

Observational scalings testing modified gravity

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Abstract We consider different observational effects to test a modified gravity approach involving the cosmological constant in the common description of dark matter and dark energy. We obtain upper limits for the cosmological constant by studying the scaling relations for 12 nearby galaxy clusters, the radiated power from gravitational waves and the Tully-Fisher relation for super spiral galaxies. Our estimations reveal that, for all these cases, the upper limits for Λ are consistent with its actual value predicted by cosmological observations.

Key words: Cosmology: dark energy — Cosmology: dark matter — Galaxies: clusters: general — Galaxies: general

1 INTRODUCTION

A new perspective for describing the dark sector - dark matter and dark energy - is provided by the modification of gravity based on Newton's theorem (Gurzadyan 2019; Gurzadyan & Stepanian 2018, 2019b). Namely, within that approach, the weak-field modification of General Relativity (GR) for a spherically symmetric case is written as

$$g_{00} = 1 - \frac{2Gm}{c^2 r} - \frac{\Lambda r^2}{3}; \quad g_{rr} = \left(1 - \frac{2Gm}{c^2 r} - \frac{\Lambda r^2}{3}\right)^{-1}, \quad (1)$$

where the cosmological constant Λ , as a fundamental constant (Gurzadyan & Stepanian 2019a), is entered self-consistently in the gravitational equations. In this sense, this metric can explain the accelerated expansion of the Universe and the dynamics of DM in astrophysical configurations (Gurzadyan & Stepanian 2018) simultaneously without any further free parameter.

The above metric is obtained by considering the most general function for force $\mathbf{F}(r)$ satisfying Newton's theorem which is equal to (Gurzadyan 2019; Gurzadyan & Stepanian 2018; Gurzadyan 1985)

$$\mathbf{F}(r) = \left(-\frac{A}{r^2} + Br\right) \hat{\mathbf{r}}. \quad (2)$$

Importantly, within the McCrea-Milne cosmology, the constant B in the second term in Equation (2)

Table 1 Background Geometries of GR According to the Sign of Λ

Sign	Background geometries		
	Spacetime	Isometry group	Curvature
$\Lambda > 0$	de Sitter (dS)	O(1,4)	+
$\Lambda = 0$	Minkowski (M)	IO(1,3)	0
$\Lambda < 0$	Anti de Sitter (AdS)	O(2,3)	-

corresponds to the cosmological constant Λ (Gurzadyan 2019; Gurzadyan & Stepanian 2018). Then, the sign of the cosmological constant Λ corresponds to the vacuum solutions for GR equations and their isometry groups as displayed in Table 1.

One of the remarkable features of Equation (2) is that the force, in contrast the pure Newtonian gravity, defines a non-force-free field inside a spherical shell. In fact, this feature agrees with observational indications that galactic halos do determine features of galactic disks (Kraivtsov 2013). In this sense the weak-field GR will be able to describe the observational features of galactic halos (Gurzadyan 2019; Gurzadyan et al. 2018) and of groups and clusters of galaxies (Gurzadyan & Stepanian 2019b).

The metric in Equation (1) is already known as the Schwarzschild-de Sitter solution. However, its deduction according to Newton's theorem enables one to describe the dynamics of astrophysical structures such as galaxy binaries, groups and clusters in the context of Λ -gravity (Gurzadyan 2019; Gurzadyan & Stepanian 2019b). The

current reported value of Λ (Planck Collaboration et al. 2016) is

$$\Lambda = 1.11 \times 10^{-52} \text{ m}^{-2}. \quad (3)$$

The previous studies have shown the possibility of the description of galaxy scale effects with metric (1) (Gurzadyan 2019; Gurzadyan et al. 2018; Gurzadyan & Stepanian 2019b). We now will aim to test certain observational scalings for galaxies and their clusters, including those for super spirals (SSs). The extreme observational samples are known for their efficiency, at least for ruling out certain models. We also consider data on gravitational waves (GWs).

