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Mass transfer and possible third-body effect on the period variations of the near-contact binary CN And

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Abstract We present a period analysis of the near-contact binary CN And using all available times of light minima. It is revealed that the orbital period exhibits a long-term decrease as well as a small-amplitude cyclic oscillation. This result suggests that the secular period decrease at the rate of $dP/dt = -1.4017 \times 10^{-7}\,\mathrm{d\,yr^{-1}}$ is caused by a combination of mass transfer and angular momentum loss due to magnetic braking. The periodic variation with an amplitude of $A = 0.0036\,\mathrm{d}$ and a period of $P_{\mathrm{mod}} = 28.3542\,\mathrm{yr}$ should be rooted in the light-time effect of a third body, rather than cyclic magnetic activity.

Key words: stars: binaries: close — stars: binaries: eclipsing — stars: individual: CN And

1 INTRODUCTION

CN Andromedae (CN And, GSC 02787–01815, BD +39 59, AAVSO 0015+39, SON 04704, PPM 42831) was first detected as a variable by Hoffmeister (1949). The photographic observations of this system were acquired by Tsesevich (1956) who classified it as an Algol-type eclipsing binary with an orbital period of 2.2599 d. Later, Löchel (1960) reobserved the binary and found that its light curves were EW-type. Moreover, he derived a period of 0.462798 d. Photoelectric light curves on BV bands and a single V band were reported by Bozkurt et al. (1976) and Seeds & Abernethy (1982), respectively. They confirmed that the light curves were of EW type and showed significant asymmetries. Kaluzny (1983) performed the photoelectric observations again. Their analysis suggested that the light curves of CN And should be EB type rather than EW type due to a significant difference in depth between the primary and secondary minima. In view of the controversial morphology or possible alterations in the light curves, CN And has been frequently targeted and studied by many researchers (e.g., Michaels et al. 1984, Rafert et al. 1985, Evren et al. 1987, Keskin 1989 and Samec et al. 1998). In the study of Michaels et al. (1984), the reported light curves on BV bands were more asymmetric than those of Bozkurt et al. (1976) and Seeds & Abernethy (1982). Rafert et al. (1985) recorded this system photoelectrically and obtained complete light curves. Their photometric results were similar to those of Kaluzny (1983), where CN And was classified as a contact system with components in poor thermal contact. However, Evren et al. (1987), Keskin (1989) and Samec et al. (1998) suggested that CN And should be a β Lyrae-type system with a semi-detached configuration. In addition, Yang & Liu (1985) observed CN And in the V band, and found two flares with a total duration of about 22 min. Shaw et al. (1996) detected its X-ray emission and revealed its X-ray luminosity of $\log L_{\mathrm{X}} = 30.55\,\mathrm{erg\,s^{-1}}$. This information indicated the activity of its light level.

The first spectroscopic observations of CN And were performed by Rucinski et al. (2000). From their spectroscopic solution, CN And was expected to be a very close semi-detached binary. Van Hamme et al. (2001) observed this system with UBV filters, and solved the UBV light curves simultaneously with the radial velocity curves of Rucinski et al. (2000). They concluded that CN And should be semidetached with the primary star filling its Roche lobe and the secondary one less than 1% underfilling it. Çiçek et al. (2005) acquired photoelectric observations and obtained the BVR-band light curves. Their photometric study hinted that CN And was a semi-detached binary which is similar to the result of Rucinski et al. (2000). In view of the rapid mass transfer, they expected that CN And was evolving from a semi-detached to a contact state, and finally evolved into an A-subtype W UMa

Table 1 Summary of the Orbital Period Changes of CN And from Literature

Year	Revised period (d)	Continuous change (d yr^{-1})	Reference
1956	2.2599	–	Tsesevich (1956)
1960	0.462798	–	Löchel (1960)
1976	0.462798	–	Bozkurt et al. (1976)
1982	0.4627946	–	Seeds & Abernethy (1982)
1983	0.46279661	4.7×10^{-11}	Kaluzny (1983)
1984	0.46279475	–	Michaels et al. (1984)
1985	0.4627945	–	Rafert et al. (1985)
1987	0.46279731	7.1×10^{-11}	Evren et al. (1987)
1998	0.46279092	9.8×10^{-11}	Samec et al. (1998)
2001	0.462790073	1.325×10^{-10}	Van Hamme et al. (2001)
2005	0.46279254	1.45×10^{-10}	Çiçek et al. (2005)
2006	0.462792448	1.153×10^{-10}	Lee & Lee (2006)
2006	0.462798329	1.048×10^{-10}	Jassur & Khodadadi (2006)

