sky locations, therefore, even if the GRB is detected, the pinpointing of the location or identifying the host galaxy is still a needle-in-a-haystack search. Instead, the successful identification of a host galaxy for GRB170817A relied on the much later stage emission referred to as the kilonova or macronova, where optical and infrared emission is powered by decay of radioactive materials produced in the so-called r-process after the binary neutron star merger (Li & Paczynski 1998; Barnes & Kasen 2013). Compared with the prompt emission of GRB, macronovae have the advantage of a much wider viewing angle (Metzger & Berger 2012), however, they suffer from lower luminosity. The fact that no EM counterpart of GW190425 was detected reflected the difficulty in the related searching.

In this work, we explore the X-ray afterglow emission, to assist the rapid localization of a binary neutron star merger. If on axis, the X-ray afterglow is expected to happen much sooner compared with the macronova, start emission $\lesssim 10\,\mathrm{s}$ after the merger, enabling the observation of earlier stage phenomena (Sari et al. 1998). More importantly, the duration for GRB prompt emission is too short to perform target of opportunity observation, while the X-ray afterglow can last long enough to perform such observations triggered by GW alerts. We aim to study these target of opportunity observations under the assumption that a trigger from GW observatories has been issued, and no short GRB has been observed, in order to investigate what observation strategy X-ray telescopes should adopt, so that one can increase the probability of observing the EM counterpart and pinpointing its sky location. Specifically, we consider the Einstein Probe as an example, which is scheduled to be launched by the end of 2022, and the WXT module has a 3600 deg^2 field of view, making it a encouraging facility to facilitate such multi-messenger observations.

This paper is organized as follows. Section 2 describes the statistical framework we adopt. Section 3 illustrates the two algorithms we use for the observation strategy. We show the results in Section 4, and discuss future work and provide a summary in Section 5.

2 STATISTICAL FRAMEWORK

In order to optimize the observation strategy, we need to first establish the appropriate statistical framework.

Throughout this work, we make certain assumptions to simply the calculation. Readers are reminded that some simplifications are designed for accelerating the calculation, while others can be lifted while implementing for a specific telescope. The later kind of simplifications are assumed to preserve the generality of the outcome. For example, we assume that the sky position and distance of the GW events are independent of each other, so that their joint probability distribution is simply the product of each distribution. Adopting this assumption could boost the computation efficiency, which is critical for the science problem considered. We further highlight that triggers from O3a demonstrated that many GW events do meet this separation condition. We also assume that the telescopes can point to any direction, ignoring the potential influence from the Sun, the Moon, and the Earth, as was done in Chan et al. e.g., 2017; Coughlin & Stubbs e.g., 2016; Coughlin et al. e.g., 2016, 2018. Notice that there are works in the field that targets this issue, like Singer et al. e.g., 2016; Coughlin et al. e.g., 2018; Ghosh et al. e.g., 2017; Rana et al. e.g., 2019; Coughlin et al. e.g., 2019, and we refer the interested readers to these works. We haven't included these potential influence explicitly because we aim to explore a totally different dimension of the work, and we want to preserve the generality of the work. But the actual implementation for a specific telescope should take such effects into consideration. Currently, we will restrict our attention to the proof of principle of the X-ray afterglow followup strategy. Furthermore, since we discuss the strategy for X-ray telescopes, which will operate in space, the constraints from rising and setting, as seen from different observing sites, is much less stringent. Finally, we do not account for the overlap of different fields, which will cause a multiple counts for certain areas. By not considering this overlap, we can simplify the calculation. Since this choice will make our conclusion more conservative, we choose not to lift this simplification.

