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***R**esearch in*
***A**stronomy and*
***A**strophysics*

From multicolor-photometric observations to a guaranteed mass of AL Cas

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Abstract We utilize the PAdova and TRieste Stellar Evolution Code (PARSEC) combined with photometric observations to determine a guaranteed mass of AL Cas and re-examine its related physical parameters. Multicolor-photometric observations of AL Cas have been performed in 2016 and 2017. We use the Wilson-Devinney (W-D) code to analyze the light curves and find that AL Cas is probably an A-subtype contact binary ($f = 35.7 \pm 0.9\%$) with a mass ratio $q = 0.6399 \pm 0.0230$ and an effective temperature difference $\Delta T = 78$ K. The mass-radius relation of a higher luminosity component for AL Cas is obtained by two methods: depending on calculation of the Roche lobe (DCRL method) and depending on calculation of the W-D code (DCWD method). Using this relationship with the PARSEC model, we investigate the component masses of AL Cas as $M_1 = 1.19 \pm 0.23\,M_\odot$ with $M_2 = 0.76 \pm 0.18\,M_\odot$ by the DCRL method and $M_1 = 1.22 \pm 0.26\,M_\odot$ with $M_2 = 0.78 \pm 0.20\,M_\odot$ by the DCWD method. By means of the photometric studies, we examine the related physical properties of AL Cas with the latest findings. We update the orbital period ($P_{\rm orb} = 0.50055593$ d) of AL Cas according to six new times of light minimum together with those collected from the literature. Meanwhile, the $(O - C)_2$ curve analysis suggests that the orbital period of AL Cas has a cyclic variation with a period of 81.25 yr and an amplitude of 0.01415 d. This cyclic change would be caused by the light-travel time effect from a third body. A similar mass of the third body ($M_3 \sin i' = 0.279\ M_\odot$) is derived from our two methods.

Key words: stars: binaries: close — stars: binaries: eclipsing — stars: individual: AL Cassiopeiae

1 INTRODUCTION

AL Cas (2MASS J02134464+7008429) was first discovered as a variable star by Hoffmeister (1928). It was classified as an EW-type eclipsing binary with a spectral type of B according to the several catalogs of variable stars (Reed 2003; Malkov et al. 2006; Samus & Kazarovets 2011). Kreiner et al. (2001) obtained the first linear ephemeris (Min.I= $2425303.5729 + 0.5005555^{\rm d} \times E$). Based on some light minimum times (Safar & Zejda 2002; Zejda 2002, 2004; Hubscher 2005; Hubscher et al. 2005; Kotkova & Wolf 2006; Kim et al. 2006; Hubscher 2007; Brát et al. 2007; Brat et al. 2009; Hubscher et al. 2009; Hubscher 2011; Qian et al. 2014), Qian et al. (2014) updated the linear ephemeris as follows,

$$\rm Min.I = 2425303.55676 + 0.50055604^{d} \times \mathit{E}. \qquad (1)$$

Qian et al. (2014) considered that AL Cas was a contact binary with a contact degree of 39.3% and a mass ratio of 0.61. Their $O - C$ analysis showed that AL Cas might have a third body with a period of 86.6 yr, and the third body mass was computed according to an assumed total mass of $2.14\,M_\odot$. They also suggested that the spectral type of AL Cas is F7-type rather than B-type. The relative parameters of AL Cas had been investigated by Qian et al. (2014), but few studies focused on the absolute parameters of AL Cas using the stellar-evolution models.

In this work, the photometric observations for AL Cas are carried out, and the BVR light curves are analyzed by using the Wilson-Devinney (W-D) code (Wilson & Devinney 1971). Meanwhile, according to the multicolor-photometric analysis we use the PAdova and TRieste Stellar Evolution Code (PARSEC) (e.g., Bressan et al. 2012) to calculate the absolute parameters of AL Cas. In addition, the orbital period variation of AL Cas is also investigated based on some new minimum times combined with those collected from literature.

Table 1 The Observed Information of AL Cas

Date	Band	Frame	Exposure time (s)
20161106	B	215	35
	V	229	20
	R	162	15
20161107	B	218	35
	V	166	22
	R	85	30
20161225	B	159	20
	V	194	9
	R	229	6
20170115	B	62	20
	V	62	9
	R	64	6

2 OBSERVATIONS AND DATA REDUCTION

AL Cas was observed in 2016 and 2017 using the Nanshan One-meter Wide-field Telescope (hereafter NOWT, Liu et al. 2014). NOWT is located at the Nanshan station of Xinjiang Astronomical Observatory. This optical telescope is equipped with the Johnson multi-color filter system (e.g., Cousins 1976). During our observations, the BVR filters were used. The field of view was $18.75' \times 18.75'$. The basic observation information of AL Cas is listed in Table 1. In general, we have obtained 654 CCD images in B filter, 651 in V filter, 540 in R filter. One of the observed CCD images is shown in Figure 1, where the "V", "C" and "K" are AL Cas, the comparison star and the check star, respectively. The photometric precisions of the BVR bands at the four nights are revealed in Figure 2, where the red, blue and black histograms refer to the B, V and R band, respectively, and the four nights are marked as Arabic numerals "1, 2, 3, 4". It can be seen that the precisions are better than 0.015 mag for more than about 90 percent of observation nights. The observed CCD images are reduced using the standard aperture photometry package of the Image Reduction and Analysis Facility (IRAF[1]).

The essential information of AL Cas, the comparison and the check star are given in Table 2. To obtain the complete light curves of AL Cas, we employ the difference aperture photometry for all CCD images. In Figure 3, the upper panel displays the complete light curves of AL Cas in the BVR bands, and the magnitude differences between the comparison and the check stars are displayed in the bottom panel. The corresponding photometric data of AL Cas in the *B*, *V* and *R* passbands on 2016 November 6 and 7, 2016 December 25 and 2017 January 15 are listed in Table 3. From Figure 3, one can find that the light curves of AL Cas are typically the EW-type (Molík 1998).

3 ORBITAL PERIOD CHANGE OF AL CAS

We analyze the orbital period change of AL Cas by the $O-C$ method. All times of light minima, including six new minimum times (by a parabolic fitting method for those CCD photometric data) and 74 collected from the literature, are listed in Table 4.

Based on the above times of light minima, the $(O-C)_1$ value and E are calculated by Equation (1). The $O-C$ diagram is exhibited in Figure 4. The smaller open circles in this figure stand for the visual or photographic observation data (hereafter VP), and the bigger open circles refer to the photoelectric or CCD observed data (hereafter PC). The solid points are the photometric data by the NOWT. During the $O-C$ analysis, the weight of the VP and PC data are selected as 1 and 8, respectively (Qian et al. 2008).