system. Lee & Lee (2006) published four-color ($BVRI$) light curves. Although their photometric solution did not significantly deviate from those of Çiçek et al. (2005), they thought that the system was just at the stage of broken contact. From these photometric results of Rucinski et al. (2000), Çiçek et al. (2005) and Lee & Lee (2006), a semi-detached configuration for CN And seems to be well determined. However, in the photometric study of Jassur & Khodadadi (2006), the simultaneous solution combining their own BV light curves and the radial velocity curves of Rucinski et al. (2000) suggested that the CN And system is an A-subtype W UMa binary with an overcontact configuration (the degree of overcontact is $f = 17\%$). Additionally, Siwak et al. (2010) reobserved this system using a CCD camera and wide-band $BVRI$ filters, and obtained new high-precision, multicolor ($BVRI$) light curves. Combining the multicolor light curves with the radial velocity curves of Rucinski et al. (2000), their solution indicated that CN And is in a contact configuration with a large temperature difference between its two components. Koju & Beaky (2015) reported $BVRI$ light curves obtained in 2010 and collected all previous photometric data to analyze the relation between the O'Connell effect and the eclipse timing variations (O'Connell 1951). Their result revealed no correlation between them.

Similar to the light-curve analyses, the orbital period variations of CN And have been also intensively investigated. Firstly, the orbital period of CN And has been gradually revised from 2.2599 d reported by Tsesevich (1956) to 0.4627946 d derived by Seeds & Abernethy (1982). Subsequently, secular decreases in its orbital period have been found and different decreasing rates were derived by Kaluzny (1983), Evren et al. (1987), Keskin (1989), Samec et al. (1998), Van Hamme et al. (2001), Çiçek et al. (2005), Lee & Lee (2006) and Jassur & Khodadadi (2006). All previous studies on the orbital period variations of CN And are summarized in Table 1. Following these studies, a large number of new and high-quality times of light minima for the last 12 years have been reported and provided an opportunity to further explore the period changes of CN And. In the present paper, we compiled all available light-minimum times ranging over 66 yr from 1950 to 2016, and a distinct period change is uncovered, i.e., a long-term period increase superposed on a cyclic oscillation. The physical mechanisms causing the period variations are discussed.

2 ORBITAL PERIOD VARIATIONS OF CN AND

For an eclipsing binary, one of the best methods to investigate orbital period variations is to search for the variations of its eclipsing times. Usually, one may analyze the $(O-C)$ diagram to determine the orbital period changes. In order to construct the $(O-C)$ curve of CN And, a careful search for all available light-minimum times has been performed. Two-hundred seven samples were collected from two well-known databases[1] and 39 photoelectric and CCD sources, which are listed in the first column of Table 2, were obtained from the literature.

In Table 2, the fourth column tells the observational methods, where "ccd" refers to charge-coupled device photometry and "pe" signifies photoelectric photometry. Among all light-minimum times, 186 data values are photoelectric or CCD observations (hereafter "PC") and 60 ones are visual or photographic cases (hereafter "VP"). In the following calculations, weights of 1 and 8 were

[1] the $O-C$ gateway: `http://var.astro.cz/ocgate/` and the Lichtenknecker database of the BAV: `http://www.bav-astro.de/LkDB/index.php`

