

A strange phenomenon in XZ Andromedae: two Keplerian periods with a 1:3 ratio

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Abstract Six mid-eclipse times of the eclipsing binary XZ And are obtained, which are analyzed together with others collected from the literature. Two sets of cyclic variations with periods of 33.43 and 100.4 yr are found if a double-Keplerian model is used to fit the data. The 1:3 ratio of the periods suggests that both cyclic variations arise from dynamic motions of two companions rather than magnetic activity of the eclipsing pair. According to the double-Keplerian model, the companions have masses of $\sim 1.32 M_{\odot}$ and $\sim 1.33 M_{\odot}$, respectively. Comparing with the total mass of the eclipsing pair of $3.12 M_{\odot}$, it is obvious that XZ And is a general N-body system. The strong gravitational perturbation between the two companions invalidates the double-Keplerian model. It is strange that two Keplerian periods with a 1:3 ratio are derived from the best fits with an “inappropriate” model. This illogical, but interesting phenomenon also appears in two other Algol systems, suggesting that our discoveries deserve attention from astronomers.

Key words: binaries: close — stars: individual: XZ Andromedae

1 INTRODUCTION

XZ Andromedae (BD+41°367) is a classic Algol-type binary (hereafter XZ And AB). The primary is a main sequence star and the more evolved secondary fills its Roche lobe (Manzoori 2016). Dugan & Wright (1939) found that primary eclipsing times and secondary eclipsing times follow the same linear ephemeris roughly, but they also pointed out quite intricate variations in the residuals (i.e., $O - C$) between the observed (O) and computed (C) mid-eclipse times. Odinskaya & Ustinov (1952) ascertained that these irregular variations contain two sets of cyclic modulation. Todoran (1967) fitted the data between the years 1891–1919 with a sinusoidal curve with a period of 21.3 yr, and the data between 1924–1966 with another sinusoidal curve with a period of 44.6 yr. Todoran (1967) interpreted these variations as apsidal motion, but Kreiner (1976) disproved this explanation. Demircan et al. (1995) reported three cyclic variations with periods of 11.2 yr, 36.8 yr and 137.5 yr, respectively, and attributed the cyclic variations to magnetic activity of the secondary and light-travel time (LTT) effect due to one or two underluminous star(s) around the eclipsing pair. Frieboes-Conde & Herczeg (1973) and Borkovits & Hegedüs (1996) claimed that an unambiguous identification of a third com-

ponent was not possible for XZ And, but the latter still presented the orbital parameters of third and fourth bodies, including the periods of 35.6 yr and 69.8 yr, respectively. Selam & Demircan (1999) reported two periods of 36.79 and 126.35 yr for these two companions. Recently, Yang (2013) found only a quasi-cyclic period of 32.60 yr, while Manzoori (2016) obtained two periods, i.e., 23.3 and 34.8 yr. Just as before, both authors did not affirm an explicit reason for the variations. Therefore, it is necessary to reanalyze the behavior of the mid-eclipse times.

All available mid-eclipse times collected from the literature and several new data obtained in this paper are used to plot the $O - C$ diagram in Section 2. In Section 3, the fitting procedures are described, and the best-fit solution is given. In Section 4, we summarize our results and give our conclusions in Section 5.

2 ECLIPSE-TIMING VARIATIONS

CCD photometric observations have been carried out in the past six years. The 85-cm telescope at the Xinglong Station of National Astronomical Observatories, Chinese Academy of Sciences (NAOC) (NAOC-85), equipped with a primary-focus multicolor CCD photometer (Zhou et al. 2009), was used in February and December of

Table 1 Six New Mid-eclipse Times of XZ And

HJD (UTC) 2400000+	BJD (TDB) 2400000+	Error (d)	Filter	Origin
56313.1564	56313.15716	±0.0002	<i>R</i>	YNAO-100
56338.94475	56338.94552	±0.00005	<i>V</i>	NAOC-85
56655.19173	56655.19250	±0.00005	<i>R</i>	NAOC-85
57061.0204	57061.02119	±0.0002	<i>R</i>	YNAO-100
57422.05866	57422.05947	±0.00009	<i>V</i>	YNAO-100
57422.05866	57422.05947	±0.00011	<i>R</i>	YNAO-100
58159.0693	58159.07013	±0.0001	<i>V</i>	YNAO-60
58159.0693	58159.07013	±0.0001	<i>R</i>	YNAO-60