In this work, we apply the linear fitting (a dashed line in the $(O-C)_1$ diagram) for all the $O-C$ data to obtain a new linear ephemeris. Meanwhile, the sinusoidal fitting (a blue solid line in the $(O-C)_2$ diagram) is adopted for the residuals that remove the linear trend of the $(O-C)_1$ curves. The final fitting (a black solid line in the $(O-C)_1$ diagram) result is given as

$$\begin{aligned}\mathrm{Min.I} =& 2425303.55605(\pm 0.00425)\\ &+ 0.50055593^{\mathrm{d}}(\pm 0.00000008) \times E\\ &+ 0.01415(\pm 0.00339)\sin[0.00607^\circ(\pm 0.00022^\circ) \times E\\ &+ 1.52177^\circ(\pm 0.65021^\circ)].\end{aligned} \tag{2}$$

From Figure 4, we can see that our fitting result is reasonable, and the value of sum of square residuals ($\Sigma_i(O-C)_i^2 = 0.0066\,\mathrm{d}^2$) could be adopted. The result of Equation (2) indicates that a cyclic variation exists in the $O-C$ curve of AL Cas with a period of 81.25 yr and an amplitude of 0.01415 d.

4 PHOTOMETRIC SOLUTION WITH THE W-D CODE

In this work, we adopt the W-D code of the 2013 version (Wilson et al. 2010; Wilson 2012) to analyze the light curves of AL Cas.

The effective temperature of star 1 (the star eclipsed at the primary light minimum) is fixed at $T_1 = 6400\,\mathrm{K}$ (Qian et al. 2014). The fixed parameters, including the gravity-darkening coefficients (g_1 and g_2: Lucy 1967), the bolometric albedo (A_1 and A_2: Ruciński 1969), the circular orbit (e), the synchronous rotation (F_1 and F_2: Jiang et al.

[1] IRAF is distributed by the National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc. (AURA) under cooperative agreement with the National Science Foundation.

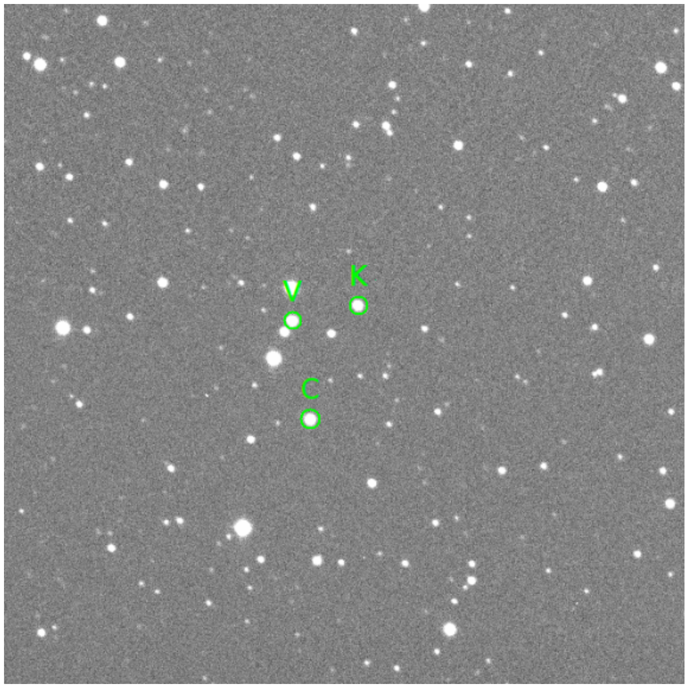

Fig. 1 CCD image of AL Cas (V), the comparison star (C) and the check star (K).

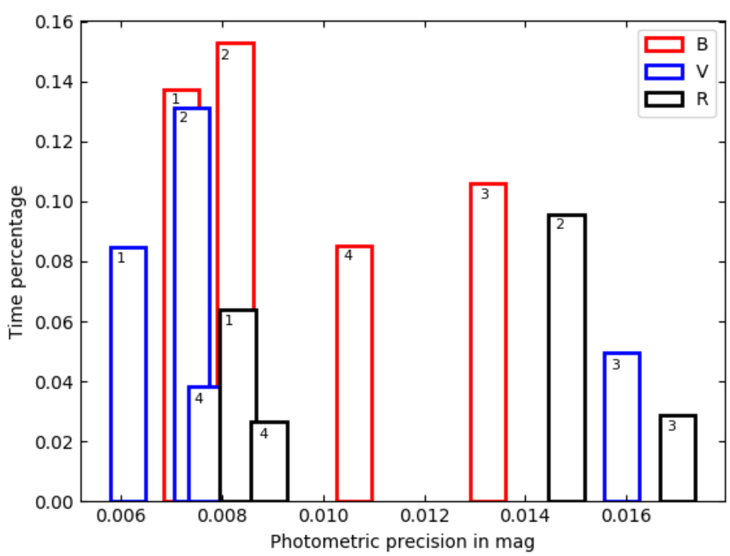

Fig. 2 The photometric precisions of the BVR bands at four nights.

Table 2 Essential Information for AL Cas, and the Comparison and Check Stars

Target	Name	α_{2000} (h m s)	δ_{2000} (° ′ ″)	$B_{\rm mag}$	$J_{\rm mag}$	$H_{\rm mag}$
Variable	AL Cas	02 13 44.652	+70 08 42.976	12.300	11.037	10.789
The comparison	2MASS 02134031+7010300	02 13 40.318	+70 10 30.094	–	11.593	11.354
The check	GSC 04315–00032	02 13 30.660	+70 08 24.581	13.730	–	–

Notes: Simbad and Vizier: `http://simbad.u-strasbg.fr/simbad/`.

Table 3 The photometric data of AL Cas in the *B*, *V* and *R* passbands on 2016 November 6 and 7, 2016 December 25 and 2017 January 15.

