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Photometric light curve solutions of three ultra-short period eclipsing binaries

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Abstract   We present the results of our study of the eclipsing binary systems CSS J112237.1+395219, LINEAR 1286561 and LINEAR 2602707 based on new CCD B, V, R_c and I_c complete light curves. The ultra-short period nature of these stars, as reported by Drake et al., is confirmed and the system's periods are revised. The light curves were modeled using the 2003 version of the Wilson-Devinney code. When necessary, cool spots on the surface of the primary component were introduced to account for asymmetries in the light curves. As a result, we found that CSS J112237.1+395219 is a W UMa type contact binary system belonging to W subclass with a mass ratio of $q = 1.61$ and a shallow degree of contact of 14.8% where the primary component is hotter than the secondary one by $500\,\mathrm{K}$. LINEAR 1286561 and LINEAR 2602707 are detached binary systems with mass ratios $q = 3.467$ and $q = 0.987$ respectively. These detached systems are low-mass M-type eclipsing binaries with similar temperatures. The marginal contact, fill-out factor and temperature difference between components of CSS J112237.1+395219 suggest that this system may be at a key evolutionary state predicted by thermal relaxation oscillation (TRO) theory. From the estimated absolute parameters, we conclude that our systems share common properties with other ultra-short period binaries.

Key words: techniques: photometric — stars: variables: Contact Binary — stars: individual: CSS J112237.1+395219, LINEAR 1286561 and LINEAR 2602707

1 INTRODUCTION

The term ultra-short period binaries (USPBs) refers to binaries with orbital periods shorter than $\sim 0.22\,\mathrm{d}$ (Ruciński 1992; 2007). Although several theories to explain the observed short-period cut-off have been proposed (Stępień 2011; Jiang et al. 2012), the explanation of this abrupt short period limit is still an open question. Since Ruciński's works, several hundred USPBs have been discovered, with the first one being BW3V38 – a detached system (Udalski et al. 1995) while the list continues to grow (e.g., Norton et al. 2011; Prša 2011; Lohr et al. 2013; Drake et al. 2014; Soszyński et al. 2015 and Li et al. 2017). The short-period contact systems, especially those on the short-period end, are of great interest for the study of the structure and evolution of eclipsing binaries. These systems are expected to be composed of two K or later-type components, according to the period-color relation for contact binaries (Zhu et al. 2015). CSS J112237.1+395219 (hereafter J112237), LINEAR 1286561 (hereafter L1286561) and LINEAR 2602707 (hereafter L2602707) were reported as

USPB systems with periods below $0.2\,\mathrm{d}$ by Drake et al. (2014). The spectral types of these systems were obtained by matching all the USPBs in their list with objects having spectra within SDSS Data Release 10 (Ahn et al. 2014). Table 1 provides the coordinates, periods and spectral types of our target stars.

J112237, being of K-type, is an important object for explaining the period cut-off phenomenon (Liu et al. 2014a). At present, however, only a few binaries with periods shorter than $0.25\,\mathrm{d}$ have been studied in detail. This makes our study of this $<0.2\,\mathrm{d}$ period K-type system interesting. The other two systems are of M spectral type. Since short period M dwarf binaries are relatively faint objects, they are difficult to detect (Davenport et al. 2013). Therefore, any new detection of such a system constitutes a valuable contribution to the understanding of eclipsing binary formation.

In this paper, multicolor light curves are analyzed simultaneously using the 2003 version of the Wilson-Devinney (W-D) Code, which was revised in October 2005

Table 1 List of Targets

Object	RA (2000)	DEC (2000)	P(d)	Spec. Type
J112237	11:22:37.06	+39:52:19.9	0.184749	K2V
L1286561	11:22:43.44	+37:21:30.2	0.168744	M4.5V
L2602707	11:55:33.44	+35:44:39.3	0.199725	M2V

(Wilson & Devinney 1971; Wilson 1990; Wilson 1994; Wilson & van Hamme 2004). Asymmetries in the light curves of short-period binaries have been commonly reported and are attributed to spot activity on the stellar photospheres which can be modeled very well by hot or cool spots on the components of the systems.

