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Stability of LRS Bianchi type-I cosmological models in $f(R,T)$-gravity

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Abstract This paper examines the stability of the transition from the early decelerating stage of the Universe to the recent accelerating stage for the perfect fluid cosmological locally rotationally symmetric (LRS) Bianchi-I model in $f(R,T)$ theory. To determine the solution of field equations, the idea of a time-varying deceleration parameter (DP) which yields a scale factor, for which the Universe attains a phase transition scenario and is consistent with recent cosmological observations, is used. The time-dependent DP yields a scale factor $a = \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right]$, where β and k are respectively arbitrary and integration constants. By using the recent constraints ($H_0 = 73.8$, and $q_0 = -0.54$) from Type Ia Supernova (SN Ia) data in combination with Baryonic Acoustic Oscillations (BAO) and Cosmic Microwave Background (CMB) observations (Giostri et al.), we obtain the values of $\beta = 0.0062$ and $k = 0.000016$ for which we have derived a cosmological model from the early decelerated phase to the present accelerating phase. By applying other recent constraints ($H_0 = 73.8$, $q_0 = -0.73$) from SNe Ia Union data (Cunha), we obtain the values of $\beta = 0.0036$ and $k = 0.000084$ for which we have derived a cosmological model in the accelerating phase only. We have compared both models with experimental data. The stability of the background solution has been examined also for the metric perturbations alongside the properties of future singularities in a Universe ruled by dark energy with phantom type fluid. We demonstrate the presence of a stable fixed point with a condition of state $\omega < -1$ and numerically affirm this is really a late-time attractor in the ghost overwhelmed Universe. Some physical and geometric properties of the model are found and examined.

Key words: LRS Bianchi-I Universe — Time-dependent deceleration parameter — $f(R,T)$ gravity theory — Transit universe

1 INTRODUCTION

The hypothetical contentions for the cosmic acceleration (late-time) are becoming a noteworthy matter between the cosmological studies originating from the twentieth century. In recent years, the importance of observational cosmology cannot be ignored. Recently, observed astronomical phenomena have revolutionized the understanding of cosmology. About twenty years ago, the idea of a Universe which is in an accelerated expansion phase was discovered by various observations (Riess et al. 1998; Perlmutter et al. 1999; Garnavich et al. 1998; Spergel et al. 2003, 2007; Huang et al. 2006; Tegmark et al. 2004; Bennett et al. 2003, 2013). Later, numerous test confirmations additionally demonstrated that the expansion of the Universe is accelerating in nature (Ade et al. 2016; Naess et al. 2014) These perceptions demonstrate that our Universe is overwhelmed by a bizarre cosmic fluid with huge pressure (negative), named dark energy (DE), which constitutes $\simeq 3/4$ (Padmanabhan 2003; Sahni 2004) of the critical density. It is believed that the DE might be a dependable contender for the present cosmic acceleration. Yet, the nature of DE is secretive and important questions about DE, for example, where, how, why and when, are fascinating. The reason for the sudden change from the prior deceleration stage to the ongoing speeding up stage and the wellspring of accelerated expansion is as yet obscure. Be that as it may, the nature of DE is baffling and related bewildering questions, such as where, how, why and when about DE, are intriguing.

In an essentially indistinguishable way, another modified theory has been started like Gauss-Bonnet gravity investigated by Nojiri et al. (2005, 2006) and Bamba et al. (2010, 2017) where the component of the invariant term (Gauss-Bonnet) G is $f(G)$. Recently, some authors (Bamba et al. 2013; Odintsov et al. 2015; Cai et al. 2016; Paliathanasis et al. 2016) have also reviewed $f(T)$ theories, a different approach to modified gravity consisting of a deformation in the teleparallel equivalent of general relativity.

Various candidates have been suggested for this strange DE, such as the constant (cosmological, originally introduced by Einstein), quintessence, phantom, quantum and so on. A more thorough survey is given in Copeland et al. (2006). Soon after the development of General Relativity (GR) by Einstein, many alternatives to GR had been proposed. Most of them, however, were lacking simplicity as well as observational fitting. Nevertheless, modified proposals have been published by researchers ever since. Numerous models have been proposed to clarify this present accelerated expansion. Basically, there are two approaches that allow one to modify gravity. The first is to adjust the right-hand side of Einstein's field equations by considering particular structures for the energy-momentum tensor ($T_{\mu\nu}$) having a negative pressure, which assumes there is an "exotic cosmic fluid," however the experimental information does not totally clarify the expansion (Sharif & Zubair 2010a,b). Secondly, from a mathematical perspective, there is no a priori demand that forces one to consider the Einstein-Hilbert action (linear in R) as the fundamental action of gravity. The modified hypothesis of gravity has turned into a well-known contender to clarify the inception of DE. The main outline is supplanting the Hilbert-Einstein term by a random function of scalar curvature R, named $f(R)$ gravity (Capozziello & Vignolo 2009, 2011, 2012; Nojiri et al. 2017; Sepehri et al. 2016, 2017).

According to Sharma et al. (2018), the torsion scalar T is also a case of modified gravity. Utilizing diverse mixes of scalars, these altered hypotheses of gravity summed up to $f(R, P, Q)$ and $f(R, G)$ (Nojiri et al. 2005; De la Cruz-Dombriz & Sáez-Gómez 2012) where $P = R^{\mu\nu}R_{\mu\nu}$, and in addition $Q = R^{\mu\nu\sigma\tau}R_{\mu\nu\sigma\tau}$ (here $R_{\mu\nu\sigma\tau}$ addresses the Riemann tensor and $R_{\mu\nu}$ addresses the Ricci tensor). Another kind of alternative hypothesis of gravity is $f(R, T, R_{\mu\nu}T^{\mu\nu})$ (Sharif & Zubair 2013; Yousaf et al. 2017). In this research, we concentrate on our regard for $f(R, T)$ gravity which was first developed by Harko et al. (2011) and models from this theory, focusing on energy-momentum tensor $T_{\mu\nu}$ variation concerning the metric, which relies on a source term. The clarification of picking this source term, in general, is a part of the Lagrangian L_m. Thus, for every decision of L_m there ought to be a particular solution of equations. The field equations of this theory by the Hilbert-Einstein variational principle were obtained by Harko et al. (2011) to furthermore find the covariant divergence of $T_{\mu\nu}$. In this way, alternative gravity speculations give a distinct strategy for understanding the issue of DE and the likelihood of reproducing the gravitational field hypothesis that would have the ability to explain that the Universe is presently in an accelerated expansion phase. There are many researchers who have thought about cosmology in this theory of gravity. It is not conceivable to say every one of them has yet been considered; we give some pertinent and most recent references which are specifically identified with our present work (Adhav 2012; Shamir 2014; Chaubey & Shukla 2013; Chandel & Ram 2013; Velten & Caramês 2017; Chakraborty 2013; Moraes 2015; Singh et al. 2016; Sahu et al. 2017; Sahoo et al. 2017; Ahmed & Pradhan 2014; Pradhan et al. 2015; Mishra et al. 2016; Ahmed et al. 2016; Tiwari et al. 2017; Sharma & Pradhan 2018; Pradhan & Jaiswal 2018; Sahoo et al. 2018; Nagpal et al. 2018; Pullen & Kamionkowski 2007; Samal et al. 2008 and references therein).