2 SCALING RELATIONS

We use scaling relations from the data of Gopika & Desai (2020) for 12 galaxy clusters. Since all of them are considered to be virialized, one can apply the virial theorem in the context of metric (1), i.e.

$$\sigma^2 = \frac{GM}{r} - \frac{\Lambda c^2 r^2}{3}, \quad (4)$$

where σ is the velocity dispersion and $M = \frac{4\pi}{3} r_{\text{vir}}^3 \rho$ is total dynamical mass of the cluster, to obtain a possible constraint over the value of Λ . Thus, in the context of Λ -gravity, the scaling relations obtained in Gopika & Desai (2020) will be written as

$$\ln \left(\frac{\rho_c r_c}{M_{\odot} \text{pc}^{-2}} \right) = (-0.07^{+0.05}_{-0.06}) \ln \left(\frac{M_{200}}{M_{\odot}} \right) + (9.41^{+2.07}_{-1.80}). \quad (5)$$

Here, ρ_c and r_c stand for the central density and core radius respectively. In addition, M_{200} is the total mass enclosed in the radius r_{200} within which the total density is 200 times denser than the critical density, i.e. $H^2 = \frac{8\pi G \rho_{\text{crit}}}{3}$. Thus, by considering Equation (4) and Equation (5) we will get the following upper limit relation for Λ

$$\Lambda \leq \frac{1}{c^2} (600 \times H^2) \left(1 - \frac{1}{\left(\frac{\rho_c r_c}{(\rho_c + \mathbb{E}(\rho_c))(r_c + \mathbb{E}(r_c))} \right)^{0.07}} \right), \quad (6)$$

where \mathbb{E} stands for reported error. The results of calculations are presented in the following Table 2.

As one can see from the table, the obtained upper limits of Λ for each given cluster are in agreement with its current numerical value (Planck Collaboration et al. 2016).

Here, we want to stress that in general it is possible to study different samples of clusters with different parameters for scaling relations. However, the particular importance of the studied sample is the fact that being virialized structures we can rely on the basic fundamental and theoretical relations such as Equation (4) instead of different empirical models.

Table 2 Constraints Obtained for Λ by Studying the Scaling Relations

Upper limits for Λ			
Cluster	$\rho_c (10^{-3} M_{\odot} \text{pc}^{-3})$	r_c (kpc)	$\Lambda (m^{-2}) \leq$
A133	$11.68^{+0.02}_{-0.02}$	$102.01^{+0.08}_{-0.11}$	8.49×10^{-52}
A262	$5.17^{+0.87}_{-0.89}$	$136.36^{+5.40}_{-5.49}$	5.68×10^{-50}
A383	$9.63^{+0.62}_{-0.78}$	$121.45^{+3.95}_{-4.94}$	3.50×10^{-50}
A478	$3.39^{+0.72}_{-0.84}$	$286.14^{+30.41}_{-35.62}$	5.96×10^{-50}
A907	$4.15^{+0.42}_{-0.51}$	$208.96^{+10.66}_{-12.98}$	4.42×10^{-50}
A1413	$6.27^{+0.49}_{-0.53}$	$154.68^{+6.06}_{-6.61}$	3.27×10^{-50}
A1795	$7.15^{+0.68}_{-0.79}$	$131.89^{+6.32}_{-7.33}$	4.07×10^{-50}
A1991	$111.22^{+0.83}_{-0.92}$	$11.15^{+0.04}_{-0.04}$	4.20×10^{-50}
A2029	$9.39^{+0.66}_{-0.76}$	$134.31^{+4.72}_{-5.45}$	3.27×10^{-50}
A2390	$5.83^{+0.22}_{-0.23}$	$137.18^{+2.60}_{-2.81}$	1.69×10^{-50}
RX J1159+5531	$41.06^{+1.33}_{-1.19}$	$34.07^{+0.55}_{-0.49}$	1.12×10^{-50}
MKW 4	$102.4^{+0.92}_{-0.98}$	$10.31^{+0.04}_{-0.04}$	5.06×10^{-51}

