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Time-dependent diffusive interactions between dark matter and dark energy in the context of k-essence cosmology

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Abstract We investigated the scenario of time-dependent diffusive interaction between dark matter and dark energy and showed that such a model can be accommodated within the observations of luminosity distance-redshift data in Supernova Ia (SN Ia) observations. We obtain constraints on different relevant parameters of this model from the observational data. We consider a homogeneous scalar field $\phi(t)$ driven by a k-essence Lagrangian of the form $L = V(\phi)F(X)$ with constant potential $V(\phi) = V$, to describe the dynamics of dark energy in this model. Using the temporal behaviour of the FRW scale factor, the equation of state and total energy density of the dark fluid, extracted from the analysis of SN Ia (JLA) data, we have obtained the time-dependence of the k-essence scalar field and also reconstructed the form of the function $F(X)$ in the k-essence Lagrangian.

Key words: Dark Energy — Dark Matter — k-essence

1 INTRODUCTION

In this work, we explore a scenario of interacting dark matter (DM) and dark energy (DE), where the DE is represented by a homogeneous scalar field ϕ with its dynamics driven by a k-essence Lagrangian with constant potential and DM is assumed to be a perfect fluid. In the late time phase of cosmic evolution, we can neglect the effect of radiation and baryonic matter on the total energy density of the universe. Thus for the late time phase, DE and DM are the dominant sectors to study the new consequences. Motivation behind consideration of such classes of a unified model of DM and DE is to address the issue of coincidence of observed present-day DE and DM densities utilizing a dynamical relation between DM and DE (Szydłowski & Stachowski 2016; Calogero 2011, 2012; Haba et al. 2016; Bandyopadhyay & Chatterjee 2019b). The interaction between DM and DE is assumed to happen through velocity diffusion of particles of DM fluid in the background of field ϕ. In an earlier work (Bandyopadhyay & Chatterjee 2019a), we studied implications of such a diffusive interaction with constant (time independent) diffusion coefficient. In this work, we revisit an interacting model of DE and DM with a time-dependent diffusion coefficient. We also explore the implications of measurements of luminosity distances and redshifts in supernova Ia (SN Ia) observations (Suzuki et al. 2012; Betoule et al. 2014; Wang et al. 2017; Wang 2008) in constraining different parameters for this model.

The basic framework of the model of DM-DE interactions has been discussed in detail in Bandyopadhyay & Chatterjee (2019a); Velten & Calogero (2014). We, first, indulge in a brief recollection of the model with an emphasis on the consideration of the time-dependence of the diffusion coefficient. We assume DM and DE to be perfect fluids and neglect contribution of radiation and baryonic matter during the late time phase of cosmic evolution (probed in SN Ia observations). In a flat Friedmann-Robertson-Walker (FRW) spacetime background, conservation of total energy momentum tensor for the dark fluid (DE + DM) takes the form

$$\left[\dot{\rho}_{\rm dm} + 3H\dot{\rho}_{\rm dm}\right] = -\left[\dot{\rho}_{\rm de} + 3H(\rho_{\rm de} + p_{\rm de})\right] = Q(t) \quad (1)$$

where $\rho_{\rm dm}$ and $\rho_{\rm de}$ are the energy densities of DM and DE fluid respectively. $p_{\rm de}$ is the pressure of the DE fluid and we take DM as pressureless dust. $H \equiv \dot{a}/a$ is the Hubble parameter, where $a(t)$ is the scale factor corresponding to FRW spacetime background. Here t is regarded as a dimensionless time parameter with $t = 1$ corresponding to the present epoch. The quantity $Q(t)$ in Equation (1) is a measure of rate of energy transfer between the fluid DM and DE caused by a diffusion at the instant of time t. We parametrise time dependence of the quantity $Q(t)$ in terms of a parameter k

$$Q(t) = Q_0 \big[a(t)\big]^k \tag{2}$$

where scale factor $a(t)$ has been taken to be normalised to unity at the present epoch, $a(t = 1) = 1$, and Q_0 is the value of $Q(t)$ at the present epoch. Applying the results of a comprehensive analysis of recently released "Joint Light-curve Analysis" (JLA) data (Betoule et al. 2014) consisting of 740 SN Ia events as described in Wang et al. (2017), the observed temporal behaviours of the quantities: the scale factor $a(t)$, the total energy density $(\rho_{\rm dm} + \rho_{\rm de})(t)$ and the equation of state $\omega(t)$ have been extracted over a time domain $0.44 < t < 1$ accessible in SNe Ia observations. Considering these observed time evolutions and the chosen form, Equation (2), of parametrisation of $Q(t)$ in Equation (1), we obtained the range of values of k for which the interacting DE-DM model with such a time-dependent diffusion term $Q(t)$ may be accommodated within the scheme of luminosity distance-redshift measurements of SN Ia observation. The methodology of obtaining such constraints is discussed comprehensively in Section 2.

We have assumed that the diffusive interaction, considered here, occurs in the background of a k-essence scalar field ϕ, whose dynamics are driven by a non-canonical Lagrangian $L = V(\phi)F(X)$, where $X = (1/2)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$. We take the potential $V(\phi)$ to be constant, which ensures existence of a scaling relation of the form $X(dF/dX)^2 = Ca^{-6}$, where C is a constant (Scherrer 2004; Chimento 2004). We then identify the stress energy tensor corresponding to the non-canonical Lagrangian with that of the DE fluid and consider the k-essence scalar field $\phi = \phi(t)$ to be homogeneous. Use of the scaling relation then enables us to establish relations between time derivative of scalar field and energy density and pressure of DE fluid. Utilizing the above relations along with the observed temporal behaviour of the quantities like $a(t)$, $\omega(t)$ and $\rho_{\rm dm} + \rho_{\rm de}$, we reconstructed the temporal behaviour of the k-essence scalar field ϕ as well as the form of the function $F(X)$. The results are obtained for different benchmark values of the parameter k with its allowed domain as obtained earlier. The quantity $Q(t)$ plays an important role in studying the interaction between DM and DE sectors. We have also calculated the energy transferred in these two sectors for some chosen values of the diffusion parameters. This is described in detail in Section 3.