It is fascinating to observe that in the dynamical history of our Universe the accelerating expansion of the Universe plays an important part. Evaluating the equation of state (EoS) for DE in observational cosmology at present is one of the best undertakings. DE has been depicted routinely by the EoS parameter $\omega = \frac{p}{\rho}$ which is not generally constant (Carroll et al. 2003). Displaying the data appear to imperceptibly bolster a propelling DE with EoS $\omega < -1$ at the present stage and $\omega > -1$ in the near past. Plainly, ω cannot cross -1 for phantom or quintessence alone. A couple of undertakings have been made to develop a DE behavior whose EoS may cross the phantom segment. The least complex DE part is the vacuum energy ($\omega = -1$), which is numerically similar to (Λ). The other normal decisions, which can be depicted by inconsequential coupled scalar fields, are the quintessence ($\omega > -1$) (Steinhardt & Wesley 2009), phantom $\omega < -1$ (Caldwell 2002) and quintom that has a time subordinate EoS parameter. The EoS parameter ω may be taken as constant, yet additionally it is a mapping of time or redshift z or scale factor a (Ratra & Peebles 1988; Jimenez 2003; Das et al. 2005).

The bulk of cosmological models uses the cosmological principle; that is, they assume that the Universe is homogeneous and isotropic. On the other hand, the Cosmic

Microwave Background (CMB) temperature and polarization anisotropy fundamentals (Hu 2003) and CMB polarization of complimentary information to anisotropies (Kaplan 2003; Souradeep 2011; Buzzelli et al. 2016) suggest that the assumption of statistical isotropy is broken on the largest angular scales, leading to some intriguing anomalies. To provide predictions for the CMB anisotropies, one may consider the homogeneous but anisotropic cosmologies known as Bianchi type spacetimes, which include the isotropic and homogeneous Friedmann-Robertson-Walker (FRW) models. The Bianchi type I, II, ..., IX cosmological models are spatially homogeneous spacetimes of dimension $1 + 3$ admitting a group of motions G_3 acting on space-like hyper-surfaces (Stephani et al. 2003; Wald 1984). More precisely, they are manifolds of the form $M = I \times G$ where $I \subset R$ is an interval and G is a Lie group, endowed with a Lorentzian metric of the form $-dt^2 + g_t$ where $(g_t)_{t\epsilon I}$ is a family of left-invariant Riemannian metrics on G. The physical substance of a spacetime M is paraphrased in terms of nonlinear partial differential equations (PDEs) on its Lorentz metric: the so called Einstein's equations. For Bianchi cosmological models, these PDEs reduce to a set of second order ordinary differential equations on the family of metric $(g_t)_{t\epsilon I}$. However, this is not the case in locally rotationally symmetric (LRS) Bianchi type cosmological models where Einstein's field equations lead to a set of nonlinear differential equations. The study of Bianchi type-I cosmological models creates more interest as these models contain isotropic special cases and permit arbitrarily small anisotropic levels at certain stages. When the Bianchi type-I spacetime flourishes equally in two spatial directions, it is called locally rotationally symmetric. For simplification and description of the large scale behavior of the actual Universe, LRS Bianchi type-I spacetime is extensively implemented. This model is characterized by three metric functions, $R_1(t)$, $R_2(t)$ and $R_3(t)$, such that $R_1 \neq R_2 = R_3$. LRS Bianchi type-I spacetime is a generalization of the flat FRW metric.

Inspired by the above discussion, in this paper, we have revisited the solutions acquired by Pradhan et al. (2018). The paper is dedicated to contemplating the cosmological LRS Bianchi I model in the $f(R,T)$ altered hypothesis of gravity in the presence of $\Lambda(T)$ which has the property that it shows progress from decelerating at early time to accelerating at late time. The outline of the paper follows the development of altered $f(R,T)$ hypothesis with $\Lambda(T)$ as given in Section 2. The metric and field conditions are given in Section 3. Solutions are exhibited in Section 4 and furthermore the physical and geometric characteristics of the model are portrayed with a brief discussion of the outcomes. In Section 5, the stability of corresponding solutions is discussed. The conclusion is given in Section 6.

2 CONSTRUCTION OF MODIFIED F(R,T) THEORY WITH $\Lambda(T)$ GRAVITY

For $f(R,T)$ gravity, the action integral is characterized by

$$S = \frac{c^4}{16\pi G}\int \sqrt{-g} f(R,T) d^4x + \int \sqrt{-g} L_m d^4x \ . \quad (1)$$

Here

$f(R,T) \rightarrow$ a random mapping of R and T.
$R \rightarrow$ Ricci scalar.
$T = g^{\mu\nu} T_{\mu\nu} \rightarrow$ follows $T_{\mu\nu}$.
$L_m \rightarrow$ (matter) the Lagrangian density.
$T_{\mu\nu}$ is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta\sqrt{-g} L_m}{\delta g^{\mu\nu}} \ . \quad (2)$$

Let L_m rely upon $g_{\mu\nu}$ as it were. $T_{\mu\nu}$ is characterized as (Harko & Lobo 2010)

$$T_{\mu\nu} = -2\frac{\partial L_m}{\partial g^{\mu\nu}} + g_{\mu\nu} L_m \ . \quad (3)$$

Here we use the unit for $c = G = 1$. $F(R, L_M)$ gravity has been explored by Harko & Lobo (2010). Likewise, a speculation in which the cosmological constant is made by a limit of $\Lambda(T)$ has been explored by Poplawski (2006a).

Variation in S with respect to $g^{\mu\nu}$ gives the field condition as

$$\begin{aligned}&(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) f_R(R,T) - \frac{1}{2} f(R,T) g_{\mu\nu} \\ &+ f_R(R,T) R_{\mu\nu} \\ =& - f_T(R,T)\Theta_{\mu\nu} - f_T(R,T) T_{\mu\nu} + 8\pi T_{\mu\nu} \ .\end{aligned} \quad (4)$$

By the connection $\delta\left(\frac{g^{ij}T_{ij}}{\delta g^{\mu\nu}}\right) = \Theta_{\mu\nu} + T_{\mu\nu}$, $g^{ij}\left(\frac{\delta T_{ij}}{\delta g^{\mu\nu}}\right) \equiv \Theta_{\mu\nu}$ and furthermore, $\nabla^i\nabla_i = \Box$, $f_R(R,T) \equiv \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f(R,T)}{\partial T}$ and ∇_i speaks to the covariant derivative.