Table 3 Constraints Obtained for Λ by Studying the GWs

Upper limits for Λ			
GW	P (erg s ⁻¹)	ω (Hz)	$\Lambda (m^{-2}) \leq$
GW 150914	$3.6^{+0.5}_{-0.4} \times 10^{56}$	75	2.08×10^{-14}
GW 151226	$3.3^{+0.8}_{-1.6} \times 10^{56}$	210	2.85×10^{-13}
GW 170104	$3.1^{+0.7}_{-1.3} \times 10^{56}$	80 – 99.5	$(3.85 – 5.96) \times 10^{-14}$
GW 170608	$3.4^{+0.5}_{-1.6} \times 10^{56}$	226.5 – 305	$(2.01 – 3.64) \times 10^{-13}$
GW 170814	$3.7^{+0.5}_{-0.5} \times 10^{56}$	77.5 – 101.5	$(2.16 – 3.71) \times 10^{-14}$

3 GRAVITATIONAL WAVES' RADIATED POWER

The existence of GWs is among the earliest predictions of GR, introduced by Einstein himself (Einstein 1916, 1918). However, it took almost 100 years before the first GWs were discovered. In the following, the physics of GWs is discussed in the presence of Λ .

Although several astrophysical scenarios can lead to the production of GWs, all the recently detected GWs have been produced by binary systems. Among them, five detections have confirmed that GWs are produced during the merging of two black holes (Abbott et al. 2016b,a, 2017a; LIGO Scientific Collaboration & Virgo Collaboration 2017; Abbott et al. 2017b). As one of the observable quantities, we study the radiated power in GW events. As a first step, we have to find the quadrupole moments \ddot{Q}_{ij} which are defined as

$$\overline{h_{ij}} = \frac{2G}{c^4 r} \ddot{Q}_{ij} \left(t - \frac{r}{c} \right) \quad i, j = 1, 2, 3 \quad (7)$$

where $\overline{h_{\mu\nu}} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu}$ which is called trace-reversed perturbation. The power radiated away during the merging process at null infinity is calculated utilizing the following formula

$$P = \frac{G}{5c^5} \left(\left(\frac{d^3 J_{ij}}{dt^3} \right)^2 \left(\frac{d^3 J^{ij}}{dt^3} \right)^2 \right) \quad (8)$$

Table 4 Constraints Obtained for Λ by Studying the Super Spiral Galaxies

Upper limits for Λ by considering the error limits on velocity				
Galaxy	$\log M_{\text{stars}}(M_{\odot})$	$\log M_{\text{gas}}(M_{\odot})$	r (kpc)	Λ (m^{-2}) \leq
2MASX J09394584+0845033	11.45	10.7	14	1.85×10^{-48}
SDSS J095727.02+083501.7	11.60	10.4	31	1.84×10^{-50}
2MASX J10222648+0911396	11.42	10.5	33	1.71×10^{-49}
2MASX J10304263+0418219	11.66	10.7	30	2.54×10^{-50}
2MASX J11052843+0736413	11.59	10.8	54	3.08×10^{-51}
2MASX J11232039+0018029	11.43	10.6	45	3.28×10^{-51}
2MASX J11483552+0325268	11.42	10.5	31	1.98×10^{-49}
2MASX J11535621+4923562	11.64	10.8	19	1.42×10^{-48}
2MASX J12422564+0056492	11.24	10.2	14	1.34×10^{-48}
2MASX J12592630–0146580	11.23	10.2	20	3.98×10^{-49}
2MASX J13033075–0214004	11.37	< 10.4	23	3.05×10^{-49}
SDSS J143447.86+020228.6	11.60	10.7	26	1.70×10^{-50}
2MASX J15154614+0235564	11.74	10.7	41	9.42×10^{-51}
2MASX J15404057–0009331	11.39	10.4	30	1.45×10^{-49}
2MASX J16014061+2718161	11.63	10.6	29	4.10×10^{-50}
2MASX J16184003+0034367	11.67	10.6	40	1.27×10^{-50}
2MASX J16394598+4609058	11.74	10.9	31	1.61×10^{-49}
2MASX J20541957–0055204	11.41	10.6	37	1.29×10^{-49}
2MASX J21362206+0056519	11.47	10.4	29	2.34×10^{-49}
2MASX J21384311–0052162	11.20	9.9	15	1.06×10^{-48}
2MASX J21431882–0820164	11.13	9.9	18	1.06×10^{-49}
2MASX J22073122–0729223	11.20	10.3	31	1.24×10^{-50}
2MASX J23130513–0033477	11.20	10.3	19	3.91×10^{-49}