2013. The 60-cm (YNAO-60) and 100-cm (YNAO-100) Cassegrain telescopes at Yunnan Astronomical Observatories (YNAO) were employed in January 2013, February 2015, February 2016 and February 2018. The comparison and check stars are GSC 02824–01778 ($\alpha_{J2000.0} = 01^{\text{h}}57^{\text{m}}14.2^{\text{s}}$, $\delta_{J2000.0} = +42^{\circ}02'19.2''$) and 2MASS 01564776+4201523 ($\alpha_{J2000.0} = 01^{\text{h}}56^{\text{m}}47.7^{\text{s}}$, $\delta_{J2000.0} = +42^{\circ}01'52.8''$), respectively. We applied the aperture photometry package IRAF¹ to reduce the CCD data. Six new mid-eclipse times are obtained by using a parabolic fitting method. The new data are listed in Table 1. A mean time is given if multi-band values were obtained simultaneously.

The Lichtenknecker Database of the BAV² and the O-C Gateway Database³ list a large number of mid-eclipse times for XZ And, which come mainly from Zessewitsch (1924), Banachiewicz (1925), Dugan & Wright (1939), Kordylewska (1931), Laue (1934, 1936, 1949), Szafraniec (1950, 1952a,b, 1955, 1956, 1957), Szczepanowska (1950, 1953, 1956, 1959), Piotrowski (1950), Odinskaya & Ustinov (1952), Ashbrook (1952a,b, 1953), Domke & Pohl (1953), Pohl (1955), Rudolph (1960), Robinson (1965a,b, 1966, 1967a,b), Todoran (1967, 1968, 1973), Todoran & Popa (1967), Robinson & Ashbrook (1968), Frieboes-Conde & Herczeg (1973), Baldwin (1973, 1976, 1977, 1978), Mallama et al. (1977), Kreiner et al. (1980), Olson (1981), Baldwin & Samolyk (1993), Hegedüs et al. (1996), Agerer & Huebscher (2003), Cook et al. (2005), Hübscher et al. (2005, 2006, 2009), Nagai (2007, 2008, 2010), Samolyk (2008, 2009, 2010) and Yang (2013).

Three visual times (HJD 2423681.21, 2423694.29 and 2423699.40) are discarded due to their poor precision, and six mid-eclipse times (HJD 2423670.430, 2423756.292, 2441650.291, 2441958.429, 2444488.400 and 2450752.310) are not adopted because of their large

deviation from the $O - C$ curve. Finally, we have collected 1131 mid-eclipse times over a 127-year time span. Most photographic and visual data were published without uncertainties; the typical uncertainty of $\sigma = \pm 0.003$ d is used. For CCD data, the uncertainty of ± 0.0001 d is adopted if it is less than ± 0.0001 d.

The mid-eclipse times were usually reported in Heliocentric Julian Dates (HJDs) based on the Coordinated Universal Time (UTC) standard, which is not strictly uniform. Therefore, we adopted the Barycentric Dynamical Time (TDB) standard, and corrected all data to the solar-system barycenter, giving Barycentric Julian Dates (BJD) (Eastman et al. 2010). The relation between Universal Time (UT) and Terrestrial Time (TT) given by Duffett-Smith & Zwart (2011) was used to convert the old data from before 1950.

The calculated mid-eclipse epochs are computed with the linear ephemeris

$$\text{Min I} = \text{BJD } 2452500.51473 + 1.3572855^{\text{d}} \times E, \quad (1)$$

where the period was also used by Manzoori (2016). In Equation (1), E is the eclipse cycle number counted from BJD 2452500.51473. We can calculate the residuals $O - C$, i.e., the observed mid-eclipse times minus the calculated mid-eclipse epochs. Figure 1 shows all $O - C$ values.

