RAA 2018 Vol. **18** No. **6**, 64(8pp) doi: 10.1088/1674–4527/18/6/64

http://www.raa-journal.org *http://iopscience.iop.org/raa*

***R**esearch in*
***A**stronomy and*
***A**strophysics*

Lyra's cosmology of hybrid universe in Bianchi-V space-time

Anil Kumar Yadav[1] and Vinod Kumar Bhardwaj[2]

[1] Department of Physics, United College of Engineering and Research, Greater Noida 201306, India; *abanilyadav@yahoo.co.in*

[2] Department of Mathematics, GLA University, Mathura, India; *dr.vinodbhardwaj@gmail.com*

Received 2018 January 15; accepted 2018 March 6

Abstract In this paper we have searched for the existence of Lyra's cosmology in a hybrid universe with minimal interaction between dark energy and normal matter using Bianchi-V space-time. To derive the exact solution, the average scale factor is taken as $a = (t^n e^{kt})^{\frac{1}{m}}$ which describes the hybrid nature of the scale factor and generates a model of the transitioning universe from the early deceleration phase to the present acceleration phase. The quintessence model makes the matter content of the derived universe remarkably able to satisfy the null, dominant and strong energy condition. It has been found that the time varying displacement $\beta(t)$ co-relates with the nature of cosmological constant $\Lambda(t)$. We also discuss some physical and geometrical features of the universe.

Key words: cosmology: hybrid universe — cosmology: dark energy — cosmology: Bianchi-V space-time

1 INTRODUCTION

In the recent past, astronomical observations of SNe Ia (Riess et al. 1998; Perlmutter et al. 1999; Riess et al. 2001; Rodrigues 2008) have indicated that the present universe is undergoing an accelerated expansion. However, the cause of this acceleration is still unknown and represents an open question for theoretical physicists. In the literature, numerous cosmological models had been proposed to resolve this problem including the modified theory of gravity and possible existence of dark energy (DE).

Firstly, Caldwell et al. (2006) and later on several other authors (Yadav 2012; Yadav & Sharma 2013; Pradhan & Amirhashchi 2011; Yadav 2016) studied models of the transitioning universe in different physical contexts. The cosmological constant Λ is assumed to be the simplest candidate for DE but it suffers from two problems on theoretical grounds – fine tuning and cosmic coincidence. So, in the literature, different models of DE with various effective equations of state (EoSs) have been proposed.

A Bianchi type-V universe, being the natural generalization of the Friedmann-Robertson-Walker model of the universe, is of particular interest because it describes a homogeneous and anisotropic universe that has different scale factors along each spatial direction. Moreover, the Bianchi-V universe converts to the Bianchi-I universe by considering specific choices for parameters. In 2011, Kumar and Yadav investigated the Bianchi-V DE model governed by power law expansion. Later on, Pradhan & Amirhashchi (2011) proposed the simple form of a hybrid expansion law in Bianchi-V space-time. In Kumar & Yadav (2011); Pradhan & Amirhashchi (2011), the authors considered an isotropic distribution of DE, but here we assume the generalized form of a hybrid expansion law in Bianchi-V space-time with an anisotropic distribution of DE that gives new and different expressions for cosmological parameters. In the literature, several authors (Yadav & Yadav 2011; Akarsu & Kılınç 2010; Kumar & Yadav 2011; Kumar & Singh 2011; Pradhan et al. 2012) have considered the EoS of DE ($\omega^{(\mathrm{de})}$) to be time dependent. A time dependent $\omega^{(\mathrm{de})}$ describes the three types of accelerating universe models - quintessence, ΛCDM and phantom. According to the latest cosmological data available, the associated uncertainties are still too large to discriminate these three cases, i.e.

$$\omega^{(\rm de)} > -1\,, \quad \omega^{(\rm de)} = -1\,, \quad \text{and} \quad \omega^{(\rm de)} < -1\,,$$

respectively, which describe the quintessence, ΛCDM and phantom models of the accelerating universe. The case of $\omega^{(\rm de)} = -1$ simply represents a cosmological constant dominated universe which has two main theoretical problems - the cosmic coincidence and fine tuning puzzles. The possibility of quintom DE containing both quintessence and phantom DE has been studied in Saha et al. (2012). Carroll et al. (2003) investigated the issue of whether phantom DE density can become infinite at a finite time – the big rip condition. The possibility of the quintessence model (i.e. $\omega^{(\rm de)} > -1$) has been already mentioned in Hinshaw et al. (2009). Some other applications of Bianchi type DE models in context with recent observational data have been also discussed in Amirhashchi (2017b,a); Amirhashchi & Amirhashchi (2018). In this paper, we confine ourselves to a transitioning model of the universe within the framework of Lyra's manifold in Bianchi-V space-time. Recently, Akarsu et al. (2014) investigated observational constraints, referred to as the hybrid expansion law, which provide an elegant description of a transitioning universe from deceleration to acceleration.