2 OBSERVATIONAL CONSTRAINTS ON TIME-DEPENDENT DIFFUSIVE DE-DM INTERACTING MODEL

We use the measurements of luminosity distance and redshift in SN Ia observations for redshift values up to $z \approx 1$ to extract features of the late time phase of cosmic evolution. These data are instrumental in obtaining the temporal behaviour of FRW scale factor $a(t)$. There exist different compilations of SN Ia data from different surveys. These include high redshift ($z \sim 1$) projects *viz.* Supernova Legacy Survey (SNLS) (Astier et al. 2006; Sullivan et al. 2011), the ESSENCE project (Wood-Vasey et al. 2007), the Pan-STARRS survey (Tonry et al. 2012; Scolnic et al. 2014; Dotson et al. 2018); intermediate redshift ($0.05 < z < 0.4$) projects *viz.* Sloan Digital Sky Survey (SDSS)-II supernova surveys (Kessler et al. 2009; Sollerman et al. 2009; Lampeitl et al. 2010; Campbell et al. 2013) and small redshift programmes such as surveys like Harvard-Smithsonian Center for Astrophysics survey (Hicken et al. 2009), the Carnegie Supernova Project (Contreras et al. 2010; Folatelli et al. 2010; Stritzinger et al. 2011), the Lick Observatory Supernova Search (Ganeshalingam et al. 2013) and the Nearby Supernova Factory (Aldering et al. 2002). Various other compilations of SN Ia data may be found in Leibundgut et al. (1996); Goldhaber et al. (2001); Foley et al. (2005); Hook et al. (2005); Blondin et al. (2006); Li et al. (2000); Filippenko (2005); Wood-Vasey et al. (2004); Frieman et al. (2008); Dilday et al. (2012); Lidman (2004); Hamuy et al. (1996); Miknaitis et al. (2007); Matheson et al. (2005); Li et al. (2001, 2003); Jha et al. (2006); Percival et al. (2001); Voit (2005); Linder (2003). All such surveys include nearly one thousand SN Ia events. In this work we used the results from analysis of JLA data (Suzuki et al. 2012; Betoule et al. 2014; Wang et al. 2017; Wang 2008) consisting of 740 data points from a three-year survey from SDSS, five-year SNLS survey and 14 very high redshift $0.7 < z < 1.4$ SNe Ia from space-based observations with the Hubble Space Telescope (HST) (Riess et al. 2009). The methodology of analysis of JLA data has been described in detail in Wang et al. (2017); Bandyopadhyay & Chatterjee (2020) where a χ^2 function for the JLA data is defined as

$$\chi^2_{\rm SN}(\alpha,\beta) = \sum_{i,j}\left(\mu^{(i)}_{\rm obs} - \mu^{(i)}_{\rm th}\right)\left(\Sigma^{-1}\right)_{ij}\left(\mu^{(j)}_{\rm obs} - \mu^{(j)}_{\rm th}\right) \tag{3}$$

where i and j run from 1 to 740 corresponding to 740 SN Ia events contained in the JLA data set (Betoule et al. 2014). $\mu^{(i)}_{\rm obs}$ is the observed value of distance modulus at a redshift z_i corresponding to the $i^{\rm th}$ entry of the JLA data set and $\mu_{\rm th}^{(i)}$ is the corresponding theoretical estimate expressed through an empirical relation expressed in terms of various parameters. Σ is the total covariance matrix given in terms of statistical and systematic uncertainties (see Wang et al. 2017 for details). Wang et al. (2017) performed the marginalisation of the χ^2 function of Equation (3), over the parameters of the theory to obtain best-fit values of the parameters. The estimated value of the quantity $E(z) = H(z)/H_0$ at the best-fit has been displayed in the left panel of figure 5 of Wang et al. (2017), where $H = \dot{a}/a$ is the Hubble parameter at redshift z and H_0 is its value at the present epoch ($z = 0$). We have used the results of analysis obtained in Wang et al. (2017) along with the relations $H = \dot{a}/a$ and $1/a = 1 + z$ (value of the FRW scale factor at present epoch is normalised to unity) and write $dt = -\frac{dz}{(1+z)H_0E(z)}$ which on integration gives

$$\frac{t(z)}{t_0} = 1 - \frac{1}{H_0 t_0}\int_z^0 \frac{dz'}{(1+z')E(z')} \tag{4}$$

where t_0 is the time denoting the present epoch. We utilize the function $E(z)$ as obtained from best-fit in analysis of JLA data in Wang et al. (2017) in Equation (4), and obtain t as a function of z by performing the integration numerically. We then eliminate z from the obtained z-$t(z)$ dependence and the equation $1/a = 1 + z$ to obtain scale factor $a(t)$ as a function of t. The obtained temporal profile may be used to find the time-dependence of the Hubble parameter $H \equiv \dot{a}/a$ which directly governs the cosmological dynamics through Friedmann equations. In the right panel of Figure 1, we have plotted the observed temporal behaviour of H in terms of a newly introduced time parameter τ, defined as

$$\tau = \ln a. \tag{5}$$

Note that the present epoch corresponds to $\tau = 0$ as the scale factor at the present epoch is normalised to unity.