2 CCD PHOTOMETRIC OBSERVATIONS AND DATA REDUCTION

Observations were carried out at the San Pedro Martir Observatory either at the 2-m telescope fitted with a filter-wheel and the *Marconi 4* CCD detector (a deep depletion e2v CCD42-40 chip with gain of 2.30 e$^-$/ADU and readout noise of 5.20 e$^-$) giving a field of view of $6' \times 6'$ and at the 0.84-m telescope with another filter-wheel and a *Spectral Instruments 1* CCD detector (a deep depletion e2v CCD42-40 chip with gain of 1.39 e$^-$/ADU and readout noise of 3.54 e$^-$) yielding a $7.6' \times 7.6'$ field of view. Binning with size 2×2 was applied during all the observations. Alternating exposures of the targets were taken with different Johnson-Cousins filters. The details of the observations are provided in Table 2. Flat field and bias frames were also obtained during all the observing runs.

All images were processed by applying IRAF[1] routines. Images were bias subtracted and flat field corrected before the instrumental magnitudes of the variables and some field stars marked in Figure 1 were computed with the standard aperture photometry method. In order to choose good comparison stars (with similar colors as the variables), the fields were calibrated in the $UBV(RI)_c$ system (during very photometric nights) using standard stars from the Landolt catalog. The obtained magnitudes of the comparison stars are given in Table 3. All or any part of the data is available upon request.

The new times of minima for the three systems, presented in Table 4, are all heliocentric and determined with the polynomial fit method.

These new data allowed us to refine the ephemeris of the systems as follows:

$$\text{J112237, HJD(MinI)} = 2458213.7712(5) \atop + 0\text{d}.1823999(4)E, \tag{1}$$

$$\text{L1286561, HJD(MinI)} = 2453711.9610(2) \atop + 0\text{d}.1687441(1)E, \tag{2}$$

$$\text{L2602707, HJD(MinI)} = 2457462.7621(3) \atop + 0\text{d}.1797256(2)E. \tag{3}$$

3 MODELING THE LIGHT CURVES

Analyses of the observed light curves of the systems were carried out employing the 2003 version (October 2005 revision) of the W-D code. To determine the mean surface temperature of star 1, we considered the spectral classes provided by Drake et al. (2014) and, according to the MK spectral types reported in Allen's Astrophysical Quantities (Cox 2000), we assigned the primary stars effective temperatures of $T_{\text{eff}} = 4830\,\text{K}$, $3120\,\text{K}$ and $3520\,\text{K}$ for J112237, L1286561 and L2602707 respectively and used them as primary temperatures in the light curve analyses.

In each case, the q-search method was performed to find the best initial value of q. We assumed gravity darkening and bolometric albedo exponents appropriate for the convective envelopes ($T_{\text{eff}} < 7500\,\text{K}$). Limb-darkening coefficients of the components were interpolated from the square root law of the Claret & Bloemen (2011) tables.

The shape of the light curve of J112237 resembles that of typical W UMa-type binary stars. Therefore, for this star we started the W-D analysis directly in Mode 3 – overcontact configuration. For the other two systems, the calculations were started in Mode 2 – detached configuration. The different computation modes offered by the W-D code can be found in Wilson & van Hamme (2004).

The observed light curves of J112237 and L1286561 are asymmetric and show unequal quadrature heights, with maximum I being brighter than maximum II. A spotted model was introduced to account for asymmetry in the observed light curves, known as the O'Connell effect (O'Connell 1951), which is present in several eclipsing binary light curves and can be explained by spot activity in the component stars (Zhai et al. 1988) that have the same nature as solar magnetic spots (Mullan 1975).

A sufficient number of runs of the DC program was executed, until the corrections to the parameters became smaller than their probable errors. The corresponding relation is plotted in Figure 2.

