The Richardson-Lucy deconvolution method to extract LAMOST 1D spectra

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Abstract We use the Richardson-Lucy deconvolution algorithm to extract one-dimensional (1D) spectra from Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) spectrum images. Compared with other deconvolution algorithms, this algorithm is much faster. The application on a real LAMOST image illustrates that the 1D spectrum resulting from this method has a higher signal-to-noise ratio and resolution than those extracted by the LAMOST pipeline. Furthermore, our algorithm can effectively suppress the ringings that are often present in the 1D resulting spectra generated by other deconvolution methods.

Key words: instrumentation: spectrographs — methods: numerical — techniques: image processing — techniques: spectroscopic

1 INTRODUCTION

The Guo Shou Jing Telescope (also called the Large Sky Area Multi-Object Fiber Spectroscopic Telescope, LAMOST) (Wang et al. 1996; Su & Cui 2004; Zhao et al. 2012; Cui et al. 2012; Luo et al. 2012) can obtain 4000 spectra in one exposure, and has collected more than 10 million spectra (see http://dr7.lamost.org). So far, about 400 peer reviewed papers based on LAMOST data have been published, which help us understand our Galaxy in more depth.

The spectrum extraction method is a technique to convert a two-dimensional (2D) CCD image into onedimensional (1D) spectra, which can help astronomers explore the natures of celestial objects. The traditional extraction methods, including Aperture Extraction Method (AEM, hereafter), Optimal Extraction Method and Profile Fitting Method, are the most frequently used methods at present, and are discussed in detail in Li et al. (2019). Compared to the traditional methods, the deconvolution method is a completely different method, which tries to recover the 1D spectra by eliminating instrumental profiles (Point Spread Function, PSF) on a 2D image. The deconvolution method was first presented by Bolton & Schlegel (2010). In their work, a calibration matrix was constructed from known instrumental profiles, and then inverted to generate the resulting 1D spectra. As a result, this method can only extract a small piece of a spectrum image, but cannot be applied to extract a large spectrum image (for example, a LAMOST $4k \times 4k$ spectrum image) because of the huge storage required for the calibration matrix, let alone calculation. Furthermore, the noise in a real image would destroy the resulting 1D spectra.

To overcome these shortcomings, Guangwei et al. (2015) presented a deconvolution algorithm based on the Tikhonov regularization method (TDM, hereafter) for practical spectrum extraction. First, they gave a method to obtain all PSFs which vary with positions on a CCD image. Second, the big spectrum image is divided into many small subimages, whose calibration matrices can be easily stored and inverted during calculation. Third, Tikhonov regularization can effectively suppress the noise. This algorithm is the first practical deconvolution method to extract 1D spectra from a multi-fiber spectrum image. The signal-tonoise ratio (SNR) and resolution of the resulting 1D spectra are both higher than those generated by traditional methods. However, they did not discuss how to choose the best Tikhonov parameter. The choice of the parameter is a cru-

cial problem, because a fixed Tikhonov parameter is not always appropriate for all fibers in a multi-fiber spectrum image.

In our last paper (Li et al. 2019), we developed a deconvolution method based on adaptive Landweber iteration (ALI, hereafter), which can extract 1D spectra with the regularization parameter adaptively selected for every fiber. The SNR and resolution of 1D resulting spectra are both as high as those of the 1D resulting spectra extracted by TDM with a deliberately selected Tikhonov regularization parameter.

In this paper, the Richardson-Lucy Iteration deconvolution method is presented. This method can not only suppress the noise and improve both SNR and resolution of the resulting 1D spectrum but can also reduce ringings in the resulting 1D spectrum. Besides, the algorithm can be easily programmed and runs fast. The Richardson-Lucy Iteration formula is given in Section 2. Section 3 shows the extraction experiments on simulated 2D spectrum images. In Section 4, we apply our method on a real LAMOST spectrum image. Finally, the conclusion is provided in Section 5.

2 SPECTRUM EXTRACTION METHOD BASED ON RICHARDSON-LUCY ITERATION

Because the crosstalk on a LAMOST CCD is marginal, we only discuss how to extract a 1D spectrum from an image with only one fiber. The image model can be found in Li et al. (2019), which is

$$f(x,y) = \sum_{j=1}^{N} g_j h_j(x-j,y) + \eta(x,y), \qquad (1)$$

where N is the number of rows, f(x, y) and $\eta(x, y)$ are the count and noise at the xth row and yth column on the CCD, respectively, g_j is the flux at the *j*th row, $h_j(x, y)$ is the value of the PSF at the *j*th row on (x, y), and $G = (g_1, g_2, ..., g_N)$ is the 1D resulting spectrum. PSFs can be obtained by the method outlined in Guangwei et al. (2015) and Li et al. (2019).

The Richardson-Lucy Iteration algorithm (Richardson 1972; Lucy 1974) is a typical nonlinear iterative algorithm, which is widely applied in astronomical and medical image processing.