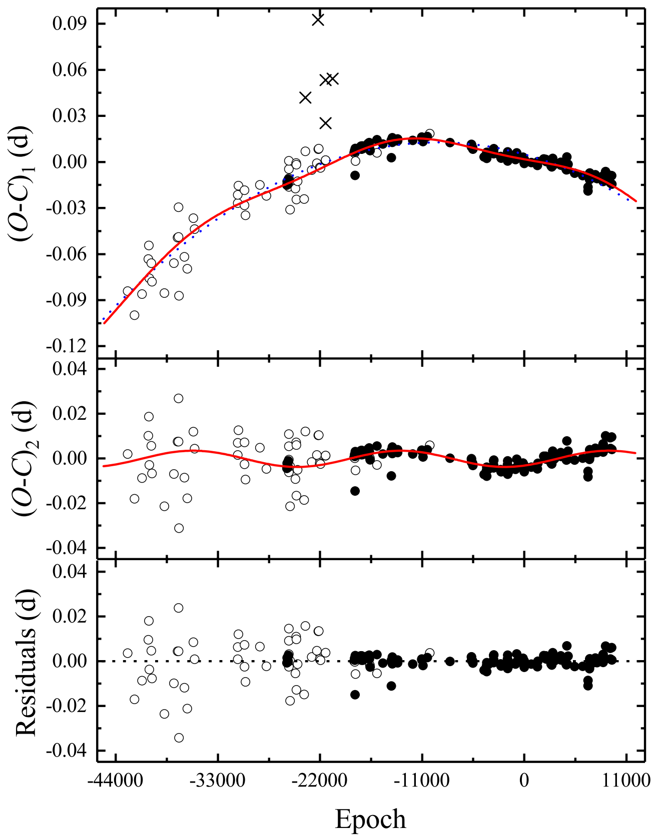

Fig. 1 *Top*: $(O-C)_1$ diagram of CN And based on Eq. (1) and the fitting curves. The *dotted* and *solid lines* represent the parabolic fit and full contribution of Eqs. (2) and (3), respectively. *Middle*: the residuals $(O-C)_2$ and the sine fitting curve of Eq. (3). *Bottom*: the final residuals and the linear fitting curve.

assigned to the VP and PC light-minimum times respectively. Based on these minima, a linear ephemeris derived by Jassur & Khodadadi (2006) was improved as follows

$$\mathrm{Min.I} = 2453339.15915 + 0.462791695E. \quad (1)$$

From the above new linear ephemeris, the $(O-C)_1$ values and their corresponding epochs for all available data were calculated and plotted in the upper panel of Figure 1, where the filled and open circles refer to PC eclipse data and VP data, respectively, and the "×" symbols mark the five unadopted VP data values [2442427.267 (Löcher 1975), 2443055.326 (Diethelm 1976), 2443431.277 and 2443432.462 (Diethelm 1977), 2443791.358 (Locher 1978)]. These display large deviation from the general trend formed by all other $(O-C)_1$ data.

In view of the apparent parabolic trend of $(O-C)_1$ values, we firstly adopt a quadratic polynomial to fit them. A weighted least-squares method yields the following nonlinear ephemeris

$$\begin{aligned}\mathrm{Min.I} = {} & 2453339.15915(37) + 0.462790035(46)E \\ & + 8.88(20) \times 10^{-11} E^2,\end{aligned} \quad (2)$$

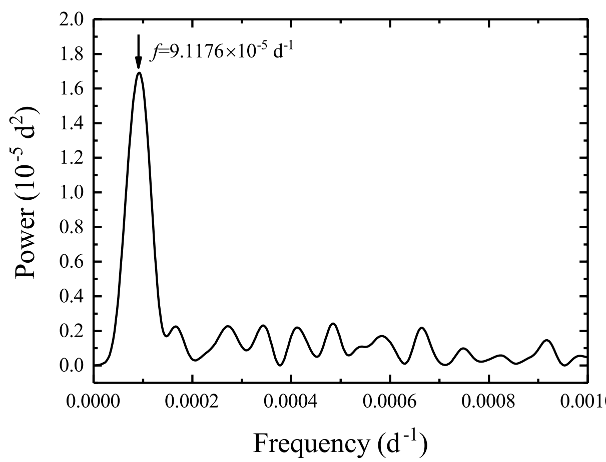

Fig. 2 Fourier power spectrum of $(O-C)_2$ residuals.

which is depicted as a dashed line in the top panel of Figure 1. From the quadratic term of Equation (2), a secular period change at the rate of $\dot{P}_{\mathrm{obs}} = -1.4017 \times 10^{-7}\,\mathrm{d\,yr^{-1}}$ could be derived. The residuals $(O-C)_2$ are plotted in the middle panel of Figure 1, which seem to show a small-amplitude cyclic oscillation. In order to detect the cyclic oscillation, Fourier analysis for the residuals $(O-C)_2$ has been performed by using the soft-

Table 2 Photoelectric and CCD Times of Light Minima for CN And Collected from Literature