Firstly in 1951, Lyra proposed a scalar-tensor theory that suggests some modifications in Riemannian geometry by introducing a time varying gauge function (Lyra 1951). This alternative theory is attractive because it produces similar effects as Einstein's theory. The time varying vector field $\beta(t)$ plays the same role as cosmological constant $\Lambda(t)$ in general relativity (Halford 1970, 1972).

In the literature, numerous authors (Sen & Vanstone 1972; Bhamra 1974; Singh & Singh 1992; Rahaman et al. 2005; Pradhan et al. 2005; Yadav 2010; Yadav & Haque 2011) have studied different cosmological models based on Lyra geometry in various physical contexts.

In this paper, we present the model of a hybrid universe that describes an accelerating universe as well as a singularity free universe, in Bianchi-V space-time within the framework of Lyra geometry. We observe that the quintessence model validates the energy conditions with suitable choice of problem parameters. The outline of the paper is as follows: In Section 2, the field equations are established. The generalized hybrid expansion law and solution of field equations along with physical significance are presented in Section 3. Validation of the stability condition for the derived model is given in Section 4. Finally, the findings are summarized in Section 5.

2 FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi-V space-time reads as

$$ds^2 = -dt^2 + X^2dx^2 + e^{\alpha x}(Y^2dy^2 + Z^2dz^2)\,, \quad (1)$$

where $X(t)$, $Y(t)$ and $Z(t)$ are the metric functions.

We define $a = (XYZ)^{1/3}$ as the average scale factor for the space-time described in Equation (1). Hence, the average Hubble parameter is expressed as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\Big(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\Big)\,, \quad (2)$$

where $a = (XYZ)^{1/3}$ is the average scale factor and an overdot denotes derivative with respect to cosmic time t.

The directional Hubble parameters along x, y and z directions, respectively, may be defined as

$$H_x = \frac{\dot{X}}{X}, \quad H_y = \frac{\dot{Y}}{Y}, \quad H_z = \frac{\dot{Z}}{Z}. \quad (3)$$

The field equations in Lyra geometry are given by

$$R^j_i - \frac{1}{2}g^j_i R + \frac{3}{2}\phi_i\phi^j - \frac{3}{4}g^j_i\phi_k\phi^k = -T^{(m)i}_j - T^{(\rm de)i}_j\,, \quad (4)$$

where $T^{(m)}_{ij}$ and $T^{(\rm de)}_{ij}$ are the energy momentum tensors of perfect fluid and DE respectively. These are given by

$$T^{(m)}_{ij} = \text{diag}\Big[-\rho^{(m)}, p^{(m)}, p^{(m)}, p^{(m)}\Big]\,, \quad (5)$$

$$\begin{aligned} T^{(\rm de)}_{ij} &= \text{diag}\Big[-\rho^{(\rm de)}, p^{(\rm de)}_x, p^{(\rm de)}_y, p^{(\rm de)}_z\Big] \\ &= \text{diag}\Big[-1, \omega+\delta, \omega+\gamma, \omega+\eta\Big]\rho^{(\rm de)}. \end{aligned} \quad (6)$$

Here, $\rho^{(m)}$ and $\rho^{(\rm de)}$ are the energy densities of normal matter and DE components respectively. $p^{(m)}$ is the pressure of normal matter. $p_x = (\omega + \delta(t))\rho$, $p_y = (\omega+\gamma(t))\rho$ and $p_z = (\omega+\eta(t))\rho$ are the EoS parameters along the spatial directions. $\delta(t)$, $\gamma(t)$ and $\eta(t)$ are called the skewness parameters which are introduced to modify the EoS parameters of the DE component that quantifies the anisotropic nature of DE.

In Equation (4), ϕ_i is the displacement field vector which is defined as

$$\phi_i = \Big(0, 0, 0, \beta(t)\Big). \quad (7)$$

In a comoving co-ordinate system, the field Equations (4) for the anisotropic Bianchi-V space-time (1), along with Equations (5) and (6), can be written as

$$\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}\dot{Z}}{YZ} - \frac{\alpha^2}{X^2} - \frac{3}{4}\beta^2 = -p^{(m)} - (\omega+\delta)\rho^{(\rm de)}\,, \quad (8)$$

$$\frac{\ddot{Z}}{Z}+\frac{\ddot{X}}{X}+\frac{\dot{Z}\dot{X}}{ZX}-\frac{\alpha^2}{X^2}-\frac{3}{4}\beta^2=-p^{(m)}-(\omega+\gamma)\rho^{(\mathrm{de})}\,, \tag{9}$$