The two independent Friedmann equations governing dynamics of late time cosmic evolution in FRW spacetime background are

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}(\rho_{\rm de} + \rho_{dm}); \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\left[(\rho_{\rm dm} + \rho_{\rm de}) + 3p_{\rm de}\right]. \end{aligned} \tag{6}$$

Here we consider a flat spacetime background (zero curvature constant) and neglect contributions from radiation and baryonic matter during late time phase of cosmic evolution. Employing Equation (6), the equation of state of total dark fluid may be expressed as

$$\begin{aligned} \omega \equiv \frac{p_{\rm de}}{\rho_{\rm de} + \rho_{\rm dm}} &= -\frac{2}{3}\frac{\ddot{a}a}{\dot{a}^2} - \frac{1}{3} \\ &= -\frac{2}{3}\frac{a(a'HH' + H^2a'')}{(a'H)^2} - \frac{1}{3} \end{aligned} \tag{7}$$

where in the last expression, the time variable has been changed from t to τ and $'$ corresponds to derivatives with respect to τ. We apply the time dependence of the scale factor $a(\tau)$ extracted from the analysis of JLA data to obtain the temporal behaviour of the equation of state $\omega(\tau)$ of the dark fluid. The derived time-dependence is depicted in the left panel of Figure 1.

Transforming the time variable from t to τ and substituting $\omega = \frac{p_{\rm de}}{\rho_{\rm de}+\rho_{\rm dm}}$ in the total continuity equation (Eq. (1)) of the dark fluid, we find

$$\frac{d}{d\tau}\ln(\rho_{\rm de} + \rho_{\rm dm}) = -3(1 + \omega(\tau)), \tag{8}$$

and performing integration over τ we arrive at

$$\begin{aligned} \rho(\tau) &\equiv \Big[\rho_{\rm de} + \rho_{\rm dm}\Big]_\tau \\ &= \Big[\rho_{\rm de} + \rho_{\rm dm}\Big]_0 \exp\left[-3\int_{\tau'=0}^{\tau}(1 + \omega(\tau'))d\tau'\right]. \end{aligned} \tag{9}$$

We use the obtained τ-dependence of the function $\omega(\tau)$ as extracted from SN Ia data in Equation (9) to reveal temporal behaviour of the total density ($\rho_{\rm de} + \rho_{\rm dm}$) of the dark fluid. We display this dependence in the middle panel of Figure 1.

As already mentioned in Section 1, we assume the quantity $Q(t)$ in Equation (1), which is a measure of rate of energy transfer between the fluid DM and DE, to be time-dependent and parametrise the time dependence as

$$Q(t) = Q_0\Big[a(t)\Big]^k \tag{10}$$

where k is a constant. In terms of the time parameter τ, the above equation reads $Q(\tau) = Q_0 e^{k\tau}$ and Equation (1) takes the following form,

$$\frac{d\rho_{\rm dm}}{d\tau} + 3\rho_{\rm dm} = Q_0\frac{e^{k\tau}}{H(\tau)}. \tag{11}$$

We assume a trial solution of Equation (11) for $\rho_{\rm dm}(\tau)$ as

$$\rho_{\rm dm}(\tau) = \Big[\rho_{\rm de} + \rho_{\rm dm}\Big]_0 \sum_{i=0}^{\infty}\alpha_i\tau^i \tag{12}$$

and express the temporal behaviour of $H(\tau)$ as extracted from JLA data in the form of a polynomial as

$$\frac{1}{H(\tau)} = \sum_i \gamma_i\tau^i \tag{13}$$

where the coefficients γ_i are obtained by fitting the above polynomial with the temporal behaviour of the reciprocal of the function $H(\tau)$ displayed in the right panel of Figure (1). The obtained best-fit values of the γ_i parameters are given in Table 1. We use Equations (12) and

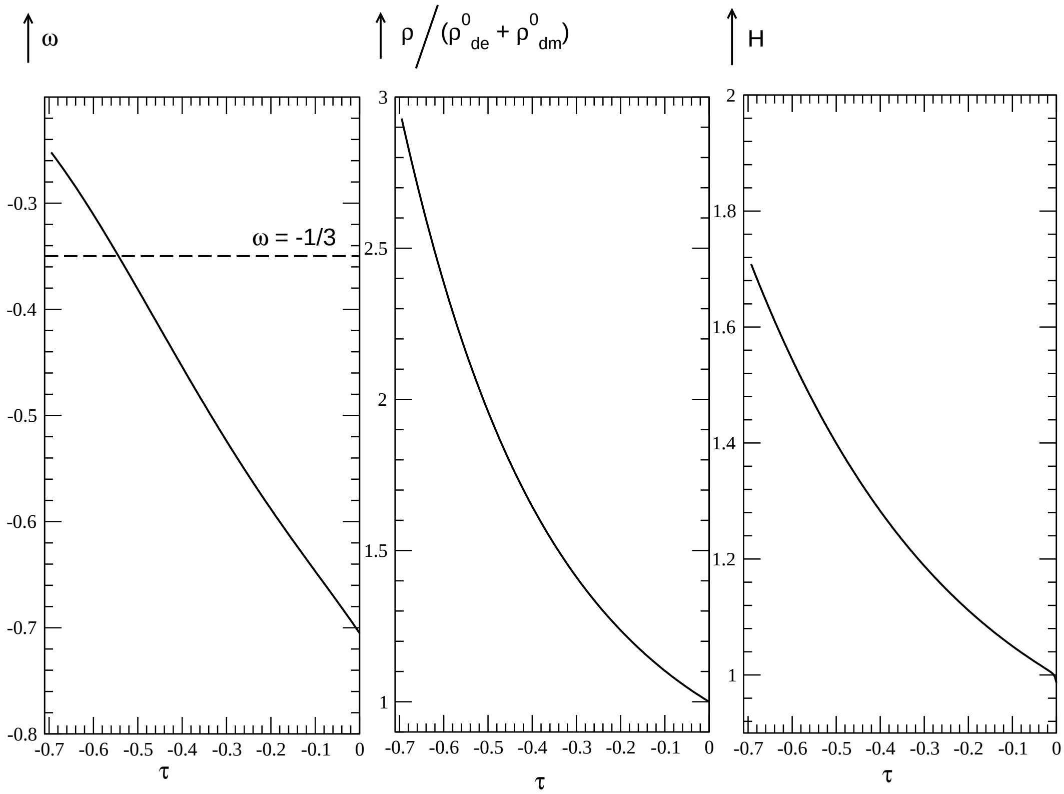

Fig. 1 Plot of ω, $\rho/(\rho^0_{\rm de}+\rho^0_{\rm dm})$ and H as a function of time parameter τ as extracted from JLA data, from *left* to *right* respectively.