Starting with the preliminary solutions for the values of q found, we performed a more detailed analysis with q being treated as an additional free parameter. Parameters of the accepted solutions are listed in Table 5, while Figure 3 displays the best fits for the model parameter from W-D code and the observed light curves of the systems. In our final solutions, we also found that the contribution of a third light is negligible.
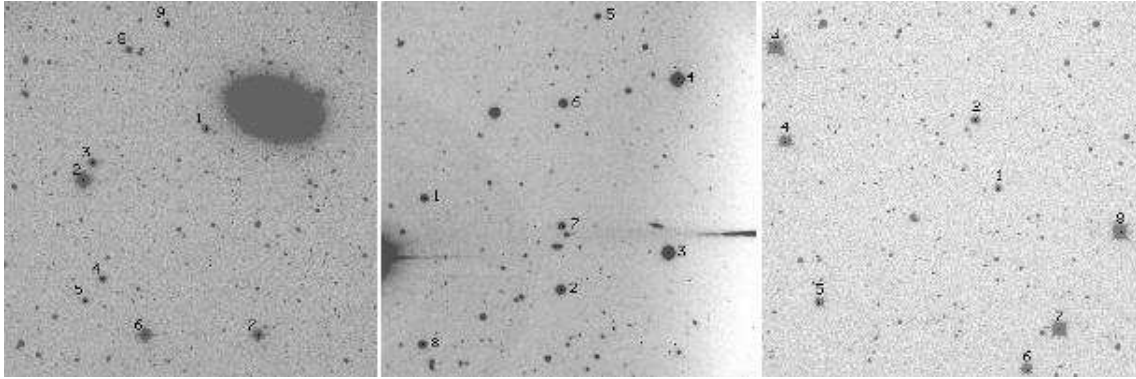
Fig. 1 Observed fields. J112237 is on the left, L1286561 in the middle and L2602707 to the right. The variable stars are marked with number 1.

Table 2 Log of Photometric Observations

Target	UT Date	Telescope	Obs. Hours	Filters	Exp. Times (s)	Number of Integrations	Mean errors (mag)
J112237	2018 Apr 8	2m	4.9	B, V, R_c, I_c	40, 20, 15, 15	105, 104, 106, 105	0.010, 0.009, 0.008, 0.011
L1286561	2018 Apr 6	2m	4.3	V, R_c, I_c	30, 15, 10	130, 132, 131	0.012, 0.010, 0.009
L2602707	2016 Mar 15	0.84m	10.2	V, R_c, I_c	120, 60, 30	122, 121, 110	0.009 , 0.009, 0.007
	2018 Jan 16	0.84m	2.0	V, R_c, I_c	120, 60, 30	25, 24, 24	0.008 , 0.007, 0.005
	2018 Apr 7	2m	0.7	V, R_c, I_c	30, 20, 15	20, 19, 18	0.008 , 0.007, 0.008

Table 3 $UBVR_cI_c$ Magnitudes of the Reference Stars

Field	Name	RA (deg)	DEC (deg)	U	B	V	R_c	I_c
J112237	2MASSJ11224702+3951334	170.696042	+39.859296	15.531±0.010	15.714±0.016	15.403±0.022	14.754±0.002	14.306±0.030
L1286561	2MASSJ11223284+3720037	170.636954	+37.334328	19.273±0.023	18.327±0.007	16.710±0.003	15.650±0.002	14.400±0.003
L2602707	2MASSJ11553585+3546032	178.899350	+35.767600	17.657±0.023	16.773±0.010	15.713±0.007	15.103±0.006	14.542±0.005

It should be noted that the errors of the parameters given in this paper are the formal errors from the W-D code and are known to be unrealistically small (Maceroni & Ruciński 1997). For a discussion, see Barani et al. (2017).

4 ESTIMATE OF THE ABSOLUTE ELEMENTS

Due to the lack of radial velocity (RV) solutions, we employed empirical relations to determine the absolute parameters of the binary systems. Dimitrov & Kjurkchieva (2015) reported a period - semi-major axis $(P - a)$ relation on the basis of 14 binary stars having $P < 0.27\,\mathrm{d}$ that had both RV and photometric solutions, which is approximated by a parabola

$$a = -1.154 + 14.633 \times P - 10.319 \times P^2 , \quad (4)$$

where P is in days and a is in solar radii.

Following the above relation, we determined a semi-major axis $a(R_\odot)$ for our three systems.

The (P, a) relation defined by Equation (4) corresponds to the following "period-mass" relation for short-period binaries

$$M = 0.0134/P^2 \times (-1.154 + 14.633 \times P - 10.319 \times P^2)^3, \quad (5)$$

where M is the total mass of the binary.