We can assume that the count of each pixel is independent and obeys a Poisson distribution. We denote

$$a(x,y) = \sum_{j=1}^{N} g_j h_j(x-j,y) \,,$$

which is the real spectrum image without noise. Then the likelihood function of the image is (Zou 2001)

$$P(f(x,y)|G) = \prod_{x,y} \frac{a(x,y)^{f(x,y)}e^{-a(x,y)}}{f(x,y)!} .$$
(2)

Then,

$$\ln P(f(x,y)|G) = \sum_{x,y} [f(x,y)\ln a(x,y) - a(x,y) - \ln(f(x,y)!)].$$
(3)

Let $\frac{\partial}{\partial g_j}[\ln P(f(x,y)|G)] = 0, j = 1, 2, 3, ..., N.$ Then

$$\sum_{x,y} \left[\frac{f(x,y)h_j(x-j,y)}{a(x,y)} - h_j(x-j,y) \right] = 0, \quad (4)$$

or

$$\sum_{x,y} \left[\frac{f(x,y)h_j(x-j,y)}{a(x,y)} \right] - 1 = 0,$$
 (5)

where j = 1, 2, 3, ..., N.

To output the spectrum G, Meinel (1986) suggested the iteration formula

$$g_{j}^{(k+1)} = g_{j}^{(k)} \left\{ \sum_{x,y} \frac{f(x,y)h_{j}(x-j,y)}{a(x,y)} \right\}^{p}, \qquad (6)$$
$$j = 1, 2, 3, ..., N$$

where k is the number of iterations.

If we set p = 1, the above formula describes the Richardson-Lucy Iteration

$$g_{j}^{(k+1)} = g_{j}^{(k)} \left\{ \sum_{x,y} \frac{f(x,y)h_{j}(x-j,y)}{a(x,y)} \right\}.$$

$$j = 1, 2, 3, ..., N$$
(7)

3 EXPERIMENTS ON SIMULATION IMAGES

We use a similar construction method as Li et al. (2019) to generate the simulation image. The simulation image is generated by an input 1D spectrum with 4000 flux points given by the LAMOST pipeline convolved with PSFs with size of 13×15 pixels. These PSFs are the linear interpolations on the basic PSFs from emission lines on an arc image. Finally, Poisson noise is added. The resulting image is 4000×15 pixels.

We extracted the 1D spectrum from the simulation image by applying Equation (7) for 10 iterations. Besides, we also performed TDM and ALI for comparison. The Tikhonov regularization parameter for TDM was set to 0.02, which is the best value in this extraction. The block sizes in TDM and ALI were both set to 20, while the computational precisions were both 100.



Fig. 1 The original noise and 2D residuals of different methods. (a) The original Poisson noise; (b), (c) and (d) are the 2D residuals of TDM, ALI and Eq. (7) for 10 iterations, respectively.



Fig. 2 From left to right, panels depict the 1D residuals of TDM, ALI and Eq. (7) for 10 iterations, respectively.



Fig. 3 From top to bottom, the spectra are extracted by AEM, TDM, ALI and Eq. (7) after five and 10 iterations, respectively. All these spectra are also overplotted at the bottom in the same colors.

We ran these three algorithms on a DELL computer with a 3.30 GHz CPU and the Windows 7 operating system, using MATLAB R2014a software. The original Poisson noise and the 2D residuals of different deconvolution methods are shown in Figure 1, while 1D residuals are displayed in Figure 2. These two figures demonstrate that their 2D residuals are Poisson noise, and their 1D residuals are all at a similar level.

 Table 1 SNRs and Computational Times of Different Methods

Method	SNR	Time (s)
TDM	46.63	11.94
ALI	47.08	6.05
Eq. (7) for 10 iterations	50.03	1.64

The SNRs and computation times of different extraction methods are shown in Table 1, where the SNR is defined by equation 11 in Li et al. (2019). From Table 1, we can see that the Richardson-Lucy Iteration algorithm is much faster than ALI and TDM, and the SNR of the resulting spectrum extracted by the Richardson-Lucy Iteration algorithm is also higher than those extracted by ALI and TDM. TDM is the slowest because it took too much time to invert calibration matrices.

4 PERFORMANCE ON A REAL LAMOST SPECTRUM IMAGE

We apply Equation (7) on a real LAMOST multi-fiber spectral image. The resulting 1D spectrum after five and 10 iterations is depicted in blue and red, respectively in Figure 3, while the spectra extracted by AEM, TMD and ALI are signified in black, green and grey, respectively. All spectra are overplotted together for direct comparison in the bottom of the figure. The Gibbs artifact, which is also called ringing artifact in signal processing, always occurs in the results of deconvolution methods. The dashed lines in Figure 3 indicate the positions of overshoots of ringings in the spectra of TMD and ALI.

From the figure, we can see:

1) The spectra extracted by TMD, ALI and Equation (7) all have higher SNRs and resolutions than that extracted by AEM. Furthermore, their emission lines are all more symmetric, which means that these three methods can correct the distortions of PSFs on a CCD.

2) The blue spectrum, which is the result of Equation (7) after five iterations, has no overshoots. Its resolution is lower than those of the spectra extracted by TMD and ALI, but is higher than the spectrum extracted by AEM.

3) After 10 iterations of Equation (7), the resolution of the resulting spectrum became similar to those of the spectra extracted by TMD and ALI, but the amplitudes of ringings are much lower.

5 CONCLUSIONS

This paper describes a deconvolution extraction algorithm based on the Richardson-Lucy Iteration to extract 1D spectra from LAMOST spectrum images. Compared with the spectrum extracted by AEM, the resolution and SNR of the spectrum extracted by the Richardson-Lucy Iteration are both higher. Compared with spectra extracted by TDM and ALI, the ringings of the spectrum extracted by the Richardson-Lucy Iteration are much weaker. Furthermore, the Richardson-Lucy Iteration is the fastest deconvolution method to extract 1D spectra from a LAMOST image.

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