Table 1 Values of γ_i's in Equation (13) Providing the Best Fit to the Values of $\frac{1}{H(\tau)}$ Extracted from Analysis of JLA Data

$\gamma_0 =$	1.00118	$\gamma_2 =$	–0.321574	$\gamma_i = 0$
$\gamma_1 =$	0.449751	$\gamma_3 =$	–0.149526	for $i > 3$

(13) in Equation (11) to produce

$$\sum_{i=0}^{\infty} i\alpha_i\tau^{i-1} + 3\sum_{i=0}^{\infty}\alpha_i\tau^i = \beta_0\sum_{i=0}^{3}\gamma_i\tau^i\sum_{j=0}^{\infty}\frac{k^j\tau^j}{j!} \quad (14)$$

where,

$$\beta_0 \equiv Q_0/[\rho_{\rm de}+\rho_{\rm dm}]_0 \quad (15)$$

is the ratio of the value of Q to that of total dark fluid energy density at the present epoch. Equating the coefficients of τ^i from both sides of Equation (14), we find

$$\alpha_{i+1} = \beta_0\sum_{n=0}^{i}\frac{\gamma_n k^{i-n}}{(i-n)!(i+1)} - \frac{3\alpha_i}{i+1}. \quad (16)$$

We note from Equation (12) that

$$\alpha_0 = \frac{\rho^0_{\rm dm}}{[\rho_{\rm de}+\rho_{\rm dm}]_0} \quad (17)$$

is the fraction of DM energy density contribution to the total dark fluid density at the present epoch which has the mathematically allowed domain of $0 \leqslant \alpha_0 \leqslant 1$. However, WMAP (Hinshaw et al. 2013) and Planck (Planck Collaboration et al. 2014) measurements established that the observed value of the fraction α_0 is ~ 0.27. For given values of α_0, k and β_0, one may find α_i's ($i > 0$) using Equation (16). The series $\{\alpha_i\}$ will always converge as γ_i's are zero for $i > 3$ (see Table 1) and $(i+1)$ appears in the denominator of the recursion relation (Eq. (16)). Utilizing the evaluated series $\{\alpha_i\}$, we apply Equation (12) to obtain the values of $\rho_{\rm dm}(\tau)$ at any τ, corresponding to any given set of values of (α_0, k, β_0). Since $|\tau| < 1$ and the series $\{\alpha_i\}$ converges, the numerical value of $\rho_{\rm dm}(\tau)$ will have a negligible contribution from the terms above certain order in the summation series of Equation (12).

The temporal behaviour of energy density $\Big[\rho_{\rm dm} + \rho_{\rm de}\Big]_\tau$ of the total dark fluid has already been obtained directly from the analysis of JLA data (middle panel of Fig. 1). The estimated value of the DM density $\rho_{\rm dm}(\tau;\alpha_0,k,\beta_0)$ computed from Equation (12) for given sets of values of (α_0, k, β_0) is subject to the constraint

$$0 < \rho_{\rm dm}(\tau;\ \alpha_0,k,\beta_0) < \Big[\rho_{\rm dm}+\rho_{\rm de}\Big]_\tau. \quad (18)$$

The above constraints put limits on the range of allowed values of α_0, k and β_0. The range of values of the parameters (α_0, k, β_0) for which the condition in Equation (18) is satisfied corresponds to the values of the parameters for which the scenario of interacting DE-DM with time-dependent diffusion coefficient may by accommodated with the luminosity distance-redshift measurements of the SN Ia events in the JLA data. The constraints on parameters (α_0, k, β_0), thus procured, are presented in Figures 2 and 3. In Figure 2, we display the allowed domain of $\alpha_0 - k$ parameter space for four different benchmark values of the parameter β_0 (*viz.* 0.1, 1.0, 5.0, 10.0). In Figure 3, the allowed domain in $\beta_0 - k$ parameter space is shown for $\alpha_0 = 0.27$.

3 REALISATION IN TERMS OF K-ESSENCE COSMOLOGY

We also try to realise the diffusive interaction between DE and DM with time-dependent diffusion coefficient, in terms of a k-essence scalar field ϕ representing the dynamics of DE. The scalar field ϕ plays the role of background medium in which diffusion takes place.

3.1 Theoretical Framework of k-essence Model

The minimally coupled (with gravitational field $g_{\mu\nu}$) action for a k-essence scalar field ϕ is written as,

$$S_k = \int d^4x\sqrt{-g}\mathcal{L}(X,\phi) \tag{19}$$

where kinetic term $X \equiv \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$, g is the determinant of the metric $g_{\mu\nu}$ and ∇_μ represents covariant derivative associated with metric $g_{\mu\nu}$. The total action for gravitational field $g_{\mu\nu}$ and k-essence scalar field is expressed as,

$$S_k = \int d^4x\sqrt{-g}\left[-\frac{R}{16\pi G_N} + \mathcal{L}(X,\phi)\right] \tag{20}$$

where G_N is Newton's gravitational constant. We derive the expression for energy momentum tensor for the k-essence field by varying the action with respect to the field $g_{\mu\nu}$

$$\begin{aligned} T_{\mu\nu} &\equiv \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \\ &= \frac{\partial\mathcal{L}}{\partial X}\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\mathcal{L}. \end{aligned} \tag{21}$$

The energy-momentum tensor Equation (21) may be written in the form of a perfect fluid as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu} \tag{22}$$

where u_μ is the effective four-velocity given by

$$u_\mu = \mathrm{sgn}(\partial_0\phi)\frac{\nabla_\mu\phi}{\sqrt{2X}}, \tag{23}$$

the pressure p of the fluid is the Lagrangian density,

$$p = \mathcal{L}(\phi, X) \tag{24}$$

and the energy density is

$$\rho = 2X\frac{\partial p}{\partial X} - p. \tag{25}$$

We use the form of k-essence model Lagrangian as

$$\mathcal{L}(\phi, X) = V(\phi)F(X). \tag{26}$$

For the k-essence Lagrangian in the form $\mathcal{L}(\phi, X) = V(\phi)F(X)$, the pressure and energy density of the fluid (whose energy momentum is equivalent to that of the k-essence field) may be written from Equations (24) and (25) as

$$p = V(\phi)F(X) \tag{27}$$
$$\rho = V(\phi)(2XF_X - F) \tag{28}$$

where $F_X = dF/dX$. In this way, we can relate the DE fluid with k-essence scalar field ϕ.