3 DATA ANALYSIS AND LTT MODELS

The secondary component is transferring mass to the primary. Therefore, the observed period increases (Yang 2013; Manzoori 2016), and the $O - C$ curve should have a parabolic trend. Figure 1 demonstrates that an additional periodic model is also required. Following the method adopted by Yuan et al. (2016), we first use a quadratic plus one-companion model

$$\begin{aligned} O - C &= T_O(E) - T_C(E) \\ &= C_0 + C_1 \times E + C_2 \times E^2 + \tau_3 \end{aligned} \quad (2)$$

to fit the $O - C$ values. The LTT term, τ_3 , arises from the variation of distance of an eclipsing binary from the

¹ IRAF is developed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

² <http://www.bav-astro.de/index.php?sprache=en>

³ <http://var.astro.cz/ocgate/>

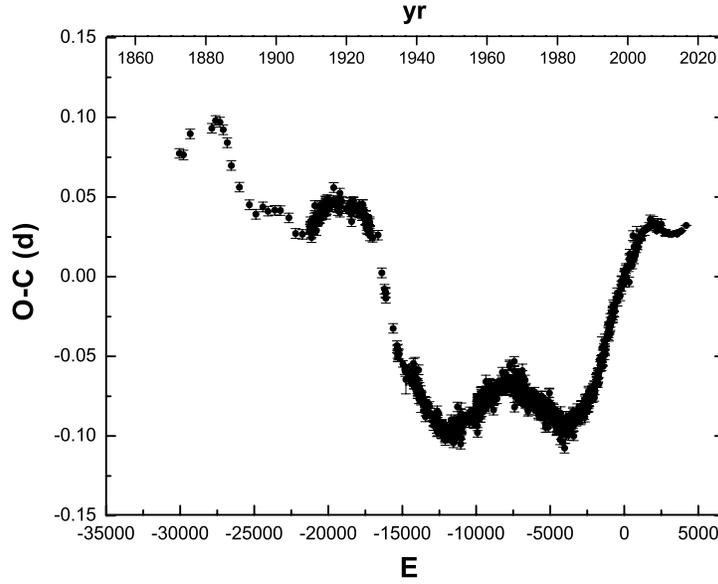


Fig. 1 The $O - C$ diagram of XZ And.

observer as a result of a distant third component, and can be calculated using the following equation (Irwin 1952)

$$\tau_3 = \frac{a_3 \sin i_3}{c} \left[\frac{1 - e_3^2}{1 + e_3 \cos \nu_3} \sin(\nu_3 + \omega_3) + e_3 \sin \omega_3 \right], \quad (3)$$

where $a_3 \sin i_3$ is the semimajor axis of the eclipsing binary around the barycenter of the triple system, projected onto the tangent plane of the sky. ω_3 is the argument of periastron measured from the ascending node and e_3 is the eccentricity. For any mid-eclipse time t , the true anomaly ν_3 can be derived from the following relation

$$\tan \frac{\nu_3}{2} = \sqrt{\frac{1 + e_3}{1 - e_3}} \tan \frac{\varphi_3}{2}, \quad (4)$$

where φ_3 is the eccentric anomaly, and can be obtained by solving Kepler's equation

$$M_3 = \varphi_3 - e_3 \sin \varphi_3. \quad (5)$$

In Equation (5), the mean anomaly $M_3 = 2\pi(t - T_3)/P_3$, where T_3 is the time of the periastron passage and P_3 is the orbital period.

For one (e_3, T_3, P_3) configuration, we fit the $O - C$ data with Equation (2) and get the goodness-of-fit statistic

$$\chi^2 = \sum_{i=1}^{1131} \left[\frac{y_i - y(t_i)}{\sigma_i} \right]^2, \quad (6)$$

where y_i is the $O - C$ value given by Equation (1) and $y(t_i)$ is the model value at mid-eclipse time t_i calculated by Equation (2). In Equation (6), σ_i is the uncertainty of the $O - C$ datum y_i (i.e., the uncertainty of the i -th mid-eclipse time). The best $a_3 \sin i_3$ and ω_3 can be obtained from the best fit. Searching e_3 from 0.0 to 0.99, and T_3

from 24500000.0 to 24500000.0 + P_3 , the local χ^2 minimum is obtained for the particular P_3 , i.e., $\chi^2(P_3)$, which is plotted in Figure 2.