HJD2400000+	Type	Error	Method	Ref.	HJD2400000+	Type	Error	Method	Ref.
41509.4942	II	-	pe	[1]	48230.6504	II	-	pe	[7]
41512.5046	I	-	pe	[1]	48231.5708	II	-	pe	[7]
41567.5753	I	-	pe	[1]	48231.5759	II	-	pe	[7]
41568.5014	I	-	pe	[1]	48231.5777	II	-	pe	[7]
41577.2969	I	-	ccd	[2]	48233.6558	I	-	pe	[7]
41577.5280	II	-	pe	[1]	48233.6565	I	-	pe	[7]
45608.6999	I	-	pe	[3]	48233.6580	I	-	pe	[7]
45620.7291	I	-	pe	[3]	51458.3786	I	-	ccd	[8]
45620.7300	I	-	pe	[3]	51469.2556	II	-	pe	[8]
45654.7387	II	-	pe	[3]	51471.3341	I	-	pe	[8]
46711.5220	I	-	ccd	[4]	51807.3239	I	-	pe	[8]
46712.4582	I	-	pe	[5]	51811.2607	II	-	pe	[8]
46714.3104	I	-	pe	[5]	51814.2692	I	-	pe	[8]
46760.3571	II	-	pe	[5]	52500.1204	I	-	ccd	[9]
46762.2100	II	-	pe	[5]	53273.2145	II	0.0003	ccd	[10]
46762.2111	II	-	pe	[5]	53338.9296	II	0.0006	ccd	[10]
46762.4391	I	-	pe	[5]	53339.1590	I	0.0002	ccd	[10]
46762.4392	I	-	pe	[6]	53341.0093	I	0.0003	ccd	[10]
48230.6479	II	-	pe	[7]	54014.8352	I	0.0005	ccd	[11]
48230.6482	II	-	pe	[7]	-	-	-	-	-

References: [1] Kaluzny (1983); [2] Dvorak (2004); [3] Michaels et al. (1984); [4] Maciejewski & Karska (2004); [5] Pohl et al. (1987); [6] Evren et al. (1987); [7] Van Hamme et al. (2001); [8] Jassur & Khodadadi (2006); [9] Siwak et al. (2010); [10] Lee & Lee (2006); [11] Vander Haagen (2013).

ware PERIOD04 (Lenz 2004, Yang et al. 2012, Li et al. 2016). The power spectrum is displayed in Figure 2, where a significant peak is located at the frequency of $f = 9.1176 \times 10^{-5}\,\mathrm{d}^{-1}$. Thus, we suggest that there is a cyclic variation in the orbital period of CN And. With $f = 9.1176 \times 10^{-5}\,\mathrm{d}^{-1}$, the period of the cyclic variation could be derived to be 30.0282 yr. By using a sine function to fit the $(O-C)_2$ data, a weighted least-squares solution yields the following equation

$$(O-C)_2 = -0.00018(6) - 0.0036(6) \times \sin[0.0161^\circ(2)E - 56.2430^\circ(\pm 9.8660^\circ)], \tag{3}$$

which is plotted as the continuous line in the middle panel of Figure 1. Equation (3) suggests that the orbital period of CN And manifests a cyclic oscillation with an amplitude of $A = 0.0036\,\mathrm{d}$ and a period of $P_{\mathrm{mod}} = 28.3542\,\mathrm{yr}$. The modulated period of 28.3542 yr is slightly shorter than the period of 30.0282 yr derived from the power spectrum. The complete fitting curve combining Equations (2) with (3) is represented as the continuous line in the top panel of Figure 1. The corresponding residuals are depicted in the bottom panel of Figure 1, where no significant regular variations can be found. The weighted residual sum of squares is $\sum_i w_i(O-C)_i^2 = 0.0150\,\mathrm{d}^2$.