Date	HJD	Phase	$\Delta m\,(B)$	HJD	Phase	$\Delta m\,(B)$	HJD	Phase	$\Delta m\,(B)$
20161106	2457699.04485	0.00347	0.290	2457699.17777	0.26902	–0.461	2457699.34822	0.60955	–0.235
	2457699.04677	0.00731	0.278	2457699.17968	0.27283	–0.461	2457699.34983	0.61276	–0.243
	2457699.04853	0.01083	0.272	2457699.18173	0.27692	–0.463	2457699.35143	0.61595	–0.254
	2457699.05044	0.01464	0.253	2457699.18352	0.28051	–0.454	2457699.35315	0.61940	–0.265
	2457699.05221	0.01818	0.241	2457699.18544	0.28435	–0.452	2457699.35475	0.62259	–0.272
	2457699.05404	0.02183	0.230	2457699.18753	0.28851	–0.454	2457699.35637	0.62582	–0.288
	2457699.05581	0.02537	0.205	2457699.18923	0.29191	–0.448	2457699.35798	0.62904	–0.290
	2457699.05772	0.02918	0.193	2457699.19093	0.29531	–0.445	2457699.35959	0.63225	–0.294
	2457699.05952	0.03277	0.167	2457699.19276	0.29896	–0.431	2457699.36131	0.63570	–0.304
	2457699.06130	0.03633	0.140	2457699.19446	0.30236	–0.428	2457699.36292	0.63891	–0.315
	2457699.06321	0.04015	0.114	2457699.19618	0.30580	–0.427	2457699.36452	0.64210	–0.322
	2457699.06823	0.05018	0.058	2457699.19789	0.30920	–0.423	2457699.36613	0.64532	–0.325
	2457699.07006	0.05383	0.031	2457699.19960	0.31262	–0.415	2457699.36774	0.64853	–0.335
	2457699.07184	0.05739	0.014	2457699.20120	0.31582	–0.412	2457699.36935	0.65174	–0.346
	2457699.07368	0.06107	–0.008	2457699.20281	0.31903	–0.412	2457699.37096	0.65496	–0.349
	2457699.07545	0.06461	–0.030	2457699.20440	0.32222	–0.406	2457699.37257	0.65817	–0.351
	2457699.07725	0.06819	–0.057	2457699.20601	0.32543	–0.397	2457699.37416	0.66136	–0.362
	2457699.07902	0.07173	–0.063	2457699.20762	0.32865	–0.393	2457699.37577	0.66458	–0.368
	2457699.08080	0.07529	–0.086	2457699.20936	0.33212	–0.389	2457699.37750	0.66802	–0.371
	2457699.08255	0.07878	–0.106	2457699.21095	0.33531	–0.376	2457699.37922	0.67147	–0.384
	2457699.08445	0.08258	–0.126	2457699.21256	0.33852	–0.372	2457699.38094	0.67491	–0.386
	2457699.08618	0.08604	–0.134	2457699.21417	0.34174	–0.369	2457699.38281	0.67864	–0.394
	2457699.08797	0.0896	–0.145	2457699.22301	0.35940	–0.325	2457699.38453	0.68208	–0.400
	2457699.08970	0.09307	–0.174	2457699.22463	0.36264	–0.321	2457699.38640	0.68580	–0.409
	2457699.09147	0.09661	–0.178	2457699.22635	0.36606	–0.312	2457699.38812	0.68925	–0.409
	2457699.09324	0.10015	–0.197	2457699.22807	0.36951	–0.302	2457699.38973	0.69246	–0.420
	2457699.09514	0.10394	–0.220	2457699.22967	0.37270	–0.303	2457699.39147	0.69593	–0.426
	2457699.09688	0.10741	–0.216	2457699.23128	0.37591	–0.292	2457699.39320	0.69940	–0.434
	2457699.09865	0.11095	–0.238	2457699.23299	0.37933	–0.283	2457699.39492	0.70282	–0.435
	2457699.10044	0.11453	–0.251	2457699.23459	0.38253	–0.275	2457699.39664	0.70627	–0.433
	2457699.10221	0.11807	–0.257	2457699.23678	0.38690	–0.257	2457699.39835	0.70969	–0.436
	2457699.10397	0.12158	–0.268	2457699.23837	0.39009	–0.244	2457699.40008	0.71314	–0.442
	2457699.10570	0.12503	–0.276	2457699.24021	0.39376	–0.234	2457699.40180	0.71658	–0.449
	2457699.10742	0.12847	–0.287	2457699.24181	0.39695	–0.219	2457699.40353	0.72003	–0.453
	2457699.10913	0.1319	–0.294	2457699.24352	0.40038	–0.215	2457699.40537	0.72370	–0.450
	2457699.11085	0.13532	–0.301	2457699.24525	0.40382	–0.209	2457699.40707	0.72710	–0.452
	2457699.11255	0.13872	–0.308	2457699.24725	0.40782	–0.189	2457699.40868	0.73032	–0.457
	2457699.11425	0.14212	–0.322	2457699.24886	0.41104	–0.173	2457699.41040	0.73376	–0.463
	2457699.11595	0.14552	–0.330	2457699.25070	0.41471	–0.155	2457699.41211	0.73718	–0.460
	2457699.11764	0.14889	–0.337	2457699.25240	0.41811	–0.150	2457699.41382	0.74058	–0.462
	2457699.11933	0.15227	–0.339	2457699.25411	0.42153	–0.133	2457699.41541	0.74377	–0.457
	2457699.12102	0.15564	–0.353	2457699.25632	0.42595	–0.114	2457699.41715	0.74724	–0.461
	2457699.12272	0.15904	–0.363	2457699.25843	0.43016	–0.108	2457699.41887	0.75069	–0.462
	2457699.12441	0.16242	–0.369	2457699.26041	0.43411	–0.038	2457699.42048	0.75390	–0.461
	2457699.12611	0.16582	–0.374	2457699.26224	0.43777	–0.081	2457699.42221	0.75735	–0.466
	2457699.12780	0.16919	–0.378	2457699.26422	0.44172	–0.005	2457699.42564	0.76421	–0.470
	2457699.12951	0.17259	–0.388	2457699.26610	0.44549	–0.014	2457699.42725	0.76743	–0.461
	2457699.13121	0.17599	–0.396	2457699.26810	0.44947	–0.001	2457699.42886	0.77064	–0.470
	2457699.13290	0.17937	–0.398	2457699.26997	0.45321	0.029	2457699.43060	0.77411	–0.461
	2457699.13459	0.18274	–0.411	2457699.27183	0.45693	0.049	2457699.43221	0.77732	–0.457
	2457699.13630	0.18617	–0.416	2457699.27369	0.46063	0.089	2457699.43393	0.78077	–0.460
	2457699.13799	0.18954	–0.425	2457699.27553	0.46431	0.091	2457699.43564	0.78419	–0.461
	2457699.13969	0.19294	–0.436	2457699.27731	0.46787	0.105	2457699.43737	0.78764	–0.452
	2457699.14140	0.19636	–0.441	2457699.28557	0.48438	0.201	2457699.01063	0.93510	–0.051
	2457699.14327	0.20009	–0.449	2457699.31836	0.54989	0.033	2457699.01500	0.94384	0.006
	2457699.14499	0.20353	–0.452	2457699.31997	0.55310	0.019	2457699.01669	0.94721	0.024
	2457699.14670	0.20695	–0.448	2457699.32158	0.55632	–0.003	2457699.01838	0.95059	0.037

The entire table is online at `http://www.raa-journal.org/docs/Supp/ms4347table3.pdf`.