The mean fractional radii of the components were obtained with the formula

$$r_{1,2\mathrm{mean}} = (r_{\mathrm{pole}} + r_{\mathrm{side}} + r_{\mathrm{back}})^{1/3}. \quad (6)$$

Using the semi-major axis, we can calculate the radii of the binary components as

$$R_{1,2} = a r_{1,2\mathrm{mean}}. \quad (7)$$

Considering a solar temperature of $T_\odot = 5780\,\mathrm{K}$, absolute parameters of bolometric magnitudes and luminosities can be calculated applying the equations

$$Mbol_{1,2} = 4.77 - 5\log(R_{1,2}/R_\odot) \\ - 10\log(T_{1,2}/T_\odot), \quad (8)$$

$$L_{1,2} = (R_{1,2}/R_\odot)^2 \times (T_{1,2}/T_\odot)^4. \quad (9)$$

Table 4 CCD Times of Minima for the Three Systems

	Filters	HJD	Epoch(1)	O–C(1)	Epoch(2)	O–C(2)	Error	Source
J112237	*BVRI*	2458213.6800	–0.5	–0.0027	–0.5	0.0000	0.0009	This paper
	BVRI	2458213.7712	0.0	–0.0039	0.0	0.0000	0.0013	This paper
L1286561		2453711.9610	0.0	0.0000	0.0	0.0000		VSX
	VRI	2458214.6438	26683.5	0.0009	26683.5	–0.0001	0.0009	This paper
	VRI	2458214.7284	26684.0	0.0011	26684.0	0.0001	0.0008	This paper
L2602707	*VRI*	2457462.6623	–0.5	0.0001	–0.5	0.0001	0.0010	This paper
	VRI	2457462.7621	0.0	0.0000	0.0	0.0000	0.0008	This paper
	VRI	2457462.8618	0.5	–0.0002	0.5	–0.0002	0.0007	This paper
	VRI	2457462.9617	1.0	–0.0001	1.0	–0.0001	0.0006	This paper
	VRI	2458135.0393	3366.0	0.0063	3366.0	0.0010	0.0006	This paper
	VRI	2458215.7266	3770.0	0.0050	3770.0	–0.0009	0.0007	This paper

Table 5 Light curve solutions for J112237 and the two LINEAR systems. Assumed parameters are marked with *.

Parameter	J112237	L1286561	L2602707
i	67.425±0.329	72.996±0.421	79.920±0.114
T_1(K)	4830*	3120*	3520*
T_2(K)	4321±15	2989±20	3465±6
Ω_1	4.612±0.056	7.791±0.041	4.092±0.038
Ω_2	4.612±0.056	7.794±0.050	4.087±0.040
$q = m_2/m_1$	1.616±0.035	3.468±0.065	0.988± 0.021
$A_1 = A_2$	0.5*	0.5*	0.5*
$g_1 = g_2$	0.32*	0.32*	0.32*
L_{1B}	0.533±0.004		
L_{1V}	0.510±0.004	0.260±0.005	0.513±0.009
L_{1Rc}	0.476±0.005	0.253±0.007	0.509±0.008
L_{1Ic}	0.455±0.004	0.247±0.007	0.508±0.008
L_{2B}	0.371±0.003		
L_{2V}	0.396±0.002	0.691±0.004	0.451±0.008
L_{2Rc}	0.442±0.004	0.699±0.005	0.455±0.008
L_{2Ic}	0.470±0.003	0.728±0.003	0.462±0.008
f	0.148±0.007	–0.06±0.003	–0.083±0.005
X_{1B}	0.987*		
X_{1V}	0.681*	0.359*	1.083*
X_{1Rc}	0.389*	0.205*	0.801*
X_{1Ic}	0.232*	–0.017*	0.444*
X_{2B}			
X_{2V}		0.660*	–0.269*
X_{2Rc}		0.785*	–0.003*
X_{2Ic}		1.002*	0.403*
l_3	0	0	0
r_1 (pole)	0.322±0.002	0.228±0.002	0.317± 0.004
r_1 (side)	0.338±0.003	0.234±0.002	0.328± 0.005
r_1 (back)	0.374±0.004	0.255±0.003	0.345± 0.006
r_2 (pole)	0.403±0.007	0.428±0.002	0.315± 0.004
r_2 (side)	0.428±0.009	0.452±0.003	0.326± 0.004
r_2 (back)	0.460±0.014	0.469±0.003	0.343± 0.007
Lat. spot	60.34±2.1	85.07±1.9	95.27±2.2
Long. spot	121.33±3.4	20.56±2.1	321±2.6
Radius	25.3±0.86	30.33±0.73	21.13±0.90
Temp. fact.	0.925±0.021	0.940±0.036	0.851±0.045
Component	1	1	1
Sum(res)^2	0.0301	0.00138	0.00010