Figure 2 shows that χ^2 reaches the minimum at $P = 34.48$ yr, suggesting a companion with a period of 34.48 yr. Hereafter, we refer to the eclipsing pair as XZ And AB, and the companion as XZ And (AB)C. The best fits corresponding to the 34.48 yr periodicity are plotted in Figure 3, and listed in the second column (Solution 1) of Table 2. As displayed in the bottom panel of Figure 3, most data have residuals larger than ± 0.01 d, which is much larger than the typical uncertainty, i.e., ± 0.003 d.

In Figure 2, the one-companion fit exhibits another periodicity at > 75 yr, suggesting another companion (XZ And (AB)D) with a longer period. However, due to the short time coverage and low precision, χ^2 remains at a very low level beyond 75 yr. We use a parabola plus two-companion model to fit the $O - C$ data. Just as the best fit with the one-companion model, we fix e_3, T_3, P_3, e_4, T_4 and P_4 during the fitting process. The parameters with the subscript '4' are similar to those with the subscript '3', but refer to the barycenter of XZ And AB and C around the barycenter of XZ And AB, C and D. After searching all possible $e_{3,4}$ and $T_{3,4}$, we obtain the local χ^2 minimum for fixed P_3 and P_4 , i.e., $\chi^2(P_3, P_4)$. $\chi^2(P_3, P_4)$ is a function of P_3 and P_4 . We search for P_3 in 20 – 40 yr and P_4 in 70 – 120 yr simultaneously. Finally, a two-dimensional periodogram results, and is shown in Figure 4. The global χ^2 minimum is located at $(P_4 \simeq 97.8$ yr, $P_3 \simeq 33.4$ yr), which confirms XZ And (AB)C and D.

Since we search for P_3 and P_4 in finite steps (i.e., 0.2 d), the global χ^2 minimum derived from the two-dimensional periodogram is not the true minimum, but is

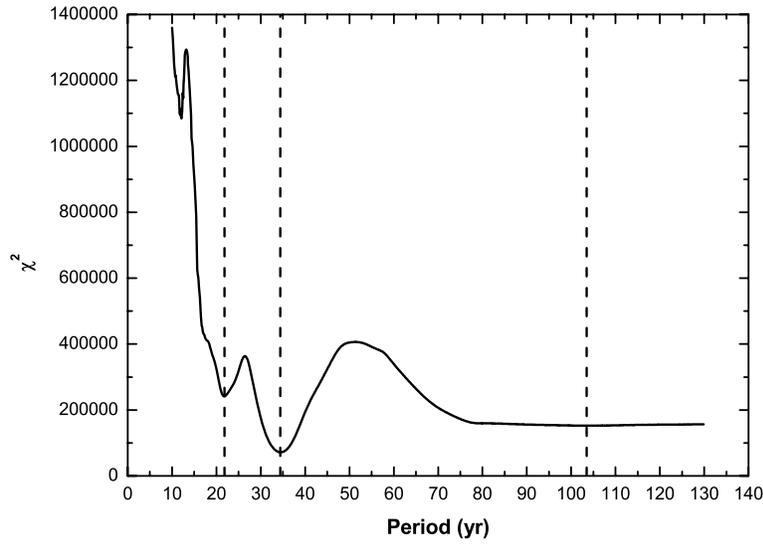


Fig. 2 The Keplerian periodogram of XZ And. The *dashed vertical lines* mark three χ^2 minima.