$$\frac{\ddot{X}}{X}+\frac{\ddot{Y}}{Y}+\frac{\dot{X}\dot{Y}}{XY}-\frac{\alpha^2}{X^2}-\frac{3}{4}\beta^2=-p^{(m)}-(\omega+\eta)\rho^{(\mathrm{de})}\,, \tag{10}$$

$$\frac{\dot{X}\dot{Y}}{XY}+\frac{\dot{Y}\dot{Z}}{YZ}+\frac{\dot{Z}\dot{X}}{ZX}-\frac{3\alpha^2}{X^2}+\frac{3}{4}\beta^2=\rho^{(m)}+\rho^{(\mathrm{de})}\,, \tag{11}$$

$$2\frac{\dot{X}}{X}-\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}=0\,. \tag{12}$$

One may consider that the perfect fluid and DE component interact minimally (Akarsu & Kılınç 2010; Yadav 2011, 2016). Hence, the energy conservation equation of perfect fluid and DE component can be conserved separately.

The energy conservation equation $T_j^{(m)i}=0$ of the perfect fluid leads to

$$\dot{\rho^{(m)}}+3(\rho^{(m)}+p^{(m)})H=0\,, \tag{13}$$

where the energy conservation equation $T_j^{(\mathrm{de})i}=0$.

The DE component yields

$$\begin{aligned}&\dot{\rho}^{(\mathrm{de})}+3\rho^{(\mathrm{de})}(\omega+1)H\\&\quad+\rho^{(\mathrm{de})}(\delta H_x+\gamma H_y+\eta H_z)=0\,,\end{aligned} \tag{14}$$

and conservation of the right hand side of Equation (4) leads to

$$\left(R_i^j-\frac{1}{2}g_i^jR\right)_{;j}+\frac{3}{2}\left(\phi_i\phi^j\right)_{;j}-\frac{3}{4}\left(g_i^j\phi_k\phi^k\right)_{;j}=0\,. \tag{15}$$

Equation (15) reduces to

$$\begin{aligned}&\frac{3}{2}\phi_i\Big[\frac{\partial\phi^j}{\partial x^j}+\phi^l\Gamma^j_{lj}\Big]+\frac{3}{2}\Big[\frac{\partial\phi_i}{\partial x^j}-\phi_l\Gamma^l_{ij}\Big]\\&-\frac{3}{4}g_i^j\phi_k\Big[\frac{\partial\phi^k}{\partial x^j}+\phi^l\Gamma^k_{lj}\Big]\\&-\frac{3}{4}g_i^j\phi^k\Big[\frac{\partial\phi_k}{\partial x^j}-\phi_l\Gamma^l_{kj}\Big]=0.\end{aligned} \tag{16}$$

Equation (16) is identically satisfied for $i=1,2,3$. For $i=4$, Equation (16) reduces to

$$\begin{aligned}&\frac{3}{2}\beta\Big[\frac{\partial(g^{44}\phi_4)}{\partial x^4}+\phi^4\Gamma^4_{44}\Big]+\frac{3}{2}g^{44}\phi_4\Big[\frac{\partial\phi_4}{\partial t}-\phi_4\Gamma^4_{44}\Big]\\&-\frac{3}{4}g_4^4\phi_4\Big[\frac{\partial\phi^4}{\partial x^4}+\phi^4\Gamma^4_{44}\Big]\\&-\frac{3}{4}g_4^4g^{44}\phi^4\Big[\frac{\partial\phi_4}{\partial t}-\phi^4\Gamma^4_{44}\Big]=0\,.\end{aligned} \tag{17}$$

Equation (17) leads to

$$\frac{3}{2}\beta\dot{\beta}+\frac{3}{2}\beta^2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0. \tag{18}$$

3 THE GENERALIZED HYBRID EXPANSION LAW

Following Yadav (2016) and references therein (Yadav & Sharma 2013; Akarsu et al. 2014; Yadav et al. 2015), we consider the generalized form of a hybrid expansion law for scale factor as follows:

$$a=\Big[t^n\exp(\xi t)\Big]^{\frac{1}{m}}\,, \tag{19}$$

where $n\geq 0$, $\xi\geq 0$ and $m>0$ are constants.

Equation (14) may be split into two parts – one corresponds to deviation of EoS parameters for the DE component and the other represents parameters where no deviation is present (Yadav & Yadav 2011).

$$\dot{\rho}+3\rho^{(\mathrm{de})}(\omega+1)H=0, \tag{20}$$

$$\rho^{(\mathrm{de})}(\delta H_x+\gamma H_y+\eta H_z)=0. \tag{21}$$

The dynamics of skewness parameters on the x-axis, y-axis and z-axis are considered as

$$\delta(t)=\alpha_1(H_y+\eta H_z)\,, \tag{22}$$

$$\gamma(t)=\eta(t)=-\alpha_1 H_x\,. \tag{23}$$

Here α_1 is the proportionality constant.