The mean densities of the binary components were derived from the following equation given by Mochnacki (1981)

$$\rho_1 = 0.0189/r_{1,\mathrm{mean}}^3 P^2(1+q), \qquad (10)$$

$$\rho_2 = 0.0189q/r_{2\mathrm{mean}}^3 P^2(1+q). \qquad (11)$$

The mass of the primary component M_1 is computed via Equation (5), while the mass of the secondary compo-

nent is directly calculated from the estimated mass ratio of the system.

All the above resulting values are listed in Table 6.

Following the equation on page 131 of Popper & Ulrich (1977), we calculated the orbital angular momentum of the targets

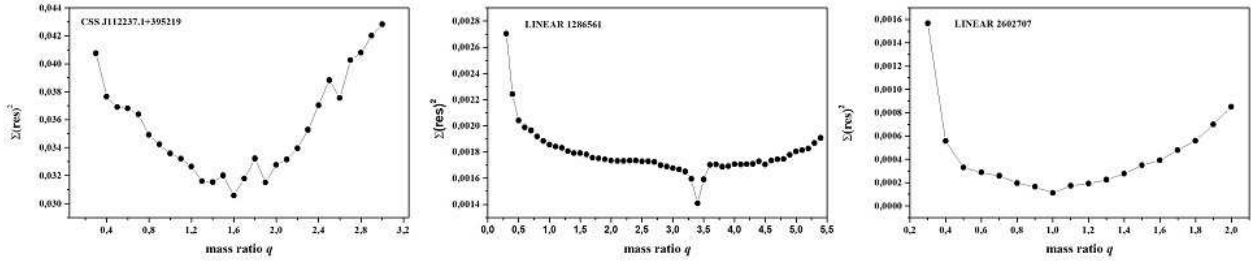$$J_{\mathrm{rel}} = M_1 M_2 (P/M_1 + M_2)^{1/3}, \qquad (12)$$

Fig. 2 The relation $\Sigma(\mathrm{res})^2$ versus mass ratio q in Mode 3 for J112237 and in Mode 2 for the two LINEAR systems in the W-D code.
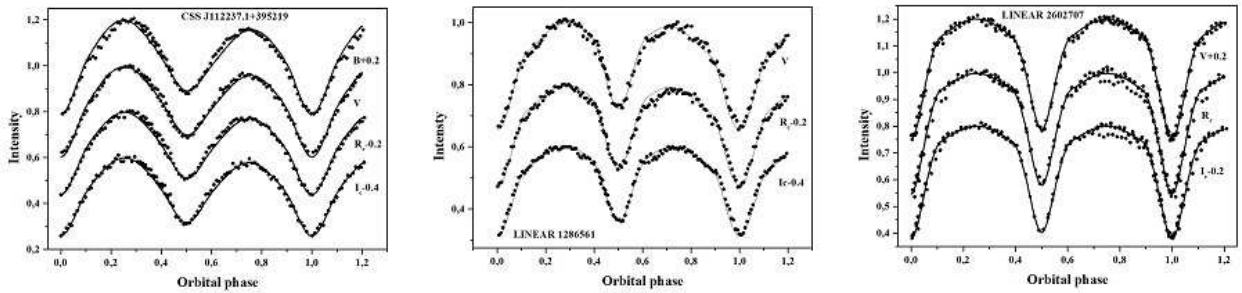