where J_{ij} is the trace-free part of \ddot{Q}_{ij} . Thus, according to [Bonga & Hazboun \(2017\)](#), for two compact objects with mass M_1 and M_2 orbiting around a circle with radius r with an angular velocity ω , the radiated power P , up to first order in Λ , will be

$$P = \frac{32G}{5c^5} \left(\frac{M_1 M_2}{M_1 + M_2} \right)^2 a^4 \omega^6 \left(1 + \frac{5\Lambda c^2}{12\omega^2} \right). \quad (9)$$

Thus according to Ligo¹ for each case, we will have the limits for Λ illustrated in Table 3. In this case the obtained upper limits are much larger compared to other measurements which are due to the sensitivity and difficulty of detecting GWs. Namely, it should be recalled that in contrast to other measurements, the first evidence for the existence of GWs was observed several years ago. Accordingly, the accuracy of reported values as well as the corresponding errors are smaller than those of other experiments and observations. However, it is worth mentioning that despite the current obtained limits for Λ , the prospect of increasing the accuracy of measurements for detection and analysis of GWs can be regarded as an important and essential test for checking the validity of different modified theories of gravity.

For all of these cases, the error limits of Λ cover the current observed value. The importance of such

analysis lies in the fact that being totally independent from relativistic cosmology, it confirms the validity of the current value of Λ .

4 TULLY-FISHER RELATION FOR SUPER SPIRALS

In this section, we check the Tully-Fisher (TF) relation ([Tully & Fisher 1977](#)) for a recently analyzed group of galaxies called SSs ([Ogle et al. 2019](#)). Indeed, among these galaxies, those with stellar mass $M_s > 10^{11.5} M_{\odot}$ manifest a non-conventional behavior regarding the baryonic TF (BTF) index b . Namely, the established BTF index breaks from 3.75 ± 0.11 to 0.25 ± 0.41 above the rotational velocity of $\approx 340 \text{ km s}^{-1}$. Before starting the analysis, it should be stated that generally objects with some extreme nature/behavior are regarded as useful tools to pose constraints on the different parameters of various modified theories of gravity and even rule them out ([Islam & Dutta 2019](#); [Gurzadyan & Stepanian 2021](#)).

The BTF relation states that

$$M_{\text{baryonic}} \propto V_c^b \quad (10)$$

where V_c is the circular velocity of the galaxy. Thus, by replacing Newtonian gravity with Λ -gravity, we will have