Table 4 Times of Minimum Light for AL Cas

J.D.(Hel) (d) 2400000+	Error	Method	E	$(O-C)_1$	$(O-C)_2$	Reference
25301.53300		pg	–4.0	–0.021535	–0.020821	Kreiner et al. (2001)
25303.55500		pg	0.0	–0.001759	–0.001045	Kreiner et al. (2001)
25374.38100		pg	141.5	–0.004439	–0.003709	Kreiner et al. (2001)
25681.49500		pg	755.0	0.018429	0.019226	Kreiner et al. (2001)
25687.51600		pg	767.0	0.032757	0.033555	Kreiner et al. (2001)
25712.49700		pg	817.0	–0.014044	–0.013241	Kreiner et al. (2001)
26747.66300		pg	2885.0	0.002064	0.003094	Kreiner et al. (2001)
26767.45300		pg	2924.5	0.020101	0.021135	Kreiner et al. (2001)
26771.44300		pg	2932.5	0.005652	0.006687	Kreiner et al. (2001)
28373.45400		pg	6133.0	–0.012953	–0.011568	Kreiner et al. (2001)
28542.68800		pg	6471.0	0.033105	0.034526	Kreiner et al. (2001)
28626.52000		pg	6638.5	0.021968	0.023408	Kreiner et al. (2001)
29216.42000		pg	7817.0	0.016675	0.018244	Kreiner et al. (2001)
29231.43500		pg	7847.0	0.014994	0.016566	Kreiner et al. (2001)
32173.41600		pg	13724.5	–0.022130	–0.019916	Kreiner et al. (2001)
33023.39600		pg	15422.5	0.013713	0.016113	Kreiner et al. (2001)
33330.49700		pg	16036.0	0.023582	0.026049	Kreiner et al. (2001)
36686.47600		vis	22740.5	0.024612	0.027812	Kreiner et al. (2001)
36700.47700		vis	22768.5	0.010043	0.013246	Kreiner et al. (2001)
36895.44800		vis	23158.0	0.014465	0.017711	Kreiner et al. (2001)
36896.44200		vis	23160.0	0.007353	0.010599	Kreiner et al. (2001)
36899.20400		vis	23165.5	0.016295	0.019541	Kreiner et al. (2001)
36904.45000		vis	23176.0	0.006456	0.009704	Kreiner et al. (2001)
44486.37000		vis	38323.0	0.004119	0.009022	Kreiner et al. (2001)
44489.37200		vis	38329.0	0.002782	0.007686	Kreiner et al. (2001)
44490.36600		vis	38331.0	–0.004329	0.000574	Kreiner et al. (2001)
44498.36400		vis	38347.0	–0.015225	–0.010319	Kreiner et al. (2001)
47727.45300		vis	44798.0	–0.013239	–0.007628	Kreiner et al. (2001)
50053.28300		ccd	49444.5	–0.016879	–0.010760	Kreiner et al. (2001)
51165.26950		ccd	51666.0	–0.015622	–0.009260	Kreiner et al. (2001)
51177.28160		ccd	51690.0	–0.016867	–0.010503	Kreiner et al. (2001)
51343.96800		ccd	52023.0	–0.015628	–0.009228	O−C Getway
51433.56600	0.00240	ccd	52202.0	–0.017160	–0.010739	Safar & Zejda (2002)
51771.44360	0.00560	ccd	52877.0	–0.014887	–0.008392	Zejda (2002)
51772.44350	0.00150	ccd	52879.0	–0.016099	–0.009604	Zejda (2002)
51834.51310	0.00150	ccd	53003.0	–0.015448	–0.008940	Hubscher (2005)
51835.51350	0.00320	ccd	53005.0	–0.016160	–0.009652	Hubscher (2005)
52041.49000		ccd	53416.5	–0.018470	–0.011917	O−C Getway
52065.52180		ccd	53464.5	–0.013360	–0.006802	O−C Getway
52151.36630	0.00250	ccd	53636.0	–0.014221	–0.007644	Brát et al. (2007)
52274.50480	0.00330	ccd	53882.0	–0.012507	–0.005903	Zejda (2004)
52944.24830	0.00030	ccd	55220.0	–0.012988	–0.006238	Kim et al. (2006)
52950.25480	0.00020	ccd	55232.0	–0.013161	–0.006409	Kim et al. (2006)
52966.77500	0.00100	ccd	55265.0	–0.011310	–0.004555	Dvorak (2004)
53252.59220	0.00070	ccd	55836.0	–0.011609	–0.004791	Hubscher et al. (2005)
53334.18330	0.00020	ccd	55999.0	–0.011143	–0.004308	Kim et al. (2006)
53335.43490	0.00010	R	56001.5	–0.010934	–0.004098	Kotkova & Wolf (2006)
53347.19700	0.00010	ccd	56025.0	–0.011901	–0.005062	Kim et al. (2006)
53663.04990	0.00040	ccd	56656.0	–0.009862	–0.002954	Kim et al. (2006)
53667.05420	0.00010	ccd	56664.0	–0.010010	–0.003102	Kim et al. (2006)
53670.05830	0.00060	ccd	56670.0	–0.009246	–0.002337	Kim et al. (2006)
53749.39500	0.00040	-Ir	56828.5	–0.010679	–0.003753	Hubscher (2007)
54753.51440	0.00020	R	58834.5	–0.006695	0.000450	Brat et al. (2009)
54798.31550	0.00140	ccd	58924.0	–0.005360	0.001794	Hubscher et al. (2009)
54798.57120	0.00100	ccd	58924.5	0.000061	0.007216	Hubscher et al. (2009)
55100.90530	0.00060	V	59528.5	–0.001687	0.005534	Diethelm (2010)
55473.31840	0.00190	-Ir	60272.5	–0.002280	0.005021	Hubscher (2011)
55473.56820	0.00250	-Ir	60273.0	–0.002758	0.004543	Hubscher (2011)
55532.63220	0.00040	V	60391.0	–0.004371	0.002943	Diethelm (2011)
55804.43640	0.00100	-Ir	60934.0	–0.002101	0.005273	Hubscher & Lehmann (2012)