Fig. 3 CCD light curves of J112237 and the two LINEAR systems. *Points* are the original observations and *lines* are the theoretical fits with the spot contribution (the spot contribution is included where necessary, elsewhere not).
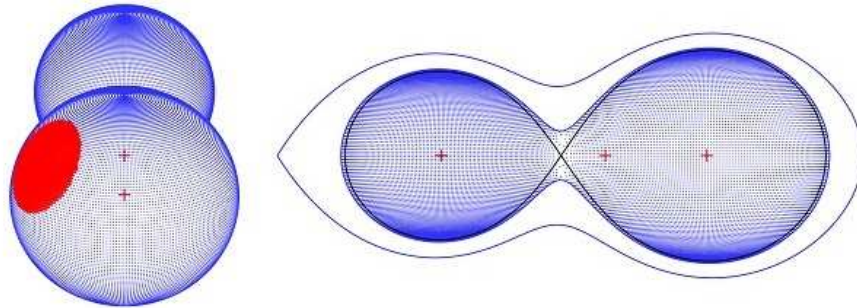

Fig. 4 Geometric representation of CSS J112237 at the primary minimum (*left*) and the configuration of the components of the system in the orbital plane at phase 0.25 (*right*), according to our solution.

where P is in days and M_i are in solar units.

The obtained values $\log J_{\mathrm{rel}}$ (Table 6) of our two LINEAR systems are considerably smaller than those of detached systems, which have $\log J_{\mathrm{rel}} > +0.08$.

The orbital angular momentum of J112237 is smaller even than that of contact systems, which have $\log J_{\mathrm{rel}} > -0.5$.

The small orbital angular momentum of J112237 implies the existence of a past episode of angular-momentum loss during binary evolution. It also means that J112237 is not a pre-main sequence object.

5 DISCUSSION ON THE SYSTEMS

Here we have presented the analysis of filtered CCD light curves of three USPBs. One of these, J112237, is a contact system of spectral type K while the other two are rare detached systems in which the components are non-degenerate M dwarfs.

5.1 J112237

The values of mass ratio found for J112237 indicate that the system is a typical W-subtype contact binary in the Binnendijk classification (Binnendijk 1965; Binnendijk
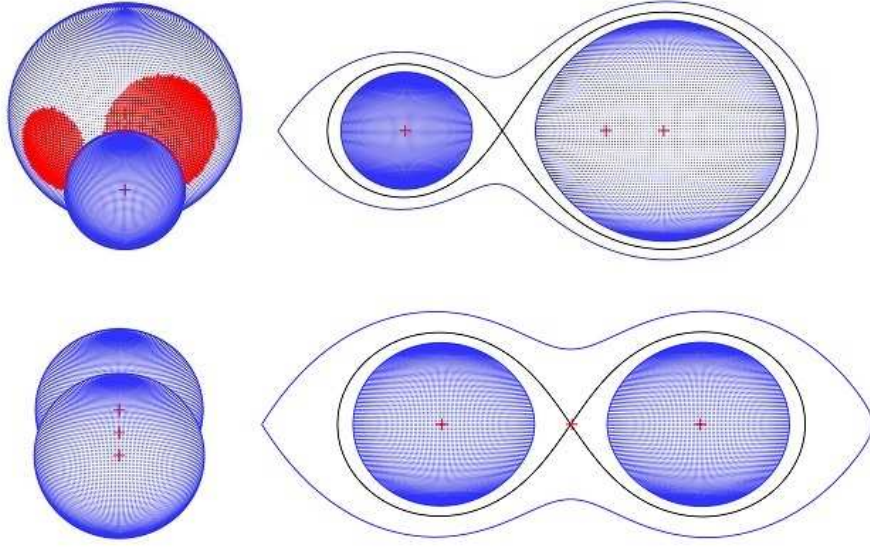
Fig. 5 Same as Fig. 4 but for L1286561 (*top*) and L2602707 (*bottom*).