Utilizing the connection $\frac{\delta g_{ij}}{\delta g^{\mu\nu}} = -g_{i\gamma} g_{j\sigma} \delta^{\gamma\sigma}_{\mu\nu}$ with $\delta^{\gamma\sigma}_{\mu\nu} = \frac{\delta g^{\gamma\sigma}}{\delta g^{\mu\nu}}$, which follows from $g_{i\gamma} g^{\gamma j} = \delta^j_i$, we acquire $\Theta_{\mu\nu}$ as given by

$$\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu} L_m - 2g^{ij} \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{ij}} \ . \quad (5)$$

The energy momentum-tensor of a perfect fluid for L_m is characterized as

$$T_{\mu\nu} = (p+\rho) u_\mu u_\nu - p g_{\mu\nu} \ , \quad (6)$$

where the four-velocity is $u^{\mu} = (0,0,0,1)$ in the moving coordinates such that $u^{\mu}u_{\nu} = 1$ and $u^{\mu}\nabla_{\nu}u_{\mu} = 0$. Here p is the pressure and ρ is the energy density of the fluid. By utilizing condition (5), we acquire

$$\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu} \ . \tag{7}$$

Since $f(R,T)$ field equations rely on the physical behavior of the matter field (through the tensor $\Theta_{\mu\nu}$), many hypothetical models may be found for every choice of f. Harko et al. (2011) suggested three forms of the function f as given below:

- $2f(T) + R$,
- $f_2(T) + f_1(R)$,
- $f_2(R)f_3(T) + f_1(R)$.

The results, cosmologically for the first case, have been discussed by various scientists (Adhav 2012; Samanta & Dhal 2013; Samanta & Dhal 2013 and references therein) and for the second case by Shamir et al. (2012); Chaubey & Shukla (2013). We are considering the cosmological results for $f(R,T) = \lambda_1 R + \lambda_2 T$. Our interpreted model is extraordinary, new and different from that of the researchers described above. So far, Λ, which is responsible for DE, is less frequently considered.

Now Equation (4) may be composed as

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_{\mu}\nabla_{\nu})f_1'(R) = 8\pi T_{\mu\nu} + f_2'(T)T_{\mu\nu} + \left(f_2'(T)p + \frac{1}{2}f_2(T)\right)g_{\mu\nu} \ , \tag{8}$$

where differentiation is represented by prime indices with respect to the argument. By choosing $\lambda = \lambda_1 = \lambda_2$ in this article with the objective that $f(R,T) = \lambda R + \lambda T$, here λ_1 and λ_2 are random parameters. Condition (8) would now have the capacity to change as

$$\lambda R_{\mu\nu} - \frac{1}{2}\lambda(R+T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_{\mu}\nabla_{\nu})\lambda = 8\pi T_{\mu\nu} - \lambda T_{\mu\nu} + \lambda(2T_{\mu\nu} + pg_{\mu\nu}) \ . \tag{9}$$

Setting $(-\nabla_{\mu}\nabla_{\nu} + g_{\mu\nu}\Box)\lambda = 0$, we get

$$\lambda G_{\mu\nu} = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \left(\lambda p + \frac{1}{2}\lambda T\right)g_{\mu\nu} \ , \tag{10}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is simply the Einstein tensor. This could be revised as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \left(\frac{8\pi+\lambda}{\lambda}\right)T_{\mu\nu} + \left(p + \frac{1}{2}T\right)g_{\mu\nu} \ . \tag{11}$$

Originally Einstein reasoned that the cosmological constant is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi T_{\mu\nu} + \Lambda g_{\mu\nu} \ . \tag{12}$$

Picking a small negative value for the random λ with the objective of having a comparable sign on the right hand side of (10) and (11), and keeping this decision of λ all through, the $\left(p + \frac{1}{2}T\right)$ term would now have the capacity to be seen as a Λ. In this manner we form

$$\Lambda(T) \equiv \Lambda = p + \frac{1}{2}T \ . \tag{13}$$

The T is dependent on the cosmological constant Λ, that has been examined before by Poplawski (2006a) where Λ in the gravitational Lagrangian which is a function of the T, furthermore, in this way we demonstrate "$\Lambda(T)$ gravity." By rejecting the matter pressure, $\Lambda(T)$ gravity which is more widely used in comparison to Palatini $f(R)$ can be obtained (Magnano 1996;, Poplawski 2006b,c). $T = \rho - 3p$, for our model, considering a perfect fluid. In this manner Equation (13) reduces to

$$\Lambda = \frac{1}{2}\rho - \frac{1}{2}p \ . \tag{14}$$

3 METRIC AND BASIC EQUATIONS

Considering the anisotropic LRS Bianchi type-I (spatially homogeneous) metric of the frame as

$$ds^2 = dt^2 - A^2(t)dx^2 - A^2(t)dy^2 - B^2(t)dz^2, \tag{15}$$

the LRS Bianchi type-I (the locally rotationally symmetric) has a symmetric plane compared to the xy-plane and $e = \sqrt{1 - \frac{B^2}{A^2}}$ is its eccentricity.

Other physical parameters like spatial volume, average scale factor and summed mean Hubble parameter H for Equation (15) are defined as

$$V = A^2 B \,, \tag{16}$$

$$a = (A^2 B)^{\frac{1}{3}} = V^{\frac{1}{3}} \,, \tag{17}$$

$$H = \frac{1}{3}(2H_x + H_z) \,, \tag{18}$$

where the Hubble parameters (directional) are $H_x = H_y = \frac{\dot{A}}{A}, H_z = \frac{\dot{B}}{B}$ in terms of x and z individually and an overhead dot from now on represents ordinary differentiation with respect to cosmic time "t".

Using Equations (17) and (18), we get an essential connection

$$H = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = \frac{\dot{a}}{a} \,. \tag{19}$$

Enunciations for the dynamic scalars, for instance, expansion scalar (θ), anisotropy parameter (A_m) and shear scalar (σ) are described clearly

$$\theta = u^i_{;i} = \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \,, \tag{20}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \,, \tag{21}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[2\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2\right] - \frac{\theta^2}{6} \,. \tag{22}$$

We characterize the deceleration parameter (DP) q as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) \,. \tag{23}$$

Assuming the coordinate system is co-moving, the field Equations (11) for (15) and $T_{\mu\nu}$ by Equation (6) may be obtained in terms of directional Hubble parameters as (Pradhan et al. 2018)

$$\dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_x H_z = -\left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \,, \tag{24}$$

$$2\dot{H}_x + 3H_x^2 = -\left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \,, \tag{25}$$

$$H_x^2 + 2H_x H_z = \left(\frac{8\pi + \lambda}{\lambda}\right)\rho - \Lambda \,. \tag{26}$$

4 COSMOLOGICAL SOLUTIONS OF THE FIELD EQUATIONS AND THEIR PROPERTIES

Now we have a system of three equations given by Equations (24)-(26) with four unknowns, viz. p, ρ, A and B. Therefore, we need one more physical condition to find the deterministic solution of these equations.