3.2 Diffusion in the Background of k-essence Scalar Field

We assume the field ϕ to be spatially homogeneous ($\phi = \phi(t)$) and the potential $V(\phi)$ to be constant (V). These respectively imply

$$X = (1/2)\dot{\phi}^2 \tag{29}$$

and existence of a scaling relation in the form

$$XF_X^2 = Ca^{-6}, \quad \text{where } C \text{ is a constant.} \tag{30}$$

Considering the time-dependences of various cosmological parameters extracted from SN Ia data as described in Section 2, we obtained the constraints on the temporal behaviour of the field ϕ and the form of the function $F(X)$ for different modes of time dependences of the diffusion coefficient, characterised in terms of the parameter k. The methodology of obtaining such results is described below.

Recalling Equations (27), (28) and (30), we can write,

$$X = \frac{a^6(\rho_{\rm de} + p_{\rm de})^2}{4CV^2}. \tag{31}$$

Using Equation (29) in Equation (31) and changing the time parameter from t to τ (as given in Eq. (5)) we obtain

$$\begin{aligned} &\left[\frac{\sqrt{2C}V}{(\rho^0_{\rm dm} + \rho^0_{\rm de})}\right]\left(\frac{d\phi}{d\tau}\right) \\ &= \frac{a^3}{H}\left[\frac{\rho_{\rm de}}{(\rho^0_{\rm dm} + \rho^0_{\rm de})} + \frac{p_{\rm de}}{(\rho^0_{\rm dm} + \rho^0_{\rm de})}\right] \end{aligned} \tag{32}$$

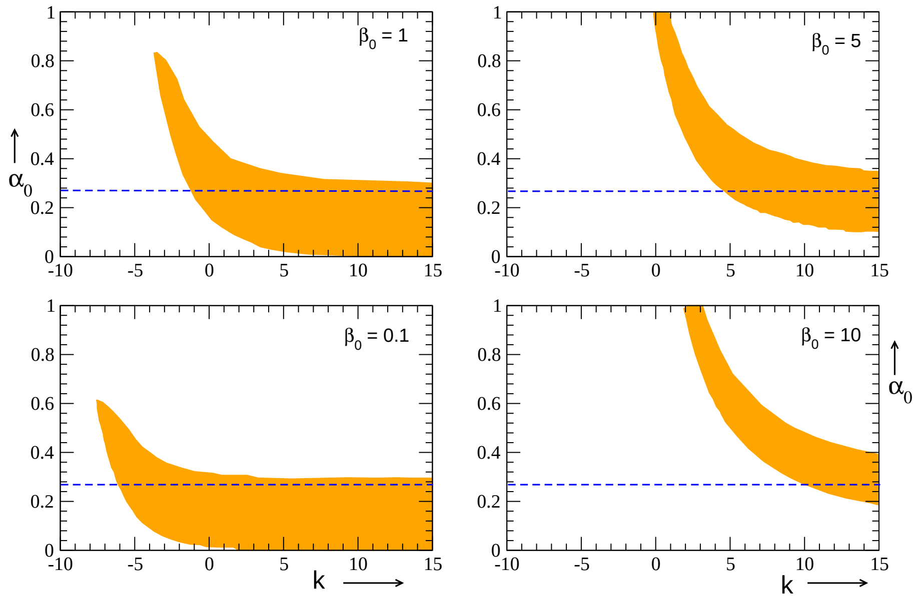

Fig. 2 Allowed regions in $\alpha_0 - k$ parameter space for different benchmark values of β_0.

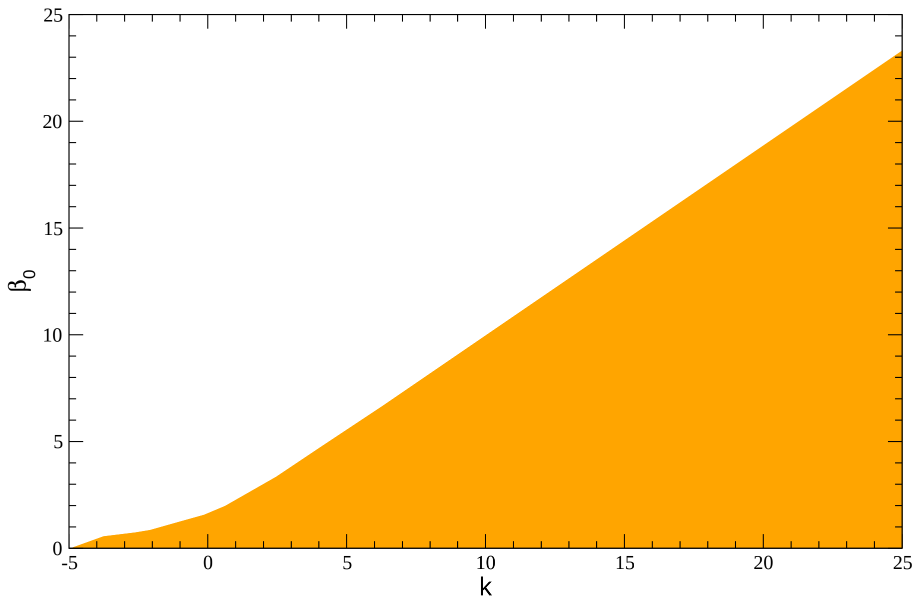

Fig. 3 Allowed region in $\beta_0 - k$ parameter space for $\alpha_0 = 0.27$.