Since the luminosity of X-ray afterglows changes rapidly, we define the detection as when multiple observations reveal an obvious luminosity difference. Notice that although the X-ray sky is much sparser than the optical sky, one has to be cautious dealing with the EM counterparts that merely pass the detection threshold. Due to the large variance of the sources' luminosity, we regard multiple exposures to be necessary in order to lower the chance of false alarm. We use D_{ag} to denote the successful detection of an afterglow. In order to confirm the existence of an afterglow, we need to observe a significant change in luminosity, so the probability of D_{ag} is defined as when the inferred flux has an obvious difference, or $\Delta f > 0$, and is only meaningful when multiple observations are executed. We note that the probability of detection $P(D_{ag})$ depends on the field of view (FOV) ω , the pointing of the telescope which determines the observed sky location (α , δ), and the corresponding exposure time of the multiple observations τ_1 and τ_2 . The posterior probability of successful detection can then be written as the probability that the first exposure accumulates enough signal-to-noise ratio (SNR), times the probability of observing an obvious change in flux in the second exposure:

$$P(D_{ag}|\omega, \text{pointing}, \tau_1, \tau_2, I) = P(N > N^* | \omega, \text{pointing}, \tau_1, I) \times P(\Delta f > 0 | \tau_1, \tau_2, I).$$
(1)

Here, I is prior information that includes the parameters of the selected telescope, such as its photon collecting area A to name one. The threshold count of received photons N^* is the criterion for detection determined by the expected SNR, the background noise, and the sensitivity of the selected telescope. The SNR is defined as the expected photon number from signal divide by the standard deviation of noise photon number.

$$SNR = \frac{N_{signal}}{\sigma(N_{noise})} = \frac{N_{signal}}{\sqrt{N_{noise}}},$$
 (2)

hence, N^* could be expressed as

$$N^* = \text{SNR}_{\text{threshold}} \times \sqrt{N_{\text{noise}}}.$$
 (3)

The number of photons received by the telescope N depends on multiple factors. The GW event is localised by GW detectors with uncertainties, and one can compute how likely a given sky area is to contain the GW source. With the knowledge of the X-ray afterglow luminosity L and the distance R, one can estimate the distribution of the expected flux f, which can be later translated into the distribution of detected photon numbers. Then the first part of Equation (1) can be expanded as

$$P(N > N^* | \omega, \text{pointing}, \tau_1, I) = \int_{N^*}^{\infty} dN \int df \int dR \int_{\omega} d\alpha d\delta \qquad (4)$$
$$\times p(N|f, \tau_1, I) p(f|I, R) p(\alpha, \delta, R|I, \text{pointing}).$$

 $p(N|f, \tau_1, I)$ is the probability of receiving N photons, which is described by a Poisson distribution, given the flux f of the source and observation time τ_1 . Notice that the intrinsic luminosity of the source could be accompanied with large variance, but it explicitly follows the inverse square law with the distance. And the formulation we chose can treat the intrinsic variance and distance dependence separately. Since we assume that the prior distribution on the distance, R, to the target afterglow is statistically independent of the prior distribution on its sky location (α , δ), Equation (4) can be written as

$$P(N > N^* | \omega, \text{pointing}, \tau_1, I) = P_{\text{gw}}(\omega, \text{pointing}) \times P_{\text{ag}}(\tau_1),$$
(5)

where

$$P_{\rm gw}(\omega, {\rm pointing}) = \int_{\omega} p(\alpha, \delta | I, {\rm pointing}) d\alpha d\delta,$$
 (6)

$$P_{ag}(\tau_{1}) = \int df \int dR \int_{N^{*}}^{\infty} dN$$

$$\times p(N|f, \tau_{1}, I)p(f|I, R)p(R|I)$$

$$= \int df \int_{N^{*}}^{\infty} dNp(N|f, \tau_{1}, I)$$

$$\times \int dRp(f|I, R)p(R|I).$$
(7)