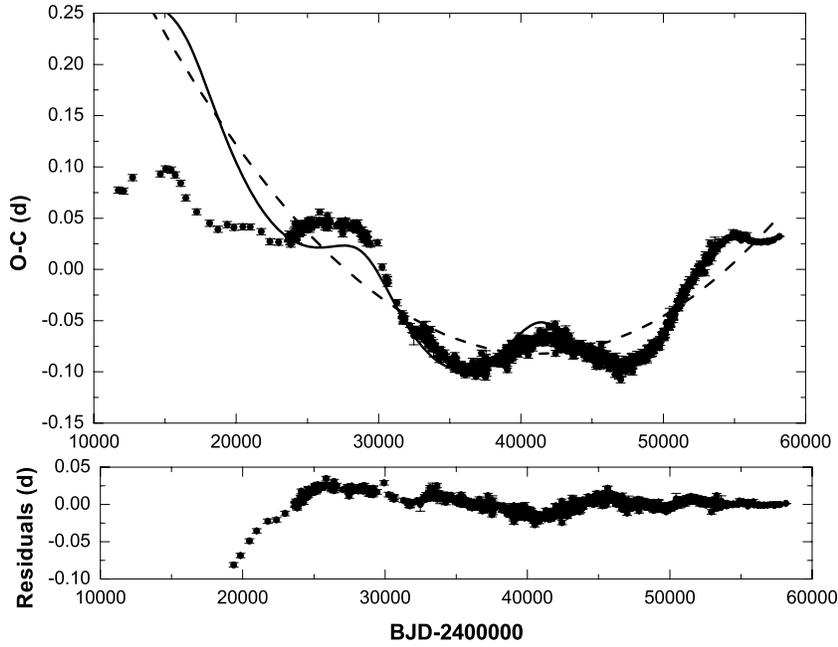


Fig. 3 The best fit to the eclipse-timing variations of XZ And with the one-companion model. The overplotted *solid line* signifies the best fit with Equation (2), and the *dashed line* only represents the second-order polynomial in the ephemeris. The residuals of the best fit are displayed in the lower panel. Note that the residuals before BJD 2420000 reach as large as -0.22 d, and do not appear in the diagram.

very close to the true minimum. Starting from the “best” solution in the two-dimensional periodogram, we fit the data by using the Levenberg-Marquardt fitting algorithm (Marquardt 2009). The Levenberg-Marquardt fits set all parameters free. The free parameters are C_0 , C_1 , C_2 , P_k , T_k , e_k , A_k and B_k ($k = 3, 4$), where A_k and B_k are related to $a_k \sin i_k$ and ω_k (see Yuan & Şenavcı (2014) for details). The best parameters and the least χ^2 are listed in the fourth column (i.e., Solution 3) of Table 2. Interestingly, P_4 and P_3 are 100.3 ± 1.5 yr and 33.43 ± 0.03 yr, re-

spectively, suggesting a possible mean-motion resonance (MMR). The improved fits are plotted in Figure 5. As shown by Figure 5, most of the residuals are within ± 0.01 , much better than that of Solution 1. The best fit fails before BJD 2420000 and around BJD 2432000, where the data are scarce. We remind the reader that the old visual data before A.D. 1900 are very low-precision, and cannot be fitted very well in most cases, such as SW Lac (Yuan & Şenavcı 2014) and Z Dra (Yuan et al. 2016). Although the χ^2 statistic is relatively poor (the reduced chi-squared

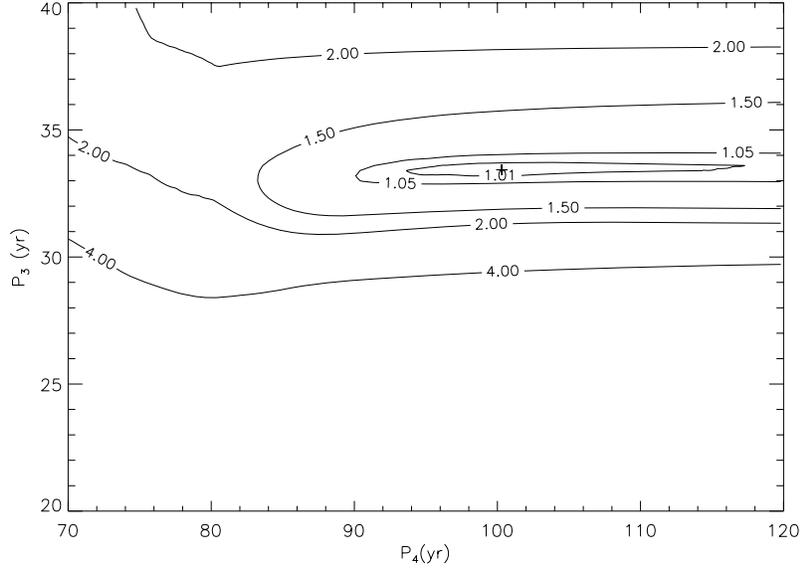


Fig. 4 Two-dimensional periodogram of XZ And derived from a parabola plus two-companion model. The χ^2 contours have been normalized by division by the global χ^2 minimum, which is marked by a cross.