By doing some calculations with Equations (24), (25) and (17) we get (Pradhan et al. 2018)

$$A = k_2^{\frac{1}{3}}\, a \exp\left[\frac{k_1}{3}\int\frac{dt}{a^3}\right] \,, \tag{27}$$

and

$$B = k_2^{-\frac{2}{3}}\, a \exp\left[-\frac{2k_1}{3}\int\frac{dt}{a^3}\right] \,, \tag{28}$$

where k_1 and k_2 are constants of integration.

The expressions for pressure (p), energy density (ρ) and cosmological constant (Λ) for the Universe can be obtained in terms of Hubble parameters as

$$p = \frac{2(2\kappa+1)\dot{H}_x + 2(3\kappa+1)H_x^2 - H_xH_z}{2\kappa(\kappa+1)} \,, \tag{29}$$

$$\rho = \frac{2\dot{H}_x + 2(1-\kappa)H_x^2 - (2\kappa+1)H_xH_z}{2\kappa(\kappa+1)} \,, \tag{30}$$

$$\Lambda = -\frac{2\dot{H}_x + 4H_x^2 + H_xH_z}{2(\kappa+1)} \,, \tag{31}$$

where $\kappa = \frac{8\pi+\lambda}{\lambda}$.

The EoS parameter is obtained as

$$\omega = \frac{2(2\kappa+1)\dot{H}_x + 2(3\kappa+1)H_x^2 - H_xH_z}{2\dot{H}_x + 2(1-\kappa)H_x^2 - (2\kappa+1)H_xH_z} \,. \tag{32}$$

The Ricci scalar R is given by

$$R = -2\left[2\dot{H}_x + \dot{H}_z + 3H_x^2 + H_z^2 + 2H_xH_z\right] \,. \tag{33}$$

From Equations (27) and (28), we observe that by knowing the value of a (scale factor), one can assess the estimations of A and B and thus the field equations can be solved. So, we consider a period subordinate DP (q) which is supported by observations of Type Ia supernovae (SNe Ia, Riess et al. 2004; Clocchiatti et al. 2006; Tonry et al. 2003) and CMB anisotropies (Hanany et al. 2000; de Bernardis et al 2007). The inspiration to pick such a period subordinate DP is that the Universe is at present experiencing accelerated expansion and in the past decelerated expansion as can be seen by ongoing observations. Furthermore, ($z < 0.5$), the current acceleration and ($z > 0.5$) the past deceleration are verified by the SNe information. Additionally corrected redshift $z_t = 0.43 \pm 0.07$ by (1σ) c.1. (Riess et al. 2007) from $z_t = 0.46 \pm 0.13$ at (1σ) c.1. (Riess et al. 2004) as recently found by the High-Z Supernova Search (HZSNS) group. The Supernova Legacy Survey (SNLS) (Astier et al. 2006) and additionally the one recently incorporated by Knop et al. (2003) yield $z_t \sim 0.6(1\sigma)$, demonstrating better concurrence with flat ΛCDM ($z_t = (2\Omega_\Lambda/\Omega_m)^{\frac{1}{3}} - 1 \sim 0.66$). In this way, the DP in terms of theory is the rate with which the Universe decelerates, which must show signature flipping (Riess et al. 2001; Padmanabhan 2003; Amendola 2003).

We consider the DP (Equation (23)) as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \beta H + \alpha = \beta\frac{\dot{a}}{a} + \alpha \,, \tag{34}$$

where α, β are arbitrary constants.

From the above equation, we have $\frac{a\ddot{a}}{\dot{a}^2} + \beta\frac{\dot{a}}{a} + \alpha = 0$, which on solving yields

$$a = \exp\left[-\frac{(1+\alpha)}{\beta}t - \frac{1}{(1+\alpha)} + \frac{l}{\beta}\right], \quad \text{provided } \alpha \neq -1 \,, \tag{35}$$

where the constant of integration is l.

From Equation (35), we calculate

$$\dot{a} = -\left(\frac{1+\alpha}{\beta}\right)\exp\left[-\left(\frac{1+\alpha}{\beta}\right)t - \frac{1}{(1+\alpha)} + \frac{l}{\beta}\right],$$

$$\ddot{a} = \left(\frac{1+\alpha}{\beta}\right)^2\exp\left[-\left(\frac{1+\alpha}{\beta}\right)t - \frac{1}{(1+\alpha)} + \frac{l}{\beta}\right] . \tag{36}$$

Putting the above values in Equation (34), we obtain the DP value as $q = -1$. Similarly we also observe that $q = -1$ for $\alpha = 0$.

For $\alpha = -1$, we have to find another solution. In this case Equation (34) reduces to

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \beta H \ , \tag{37}$$

which yields the following differential equation

$$\frac{a\ddot{a}}{\dot{a}^2} + \beta\frac{\dot{a}}{a} - 1 = 0 \ . \tag{38}$$

The solution of the above equation is found to be

$$a = \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right] , \tag{39}$$

where the constant of integration is k. Equation (39) was recently used by Tiwari et al. (2015).

Since we are interested in studying the decelerated-accelerated transiting Universe, we only consider the later case for which $\alpha = -1$.

Using Equation (39) in Equations (27) and (28), we obtain

$$A = k_2^{\frac{1}{3}} \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right] \exp\left[\frac{k_1}{3}\int \exp\left\{-\frac{3}{\beta}\sqrt{2\beta t + k}\right\}dt\right], \tag{40}$$

and

$$B = k_2^{-\frac{2}{3}} \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right] \exp\left[-\frac{2k_1}{3}\int \exp\left\{-\frac{3}{\beta}\sqrt{2\beta t + k}\right\}dt\right] . \tag{41}$$

Using Equations (40) and (41) in (15), the model of the Universe takes the form

$$ds^2 = dt^2 - k_2^{\frac{2}{3}} \exp\left[\frac{2}{\beta}\sqrt{2\beta t + k}\right] \exp\left[\frac{2k_1}{3}F(t)\right](dx^2 + dy^2) - k_2^{-\frac{4}{3}} \exp\left[\frac{2}{\beta}\sqrt{2\beta t + k}\right] \exp\left[-\frac{4k_1}{3}F(t)\right]dz^2, \tag{42}$$

where

$$\begin{aligned} F(t) &= \int \exp\left\{-\frac{3}{\beta}\sqrt{2\beta t + k}\right\}dt \\ &= \frac{e^{-\frac{8}{n}}n}{2k}\left[e^{\frac{16}{n}} \exp\left\{\int E_i\left[\frac{8\left\{-1 + \coth\left(1 - \frac{kt}{n}\right)\right\}}{n}\right]\right\} - \exp\left\{\int E_i\left[\frac{8\left\{1 + \coth\left(1 - \frac{kt}{n}\right)\right\}}{n}\right]\right\}\right]. \end{aligned} \tag{43}$$