Because the flux f depends on not only the source distance R but also the underlying models as well relevant parameters like interstellar medium density (Sari et al. 1998), it is not straightforward to derive the p(f|I,R)theoretically. We can however obtain the distribution of p(f|I, R) through the light-curve fitting of the observed Xray afterglow data¹. Given the expected source distance R, we can approximate the likelihood function of the distance distribution as Gaussian (Singer et al. 2016). We here just use a delta function for both p(f|I, R) and p(R|I) in order to simplify the calculation. The first part of Equation (1) only considers a single observation. As for the second part of Equation (1), Δf is the difference in flux of the multiple observations (denoted f_1 and f_2) at different times. These fluxes can be approximated by a distribution depending on the prior known flux f', which is based on the afterglow light curve model. The equation of this distribution can be written as

$$P(f|f',\tau) = \sum_{N=0}^{\infty} P(N|f',\tau) \times P(f|N,\tau)$$

= $\sum_{N=0}^{\infty} P(N|f',\tau) \times \frac{P(N|f,\tau)P(f)}{\int_{0}^{\infty} P(N|f_{0},\tau)P(f_{0})df_{0}}.$ (8)

Here P(f) is the prior probability of the flux emitted by afterglow. $P(N|f,\tau)$ can be approximated by Poisson distribution with the mean value $\bar{f}\tau$, and \bar{f} is the average flux during the period of τ . Considering both N and

https://www.swift.ac.uk/xrt_curves/

 $\bar{f}\tau$ to be very large, we can approximate it with a Gaussian distribution using the central limit theorem. For convenience of description, we refer to the probability in Equation (7) as P_1 ,

$$P_{1} = P_{ag}(\tau_{1}) = \int df \int dR \int_{N^{*}}^{\infty} dN$$

$$\times p(N|f, \tau_{1}, I)p(f|I, R)p(R|I)$$

$$= \int df \int_{N^{*}}^{\infty} dNp(N|f, \tau_{1}, I)$$

$$\times \int dRp(f|I, R)p(R|I).$$
(9)

Similarly we refer to the second part of Equation (1) as P_2 ,

$$P_2 = P(\Delta f > 0 | \tau_1, \tau_2, I).$$
(10)

Thus Equation (1) could be written as

 $P(D_{\rm ag}|\omega, {\rm pointing}, \tau_1, \tau_2, I) = P_{\rm gw} \times P_1 \times P_2.$ (11)

Equation (11) describes the probability of detecting the afterglow of one field. In realistic observations where multiple fields might be observed, one needs to also include the number of fields into the calculation. Assuming that the GW sky localization error region covers S deg^2 and the size of the telescope FOV is $\Omega \deg^2$, the number of fields n needed for search can be roughly estimated as $n \leq n$ S/Ω in the case of small FOV (Chan et al. 2017). However, the GW sky localization error region is not generally a regular shape, and more fields than S/Ω may actually be needed to properly cover the whole region. We will not set a constrained total observation time at first, but the observation time does have a natural constraint. For Xray afterglows, their luminosity will decay rapidly, and soon we can no longer observe the object with enough SNR. In other words, P_1 will not increase after a certain time. We mark this moment as $T_{\rm threshold}$. When the time has exceeded $T_{\rm threshold}$, we should no longer consider performing any more first time observations for new fields.

3 MODELS

3.1 Tiling

We follow Chan et al. (2017) and use a greedy algorithm to optimize the tiling of the observing fields. The pixels within a certain confidence level, say 90%, are selected so that the following computation is largely simplified. Then, the observation field is optimized step-wise. Each time, the field with the maximum GW event posterior probability will be output and labeled in order from 1 to n. Hence this label indicates the rank of each field in terms of enclosed GW probability. In the following, we consider the proposed Einstein Probe (EP, Yuan et al. 2015^2) as an example. Specifically, we use its Wide-field X-ray Telescope (WXT module³) for calculation, with a corresponding FOV of about 3600 deg² (Yuan et al. 2015). The exceedingly large FOV makes it an ideal instrument to search for X-ray counterparts of GW events.

3.2 Sequential Observation Algorithm

The tiling algorithm adopted determines where to look, while we determine the observing time allocated to different tiles with an observing algorithm. Firstly, we consider a simple Sequential Observation (SO) algorithm. As shown in Equation (5), the probability of detecting the X-ray transient can be separated to two parts, depending on the pointing directions and observation time τ respectively. Since the label of fields indicates their rank in terms of enclosed GW probability, every time we look at a new field, we start with the smallest label number.