Table 4 —*Continued.*

J.D.(Hel)(d) 2400000+	Error	Method	E	$(O-C)_1$	$(O-C)_2$	Reference
56178.35220		ccd	61681.0	–0.001663	0.005793	O−C Getway
56203.87920	0.00020	V	61732.0	–0.003021	0.004440	Diethelm (2013)
56590.31050	0.00010	ccd	62504.0	–0.000984	0.006562	Hubscher (2017)
54056.23790	0.00018	ccd	57441.5	–0.008614	–0.001621	Qian et al. (2014)
54057.48920	0.00013	ccd	57444.0	–0.008746	–0.001753	Qian et al. (2014)
54816.33510	0.00046	ccd	58960.0	–0.005798	0.001360	Qian et al. (2014)
54828.59880	0.00021	ccd	58984.5	–0.005671	0.001490	Qian et al. (2014)
54829.35017	0.00019	ccd	58986.0	–0.005165	0.001996	Qian et al. (2014)
55916.06061	0.00019	ccd	61157.0	–0.001888	0.005510	Qian et al. (2014)
55916.56057	0.00003	ccd	61158.0	–0.002484	0.004915	Qian et al. (2014)
55917.06080	0.00013	ccd	61159.0	–0.002810	0.004589	Qian et al. (2014)
56296.98397	0.00024	ccd	61918.0	–0.001674	0.005807	Qian et al. (2014)
56644.12027	0.00011	ccd	62611.5	–0.000988	0.006569	Qian et al. (2014)
56645.12150	0.00019	ccd	62613.5	–0.000870	0.006687	Qian et al. (2014)
57699.04390	0.00014	ccd	64719.0	0.000787	0.008575	This work
57699.29433	0.00051	ccd	64719.5	0.000942	0.008731	This work
57700.04499	0.00015	ccd	64721.0	0.000771	0.008560	This work
57700.29598	0.00016	ccd	64721.5	0.001480	0.009269	This work
57748.34876	0.00005	ccd	64817.5	0.000873	0.008673	This work
57769.12040	0.00031	ccd	64859.0	–0.000561	0.007242	This work

Notes: $O-C$ Gateway: `http://var2.astro.cz/ocgate/`.

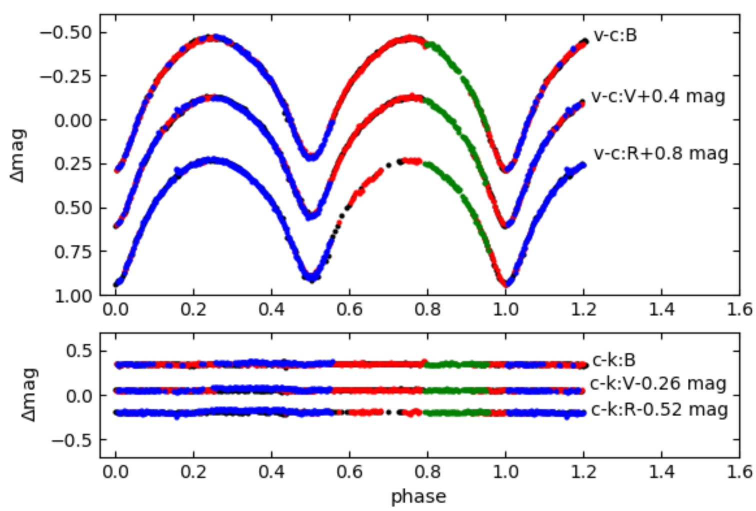

Fig. 3 Top panel shows the light curves of AL Cas in the BVR bands. The magnitude differences between the comparison and the check stars are revealed in the bottom panel. The *black*, *red*, *blue* and *green* color stand for 2016 November 6 and 7, 2016 December 25 and 2017 January 15, respectively.

2010), and the limb darkening (van Hamme 1993), are presented in Table 5. Meanwhile, the adjustable parameters are employed with a similar standard (e.g., Drechsel et al. 2001; Zhang et al. 2016; Liu et al. 2017).

The mass ratio q can be determined by two methods: one is the radial velocity-curve analysis and the other is a mass ratio search (q-search) method. The q-search method is applied in this work to obtain a guaranteed mass ratio. We use a series of fixed q values with the range from 0.0 to 3.0 and the step of 0.02, and obtain a series of weighted square deviation $\sum W(O-C)^2$ (hereafter $\sum$). For each assumed q, we find that the photometric solutions converged to the contact configuration. The q and its corresponding $\sum$ values are shown in Figure 5. From this figure, a minimum value of $\sum$ is obtained at $q\ =0.550$, which is an adjustable parameter in subsequent computa-

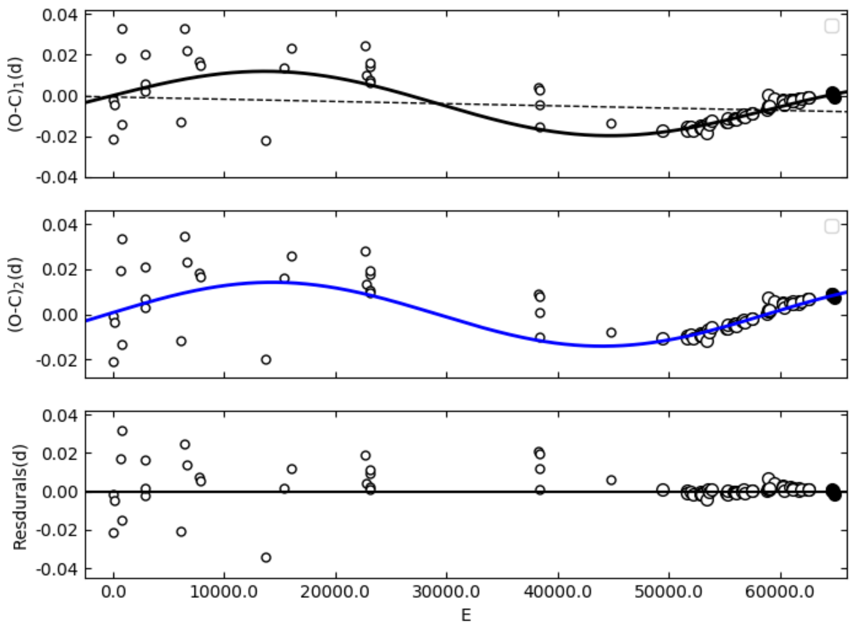

Fig. 4 *Top panel*: smaller *open circles* and *bigger open circles* refer to the visual or photographic and the photoelectric or CCD observed data, respectively. Our data are marked as the *solid points*. The *dashed line* is the linear fitting. A combination between this linear fitting and the sinusoidal fitting in the middle panel is given by the *solid line*. *Middle panel*: the *solid line* stands for the sinusoidal fitting for the residuals that remove the linear trend from the upper panel. *Bottom panel*: it is the residual of the final-fitting result.

tion. Finally, the convergent solution is ensured when the value of $q = 0.6399$. To measure the reliability of mass ratio, we apply two principles: the degree of symmetry of the light curve and the degree of sharpness for the q-search curve (Liu et al. 2017; Zhang et al. 2017). In this paper, the light curves of AL Cas are symmetric and the bottom of the q-search curve is sharp. This result implies that the q is generally reliable. The calculated parameters of the photometric solution are listed in Table 6. It is possible that AL Cas is an A-subtype contact binary since the effective temperature of the primary star is 78 K higher than that of the secondary star.