3 ORIGIN OF ORBITAL PERIOD VARIATION

3.1 Magnetic Braking vs. Mass Transfer

The present study and previous results on the orbital period variation of CN And give strong evidence that the period decrease is real. In general, a continuous period decrease may be caused by mass transfer from the primary component to the secondary one and/or angular momentum loss due to magnetic braking. If angular momentum loss due to magnetic braking fully contributes to the continuous period decrease of CN And, the decreasing rate of its orbital period can be estimated with the following formula derived by Guinan & Bradstreet (1988)

$$\dot{P}_{\mathrm{mb}} \approx -1.1 \times 10^{-8} q^{-1}(1+q)^2(M_1+M_2)^{-5/3}\mathrm{k}^2 \times (M_1R_1^4 + M_1R_1^4)P^{-7/3}, \tag{4}$$

where M_1 and M_2 are the masses of the components, and R_1 and R_2 denote their effective radii. q is the mass ratio and is defined as $q = M_2/M_1$. The gyration constant $\mathrm{k}^2 = 0.1$ for cool main-sequence stars. By setting the physical parameters in Equation (4), we obtain a period decrease rate of $\dot{P}_{\mathrm{mb}} = -7.6 \times 10^{-8}\,\mathrm{d\,yr}^{-1}$, which is significantly less than the observed period decrease rate $\dot{P}_{\mathrm{obs}} = -1.4017 \times 10^{-7}\,\mathrm{d\,yr}^{-1}$. Thus, the angular momentum loss due to magnetic braking is not enough to cause the observed period decrease of CN And.

Table 3 Near-contact Binaries with Their Primary Components Filling the Critical Roche Lobe

Year	Revised period (d)	Continuous change ($\mathrm{d\,yr^{-1}}$)	Reference
RT Scl	0.51156	-1.29×10^{-7}	Hilditch & King (1986)
FT Lup	0.47008	-1.77×10^{-7}	Djuraševic (1993)
V361 Lyr	0.30961	-0.29×10^{-7}	Hilditch et al. (1997)
V1010 Oph	0.66144	-3.97×10^{-7}	Hamdy et al. (1985)
TT Her	0.91208	-3.53×10^{-7}	Kwee & van Genderen (1983)
BL And	0.72238	-2.26×10^{-7}	Van Hamme et al. (2001)
V388 Cyn	0.85905	-2.06×10^{-7}	Kang et al. (2001)
AX Vir	0.70253	-1.72×10^{-8}	Barone et al. (1991)
HL Aur	0.62251	-1.72×10^{-8}	Gray et al. (1997)
DO Cas	0.68467	-1.04×10^{-7}	Ahn et al. (2000)
GR Tau	0.42985	-4.23×10^{-8}	Qian (2002b)
BO Peg	0.58043	-1.26×10^{-7}	Yamasaki & Okazaki (1986)

Table 4 Spot Parameters of CN And

Date	Band	Star	Co-Latitude (deg)	Longitude (deg)	Ang.radius (deg)	$T_\mathrm{s}/T_\mathrm{ph}$	$L_\mathrm{s}/L_\mathrm{ph}$	Reference
1982	B,V	1	18.0 ± 14.0	13.0 ± 11.0	32.0 ± 11.0	0.65	0.00346	[1]
		2	79.1 ± 5.4	-8.6 ± 3.2	19.5 ± 4.7	1.151 ± 0.041	0.01267	
1983	B,V	1	18.0 ± 27.0	11.1 ± 6.6	33.0 ± 28.0	0.65	0.00368	[2]
		2	82.0 ± 15.0	13.0 ± 44.0	23.7 ± 8.2	1.114 ± 0.072	0.01807	
1986	B,V	1	25.0 ± 17.0	13.5 ± 1.5	32.0 ± 11.0	0.65	0.00346	[3]
		2	84.0 ± 11.0	-8.6 ± 4.6	32.5 ± 9.1	1.082 ± 0.029	0.02738	
1992	B,V,R	1	25.0 ± 31.0	8.5 ± 8.7	29.0 ± 16.0	0.65	0.00284	[4]
		2	75.3 ± 7.0	1.3 ± 4.9	20.1 ± 5.2	1.150 ± 0.064	0.01346	
1997	U,B,V	1	22.3 ± 2.4	16.9 ± 2.0	32.9 ± 2.0	0.65	0.00365	[5]
		2	84.0 ± 5.0	1.1 ± 2.0	19.5 ± 1.8	1.104 ± 0.013	0.01070	
2000	B,V	1	70.0	294.0	25.0	0.85	0.00619	[6]
		2	90.0	174.0	18.5	1.19	0.01304	
2001	B,V,R	1	22.6 ± 4.5	17.5 ± 3.5	32.1 ± 6.4	0.65 ± 0.13	0.00348	[7]
		2	83.1 ± 0.2	1.3 ± 0.3	20.1 ± 0.4	1.093 ± 0.022	0.01094	
2004	B,V,R,I	1	19.8 ± 0.2	11.5 ± 0.2	34.9 ± 0.1	0.65	0.00406	[8]
		2	87.5 ± 2.0	3.6 ± 2.6	23.3 ± 2.1	1.092 ± 0.014	0.01465	
2004	B,V,R,I	1	–	–	–	–	–	[9]
		2	90.0	11.49 ± 0.36	113.9 ± 1.9	1.214 ± 0.072	0.49350	