In the SO algorithm, we intend to cover as many first time observations as possible before $T_{\text{threshold}}$. After the first round, we perform the second time observation, following the same sequence as of the first time observation. As for the time allocation, we take a step-wise adjustment with a 1 second step. Notice that the shifting between different tiles requires a certain slew time T_s for the telescope, so we then make the comparison between the two choices: additional probability gained ΔP_1 by observing $1 + T_s$ more seconds in the first time observation, and the probability gained by observing 1 second in a new field. We continue observing the current field until the the probability increment ΔP_1 is smaller than the probability gained by observing a new field. For the second time observation, we simply adopt the same exposure time used for the first time observation.

Under such a construction, we find that the SO algorithm prefers to accumulate large P_1 for the most fields, and it will search as many new fields as possible, when the signal is strong enough. The source is expected to be much dimmer for the second time observation, but since P_2 depends on the difference between the two observations, as long as the intensity of the first observation and the second observation are different enough the P_2 would still be significant. We illustrate the SO algorithm in Figure 1.

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² http://ep.nao.cas.cn

³ http://ep.nao.cas.cn/epmission/epinstruments/ 201909/t20190916_516240.html



Fig.1 Illustration of observation sequence for the SO algorithm. The algorithm try to cover as many first time observations as possible.



Fig. 2 Illustration of observation sequence for Local Optimization algorithm. Notice that the actual observation order is not fixed and depends on choice of light curve model as well as the sky localisation.

3.3 Local Optimization Algorithm

The best way to allocate the observation time is certainly through a global fitting scheme, as *e.g.* demonstrated in Chan et al. (2017). However, it involves the optimization over both observation order as well as observation time, which makes it very time-consuming to perform. We are therefore motivated to examine the Local Optimization (LO) algorithm as a compromise of complexity and speed.

We illustrate the LO algorithm in Figure 2. In this algorithm, we need to optimize not only the observation time, but also the order of observation. We first enumerate all possible options for the next field to observe, which could be the first-time observation of a new field, or the second-time observation of an old field, including the current tile so that the slew of telescope is no longer required. Notice that we always look at tiles with the smallest label number, to maximize the $P_{\rm gw}$. For each option, a step-wise optimization is then performed, to pick up the next tile to observe based on the gained probability with extra time.

In principle, such freedom of choice for the next fields introduces a great amount of possibility for the order of observation. But certain cases can simplify the optimization. For example, if $T_{\rm threshold}$ is passed, no new field would be considered to be explored and we would stick to the second-time observation of the old fields.



Fig.3 GRB afterglow observations and X-ray transient observations are shown with data and fitted model curves. The x-axis is the time after merger and the y-axis is the luminosity of the source.

3.4 Light Curve Models

We use two different light curve model to test the robustness of our algorithms. The first light curve model shown in Figure 3 is referred to as the GRB Afterglows Model (GAM). It is fitted based on the observed X-ray afterglow associated with short GRBs with known distance⁴. These samples are expected to be associated with relatively small viewing angles to the jet direction. For later calculation, we fit the binned data with a two-step linear function in logarithmic space.

The second model is shown in figure 3 from Xue et al. (2019) and Sun et al. (2019), which we refer to as the Xray Transient Model (XTM), which can also be produced by binary neutron star mergers (Dai et al. 2006; Gao 2006). These X-ray transients are more isotropic and may not be related with GRBs (Zhang 2013; Sun et al. 2017). Both the X-ray afterglows and the X-ray transients can be observed if the merger leaves a magnetar and we are close to the jet direction, while only X-ray transients are seen if the line of sight is off the jet axis. Generally speaking, both the two types of X-ray emissions can be regarded as the X-ray counterparts of gravitational events, and we apply the observation strategies to both models to test their robustness. We remark that GRB X-ray afterglows can be explained with various of mechanisms. The adoption of this model is mainly motivated to demonstrate the applicability of the method. Specifically the XTM model demonstrates a relatively low luminosity plateau followed by a fall, and it is interesting to compare the results on vastly different afterglow light curve models.