Table 2 The Best-fit Parameters of the Companions around XZ And

Parameter (1)	Solution 1 (2)	Solution 2 (3)	Solution 3 (4)	Solution 4 (5)
C_0 (d)	-0.0222 ± 0.0001	-0.0220 ± 0.0010	-0.0088 ± 0.0013	-0.0046 ± 0.0006
C_1 ($\times 10^{-5}$ d)	1.43 ± 0.00	1.47 ± 0.03	0.65 ± 0.01	0.70 ± 0.02
C_2 ($\times 10^{-10}$ d)	8.48 ± 0.01	8.63 ± 0.01	3.65 ± 0.03	3.75 ± 0.02
P_5 (yr)		24.35 ± 0.03		
T_5 (BJD)		2402255.0 ± 74.2		
e_5		0.638 ± 0.006		
$a_5 \sin i_5$ (au)		1.25 ± 0.02		
ω_5 (deg)		149.2 ± 1.2		
m_5 (M_\odot , $i_5 = 89.8^\circ$)		0.34 ± 0.01		
A_5 (au, $i_5 = 89.8^\circ$)		12.71 ± 0.01		
P_3 (yr)	34.48 ± 0.01	34.48 ± 0.03	33.43 ± 0.03	33.34 ± 0.03
T_3 (BJD)	2403941.0 ± 30.4	2402431.6 ± 46.2	2405738.2 ± 48.7	2405981.9 ± 52.4
e_3	0.256 ± 0.002	0.174 ± 0.004	0.228 ± 0.002	0.221 ± 0.002
$a_3 \sin i_3$ (au)	5.30 ± 0.00	4.45 ± 0.04	5.09 ± 0.03	5.15 ± 0.04
ω_3 (deg)	102.8 ± 0.8	62.5 ± 1.4	114.4 ± 1.1	118.4 ± 1.2
m_3 (M_\odot , $i_3 = 89.8^\circ$)	1.36 ± 0.00	1.16 ± 0.02	1.33 ± 0.01	1.35 ± 0.01
A_3 (au, $i_3 = 89.8^\circ$)	$17.460.01$	17.65 ± 0.02	17.06 ± 0.02	17.06 ± 0.03
P_4 (yr)			100.3 ± 1.5	102.9 ± 2.4
T_4 (BJD)			2426568.8 ± 43.9	2426505.7 ± 51.5
e_4			0.49 ± 0.01	0.49 ± 0.01
$a_4 \sin i_4$ (au)			8.84 ± 0.10	9.09 ± 0.16
ω_4 (deg)			115.4 ± 1.7	114.8 ± 2.8
m_4 (M_\odot , $i_4 = 89.8^\circ$)			1.32 ± 0.01	1.34 ± 0.01
A_4 (au, $i_4 = 89.8^\circ$)			38.70 ± 0.07	39.46 ± 0.11
χ^2	71646.1	58605.5	16996.5	14758.4

statistic $\chi^2_\nu = 15.2$), there is a good qualitative correspondence between the morphologies of the observed and model curves. In Figure 1, the thick $O - C$ curve demonstrates that most visual data often conflict with each other

within their typical uncertainties, i.e., ± 0.003 , and only seem consistent within ± 0.01 . Perhaps this explains why χ^2_ν is large.