Table 6 Estimated Absolute Elements

System	J112237	L1286561	L2602707
a	1.17	1.02	1.36
M_{tot}	0.648	0.501	0.839
M_1	0.400±0.010	0.112±0.001	0.422±0.004
M_2	0.248±0.030	0.389±0.013	0.417±0.013
R_1	0.502±0.004	0.244±0.003	0.448±0.084
R_2	0.404±0.007	0.459±0.009	0.445±0.09
L_1	0.123±0.001	0.005±0.010	0.028±0.010
L_2	0.051±0.003	0.015±0.001	0.026±0.01
$M\mathrm{bol}_1$	7.03	10.49	8.65
$M\mathrm{bol}_2$	7.98	9.31	8.73
$\log \rho_1$	0.651	1.037	0.821
$\log \rho_2$	0.724	0.735	0.823
$\log g_1$	4.64	4.71	4.76
$\log g_2$	4.62	4.7	4.76
$\log J_{rel}$	–1.28	–1.6	–1.07

Fig. 7 Positions of the components of J112237 (*filled cyan symbols*), L2602707 (*filled red symbols*) and L1286561 (*filled blue symbols*) on the mass-radius diagram compared with ten USPBs from Kjurkchieva et al. (2018) depicted with *open circles* for the primary components and *open triangles* for the secondary components. Also plotted are theoretical isochrones derived from Dartmouth models (Dotter et al. 2008) for solar metallicity, with ages 0.25, 1.0, 5.0 and 10 Gyr.

1970). The system has a shallow contact configuration (fill-out 14.8%).

Contact binaries below the period limit of 0.22 d, i.e., ultra-short period contact binaries (USPCBs), are expected to be composed of two K-type or later-type components, according to the period-color relation for contact binaries (Zhang et al. 2014). According to the accepted orbital solution, the spectral type of the secondary component of J112237 is estimated to be K5 following Cox (2000).

It is known that a large amount of K-type short-period contact binaries may be W-type systems (Liu et al. 2014b)
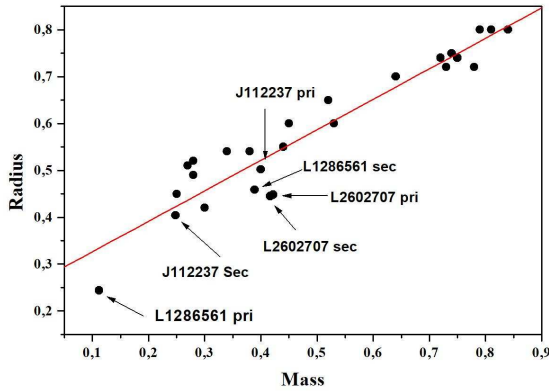

Fig. 6 The mass-radius diagram of our three systems with ten other USPBs taken from Kjurkchieva et al. (2018). The *solid line* represents the linear fit to the data.
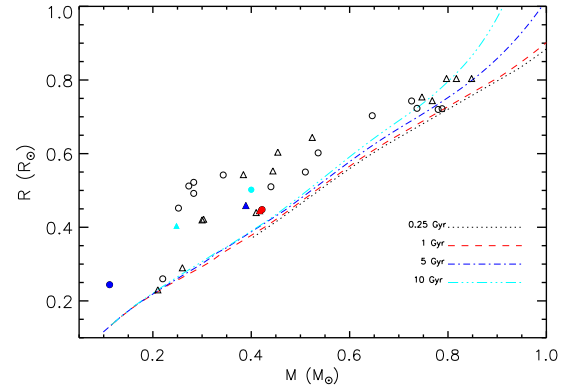
Table 7 List of Well Studied Detached M Dwarf Systems

Name	NSVS 4876238	BX Tri	BW3 V38	OGLE BLG ELC-000066	CSS J171508.5+350658	L1286561	L2602707
Period	0.22184	0.19264	0.19839	0.0984	0.17855	0.1824	0.19973
Sp. Type	K9	dM	M3	M4	M1	M4.5V	M2 V
T_1	3860	3735	3500		3700	3120	3520
T_2	3826	3106	3448		3600	2989	3465
q	0.379	0.519	0.95	0.95	0.895	3.47	0.98791
i	68	72.5	85.51	86	67.9	72.9	79.9
r_1	0.453	0.431	0.372			0.239	0.33
r_2	0.264	0.228	0.323			0.449	0.328
a	1.58	1.28	1.355			1.02	1.36
M_1	0.78	0.51	0.44	0.22		0.112	0.422
M_2	0.3	0.26	0.41	0.21		0.389	0.417
R_1	0.72	0.55	0.51	0.26		0.244	0.448
R_2	0.42	0.29	0.44	0.23		0.459	0.445
L_1	0.102	0.053	0.035			0.005	0.028
L_2	0.034	0.007	0.025			0.015	0.026
Reference	[1]	[2]	[3]	[4]	[5]	[6]	[6]

Notes: [1] Kjurkchieva et al. (2018); [2] Dimitrov & Kjurkchieva (2010); [3] Maceroni & Montálban (2004) and Dimitrov & Kjurkchieva (2010); [4] Soszyński et al. (2015); [5] Kjurkchieva et al. (2016); [6] This paper.

and that the majority of W-type contact binaries show characteristics of being in shallow contact (Zhu et al. 2010).