Definitions for physical parameters, for example, the spatial volume (V), directional Hubble parameters (H_i), mean Hubble's parameter (H), extension scalar (θ), shear scalar (σ) and anisotropy parameter (A_m) for the Universe (42) are given by:

$$V = \exp\left[\frac{3}{\beta}\sqrt{2\beta t + k}\right], \tag{44}$$

$$H_x = H_y = \frac{k_1}{3}\exp\left[-\frac{3}{\beta}\sqrt{2\beta t + k}\right] + \frac{1}{\sqrt{2\beta t + k}}, \tag{45}$$

$$H_z = -\frac{2k_1}{3}\exp\left[-\frac{3}{\beta}\sqrt{2\beta t + k}\right] + \frac{1}{\sqrt{2\beta t + k}}, \tag{46}$$

$$H = \frac{1}{\sqrt{2\beta t + k}}, \tag{47}$$

$$\theta = \frac{3}{\sqrt{2\beta t + k}}, \tag{48}$$

$$A_m = \frac{2}{9}\frac{k_1^2(2\beta t + k)}{\exp\left[\frac{6}{\beta}\sqrt{2\beta t + k}\right]}, \tag{49}$$

$$\sigma^2 = \frac{k_1^2}{3\exp\left[\frac{6}{\beta}\sqrt{2\beta t + k}\right]}. \tag{50}$$

The DP is calculated from Equation (37) as

$$q = -1 + \frac{\beta}{\sqrt{2\beta t + k}}. \tag{51}$$

From Equation (51), we see that $q > 0$ for $t < \frac{\beta^2 - k}{2\beta}$ while $q < 0$ for $t > \frac{\beta^2 - k}{2\beta}$. So, in our determined model, one can choose the values of constants β and k in such a manner that we obtain the value of q consistent with observation range $-1 < q < 0$.

From Equation (51), the present value of the DP can be calculated as

$$q_0 = -1 + \frac{\beta}{\sqrt{2\beta t_0 + k}} = -1 + \beta H_0, \tag{52}$$

where H_0 is the present value of the Hubble parameter and t_0 is the present age of the Universe.

We consider the following two cases based on two different sets of data:

Case 1: Model Based on SN Ia data in combination with Baryonic Acoustic Oscillations (BAO) and CMB observations

Putting $H_0 = 73.8$ and $q_0 = -0.54$ (Giostri et al. 2012) we get $\beta = 0.0062$.

Also from Equation (45) we have $H_0 = \frac{1}{\sqrt{2\beta t_0 + k}} = \frac{1}{\sqrt{2\frac{\beta}{H_0} + k}}$, which on solving for k results in $k = \frac{1}{H_0}(\frac{1}{H_0} - 2\beta)$ and using the values of H_0 and β, we obtain $k = 0.000016$. These values of β and k are used in plotting Figures 1–9.

Case 2: Model Based on SN Ia Union data

Putting $H_0 = 73.8$ and $q_0 = -0.73$ (Cunha 2009) we get $\beta = 0.0036$ and $k = 0.000084$.

Figure 1 corresponding to Equation (51) demonstrates the variation of DP q with time t for both cases. In case 1, for $\beta = 0.0062$ and $k = 0.000016$, we observe that the model exhibits expansion from the decelerating to accelerating eras, whereas for $\beta = 0.0036$ and $k = 0.000084$, the model is in the accelerating phase only. In case 1, at the early phase of the evolution of the Universe, q was positive, which indicates that in the early Universe the model was expanding but the expansion rate was slowing down with time. It passed through a transition phase from positive to negative and currently it is negative, indicating that the Universe is expanding with an accelerated rate of expansion. Recent observations have also established that the current Universe is undergoing a cosmic acceleration. Hence our models in both cases are consistent with observations of SNe Ia described in the Introduction.

Putting the values from Equations (45) and (46) in Equations (24)$-$(26), the pressure, energy density and cosmological constant for model (42) are obtained as

$$p = \frac{1}{2\kappa(\kappa+1)}\left[\frac{2k_1^2(3\kappa+2)}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} - \frac{k_1}{3\sqrt{2\beta t+k}e^{\frac{3}{\beta}\sqrt{2\beta t+k}}} + \frac{(6\kappa+1)}{(2\beta t+k)} - \frac{2\beta(2\kappa+1)}{(2\beta t+k)^{\frac{3}{2}}}\right], \tag{53}$$

$$\rho = \frac{1}{2\kappa(\kappa+1)}\left[\frac{2k_1^2(\kappa+2)}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} - \frac{k_1(2\kappa+1)}{3\sqrt{2\beta t+k}e^{\frac{3}{\beta}\sqrt{2\beta t+k}}} + \frac{(1-4\kappa)}{(2\beta t+k)} - \frac{2\beta}{(2\beta t+k)^{\frac{3}{2}}}\right], \tag{54}$$

$$\Lambda = -\frac{1}{2\kappa(\kappa+1)}\left[\frac{2k_1^2}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} + \frac{5k_1}{3\sqrt{2\beta t+k}e^{\frac{3}{\beta}\sqrt{2\beta t+k}}} + \frac{5}{(2\beta t+k)} - \frac{2\beta}{(2\beta t+k)^{\frac{3}{2}}}\right], \tag{55}$$

$$\omega = \frac{\frac{2k_1^2(3\kappa+2)}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} - \frac{k_1}{3\sqrt{2\beta t+k}e^{\frac{3}{\beta}\sqrt{2\beta t+k}}} + \frac{(6\kappa+1)}{(2\beta t+k)} - \frac{2\beta(2\kappa+1)}{(2\beta t+k)^{\frac{3}{2}}}}{\frac{2k_1^2(\kappa+2)}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} - \frac{k_1(2\kappa+1)}{3\sqrt{2\beta t+k}e^{\frac{3}{\beta}\sqrt{2\beta t+k}}} + \frac{(1-4\kappa)}{(2\beta t+k)} - \frac{2\beta}{(2\beta t+k)^{\frac{3}{2}}}}. \tag{56}$$

Figure 2, corresponding to Equation (53), depicts the variation of isotropic pressure p with time t. We find that p is negative and it nearly approaches zero as $t \to \infty$ for both cases. This negative pressure actually causes the accelerated expansion of the Universe.

Figure 3, corresponding to Equation (54), describes the energy density ρ variation with time t. It is observed that when $t \to 0$, $\rho \to \infty$, indicating the Big-Bang scenario, i.e. the density was very high in the early Universe. As time progressed, the concentrated matter and radiation

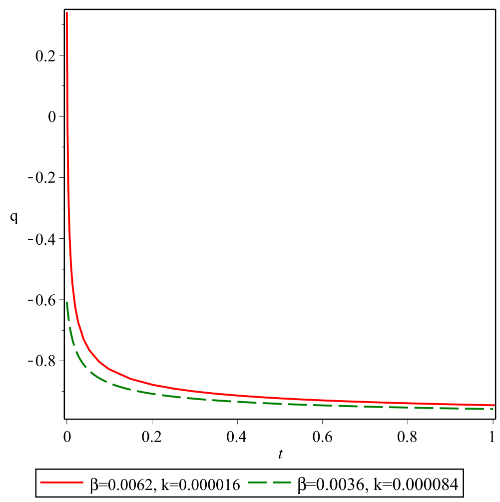

Fig. 1 Graph of DP q with t for two sets of (β, k).