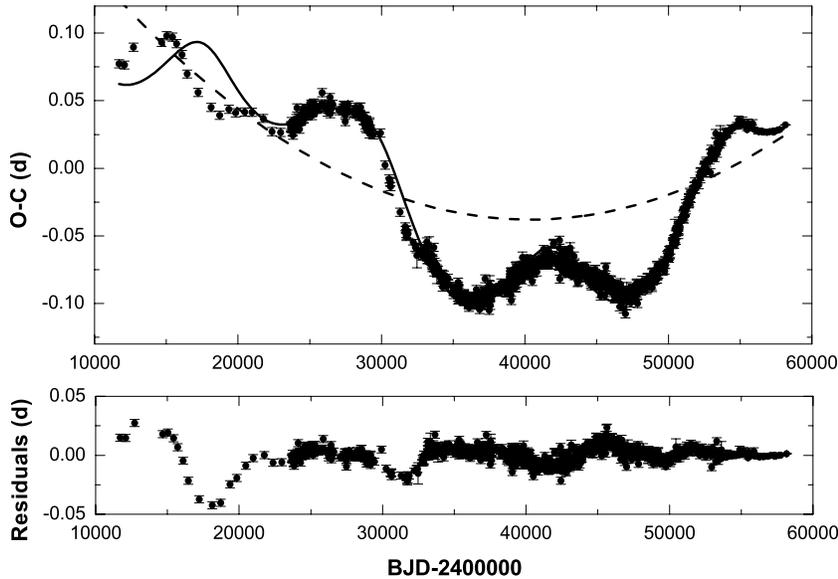


Fig. 5 The two-companion fit to the eclipse-timing variations of XZ And. The residuals of the best fit are displayed in the lower panel. The overplotted *solid line* signifies the best fit with a parabola plus two-companion model, and the *dashed line* only represents the parabola.

The uncertainty of $\sigma = \pm 0.004$ d is also used for the photographic and visual data which were published without uncertainties. We refit the $O - C$ data, and obtain similar results (see Solution 4 in Table 2). XZ And (AB)D has an orbital period of $P_5 = 102.9 \pm 2.4$ yr, while $P_4 = 33.34 \pm 0.03$ yr for XZ And (AB)C.

Figure 2 indicates that a period of ~ 23 yr is also possible. It is likely that such an LTT signal also appears in the bottom panel of Figure 5. It seems that a short-period companion (XZ And (AB)E) exists. For verification, the two-companion model is used again. This time, P_4 is still searched for at around 33 yr, but P_3 is searched for at around 23 yr. To avoid confusion, the subscript ‘5’ is used for XZ And (AB)E, while ‘3’ is for XZ And (AB)C. The Levenberg-Marquardt fit gives Solution 2, which is listed in the third column of Table 2. XZ And (AB)E has an orbital period of $P_5 = \sim 24.35$ yr and a mass of $0.34 M_\odot$. XZ And (AB)E produces a cyclic $O - C$ variation with a semi-amplitude of $a_5 \sin i_5 = 1.25$ au, which is much smaller than $a_3 \sin i_3$ and $a_4 \sin i_4$. Compared to Solution 2, the χ^2 in Solution 3 is much smaller, suggesting that Solution 3 is better. In Figure 4, the two-dimensional periodogram also reveals that the configuration of ($P_4 \simeq 100$ yr, $P_3 \simeq 33$ yr) is more likely than that of ($P_4 \simeq 100$ yr, $P_5 \simeq 24$ yr). Therefore, we infer that such a small signal may arise from unavoidable and slight imperfection in the double-Keplerian model.

4 RESULTS AND DISCUSSIONS

In this paper, new CCD observations of the Algol-type binary XZ And and all available mid-eclipse times

in the literature are investigated. The results are listed as Solution (3) in Table 2. The $O - C$ diagram shows a quadratic trend, suggesting that the orbital period of the eclipsing binary increases with a rate of $dP/dt = 1.96 \times 10^{-7} \text{ d yr}^{-1}$. By coincidence, Z Dra has a similar orbital period and increasing trend (Yuan et al. 2016). The increasing trend is attributed to mass transfer from the secondary component to the primary one. The mass transfer rate can be derived from the following equation

$$\dot{m}_1 = \frac{m_2 q}{3(1-q)} \frac{\dot{P}}{P}. \quad (7)$$

For XZ And, Manzoori (2016) reported that $m_1 = 2.10 M_\odot$, $m_2 = 1.02 M_\odot$ and the mass ratio of the eclipsing pair $q = m_2/m_1 = 0.485$, giving the mass transfer rate of $dm_1/dt = 4.6 \times 10^{-8} M_\odot \text{ yr}^{-1}$. The mass transfer rate is larger than that of Z Dra ($dm_1/dt = 9.2 \times 10^{-9} M_\odot \text{ yr}^{-1}$), but often lower than those of contact binaries. Z Dra is an Algol-type binary with a period similar to XZ And (Yuan et al. 2016). For contact binaries, such as AD Cnc (Qian et al. 2007a), V382 Cyg (Qian et al. 2007b) and TU Mus (Qian et al. 2007b), the typical value is $\sim 10^{-7} M_\odot \text{ yr}^{-1}$.