¹ www.ligo.org/detections.php

Table 5 Constraints Obtained for Λ by Studying the Super Spiral Galaxies

Upper limits for Λ by considering the error limits on BTFR index				
Galaxy	$\log M_{\text{stars}}(M_{\odot})$	$\log M_{\text{gas}}(M_{\odot})$	r (kpc)	$\Lambda (m^{-2}) \leq$
2MASX J09394584+0845033	11.45	10.7	14	1.48×10^{-51}
SDSS J095727.02+083501.7	11.60	10.4	31	7.25×10^{-52}
2MASX J10222648+0911396	11.42	10.5	33	1.46×10^{-51}
2MASX J10304263+0418219	11.66	10.7	30	1.90×10^{-51}
2MASX J11052843+0736413	11.59	10.8	54	6.76×10^{-52}
2MASX J11232039+0018029	11.43	10.6	45	5.60×10^{-52}
2MASX J11483552+0325268	11.42	10.5	31	1.29×10^{-51}
2MASX J11535621+4923562	11.64	10.8	19	2.63×10^{-51}
2MASX J12422564+0056492	11.24	10.2	14	1.30×10^{-51}
2MASX J12592630–0146580	11.23	10.2	20	8.63×10^{-52}
2MASX J13033075–0214004	11.37	< 10.4	23	1.32×10^{-51}
SDSS J143447.86+020228.6	11.60	10.7	26	1.64×10^{-51}
2MASX J15154614+0235564	11.74	10.7	41	4.91×10^{-52}
2MASX J15404057–0009331	11.39	10.4	30	1.44×10^{-51}
2MASX J16014061+2718161	11.63	10.6	29	7.52×10^{-52}
2MASX J16184003+0034367	11.67	10.6	40	1.34×10^{-51}
2MASX J16394598+4609058	11.74	10.9	31	8.37×10^{-52}
2MASX J20541957–0055204	11.41	10.6	37	1.39×10^{-51}
2MASX J21362206+0056519	11.47	10.4	29	1.26×10^{-51}
2MASX J21384311–0052162	11.20	9.9	15	9.29×10^{-52}
2MASX J21431882–0820164	11.13	9.9	18	6.70×10^{-52}
2MASX J22073122–0729223	11.20	10.3	31	1.86×10^{-51}
2MASX J23130513–0033477	11.20	10.3	19	1.07×10^{-51}

the following relation

$$\left(\frac{V_c - \mathbb{E}(V_c)}{V_c}\right)^b \leq 1 - \frac{\Lambda c^2 r^3}{3GM_{\text{baryonic}}} \quad (11)$$

in which $\mathbb{E}(V_c)$ is the error limit of circular velocity reported from observations. Considering the above relation, we can obtain the upper limits of Λ . The results are illustrated in Table 3.

Meanwhile, it is also possible to obtain the upper limits of Λ by considering the maximum error limit of the BTFR index reported for SS galaxies, i.e.

$$\Lambda \leq (1 - V_c^{\mathbb{E}(b)}) \frac{3GM_{\text{baryonic}}}{c^2 r^3} \quad (12)$$

where $\mathbb{E}(b) = \pm 0.41$. Here it is important to mention that, since $\mathbb{E}(b) = 0.41$ is the maximum error for all SS galaxies and in principle it could have smaller error for each case, Equation (12) will give us the absolute upper limit for Λ ever possible to obtain based on the SS data. In this case, the obtained limits are expressed in Table 3.

Considering the obtained upper limits of Λ based on both analyses, we can conclude that they are fully consistent with the cosmological observations (Planck Collaboration et al. 2016).

5 CONCLUSIONS

We have studied the compatibility of modified gravity, i.e. weak-field GR with a cosmological constant, by

considering several types of observational data, i.e. the scaling relations of galaxy clusters, the radiated power from GWs and the BTFR relation. This paper continues previous analyses in which, by considering both relativistic and non-relativistic effects, various upper limits have been reported for the cosmological constant Λ (Kerr et al. 2003; Sereno & Jetzer 2006; Jetzer & Sereno 2007; Kagramanova et al. 2006; Stepanian & Khlgatyan 2020; Stepanian et al. 2021).

Since the formalism of Λ -gravity enables us to study both relativistic and non-relativistic effects, we have considered different tests corresponding to each of them. First, we have obtained constraints for Λ by analyzing the data of 12 galaxy clusters reported by the Chandra X-ray satellite. In this case, although the upper limits are very close to the current reported value of Λ , they mark no inconsistency with cosmological observations.

Next, we calculated the radiated power from GWs and obtained the limits of Λ accordingly. Although the obtained limits are less tight than the previous test, for all of the analyzed cases, the current reported value of Λ is smaller than the obtained upper limits and hence they fit the predictions of the considered modified gravity.

Finally, we have checked the recently analyzed SS galaxies. It should be stressed that due to their non-typical properties, the SSs are considered as one of the efficient

tests to check the validity of different theories of gravity, as also affirmed by our analysis.

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