In recent years, the list of known USPCBs, as well as their study, has been substantially extended, signifying that most USPCBs have shallower fill-out factors ($f < 20\%$) and indicating that USPCBs have just evolved to a contact phase.

According to thermal relaxation oscillation (TRO) theory (Lucy 1976; Lucy & Wilson 1979; Flannery 1976; Robertson & Eggleton 1977; Yakut & Eggleton 2005 and Li et al. 2008), a cycle of contact-semidetached-contact states will be formed, via mass transfer between the components.

The relatively small temperature difference between components of J112237 ($\Delta T = T_h - T_c = 510$ K) is accepted for overcontact systems.

The different light levels at quadrature were reproduced by a cool spot on the primary component.

A graphical representation and the Roche geometry of J112237 is depicted in Figure 4.

5.2 L1286561 and L2602707

The LINEAR systems are two rare M dwarf detached USPB systems with non-degenerate components. Among them, only five other systems are quite well studied to date (see Table 7).

As discussed in Becker et al. (2011), the sample of known binary systems composed of two M dwarfs is very small.

For many years the known M dwarf binary system in the literature with the shortest known period was BW3 V38 (Maceroni & Ruciński 1997; Maceroni & Montálban 2004), which is composed of two main sequence M3 dwarfs with an orbital period of 0.1984 d. The similarity be-

tween the absolute parameters of this system with those of L2602707 is really surprising. Dimitrov & Kjurkchieva (2010) reported the shortest period M dwarf binary yet characterized, GSC 2314-0530 (BX Tri), with a 0.192636 d period (Soszyński et al. 2015), found among 242 USPBs, and OGLE-BLG-ECL-000066 with an orbital period below 0.1 d in which the two components are M dwarfs in a nearly contact configuration.

Subsequently, Kjurkchieva et al. (2016) published observations of CSS J171508.5+350658, a semidetached system in which both the components consist of M dwarfs with a period of 0.178 d. Kjurkchieva et al. (2018) found NSVS 4876238, a detached type of USPB in which both components are late dwarfs (K9 spectral type) and the temperature difference between the components does not exceed 400 K.

For both our systems, the difference between the temperatures of the components is quite small and, while L2602707 shows symmetric light curves, to represent the asymmetries of light curves for L1286561, it was necessary add a couple of cool spots on the massive component.

Light asymmetry has been reported commonly for light curves of short-period binaries and may be due to the spot activity on stellar photospheres. They can be reproduced by surface temperature inhomogeneities (spots). It is reasonable to assume the existence of cool spots by analogy with our Sun. An interesting characteristic of L2602707 is that it is a twin binary system in which the mass ratio is near 1 ($q = 0.9879$). This kind of binary was first noted by Lucy & Ricco (1979).

Statistical studies on the mass ratio distribution of binaries (e.g., Lucy 2006; Simon & Obbie 2009) demonstrated that the frequency of existing twins within the mass ratio $0.98-1.00$ is about 3% among all binaries, at which F, G and K spectral type systems dominate.

Among the systems in Table 7, L2602707 is not the only twin binary.

A graphical representation and the Roche geometry of L1286561 and L2602707 are depicted in Figure 5.

The components of all our three systems are plotted in the mass-radius diagram (Fig. 6) with ten other USPB systems whose data are adopted from Kjurkchieva et al. (2018).

Additionally, the positions of the components of the three systems are compared in the mass-radius diagram in Figure 7 with the USPBs listed by Kjurkchieva et al. (2018), together with theoretical isochrones derived from Dartmouth models (Dotter et al. 2008) for solar metallicity and ages 0.25, 1.0, 5.0 and 10.0 Gyr.

Our systems follow the general pattern of USPB systems.

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