which on integration yields

$$\left[\frac{\sqrt{2C}V}{(\rho^0_{\rm dm}+\rho^0_{\rm de})}\right](\phi(\tau)-\phi_0)$$
$$=\int_{\tau'=0}^{\tau} d\tau' \left[\frac{a^3(\tau')}{H(\tau')}\left(\frac{\rho_{\rm de}(\tau')}{\rho^0_{\rm dm}+\rho^0_{\rm de}}+\frac{p_{\rm de}(\tau')}{\rho^0_{\rm dm}+\rho^0_{\rm de}}\right)\right]. \tag{33}$$

The right hand side of Equation (33) contains the quantities a and H whose observed τ-dependences have already been found from the analysis of JLA data as discussed in Section 2. Since $\omega = p_{\rm de}\big/(\rho_{\rm dm}+\rho_{\rm de})$, the last term on the right hand side of the above equation, $p_{\rm de}(\tau)\big/(\rho^0_{\rm dm}+\rho^0_{\rm de})$, may be expressed as $\omega(\tau)\rho(\tau)\big/(\rho^0_{\rm dm}+\rho^0_{\rm de})$. This τ-dependence is available from observation as the dependences $\omega(\tau)$ and $\rho(\tau)\big/(\rho^0_{\rm dm}+\rho^0_{\rm de})$ are separately known from observation as shown earlier (middle panel of Fig. 1). To evaluate the other remaining term in Equation (33), $\rho_{\rm de}(\tau)\big/(\rho^0_{\rm dm}+\rho^0_{\rm de})$, we may

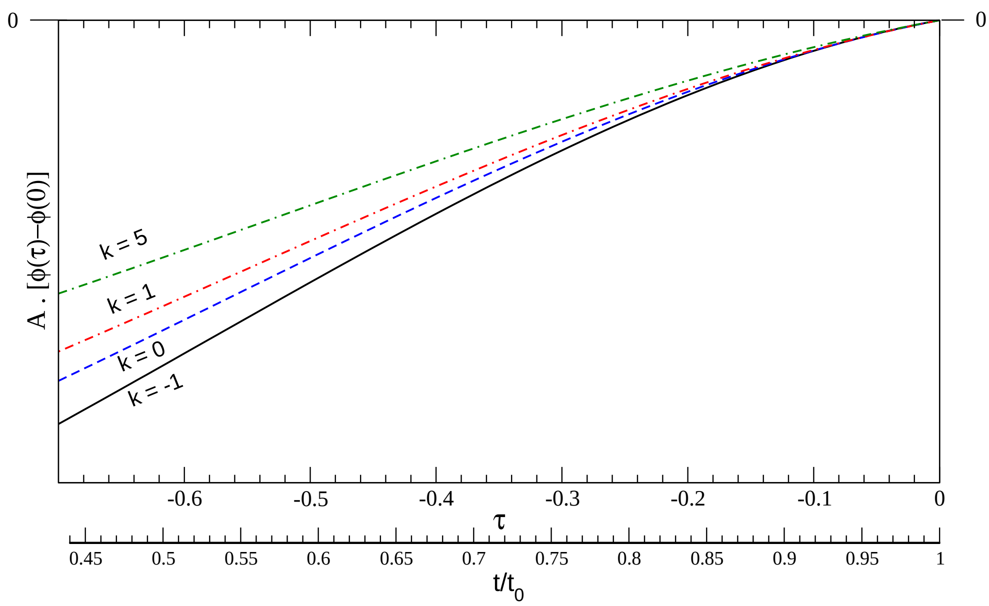

Fig. 4 Variation of the scalar field ϕ with time parameter τ and t/t_0.

compute DM density $\rho_{\rm dm}(\tau;\alpha_0,k,\beta_0)\Big/(\rho^0_{\rm dm}+\rho^0_{\rm de})$ from Equation (12), corresponding to a set of values of parameters (α_0,k,β_0) within their corresponding domains allowed from JLA data as depicted in Figure 2. The DE density $\rho_{\rm de}$ may also be evaluated at a given (α_0,k,β_0) value as

$$\frac{\rho_{\rm de}(\tau;\alpha_0,k,\beta_0)}{(\rho^0_{\rm dm}+\rho^0_{\rm de})}=\frac{\big[\rho_{\rm dm}+\rho_{\rm de}\big]_\tau}{(\rho^0_{\rm dm}+\rho^0_{\rm de})}-\frac{\rho_{\rm dm}(\tau;\alpha_0,k,\beta_0)}{(\rho^0_{\rm dm}+\rho^0_{\rm de})}. \tag{34}$$

The τ-dependence of the k-essence field, thus computed performing the integration in Equation (33), would be dependent on the parameter values α_0, k and β_0. To depict the temporal behaviour of the scalar field ϕ, we have set $\beta_0 = 1$ and fix α_0 at its experimentally observed value at 0.27 (see discussion after Eq. (17)). The obtained τ dependence of the scalar field ϕ for different benchmark values of the parameter k is shown in Figure 4. We have considered the time dependence in terms of both the time parameters $\tau = \ln a(t)$ and t. We find that, for any value of the diffusion parameter k, the time dependence of the k-essence scalar field ϕ may be fitted in terms of polynomial of degree 2 as

$$\phi(t/t_0,k)-\phi_0(k)=\varepsilon_1(k)\,(t/t_0-1)+\varepsilon_2(k)\,(t/t_0-1)^2 \tag{35}$$

where $\phi_0(k)$ is the value of the field at the present epoch ($t = t_0$), and the coefficients $\varepsilon_1(k)$ and $\varepsilon_2(k)$ are functions of k. We have also computed the values of these coefficients at different values of the parameter k and k-dependences of the coefficients $\varepsilon_{1,2}$ are illustrated in Figure 5.