⁴ https://www.swift.ac.uk/xrt_curves/, more details about how the data were produced can be seen in Evans et al. (2007) and Evans et al. (2009)



Fig. 4 The tilings of event ID 10968, 14011, and 12715, with 90% confidence level covering \sim 596, 1020, and 1100 deg², respectively. Different colors represent different covering tiles and the label number in each tile indicates their rank in terms of enclosed GW probability. EP will cover these sky localizations with 3, 6 and 7 fields respectively.

4 RESULTS

In this section, we present the performance of our two different algorithms by using the proposed EP to followup a few simulated GW events. These events, simulated by Singer et al. (2014) with different error region areas as well as shapes, are selected as examples. Notice that as it has been shown that sky localisations from realistic data have trivial difference (Berry et al. 2015), and the sky localisation pipeline studied is actually responsible for generating the public alerts (Singer & Price 2016). Notice that these data were generated assuming the O1 sensitivity, which has already been surpassed by current detectors. The same events if they were observed currently would be expected to have higher SNR, and thus smaller sky



Fig. 5 Detection probability of the SO and LO algorithm under the extremely fast fading light curve. Notice that unlike the results for the GAM or XTM models, here the two algorithms returns comparable detection probability, and in one case the LO even outperform SO.

localisation error. In order to better mimic the realistic scenario, we manually amplify the distance by a factor of 2. This leads to the changes in the distance range from 30 - 100 Mpc to 60 - 200 Mpc.

Figure 4 shows the optimized tiling of observing fields obtained using the greedy algorithm approach for three typical events. They correspond to event ID 10968, 14011, and 12715 from Singer et al. (2014), with 90% confidence level covering \sim 596, 1020, and 1100 deg², respectively. Despite the fact that EP's WXT has a very large FOV of about 3600 deg², since the sky localisation derived from GW detectors is in a ring-like shape in sky, so multiple observation fields are still needed to encapsulate the 90% confidence level. The required number and location of fields varies for different events. Among the three example events, between 3 and 7 tiles would be needed to cover the 90% confidence level.

4.1 Observing Strategy

If the GRB afterglow model is adopted, we can observe that the LO algorithm will require two consecutive observations of the same field. This can be explained by a combination of two factors: firstly, the LO algorithm is dedicated to locally maximize the detection probability with the next observation. Since the fields with smaller index are associated with relatively higher P_{gw} , the strategy is more likely to observe the previous field immediately. Secondly, compared with the exposure time, the slew time for moving the viewing field of the telescope is quite long, so the algorithm would thus tend to save time by completing the second time observation before observing a new field. For the SO algorithm, on the other hand, the sequence is pre-determined, and we perform, for

Event ID	Algorithm	allocated observation time (s)							
L vent ID	Aigorithin	Field1	Field2	Field3	Field4	Field5	Field6	Field7	
	SO	1	1	1	-	-	-	-	
10968	LO(First time)	1	1	1	-	-	-	-	
	LO(Second time)	15	1	53	-	-	-	-	
	SO	1	1	1	1	1	1	-	
14011	LO(First time)	1	1	1	1	1	4	-	
	LO(Second time)	1	1	1	35	194	109	-	
	SO	1	1	1	1	1	1	1	
12715	LO(First time)	1	1	1	1	1	1	1	
	LO(Second time)	1	1	2	19	1	25	102	