Figure 6 shows a fitting result between the theoretical light curves (the red solid lines) and the observations (the open circles) in BVR bands. The result hints that our observed CCD data are consistent with the theoretical light curves. Note that we have considered a contribution of the third light during the photometric solution.

Table 5 Assumed Parameters of Photometric Solution

Parameter	Values
$g_1 = g_2$	0.32
$A_1 = A_2$	0.5
$F_1 = F_2$	1
e	0
$x_{1\rm bol}$	0.12
$x_{2\rm bol}$	0.14
$y_{1\rm bol}$	0.55
$y_{2\rm bol}$	0.28
x_{1B}	0.281
x_{2B}	0.351
y_{1B}	0.604
y_{2B}	0.538
x_{1V}	0.108
x_{2V}	0.144
y_{1V}	0.697
y_{2V}	0.673
x_{1R}	0.021
x_{2R}	0.053
y_{1R}	0.713
y_{2R}	0.695
T_1	6400 K

5 THE ABSOLUTE PARAMETERS OF AL CAS WITH THE PARSEC MODEL

We employ the stellar-evolution code PARSEC to determine the absolute parameters of AL Cas. PARSEC is an expanded and updated program (Bertelli et al. 1994; Girardi et al. 2000; Marigo et al. 2008a; Girardi et al. 2010; Bressan et al. 2012, 2013; Tang et al. 2014; Chen et al. 2015, 2014; Marigo et al. 2017). PARSEC was originated from the Padova code (Bertelli et al. 1994). Bressan et al. (2012, 2013) described its main features, including the major input physics (i.e., equation of state, opacities and solar reference abundance) and the mixing processes (e.g., microscopic diffusion in low-mass stars). The boundary conditions in low mass stars, the envelope overshooting and

Table 6 Photometric Parameters of AL Cas

Parameter	Value	Error
i (°)	80.599	±0.040
T_2	6322	±28
$\Omega_{\rm in}$	3.1162	
$\Omega_{\rm out}$	2.7547	
$q(M_2/M_1)$	0.6399	±0.0023
$L_1/(L_1+L_2+L_3)_B$	0.61040	±0.00043
$L_1/(L_1+L_2+L_3)_V$	0.60706	±0.00039
$L_1/(L_1+L_2+L_3)_R$	0.60493	±0.00038
Ω_1=Ω_2	2.98704	±0.00442
r_1 (pole)	0.4173	±0.0040
r_1 (side)	0.4466	±0.0051
r_1 (back)	0.4879	±0.0058
r_2 (pole)	0.3438	±0.0015
r_2 (side)	0.3639	±0.0025
r_2 (back)	0.4142	±0.0031
f	35.74%	±1.22%
$L_3/(L_1+L_2+L_3)_B$	0.21%	
$L_3/(L_1+L_2+L_3)_V$	0.07%	
$L_3/(L_1+L_2+L_3)_R$	0.01%	
Equal-volume radius of star 1 (r_1)	0.4506	±0.0028
Equal-volume radius of star 2 (r_2)	0.3872	±0.0014
Radius ratio(R_2/R_1)	0.8592	±0.0063

Notes: These parameter errors are obtained by the minimization algorithm and are not the true parameter uncertainties (Prša & Zwitter 2005).

Table 7 Input Parameters of the PARSEC Model

Name	Parameter	Reference
Evolutionary track	PARSEC version 1.2S	[1],[2],[3]
Photometric system	$UBVRIJHK$	[4],[5],[6]
Circumstellar dust	No dust	[7]
Interstellar extinction	R_v =3.1, A_v =0	[8]
Initial mass function	FLUM=$M^{1-\alpha}/(1-\alpha)$	[11]
Mass	$0.1 < M < 350\ M_\odot$	[2]
Age	$6.6 < \log$ (t yr^{-1}) < 10.13	[2]
Metallicity	0.0001< Z <0.07	[9]
The relation of Y and Z	$Y = 0.2485 + 1.78Z$	[10],[11]
The mixing-length parameter ($\alpha_{\rm MLT}$)	1.74	[10]
The Reimers formula ($\eta_{\rm Reimers}$)	0.2	[10],[11]

Notes: [1] Tang et al. (2014); [2] Chen et al. (2015); [3] Chen et al. (2014); [4] Maíz Apellániz (2006); [5] Bessell (1990); [6] Bessell & Brett (1988); [7] Marigo et al. (2008b); [8] Girardi et al. (2008); [9] Bressan et al. (2012); [10] Bressan et al. (2013); [11] Goossens et al. (2008).

the mass loss in intermediate- and high-mass stars were involved in PARSEC (Tang et al. 2014; Chen et al. 2015, 2014). Marigo et al. (2017) added COLIBRI to PARSEC forming new PARSEC–COLIBRI stellar isochrones. The PARSEC model can compute stellar evolution with the initial stellar masses ($0.1 < M < 350\ M_\odot$) and ages [$6.6 < \log(t/{\rm yr}) < 10.13$] when a metallicity is given. In other words, this model can provide a complete star pa-

Table 8 Absolute Parameters of AL Cas

Parameter	The DCRL method	The DCWD method	Unit
M_1	1.19(±0.23)	1.22(±0.26)	$M_\odot$
M_2	0.76(±0.18)	0.78(±0.20)	$M_\odot$
R_1	1.38(±0.09)	1.50(±0.11)	$R_\odot$
R_2	1.13(±0.08)	1.24(±0.10)	$R_\odot$
ρ_1	0.446(±0.050)	0.357 (±0.078)	$\rho_\odot$
ρ_2	0.527(±0.049)	0.399 (±0.079)	$\rho_\odot$
L_1	2.866(±0.382)	3.369(±0.698)	$L_\odot$
L_2	1.827(±0.337)	2.210(±0.704)	$L_\odot$
T_1	6400(±400)	6400(±400)	K
T_2	6322(±400)	6322(±400)	K

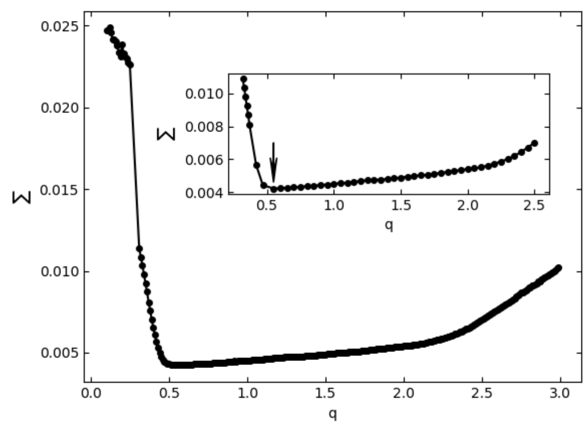

Fig. 5 The relation between $\Sigma(O-C)^2$ and q for AL Cas. The minimum value of Σ is located at $q = 0.55$.

rameter space. The main input parameters of the model in this work are listed in Table 7.