We find that the $O - C$ curve shows two sets of cyclic variations with periods of 33.43 and 100.3 yr, respectively. Interestingly, the ratio of the two periods is 1:3, or close to 1:3, which is a dynamical characteristic. Although magnetic activity can explain biperiodic variations in the mid-eclipse times of an eclipsing binary (Applegate 1992; Yuan & Qian 2007), magnetic activity cannot produce two sets of variations with commensurate periods, especially for

two periods with a (near) 1:3 ratio. The only reason for such variations is the LTT effect induced by two companions in a possible MMR.

Manzoori (2016) carried out the photometric-spectroscopic analysis, and indicated that the orbital inclination of the eclipsing pair is 89.8° , and the total mass of the eclipsing pair is $m_b = 3.12 M_\odot$. Assuming that the orbits of the two companions are coplanar with the eclipsing pair, the minimum masses of the two companions can be derived from the following mass functions

$$\frac{(m_3 \sin i_3)^3}{(m_b + m_3)^2} = \frac{4\pi^2}{GP_3^2} \times (a_3 \sin i_3)^3, \quad (8)$$

$$\frac{(m_4 \sin i_4)^3}{(m_b + m_3 + m_4)^2} = \frac{4\pi^2}{GP_4^2} \times (a_4 \sin i_4)^3, \quad (9)$$

where the subscripts ‘3’ and ‘4’ refer to XZ And (AB)C and D, respectively. The results reveal that XZ And (AB)C has a mass of $\sim 1.33 M_\odot$, and that of the outer companion XZ And (AB)D is $\sim 1.32 M_\odot$. The semimajor axes of the orbits of XZ And (AB)C and D are $A_3 = a_3 \cdot (m_b + m_3)/m_3 = 17.06$ au and $A_4 = a_4 \cdot (m_b + m_3 + m_4)/m_4 = 38.70$ au, respectively. Obviously, XZ And is a general three-body system if the central eclipsing binary is treated as a single object. According to the double-Keplerian model, we can calculate the gravitational perturbation between two companions. For the inner companion, XZ And (AB)C, the ratio of the gravitational perturbation from the outer companion to the centripetal forces from the eclipsing pair is between 0.03 and 0.19 with an average value of 0.085. For the outer companion, XZ And (AB)D, the average ratio of the gravitational perturbation from the inner companion to the centripetal forces from the eclipsing pair is 0.53. The strong gravitational perturbation invalidates the double-Keplerian model. However, it is strange that two interesting Keplerian periods are derived using an inappropriate model.

This illogical, but interesting phenomenon also appears in two other Algol systems. They are Z Dra (Yuan et al. 2016) and SW Lac (Yuan & Şenavcı 2014). Yuan & Şenavcı (2014) found that two companions are in near 1:3 MMR orbits around the eclipsing binary SW Lac with periods of 82.6 and 27.0 yr. If the orbital inclinations of the two companions of SW Lac are 90.0° , we can calculate the minimum masses of both companions ($m_3 = 0.62 M_\odot$, $m_4 = 1.90 M_\odot$) and the semimajor axes ($A_3 = 12.6$ au, $A_4 = 31.6$ au) from the best-fitting parameters (see Table 2 in Yuan & Şenavcı (2014)), while the total mass of the eclipsing pair is $m_b = 2.13 M_\odot$. The gravitational perturbation between the two companions is a little stronger than that of XZ And. Yuan et al. (2016) claimed that the Algol-type binary Z Dra has two companions with periods of 59.88 and 29.96 yr, close to a 1:2 MMR. For

Z Dra, $m_3 = 0.33 M_\odot$, $m_4 = 0.77 M_\odot$, $A_3 = 12.3$ au, $A_4 = 21.9$ au and $m_b = 1.90 M_\odot$ (see table 2 in Yuan et al. 2016). Figure 7 presented by Yuan et al. (2016) shows that the gravitational perturbation is weaker than that of XZ And, but not ignorable.

These interesting phenomena can hardly appear in three Algol systems by chance. We infer that the interesting periods and “inappropriate” double-Keplerian model reveal some unknown results. The results may be related to the dynamical characteristics of general N-body systems or the quantization of gravitation.

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