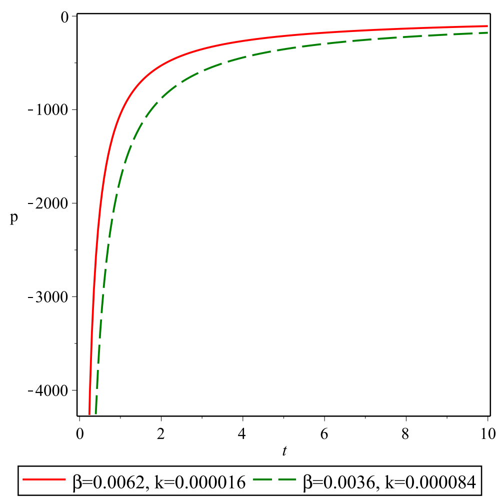

Fig. 2 Graph of pressure p with t for two sets of (β, k), $\kappa = -1.2$, $k_1 = 1$.

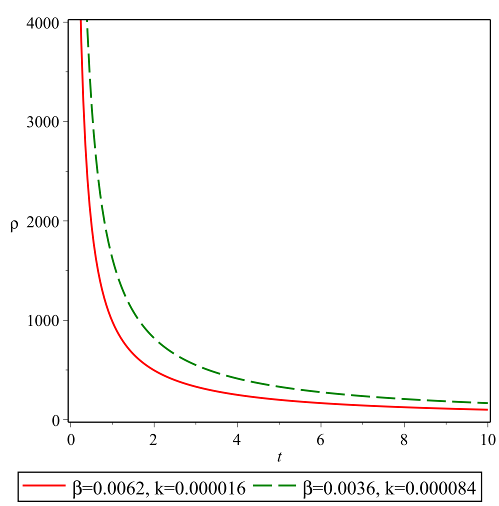

Fig. 3 Graph of energy density ρ versus t for two sets of (β, k), $\kappa = -1.2$, $k_1 = 1$.

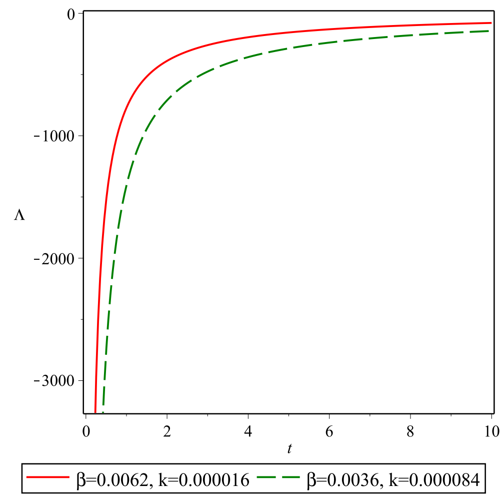

Fig. 4 Graph of cosmological constant Λ versus t for two sets of (β, k), $\kappa = -1.2$, $k_1 = 1$.

dispersed and so the density decreased. At the current time, it is also decreasing but at a moderate rate, indicating the expansion is still proceeding. This result of our model is also consistent with recent observations. ρ is a positive decreasing function of time and it nearly approaches zero as $t \to \infty$. It is worth mentioning here that ρ in case 1 decreases fast in comparison to ρ in case 2.

The behavior of the cosmological term Λ with time in both cases is shown in Figure 4. From the observations, it is clear that in the early time the cosmological constant Λ is negative and it increases quickly, becoming closer to a small negative value, almost zero. The Λ of case 1 is quickly increasing in contrast to case 2. It has been found that we get more negative pressure than the previous one every time. The reason for this is the physics. The cosmological models have been explored by Yadav (2009), Saha & Boyadjiev (2004), Pedram et al. (2008), Biswas & Mazumdar (2009) and Jotania et al. (2011) in which the cosmological constant is negative. At present, estimation of Λ is not simply trapped in any case; it is also uncertain and circumlocutory. In any case, the Einstein-Maxwell speculation indicates the other approach looks not so troublesome but rather larger, since the likelihood has appeared for $\Lambda \leq 0$, i.e. for the possibility that the presence of Λ decelerates the expansion of the Universe. Late cosmological investigations (Perlmutter et al. 1999; Riess et al. 1998, 2004) predict the presence of a positive Λ with magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These investigations of size and redshift of SNe Ia suggest that our Universe may be accelerating with prompted cosmological density through the cosmological Λ-term. Be that as it may, this does not invalidate the decelerating ones which are also dependable with these recognitions (Vishwakarma 2000). In this way, the possibility of Λ in our model is reinforced by ongoing observations.

Figure 5 corresponds to Equation (56), giving the variation of EoS parameter ω with t and it is observed that ω is a negative decreasing function of time and approaches a small negative value near zero as $t \to \infty$. Here, it is observed that ω vanishes for $t = t_c$, where t_c is a critical time given by the relation

$$\frac{2k_1^2(3\kappa+2)}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} + \frac{(6\kappa+1)}{(2\beta t+k)} = \frac{k_1}{3\sqrt{2\beta t+k}e^{\frac{3}{\beta}\sqrt{2\beta t+k}}} + \frac{2\beta(2\kappa+1)}{(2\beta t+k)^{\frac{3}{2}}} \,. \tag{57}$$

We know that $\omega = 0$, $\omega > 0$ and $\omega < 0$ represent the dusty Universe, real matter dominated Universe and DE dominated Universe respectively. Earlier (i.e. at $t < t_c$) real matter dominates, and when $t > t_c$, $\omega < 0$, indicating the DE dominated phase of the Universe. In our derived model, in the early Universe (when $t < t_c$), $\omega > 0$, indicating that the early Universe was a matter-dominated Universe. It passes through a dusty Universe phase at $t = t_c$ and at present $\omega < 0$ shows that at present the Universe is dominated by DE. The phantom dominated Universe ends up with a finite-time future singularity called the Big Rip or Cosmic Doomsday (Caldwell et al. 2003; McInnes 2002; González-Díaz 2004; Sami & Toporensky 2004; Nojiri & Odintsov 2004).

From Figure 5 we observe that in case 1, the model starts its evolution from a matter dominated era to a DE phase. In other words, we say that this model starts evolution from matter dominated phase, then reaches the quintessence DE phase and lastly ends in a phantom DE scenario. In case 2 the model is dominated by a phantom DE as is shown clearly in Figure 5.

Figure 6 corresponds to Equation (49) and plots the anisotropic parameter A_m versus time t. It is observed from the figure that in the early phase of the evolution, it is very high which means the early Universe was highly anisotropic. It decreases rapidly and approaches zero as $t \to \infty$ in both cases 1 and 2, indicating that the Universe will become isotropic in the very long term.