References: [1] Kaluzny (1983); [2] Rafert et al. (1985); [3] Keskin (1989); [4] Branly (1992); [5] Van Hamme et al. (2001); [6] Jassur & Khodadadi (2006); [7] Çiçek et al. (2005); [8] Lee & Lee (2006); [9] Siwak et al. (2010).

Another possible mechanism causing the orbital period decrease of CN And is conservative mass transfer from the primary star to the secondary one. By inserting the physical parameters $M_1 = 1.433\,M_\odot$ and $M_2 = 0.552\,M_\odot$ (Siwak et al. 2010) into the following formula derived by Pringle (1975)

$$\dot{M}_1 = \frac{M_1 M_2}{3P(M_1 - M_2)}\dot{P}_\mathrm{obs}, \tag{5}$$

the mass-transfer rate $\dot{M}_1 = -9.0646\times10^{-8}\,M_\odot\,\mathrm{yr}^{-1}$ can be determined. If the mass transfer of CN And is on the thermal timescale, i.e., $\tau_\mathrm{th} \sim \frac{GM_1^2}{R_1 L_1} = 1.2744\times10^7\,\mathrm{yr}$, the mass transfer rate may be estimated to be $\dot{M}_1 = 1.1244\times10^{-7}\,M_\odot\,\mathrm{yr}^{-1}$. It is larger than the mass-transfer rate derived from Equation (4), which suggests that conservative mass transfer could not singly cause the observed orbital period decrease. However, the observed evidence strongly suggests that the mass transfer of CN And is real. From the BFs in the phase-velocity plane, a subtle track of a brighter region has been clearly detected on the secondary component around phase 0.65. Since the surface temperature of the primary component of CN And is significantly higher than that of its secondary one, material flow via the inner Lagrange point from the primary to the secondary will create a hot spot on the surface of the secondary. All previous photometric observations and analysis suggested that such hot spots are only on the secondary component, which provide good evidence for mass transfer. Thus, for CN And, mass transfer should cause or at least partly contribute to the observed orbital period decrease.

With the high-precision photometric data, Siwak et al. (2010) obtained a relatively reliable photometric solution, where CN And is determined to be a shallow-contact binary system with a degree of contact $f = 3.0\%$. According to the theory of thermal relaxation oscillation (TRO), contact binary systems must undergo cyclic oscillations between their semi-detached and contact configurations. In the contact stage, the theory of TRO predicts that the orbital period will increase because of mass transfer from the secondary to primary component. However, the shallow-contact binary CN And shows a decreasing period. Moreover, the situation of CN And is not especially rare. So far, at least 12 binaries whose properties are similar to CN And (see Table 3) have been identified. This suggests that the evolution of contact binaries may be more intricate than the prediction of TRO. In order to explain their unusual orbital period variations, Qian (2002a) developed the theory of TRO and proposed a supplementary evolutionary scenario in which contact binaries are undergoing TRO with variable angular momentum loss. With the orbital period decreasing, their orbits will shrink and the contact degree increases, which will reduce the angular momentum loss. Gradually, they will evolve into the TRO-controlled stage and the orbital period will increase. From this perspective, the observed period decrease of CN And indicates that its evolution is just controlled by the angular momentum loss, and continuous period decrease should be caused by a combination of mass transfer and angular momentum loss due to magnetic braking.