 Table 1
 Time Allocation for the Three Example Events

Event ID	Algorithm	Probability (%)							
		Field1	Field2	Field3	Field4	Field5	Field6	Field7	
10968	$P_1(SO)$	100.00	100.00	100.00	-	-	-	-	
	$P_2(SO)$	100.00	100.00	100.00	-	-	-	-	
	$P_1(LO)$	100.00	100.00	100.00	-	-	-	-	
	$P_2(LO)$	87.23	52.77	90.06	-	-	-	-	
	P_{gw}	79.82	11.08	6.36	-	-	-	-	
14011	$P_1(SO)$	100.00	100.00	100.00	100.00	100.00	95.59	-	
	$P_2(SO)$	100.00	100.00	100.00	100.00	100.00	88.95	-	
	$P_1(LO)$	100.00	100.00	100.00	100.00	100.00	100.00	-	
	$P_2(LO)$	55.53	52.95	51.51	72.74	97.55	90.13	-	
	$P_{\rm gw}$	30.19	26.06	23.16	15.01	4.11	1.16	-	
12715	$P_1(SO)$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
	$P_2(SO)$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
	$P_1(LO)$	100.00	100.00	100.00	100.00	100.00	100.00	100.00	
	$P_2(LO)$	62.38	57.15	56.64	77.42	51.57	68.00	90.11	
	P_{gw}	45.00	23.12	18.96	8.44	1.91	1.67	0.54	

 Table 2 Probability for the Three Example Events

all fields and in order, the first time observation followed by the second.

In Table 1 we present the optimized time allocation for each simulated GW event under the two algorithms using the GAM as the light curve model. Since both observations for the SO algorithm use the same exposure time, only one row is given for each event; for the LO algorithm on the other hand, two lines are used to depict the exposure times for the first and the second observation respectively. Notice that for many fields, the observation time is assigned for as small as one second. This is due to the fact that for the earlier stage, when the afterglow is still quite bright, the telescope can accumulate high enough SNR in a relatively short time.

Notice that the isotropic equivalent X-ray afterglow fluxes for short GRBs has large uncertainty between 10^{44} – 10^{47} erg s⁻¹ (D'Avanzo et al. 2014). Therefore, for the consideration of completeness, we also consider a model where the luminosity three orders of magnitude lower than the GAM, as shown in the Figure 3, and we refer to it as GRB Afterglow Model B (GAMB). As a comparison, we adopt the GAMB to perform repeated calculations on the three events of 10968, 14011, and 12715. The results were shown in Table 3. Here we assume that observations begin 300 second after the binary merger. It can be seen that because the luminosity in GAMB is lower, the first observation requires more exposure time to obtain enough SNR. Also, due to the fast-fading light curve and limited by SNR, the number of observable fields is far less than the result obtained by using GAM and XTM.

Notice that for the second time exposure of the LO algorithm, there will be cases where a certain field is assigned with a shorter exposure time compared with the next tile, despite the fact that the algorithm always starts from tiles with higher P_{gw} , and we assume that the X-ray counterpart is continuously weakening in brightness. Under the assumption that the X-ray counterpart is continuously weakening in brightness and the LO algorithm always starts from tiles with higher $P_{\rm gw}$, the observation time for the latter field should be longer than the former. However, we could find that there are some cases where former field's second observation is assigned with a longer exposure time compared with the next fields. This can be explained when two consecutive tiles have significant decrease in $P_{\rm gw}$, therefore a possible but not decisive second time observation is outweighed by the benefit from exploring of a new field. For the SO algorithm, since the two observations of the same field

Event ID	Algorithm	allocated observation time (s)				
Lvent ID	Aigonunn	Field1	Field2			
10068	LO(First time)	119	-			
10908	LO(Second time)	94	-			
14011	LO(First time)	112	-			
	LO(Second time)	86	-			
12715	LO(First time)	4	38			
	LO(Second time)	118	26			

 Table 3 Time Allocation for the Three Example Events by

 Using GAMB

are separated by a long enough time, the models predict substantial change in the X-ray luminosity during this gap, and a short exposure time is still sufficient to tell the difference.

Let us dive into the details for the significant change of the second exposure time for the LO algorithm. The outcome might look counter-intuitive from the first glance, but since we are dealing with fast-fading yet highly uncertain light curve models, with a relatively short exposure time, one can already gain some probability in finding the X-ray signal if the source lies in the brighter end of the probability distribution. However, orders of magnitude more exposure time won't buy significantly larger probability. Move on the next field afterwards, usually the P_{gw} is again comparable, therefore we observe an increase in exposure time. Together, this could explain the observed deviation from monotonic increasing in exposure time. Indeed, if we consider an extremely fast fading light curve, with the luminosity L having a strong dependence with time after merger t, as for example $L \propto t^{-1.2}$, then the LO algorithm returns comparable results with the SO algorithm, and in one situation, even outperform the SO result, as demonstrated in Figure 5.