Energy Conditions

We examine the conceivable outcomes of energy conditions to be fulfilled or not in our model. We realize that:

- $\rho \geq 0$ and $\rho + p \geq 0$ are the energy conditions (weak).
- $\rho \geq |p|$ i.e. $\rho + p \geq 0$ and $\rho - p \geq 0$ are the energy conditions (dominant).
- $\rho + 3p \geq 0$ are the energy conditions (strong).

Utilizing definitions for ρ and p, we have plotted the diagrams for energy conditions. The left-hand side of the energy conditions has been plotted concerning cosmic time t in Figures 7 and 8 for cases 1 and 2 separately. From Figures 7 and 8, clearly all three kinds of energy conditions are not fulfilled in both cases 1 and 2. The presence of the locale with $\omega < 1$ (if such a stage in the advancement of the Universe indeed happens) opens up various major inquiries. For example, the entropy of such a Universe is negative (or the trademark temperatures ought to be negative). The DEC for ghost matter is violated, as a rule (Nojiri et al. 2005). It is worth mentioning here that our derived model is dominated by phantom DE fluid. So, the violation of SEC is consistent with a well-established law.

The Ricci Scalar and Trace

The Ricci scalar R and trace T for this model are obtained as

$$R = -6\left[\frac{k_1^2}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} + \frac{2}{(2\beta t+k)} - \frac{\beta}{(2\beta t+k)^{\frac{3}{2}}}\right], \tag{58}$$

$$T = \frac{1}{2\kappa(\kappa+1)}\left[-\frac{8(2\kappa+1)k_1^2}{9e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} + \frac{2(1-\kappa)}{3\sqrt{2\beta t+k}e^{\frac{3}{\beta}\sqrt{2\beta t+k}}} - \frac{2(11\kappa+1)}{(2\beta t+k)} + \frac{4(3\kappa+1)}{(2\beta t+k)^{\frac{3}{2}}}\right]. \tag{59}$$

The function $f(R,T) = f_1(R) + f_2(T)$ for this model is obtained as

$$f(R,T) = \lambda\left[-\frac{2k_1^2(\kappa+2)(3\kappa+1)}{3\kappa(\kappa+1)e^{\frac{6}{\beta}\sqrt{2\beta t+k}}} + \frac{2\beta(3\kappa^2+6\kappa+1)}{\kappa(\kappa+1)(2\beta t+k)^{\frac{3}{2}}} - \frac{(12\kappa^2+23\kappa+1)}{\kappa(\kappa+1)(2\beta t+k)} + \frac{(1-\kappa)e^{-\frac{3}{\beta}\sqrt{(2\beta t+k)}}}{3\kappa(\kappa+1)\sqrt{2\beta t+k}}\right]. \tag{60}$$

5 IS THE CORRESPONDING SOLUTION STABLE?

A thorough investigation by conjecturing a perturbation approach should be possible based on the stability of the related solutions. Perturbations of the fields of a gravitational framework against the background evolutionary solutions ought to be investigated to guarantee the stability of the correct or approximated background solution, which are discussed by Chen & Kao (2001) and Kao (2001). Presently we shall examine the background solution stability regarding metric perturbations. Considering perturbations for every one of the three expansion factors a_i by means of

$$a_i \rightarrow a_{Bi} + \delta a_i = a_{Bi}(1 + \delta b_i)\ , \tag{61}$$

we focus on variables δb_i rather than δa_i starting now and into the foreseeable future, for comfort. In this way, perturbations of the volume scale factor $V_B = \Pi_{i=1}^{3} a_i$, directional Hubble factors $\theta_i = \frac{\dot{a}_i}{a_i}$ and average Hubble factor $\theta = \sum_{i=3}^{3} \frac{\theta_i}{3} = \frac{\dot{V}}{3V}$ can be written as

$$\begin{aligned} V &\rightarrow V_B + V_B \sum_i \delta b_i, \\ \theta_i &\rightarrow \theta_{Bi} + \sum_i \delta b_i, \\ \theta &\rightarrow \theta_B + \frac{1}{3} \sum_i \delta b_i\ , \end{aligned} \tag{62}$$

where V_B represents the scale volume factor (background). Now one may demonstrate that the metric linear order perturbations δb_i comply with the accompanying conditions

$$\sum_i \delta \ddot{b}_i + 2 \sum \theta_{Bi} \delta \dot{b}_i = 0\ , \tag{63}$$

$$\delta \ddot{b}_i + \frac{\dot{V}_B}{V_B} \delta \dot{b}_i + \sum_j \delta \dot{b}_j \theta_{Bi} = 0\ , \tag{64}$$

$$\sum \delta \dot{b}_i = 0\ . \tag{65}$$

We can without much of a stretch find by the three conditions given above

$$\delta \ddot{b}_i + \frac{\dot{V}_B}{V_B} \delta \dot{b}_i = 0\ . \tag{66}$$

V_B for our case is taken as

$$V_B = \exp\left[\frac{3}{\beta}\sqrt{2\beta t + k}\right]\ . \tag{67}$$

Utilizing the above condition in Equation (66) and after integration we have

$$\delta b_i = -c_i \left[\frac{(\beta + 3\sqrt{2\beta t + k})}{9 \exp\left(\frac{3\sqrt{2\beta t + k}}{\beta}\right)}\right]\ , \tag{68}$$

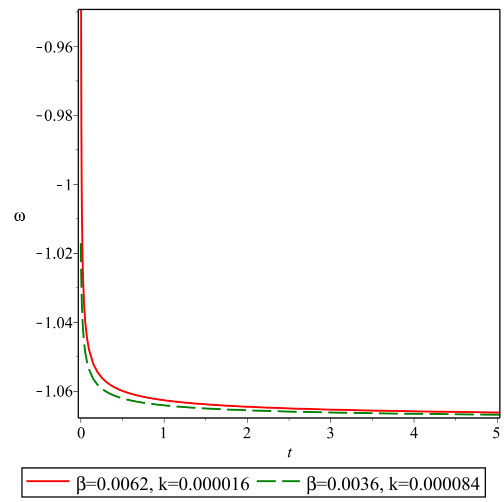

Fig. 5 Graph of EoS parameter ω with t for two sets of (β, k), $\kappa = -1.2$, $k_1 = 1$.