3.2 Magnetic Activity Cycle vs. Light Time Effect of a Third Body

Both cyclic magnetic activity and a third body orbiting a binary can lead to periodic variation in the associated orbital period. The study of Zola et al. (2005) points to CN And being an active solar-type binary with components of spectral type in the range F5 to G5. Flare events with an amplitude of 0.11 mag (Yang & Liu 1985) and moderately strong X-ray emission (Shaw et al. 1996) have also been detected from this system. In addition, almost all photometric studies hinted that the remarkable O'Connell effect in the light curves of CN And may be partially caused by a magnetically active spot, which indicates that its primary component could be a strong magnetically active star (O'Connell 1951). All this evidence suggests that the observed periodic variation in the orbital period of CN And may be generated by cyclic magnetic activity. A theoretical model to interpret this mechanism was firstly proposed by Applegate (1992) and advocated by Lanza et al. (1998) and Lanza & Rodonò (2002). According to their theory, a cyclic hydromagnetic dynamo may lead to changes in the gravitational quadrupole moment and modulate the orbital motion. In fact, under the situation of conservative angular momentum, if the gravitational quadrupole moment increases, two components of a binary will be forced to move closer to each other and move faster due to an increase in the centripetal acceleration. Therefore, the orbital period decreases. Conversely, when the gravitational quadruple moment decreases, the orbital period will increase. With the orbital period modulation $\Delta P = 2\pi A \frac{P}{P_{\rm mod}} = 1.0194 \times 10^{-6}$ d, the quadrupole moment variation can be calculated according to the following formula (Lanza et al. 1998)

$$\frac{\Delta P}{P} = -9\left(\frac{R}{a}\right)^2 \frac{\Delta Q}{MR^2}. \tag{6}$$

With the absolute physical parameters ($M_1 = 1.433\,M_\odot$, $M_2 = 0.552\,M_\odot$, $R_1 = 1.48\,R_\odot$ and $R_2 = 0.95\,R_\odot$) obtained by Siwak et al. (2010), the quadrupole moment variations in the primary and secondary components of CN And are determined to be $\Delta Q_1 = 3.3806 \times 10^{49}$ g cm^2 and $\Delta Q_2 = 1.3022 \times 10^{49}$ g cm^2, respectively. By using the typical shell mass $M_{\rm s} = 0.1\,M$, the relative luminosity variations may be conveniently computed from the following formula (Yu et al. 2015)

$$\frac{\Delta L}{L} = \frac{5G^2}{24\pi^2\sigma}\frac{M^3}{R^6}\left(\frac{a}{RT}\right)^4 \frac{(\Delta P)^2}{P_{\rm mod}}\,, \tag{7}$$

where the gravitational constant G, the Stefan-Boltzman constant σ and other physical parameters are all in the International System of Units. For CN And, we obtain the relative luminosity variations $\Delta L_1/L_1 = 0.0033$ and $\Delta L_2/L_2 = 0.0558$ for its primary and secondary components, respectively. Applegate (1992) states that the typical quadrupole moment variation is at the $\Delta Q \sim 10^{49}$ g cm^2 level and the relative luminosity variation should be lower than 0.1 (i.e., $\Delta L/L < 0.1$). The present results for CN And could just meet these requirements. Thus, cyclic magnetic activity is plausible to interpret the observed periodic variation in the orbital period of CN And.

In order to search for additional evidence of cyclic magnetic activity, we carefully checked all photometric solutions of CN And compiled so far, and collected the spot parameters of these solutions (see Table 4). Firstly, the secular hot spot on the surface of the secondary component is located in its low latitude region (i.e., its co-latitude is about 90°) with small longitude. The effective temperature of this hot spot is almost equal to the surface temperature of the primary star. These features imply that this hot spot should be generated by mass transfer from the primary to the secondary components. Secondly, aside from the hot spot on the secondary component, a large cool spot was also revealed by several early photometric solutions. However, the star did not show significant variability in either location or surface temperature in response to the possible cyclic magnetic activity. Moreover, recent photometric solutions of CN And by Siwak et al. (2010) suggested that no spot exists on the primary component and only a single hot spot on its secondary could completely contribute to the observed O'Connell effect. Thus, the periodic variation in orbital period of CN And should not originate from the cyclic magnetic activity.