We select the stars with an effective temperature (6000 K $< T <$ 6800 K) and a metallicity (0.0001$<$ $Z <$0.0700) from the star parameter space. This range of temperature is 6400 K for star 1 with 400 K error and the metallicity includes almost all possible stars. The radius of each star can be determined by $R = \sqrt[2]{GM/g}$, where the surface gravity (g) and the mass (M) can be obtained from PARSEC.

Combining Kepler's third law with the mass ratio (q, Table 6 in Section 4), we use the following two methods to obtain the mass-radius ($M-R$) relation of star 1.

(i): A dependent calculation from the effective radius of the Roche lobe (Eggleton 1983) (hereafter: DCRL method),

$$R_1/A = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1+q^{1/3})}. \quad (3)$$

So the $M-R$ relation of star 1 is given as

$$2.2399(499)\frac{M_1}{M_\odot} = \left(\frac{R_1}{R_\odot}\right)^3. \quad (4)$$

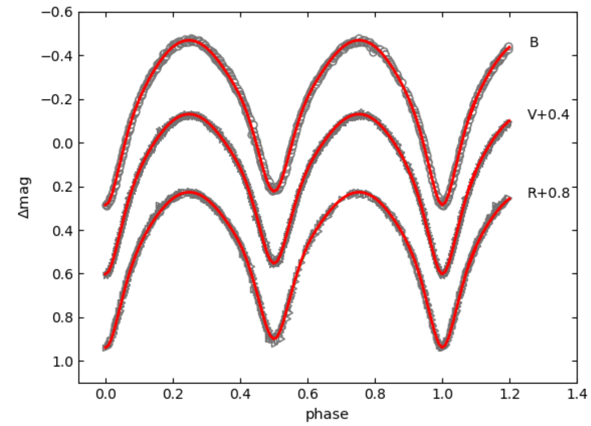

Fig. 6 Observed (*open cycles*, *pentagrams* and *triangles*) and theoretical light curves (*red solid lines*) of AL Cas in the B, V and R bands. Here, the V and R band light curves are shifted by $+0.4$ and $+0.8$ mag, respectively.

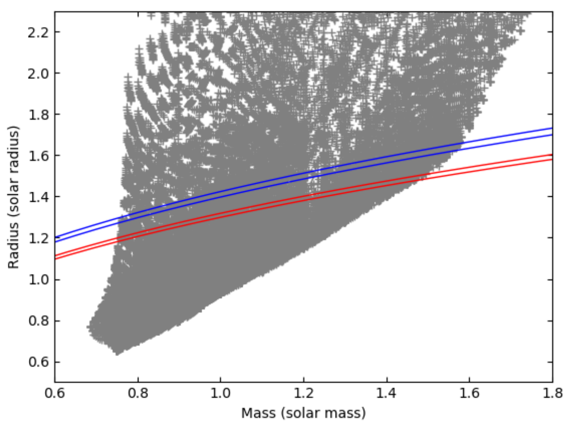

Fig. 7 The stellar M-R diagram with the M-R relation of AL Cas. The *black plus dots* stand for the selected stars by PARSEC code. The two *red lines* and two *blue lines* are determined by the DCRL method and the DCWD method, respectively.

(ii): A dependent calculation from the W-D code ($r_1 = R_1/A$, r_1 is listed in Table 6 in Section 4) (hereafter: DCWD method). So the $M - R$ relation of star 1 is given as

$$2.8043(794)\frac{M_1}{M_\odot} = \left(\frac{R_1}{R_\odot}\right)^3. \quad (5)$$

All the selected stars from PARSEC are plotted in the $M - R$ diagram of Figure 7. The red curves in this figure are described by the DCRL method and the mass and radius of star 1 can be approximately estimated as $M_1 = 1.19\ M_\odot$ and $R_1 = 1.38\ R_\odot$. The blue curves are described by the DCWD method and mass and radius of star 1 are obtained as $M_1 = 1.22\ M_\odot$ and $R_1 = 1.50\ R_\odot$. The absolute parameters of AL Cas including the information of star 2 are listed in Table 8.

6 DISCUSSION AND CONCLUSIONS

In this work, we study the orbital period variation of AL Cas with the $O - C$ method and the light curves with the W-D Code, and determine its absolute parameters by the PARSEC.

Compared to previous studies, we give a comparison between Qian et al. (2014) and our studies for main parameters of AL Cas in Table 9. From this table, we can see that the major difference is the determination of the absolute parameters of AL Cas including M_1, M_2, R_1, and R_2. Meanwhile, we not only update the new linear ephemeris, but also confirm that the orbital period of AL Cas has a cyclic oscillation from the $O - C$ method. This cyclic change may be caused by the Applegate mechanism (Applegate 1992) or the light-time effect (LTTE) via a third body. The required quadruple moment variation for two components of AL Cas can be determined based on the following equations (Rovithis-Livaniou et al. 2000; Lanza & Rodonò 2002)

$$\Delta P = \sqrt{2[1 - \cos(2\pi P/P_3)]} \times A_3, \quad (6)$$

$$\frac{\Delta P}{P} = -9\frac{\Delta Q}{M}a^2. \quad (7)$$

One can find that the values of the quadrupole moment variation ($\Delta Q_1 = 3.423 \times 10^{49}$ and $\Delta Q_2 = 2.186 \times 10^{49}$g cm^2) are smaller than the typical values ($\Delta Q = 10^{51} - 10^{52}$ g cm^2) for active-close binaries (Lanza & Rodonò 1999). It might be that the Applegate mechanism cannot be used to explain the cyclic variation. Hence, we are in favor of the *LTTE* via the third body. The parameters of the third body are calculated by the well-known equations,

$$a'_{12}\sin i' = A_3 \times c, \quad (8)$$

$$f(m) = \frac{4\pi^2}{GP_3^2} \times (a'_{12}\sin i')^3 = \frac{(M_3 \sin i')^3}{(M_1 + M_2 + M_3)^2}, \quad (9)$$

where A_3 refers to the amplitude of the $O - C$ oscillation, c is the speed of light, G is the gravitational constant and P_3 is the period of the third body. These parameters are listed in Table 10. From this table, we can see that the structural properties of the third body in the triple system is not variable too much compared with Qian et al. (2014). In this work, some parameters of the third body for A-subtype contact binaries are collected and listed in Table 11. The present third body plays a crucial role in origin and evolution of contact binaries. In other words, the present third body could remove angular momentum from the central system from Kozai oscillation or a combination of Kozai cycle and tidal friction. Meanwhile, this companion star can cause a low angular momentum and a short initial orbital period for contact binaries (e.g., Kozai