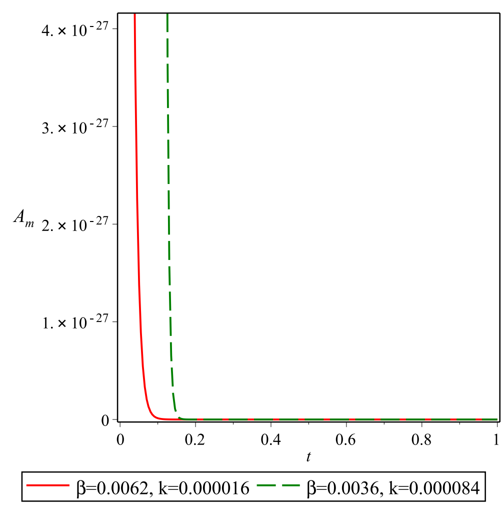

Fig. 6 Graph of anisotropy parameter A_m with t for two sets of (β, k), $k_1 = 1$.

where the constant of integration is c_i. So, for each expansion factor $\delta a_i = a_{Bi} \delta b_i$, the actual fluctuations are given by

$$\delta a_i \rightarrow -c_i \left[\frac{(\beta + 3\sqrt{2\beta t + k})}{9 \exp\left(\frac{3\sqrt{2\beta t + k}}{\beta}\right)}\right]\ . \tag{69}$$

From the above equation we see that δa_i approaches zero when $t \rightarrow \infty$. The variation of δa_i versus t is shown in Figure 9. Thus, against the perturbations of the graviton field, the background solution is stable.

6 CONCLUDING REMARKS

In the present paper, we have examined two kinds of LRS Bianchi I cosmological models inside the outline work of $f(R, T)$ theory with $\Lambda(T)$. The two models depend on the

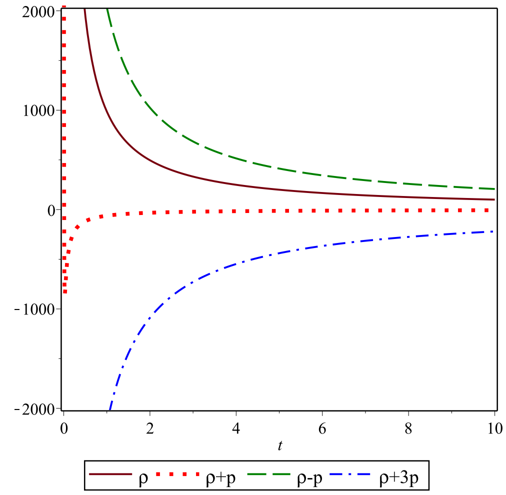

Fig. 7 Graph of energy conditions with t for case 1: $\beta = 0.0062$, $k = 0.000016$, $\kappa = -1.2$, $k_1 = 1$.

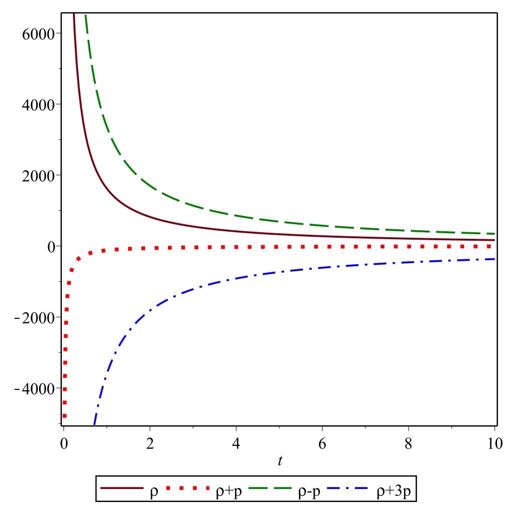

Fig. 8 Graph of energy conditions with t for case 2: $\beta = 0.0036$, $k = 0.000084$, $\kappa = -1.2$, $k_1 = 1$.

two different data sets: SN Ia data in combination with BAO and CMB observations (Giostri et al. 2012) and from SN Ia Union data (Cunha 2009) respectively. We have thought about the after effects of two cosmological models of the Universe, determined in the present work. The field conditions have been fathomed precisely with reasonable physical assumptions. The related correct solutions are found for the particular model $f(R, T) = \lambda(R + T)$, for which the Universe expands as $a = \exp\left[\frac{1}{\beta}\sqrt{2\beta t + k}\right]$. This creates a process of the Universe from the early decelerating stage to the ongoing accelerating stage. The physical parameters ρ, p and Λ evolve with an initial singularity and tend to a small value in late time which is predictable with the ongoing observations (expanding Universe). The models are anisotropic in the early Universe, however at

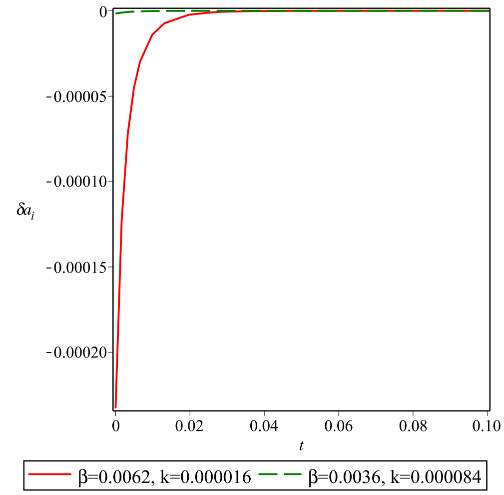

Fig. 9 Graph of δa_i with t for two sets of (β, k), $\kappa = -1.2$, $k_1 = 1$.

late time $t \to \infty$, the model behaves as an isotropic FRW model.

The primary highlights of the models are as follows:

– In outline, two expansion phases of the Universe which are naturally unified by modified gravity have been considered: early time inflation and astronomical speeding up to the current age. Our inferred models depend on two cases as stated above. In case 1, for $\beta = 0.0062$ and $k = 0.000016$, we acquired a cosmological model from the early decelerated stage to the present accelerating stage. In case 2, for $\beta = 0.0036$ and $k = 0.000084$, we have obtained a cosmological model in the accelerating stage only. In case 1, the models start evolution from a matter dominated era, then reach the quintessence DE phase and lastly end in a phantom DE era (Fig. 5). In case 2, we obtain the phantom DE era only (Fig. 5).

– The models depend on the correct solution of the $f(R, T)$ gravity field conditions for the anisotropic LRS Bianchi spacetime with perfect fluid with time subordinate DP.

– The model produces a shearing, non-rotating, expanding and transiting (from decelerating to accelerating) Universe.

– In both cases 1 and 2, the anisotropic parameter A_m tends to zero at $t \to \infty$ (Fig. 6), indicating that the Universe will become isotropic in late time. In an early phase of the evolution, anisotropy is very high which means that the early Universe was highly anisotropic. But at the present time, the model exhibits an isotropic nature.

– It is standard practice in the literature to think about constant DP. The nature of the DP for a Universe which is accelerating at present must show a flipping signature as now discussed. Subsequently, our thought of variable DP is physically supported. Our determined display is decelerating in an early period of the Universe while it is accelerating at the present stage.

– In our derived models, we observe that energy conditions are violated.

– The solution of our model is steady against the perturbation of the gravitational field as discussed in Section 5.

Consequently, the results found in this article may be useful for better comprehension of the behavior for cosmological LRS Bianchi-I DE models in the evolution of the Universe with regard to speculation about $f(R,T)$ gravity.

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