4.2 Detection Probability Comparison

In this subsection, we present a comparison of the detection probability $P(D_{ag})$ between the different algorithms and the different models. As shown in Figure 6, the detection probabilities for the two algorithms are presented for a total of 25 simulated events from Singer et al. (2014), assuming the GAM, and assuming the observations start 1000 seconds after the binary merger, to mimic the necessary time delay due to communication and processing in real life.

We note that for all events, the SO algorithm (shown as blue circles) consistently has a close-to-unity detection probability. This is because the GAM predicts a very bright signal in the early stage, and the SO algorithm can essentially cover the entire sky area with first time observations during this early stage – meaning, in other words, that it can obtain a high P_1 . Meanwhile, since



(a) X-ray detection probability versus distance for the set of events, simulated using the GRB afterglow model, considered in this study.



(b) X-ray detection probability versus 90% sky localization error region for the set of events, simulated using the GRB afterglow model, considered in this study.

Fig. 6 Detection probabilities for the GAM model. For each simulated event a *blue circle* corresponds to results from the SO algorithm while a *red square* corresponds to results from the LO algorithm. The *dashed lines* connect the results from the two algorithms for the same event.

by design there is a long time difference between the first time observation and the second time observation for the same field, it should be easy to detect an obvious luminosity change, hence a high P_2 . These two factors guarantee a high detection probability in general. On the other hand, the detection probability of the LO algorithm is consistently lower than 100%, which can be understood in terms of the short exposure time for the second time observations for certain fields, as well as the short interval between the two time observations. The other factor that prevents a high detection probability is that the LO algorithm might require a long second-time exposure, before moving to a new field. Therefore, when fields with smaller P_{gw} are observed, the expected luminosity might have been significantly decreased, and thus $T_{\rm threshold}$ might be approached before covering a sufficient number of fields.



Fig.7 A comparison of the average $P_{\rm em}$ corresponding to events with different numbers of observation fields required to achieve $P_{\rm gw} \ge 90\%$. Blue dots represent SO algorithm and the red ones represent LO algorithm. Some dots are shown in darker color due to overlap of multiple events.



Fig.8 Cumulative distribution of detection probability. Different curves represent the results of two different models using different algorithms. All four curves assume that observations begin 1000 seconds after the binary merger.

Figure 6 also plots the 90% sky localization error region and source distance for each event. We notice that for the LO results, the detection probability correlates weakly with the 90% error region, while no obvious relation with the source distance is observed. In fact, the LO algorithm can detect a very far event with error region of $\sim 600 \text{ deg}^2$ with a probability of about 80%. We further explore this relationship by looking at the detection probability with the number of fields explored by the algorithm. Due to the irregular shape of the error region, smaller error regions might require a higher number of tiles to cover them. However we can see from Figure 7 that for events that can be covered by a small number of tiles, the LO can achieve better detection probability, while more required tiles leads to a lower detection probability. Still, unlike the SO algorithm, the results of LO never reach as high as around 100% detection probabilities.



Fig.9 Detection probability results using the same time allocation and observation sequence but when the incorrect model is assumed for the light curve. The *red circles* represent results from the SO algorithm, and the *blue squares* represent results from the LO algorithm. Due to the narrow numerical range of results for the SO algorithm, these are shown in the enlarged inset in the upper right corner of the figure.

The same two algorithms are also applied to the XTM model, to check the robustness of their performance, and the change of light curve model does not dramatically affect the results. In Figure 8, cumulative distributions of detection probability over a total number of 25 events are presented. Notice that for both the GAM and XTM light curves, both the LO and SO algorithms have robust performance, with SO maintaining a high detection probability, while LO gives a detection probability ranging from $\sim 50\%$ to $\sim 90\%$. To conclude, under different models, SO consistently outperforms Local Optimization.