Finally, in the context of realisation of time dependent diffusive DE-DM interactions in terms of dynamics of a k-essence field, we have also identified the form of the function $F(X)$ appearing in the k-essence Lagrangian. We mentioned below how this is done using the inputs from the observational data. Utilizing Equations (27) and (28), and the equation $\omega(\tau) = p_{\rm de}(\tau)/\rho(\tau)$, we may write

$$F(X)V \;=\; \omega(\tau)\frac{\rho(\tau)}{(\rho^0_{\rm de}+\rho^0_{\rm dm})} \tag{36}$$

and Equation (31) may be rewritten in the form

$$XV_1 \;=\; a^6\left[\frac{\rho_{\rm de}(\tau;\alpha_0,\beta_0,k)}{\rho^0_{\rm de}+\rho^0_{\rm dm}}+\frac{\omega(\tau)\rho(\tau)}{\rho^0_{\rm de}+\rho^0_{\rm dm}}\right]^2 \tag{37}$$

where $V_1 = \frac{4CV^2}{(\rho^0_{\rm de}+\rho^0_{\rm dm})^2}$ is a constant. Considering Equation (34) and observed τ dependences of $a(\tau)$, $\omega(\tau)$ and $\rho(\tau)$ as extracted from JLA data in Equations (36) and (37), we compute values of both the quantities $F(X)V$ and XV_1 at different values of τ corresponding to any given set of values of (α_0,β_0,k). We then eliminate τ from Equations (36) and (37) to obtain dependence of $F(X)V$ as a function of XV_1. Thus, form of the function $F(X)$ has been extracted from observational data up to the undetermined constants V and V_1. We display the variation of $F(X)$ with X in Figure 6 for different values of k evaluated at $\beta_0 = 1$ and $\alpha_0 = 0.27$.

Figure 7 depicts the diffusive activity between the dark sectors. The energy has been transferred from DM to DE

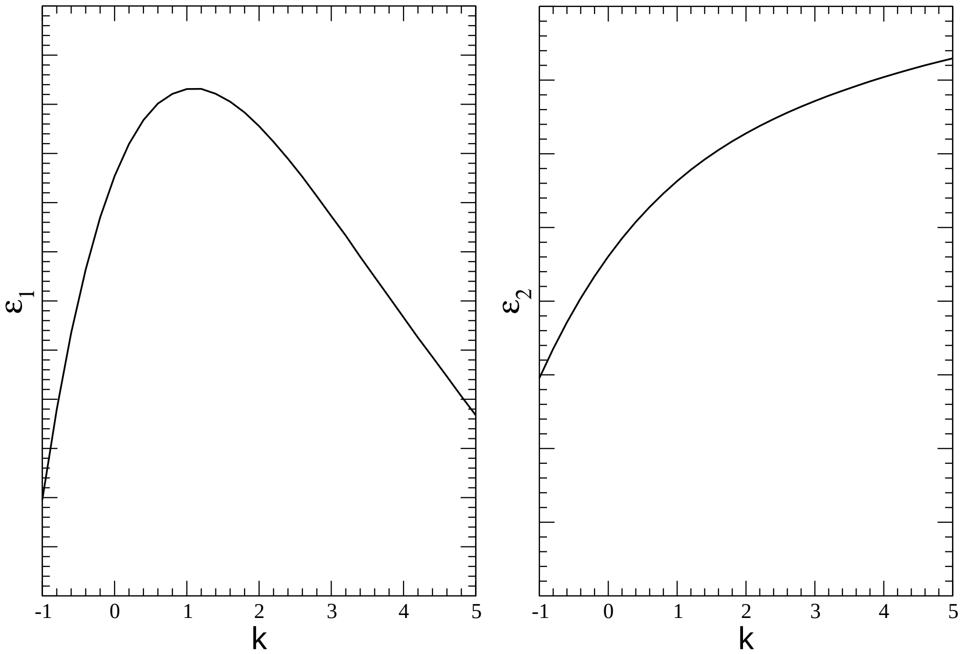

Fig. 5 Variation of the coefficients $\varepsilon_1, \varepsilon_2$ (in arbitrary units) with k.

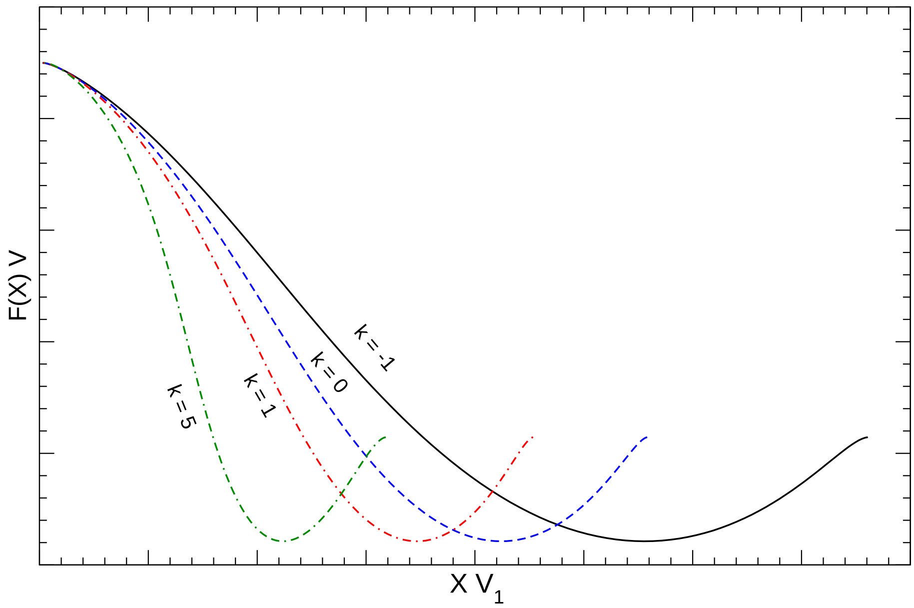

Fig. 6 Variation of reconstructed $F(X)V$ with XV_1 for different values of k. V and V_1 are constants (see text for details).

for the chosen benchmark values of the diffusion parameter β_0 (for fixed k and α_0). For $\beta_0 = 0$, the continuity equation of DM gives $\rho_{\rm dm} \propto \frac{1}{a^3}$ which corresponds to the non-interacting scenario between these two sectors. We also choose two non-zero (positive) values of β_0 to realise the interacting nature of this model. The figure demonstrates that larger values of β_0 correspond to a higher amount of transferred energy from DM to DE. The figure also affirms that DM density is always lower than DE density for the higher values of β_0 at any epoch.