Table 9 Main Parameters of AL Cas

Name	Parameter	Value (this work)	Value (Qian et al. 2014)	Unit
Orbital period information	$P_{\rm orb}$	0.50055593	0.50055604	d
	$\dot{P}$	0	0	d yr^{-1}
	$LTTE$	yes	yes	-
Photometric solution	f	35.7(±0.9)%	39.3(±1.2)%	–
	q	0.64(±0.02)	0.610(±0.003)	-
	T_2	6322(±28)	6316(±32)	K
Absolute parameters (The DCRL method)	M_1	1.19(±0.23)	1.33$^\alpha$	$M_\odot$
	M_2	0.76(±0.18)	0.81	$M_\odot$
	R_1	1.38(±0.09)	–	$R_\odot$
	R_2	1.13(±0.08)	-	$R_\odot$
Absolute parameters (The DCWD method)	M_1	1.22(±0.26)	-	$M_\odot$
	M_2	0.78(±0.20)	-	$M_\odot$
	R_1	1.50(±0.11)	–	$R_\odot$
	R_2	1.24(±0.10)	–	$R_\odot$

Notes: α refers to an assumed mass.

Table 10 Parameters of the Third Body

Parameter	Value (this work)	Value (Qian et al. 2014)	Unit
P_3(yr)	81.25(±1.230)	86.60(±1.50)	yr
A_3	0.0142(±0.0036)	0.0181(±0.0028)	d
e'	0	0	assumed
$a' \sin i'$	2.45(±0.57)	3.14(±0.49)	AU
$f(m)$	4.118(±0.241)$\times10^{-3}$	4.1(±0.8)$\times10^{-3}$	$M_\odot$
M_3 $(i' = 90°)^\alpha$	0.279(±0.032)	0.29±0.05$^\gamma$	$M_\odot$
a_3 $(i' = 90°)^\alpha$	17.085(±1.468)	23.2±2.6$^\gamma$	AU
M_3 $(i' = 90°)^\beta$	0.279(±0.035)	-	$M_\odot$
a_3 $(i' = 90°)^\beta$	17.524(±1.500)	-	AU

Notes: $^\alpha$ represents the result of the third body using the DCRL method; $^\beta$ represents the result of the third body using the DCWD method; $^\gamma$ represents the result of the third body by an assumed total mass ($M = 2.14\ M_\odot$).

Table 11 Some Parameters of the Third Body for A-subtype Contact Binaries

Star	P_3 (yr)	M_3 ($M_\odot$)	$f(m)$	Reference
DK Cyg	78.10	0.065	0.00445	Lee et al. (2015)
OO Aql	69.30	0.620	0.00566	Li et al. (2016)
V1101 Her	13.90(±1.90)	0.128	0.0005(±0.0004)	Pi et al. (2017)
AD Phe	56.20(±0.80)	0.257	0.0063(±0.0006)	Pi et al. (2017)
V566 Oph	43.40(±0.80)	0.265(±0.020)	0.0040(±0.0008)	Selam et al. (2018)
AU Ser	42.87(±3.16)	0.475(±0.001)	0.0266(±0.0001)	Amin (2015)
CK Boo	21.30	0.470	0.0599(±0.0001)	Yang et al. (2012)
UZ Leo	138.80(±2.80)	0.301(±0.006)	0.0032(±0.0001)	Lee & Park (2018)
V839 Oph	16.99(±0.15)	0.378(±0.022)	0.0083(±0.0010)	Şenavcı et al. (2006)
GR Vir	28.56(±0.15)	1.310(±0.022)	0.0333(±0.0010)	Luo & Kang (2017)
AL Cas	81.25(±1.23)	0.279(±0.032)	0.00412(±0.0002)	This work

1962; Fabrycky & Tremaine 2007; Qian et al. 2006). Under these circumstances, the original detached binary systems may evolve into contact binaries depending on magnetic torques from stellar winds (Bradstreet & Guinan 1994).

The major difference between the DCRL method and the DCWD method reflects the computation of radius of AL Cas. The DCRL method can reckon the radius by Equation (9), note that the Roche lobe critical radius is taken for this radius. The DCWD method counts the radius by the equal-volume radius in Table 6.

It is generally known that the true mass of a contact binary can be obtained from a complete spectral observation (i.e., spectrum binary). Here the target of AL Cas does not currently have spectral data corresponding to a complete orbital period. In the current paper, we try to use stellar-evolution models (i.e., PARSEC) together with the photometric observations to determine a guaranteed mass of AL Cas. We need to illustrate the following four points for a systematic analysis of AL Cas. Firstly, we assume that star 1 is regarded as a higher luminosity component of AL Cas to match the absolute parameters from the PARSEC stellar evolution tracks. Secondly, the PARSEC model supplies a good approach to determine the properties of contact binaries (e.g., Frandsen et al. 2013; Sandquist et al. 2013; Zhang et al. 2017) including the equation of state, the opacities, the nuclear reaction rates, the nuclear network, and the inclusion of microscopic diffusion. Thirdly, the higher precision of light curves in Figure 2 can provide a guarantee for calculation of those parameters. Finally, the PARSEC is a single star evolution program, and it could be assumed that the observed atmosphere parameters of stars are those of the component with higher luminosity in binary systems. Based on this assumption, we use the PARSEC model to determine the absolute parameters of AL Cas. Note that the errors of absolute parameters are much bigger than the errors from the assumption.

It is well known that stellar masses and radii are major determinants for their structure and evolution. A stellar evolutionary model is a neglected but useful tool to compute stellar masses and radii, and their small uncertainties obtained by the evolutionary model are less than 0.5% compared with the errors from observations (Kraus et al. 2011; Birkby et al. 2012). In this paper, PARSEC is used to calculate a guaranteed mass and radius of AL Cas. This will provide a method for more effective and accurate investigation of stellar physical characteristics.

In summary, we provide two methods to determine the absolute parameters of AL Cas by photometric observations. The main conclusions are as follows:

(i) The mass of AL Cas is $M = 1.95\ M_{\odot}$ using the DCRL method, and $M = 2.00\ M_{\odot}$ using the DCWD method.

(ii) No matter what above methods are used, the mass of the third body can be confirmed as $M_3 \sin i' = 0.279\ M_{\odot}$.

(iii) AL Cas is an A-subtype contact binary system with a cyclic orbital period.

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