The final robustness test we perform is to check how the algorithms perform when the wrong model is assumed. We apply the strategy obtained assuming the GAM model, while the event actually follows the XTM model. As shown in Figure 9, the SO algorithm still returns a close-to-unity detection probability, independent of the actual light curve model the source follows. Meanwhile, although the LO algorithm can reach as high as 90% detection probability when assuming the correct model, the detection probability drops to as low as only 50% - 60% when the wrong model is adopted. This illustrates the fact that the LO strategy, which is based on detecting the probability change in a step-by-step way, depends strongly on the assumed underlying model. Meanwhile, under the SO algorithm, the first-time observations of all fields are performed all at the beginning, and this can better ensure that the time interval between two observations of the same field is long enough, thus boosting the probability of P_2 . Therefore, as long as the strategy covers the most probable regions efficiently, SO can promise a high detection probability, even for a mismatched assumption about the underlying light curve model.

5 DISCUSSION AND CONCLUSION

In this work, we devised and compared two different algorithms to optimize the probability of a successful X-ray afterglow detection triggered by GW alerts. We apply the EP's WXT as a nominal telescope to assess the outcome quantitatively. The large FOV guarantees an efficient coverage of the $P_{\rm gw}$ with a handful of fields, which is very helpful in the rapid search and identification of X-ray afterglows. We find that, interestingly, the simpler strategy, SO, which is to finish the first time observation of as many fields as possible initially, consistently outperforms the more complicated algorithm, Local Optimization. Indeed, immediately after the neutron star binary merger, the X-ray afterglow is expected to be bright enough that even a very short exposure time is sufficient for detection, and the algorithm indicates that the majority of time is actually spent on the slewing of the telescope.

As a proof-of-principle study, we adopted a number of simplifications throughout the calculation. Our threshold for distinguishing the flux change used only the information of integrated photon numbers, while X-ray telescopes can register the arrival time of X-ray photons, which can further help to distinguish a bright object from random fluctuation of background flux. To some extent, the SO can be treated as doing consecutive observations twice, that is the result of LO. Here we have discussed how the SO algorithm guarantees a sufficiently long interval between the first and second time observations so that a large P_2 is obtained. There are also some caveats in our calculation. For example, we totally ignore the rising and setting of sources. Although X-ray telescopes operate in the sky and the horizon is a less stringent constraint than for groundbased telescopes, the effect of the Sun, the Moon and the Earth is still an issue, which we aim to explore in followup studies. Also, the allocation of observation time in our analysis can be as short as 1 second, which might be not practical, and more realistic consideration of the minimum observation time may be required before actual implementation of our algorithm. We also assume that the joint probability distribution of sky location and distance is independent and we simply ignore the uncertainty of the simulated source distance R as well as the flux.

We refer the interested reader to strategies that can perform prompt and automatic observation adopted by some existing and future observatories, like e.g., Ghosh et al. (2016), Arcavi et al. (2017), Andreoni et al. (2021), Ghosh et al. (2017) and Graham et al. (2020). These algorithms can prioritize observations based on existing galaxy catalogue information, or incorporating realistic constraint into the tiling optimization. Based on the search for optical counterparts during O3b, Coughlin et al. (2020) points out that the telescope networks have the advantages in increasing coverage of the localization and thereby longer exposure times can be used, which finally leads to a corresponding increase in detection efficiencies. They explore a different dimension that we did not go in depth in this work, and we aim to implement such realistic strategies for specific telescopes in the future.

We expect a bright future for the detection of EM counterpart of GW triggers, using X-ray telescopes to search for the X-ray afterglows. With multiple telescopes, one can expect to significantly improve the detection capability. Some of the techniques developed for single telescopes have already been extended to a telescope network (e.g., Coughlin et al. 2019). Also, space-borne gravitational wave detectors like LISA and TianQin have the potential of predicting the merger with very high accuracy (Sesana 2016; Hu et al. 2017; Liu et al. 2020), and a coordinated observation can better capture the very early stage evolution of the event. All of these issues can help shape a more realistic and more promising future for successful multi-messenger astronomy.

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