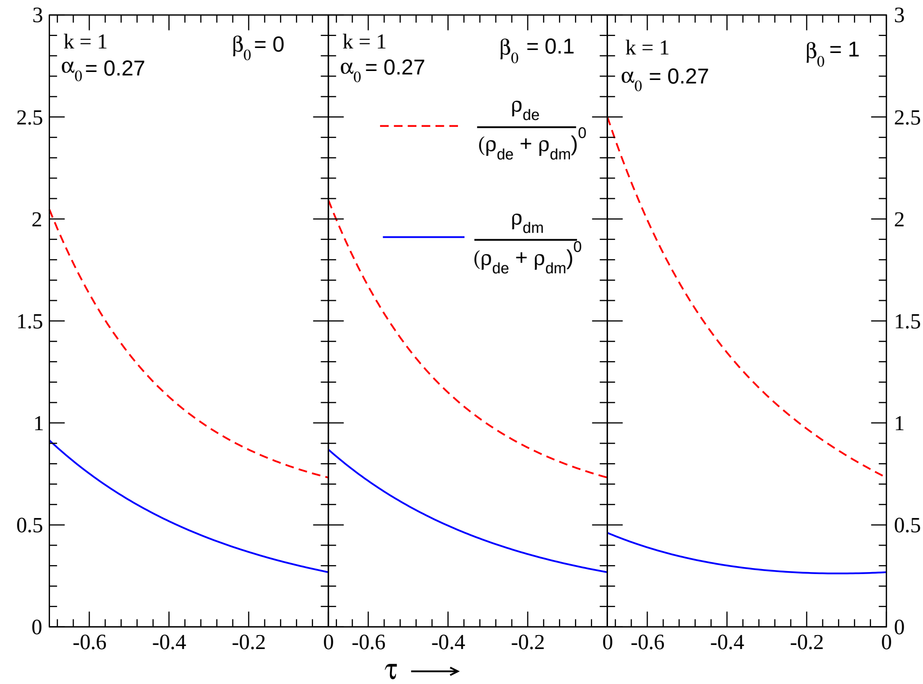

Fig. 7 Temporal behaviour of the energy density of DM and DE for the benchmark values of diffusion parameters (k, α_0, β_0).

4 CONCLUSIONS

In this work, we study the scenario of interacting DE and DM, with a time-dependent diffusive interaction between them. We have demonstrated that such a model can be accommodated within the observations of luminosity distance-redshift data in SN Ia events. We obtain the constraints on different relevant parameters of this model from the observational data. The two parameters of the model which are relevant in the context of this work are Q_0 and k. They parametrise the rate of exchange of energy due to diffusion from DM to DE in a form $Q_0[a(t)]^k$ appearing in the non-conservative Equation (1). For convenience, instead of working with the parameter Q_0, we have chosen $Q_0/(\rho^0_{\rm de} + \rho^0_{\rm dm}) \equiv \beta_0$ as the parameter. Besides β_0 and k, the parameter $\alpha_0 \equiv \rho^0_{\rm dm}/(\rho^0_{\rm de} + \rho^0_{\rm dm})$, which is approximately the fractional contribution of DM density to the total density at the present epoch, also appears in the framework of our analysis.

We have investigated the temporal behaviour of the Hubble parameter, H, total equation of state (ω) and total energy density (ρ) of the dark fluid as extracted from JLA data in Figure 1. We have exploited these dependences to obtain constraints on the above mentioned parameters β_0, k and α_0. The obtained constraints are depicted in Figures 2 and 3. The value of the parameter α_0 is, however, independently determined from WMAP and Planck experiments as $\alpha_0 \approx 0.27$. The results indicate

Table 2 Lower Limit of Allowed Values of Parameter k for Different Values of β_0 with $\alpha_0 = 0.27$

Values of β_0	0.1	1.0	5.0	10.0
Lower limit of k	–6.13	–1.11	4.76	9.89

that, if we choose this value of $\alpha_0 (\approx 0.27)$, the allowed values of the parameter k have a lower limit. For example, for different values of β_0, the obtained values of the lower limit of k are given in Table 2.

In addition to this, in the context of the interacting DE-DM model considered here along with the constraints on relevant parameters of the model from observational data, we also address certain issues related to k-essence scalar field model of DE. We consider a homogeneous scalar field ϕ driven by a k-essence Lagrangian (with constant potential) to represent dynamics of DE. We assume, in the DE-DM interacting scenario considered here, the diffusion from DM takes place in the background medium of the scalar field. Using the observational features of the cosmological parameters as extracted from the JLA data as inputs, we find constraints on the time-dependence of the field ϕ. The existence of scaling relation (Eq. (30)), owing to constancy of the potential V in the k-essence Lagrangian, also enables us to obtain the form of the function $F(X)$ appearing in the Lagrangian. We found that the temporal behaviour of the scalar field, in this context, may be very convincingly accommodated within a profile $\phi(t/t_0, k) - \phi_0(k) = \varepsilon_1(k)(t/t_0 - 1) +$

$\varepsilon_2(k)(t/t_0-1)^2$, where coefficients $\varepsilon_{1,2}$ are dependent on the (diffusion) parameter k and these dependences are also derived utilizing inputs from observation (see Fig. 5). The obtained form of the function $F(X)$ for different values of the parameter k are depicted in Figure 6. Diffusive nature of DM and DE components have been achieved through the parametrised form of $Q(t)$, and Figure 7 demonstrates the validity of our model in both non-interacting and interacting scenarios.

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