If the cyclic variation in eclipsing times of CN And is caused by the light-time effect arising from the gravitational influence of a third body, the third body could move in a circular orbit because of the aforementioned good sinusoidal fit. By assuming a circular orbit, the orbital radius of CN And rotating around the barycenter of the triple system is calculated with the following equation

$$a_{12}\sin i' = A \times c\,, \tag{8}$$

where a_{12}, i', A and c are the semimajor axis of the eclipsing pair, the orbital inclination of the triple system, the amplitude of the cyclic change and the speed of light, respectively. The mass function and the mass of the purported third body are computed with the following equation

$$\begin{aligned} f(m) &= \frac{4\pi^2}{GP_{\rm mod}^2} \times (a_{12}\sin i')^3 \\ &= \frac{M_3^3 \sin^3 i'}{(M_1+M_2+M_3)^2}\,, \end{aligned} \tag{9}$$

where M_3, G and $P_{\rm mod}$ are the mass of this third body, the gravitational constant and the modulation period, respectively. The corresponding parameters of the third body are listed in Table 5. As shown in Table 5, the third body is relatively invisible, thus the spectrum of a third body will be difficult to detect. The relations between the mass and radius of the third body and its orbital inclination are displayed in Figure 3. If the third body is coplanar with the system CN And (i.e., $i' = 69.416°$), its mass should be $M_3 = 0.1189\,M_\odot$. In this case, the third body is a cool stellar object.

4 CONCLUSIONS AND DISCUSSION

According to the above analysis of eclipsing time variations for CN And, the secular orbital period decrease should be caused by a combination of mass transfer and angular momentum loss due to magnetic braking. With the observed cyclic variations in eclipsing times of CN And, it is most probable that the light-time effect due to an unseen third star could be used interpret its nature. However, we should further explore additional evidence of its third body (e.g., any third light in photometry, a third-body spectrum in the overall spectrum, additional eclipses in a few rare cases, variations of light curve amplitude due to orbital plane precession, variation of radial velocity of the mass center of the binary system, etc.) in the future to revisit this disputable problem, although these physical effects could be difficult to detect due to the possible low mass or faint light of the third body. In particular, it should be most promising to detect the variation of radial velocity in the binary. The first spectroscopic observations of CN And performed by Rucinski et al. (2000) revealed an approximate system radial velocity of $V_{\rm c} = -24.86\,{\rm km\,s^{-1}}$ in September 1997. In order to obtain the largest observ-

Table 5 Orbital Parameters of the Postulated Third Body in CN And

Parameter	Value	Unit
A	0.0036(±0.0006)	d
P_3	28.3542(±0.3052)	yr
$f(M)$	0.0003(±0.0002)	$M_\odot$
$M_3(i'=90°)$	0.1107(±0.0342)	$M_\odot$
$M_3(i'=70°)$	0.1181(±0.0366)	$M_\odot$
$M_3(i'=50°)$	0.1462(±0.0455)	$M_\odot$
$M_3(i'=30°)$	0.2297(±0.0728)	$M_\odot$
$a_3(i'=90°)$	11.2391(±1.1488)	AU
$a_3(i'=70°)$	11.2127(±1.1499)	AU
$a_3(i'=50°)$	11.1141(±1.1539)	AU
$a_3(i'=30°)$	10.8327(±1.1646)	AU

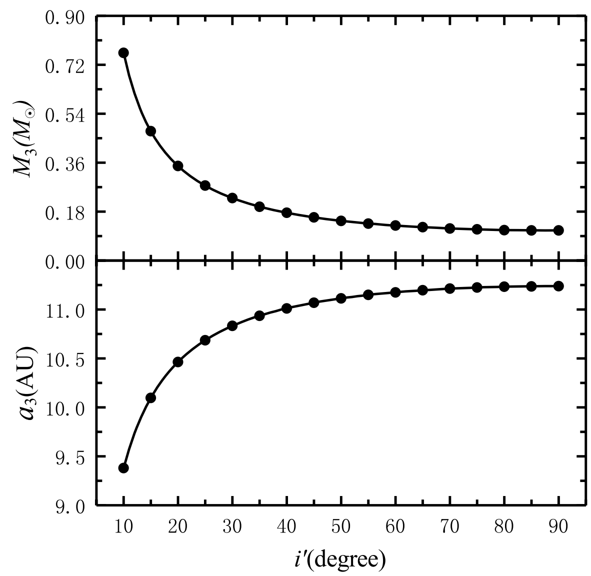

Fig. 3 *Upper*: the correlation between the third-body mass M_3 and the orbital inclination i'. *Lower*: the correlation between the orbital radius of a third body a_3 and its orbital inclination i'.

able difference in system radial velocity, we suggest that follow-up spectroscopic observation should be made in 2038, to confirm the third-body hypothesis more firmly.

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