In view of Equations (22)–(23), the solution of Equation (20) reads

$$\rho^{(\mathrm{de})}=\rho_0^{(\mathrm{de})}a^{-3(\omega+1)}, \tag{24}$$

where $\rho_0^{(\mathrm{de})}$ is a positive constant.

Integrating (12) and absorbing the constant of integration in B or C, one can obtain

$$X^2=YZ. \tag{25}$$

Subtracting Equation (7) from Equation (8), Equation (7) from Equation (9), Equation (8) from Equation (10) and taking the second integral of each, we get the following three relations respectively:

$$\begin{aligned}\frac{X}{Y}&=d_1\exp\Big[x_1\int(t^ne^{\xi t})^{\frac{-3}{m}}dt\\&\qquad-\frac{\alpha\rho_0^{(\mathrm{de})}}{\omega}\int(t^ne^{\xi t})^{\frac{-3(\omega+1)}{m}}dt\Big],\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{X}{Z}&=d_2\exp\Big[x_2\int(t^ne^{\xi t})^{\frac{-3}{m}}dt\\&\qquad-\frac{\alpha\rho_0^{(\mathrm{de})}}{\omega}\int(t^ne^{\xi t})^{\frac{-3(\omega+1)}{m}}dt\Big]\,,\end{aligned} \tag{27}$$

$$\frac{Y}{Z}=d_3\exp\Big[x_3\int(t^ne^{\xi t})^{\frac{-3}{m}}dt\Big]\,. \tag{28}$$

The parameters d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration.

3.1 Solution of the Field Equation and its Physical Significance

Solving Equations (25)–(28), we obtain the following expressions for scale factors

$$X(t) = (t^n e^{\xi t})^{\frac{1}{m}} \exp\Big[\frac{-2\alpha\rho_0^{(\mathrm{de})}}{3\omega}\int (t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}} dt\Big], \tag{29}$$

$$\begin{aligned} Y(t) =& A(t^n e^{\xi t})^{\frac{1}{m}} \times \exp\Big[l\int (t^n e^{\xi t})^{\frac{-3}{m}} dt \\ &+ \frac{\alpha\rho_0^{(\mathrm{de})}}{\omega}\int (t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}} dt\Big], \end{aligned} \tag{30}$$

$$\begin{aligned} Z(t) =& A^{-1}(t^n e^{\xi t})^{\frac{1}{m}} \times \exp\Big[-l\int (t^n e^{\xi t})^{\frac{-3}{m}} dt \\ &+ \frac{\alpha\rho_0^{(\mathrm{de})}}{\omega}\int (t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}} dt\Big], \end{aligned} \tag{31}$$

where

$$A = \sqrt[3]{d_2 d_3}, \quad l = \frac{(x_2 + x_3)}{3}, \quad d_2 = d_1^{-1}, \quad x_2 = -x_1 .$$

The deceleration parameter (DP) in the derived model is given by

$$q = \frac{d}{dt}\Big(\frac{1}{H}\Big) - 1 = \frac{mn}{(nt+\xi)^2} - 1. \tag{32}$$

The expressions for directional Hubble parameters, anisotropy parameter ($\triangle$) and shear scalar (σ^2) are respectively given by

$$H_x = \frac{1}{m}\Big(\frac{n}{t}+\xi\Big) - \frac{2\alpha\rho_0^{(\mathrm{de})}}{3\omega}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}, \tag{33}$$

$$\begin{aligned} H_y =& \frac{1}{m}\Big(\frac{n}{t}+\xi\Big) + l(t^n e^{\xi t})^{\frac{-3}{m}} \\ &+ \frac{\alpha\rho_0^{(\mathrm{de})}}{3\omega}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}, \end{aligned} \tag{34}$$

$$\begin{aligned} H_z =& \frac{1}{m}\Big(\frac{n}{t}+\xi\Big) - l(t^n e^{\xi t})^{\frac{-3}{m}} \\ &+ \frac{\alpha\rho_0^{(\mathrm{de})}}{3\omega}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}, \end{aligned} \tag{35}$$

$$\begin{aligned} \triangle =& \frac{2m^2}{3(\frac{n}{t}+\xi)^2} \times \Big[l^2(t^n e^{\xi t})^{\frac{-6}{m}} \\ &+ \frac{\alpha\rho_0^{(\mathrm{de})^2}}{3\omega^2}(t^n e^{\xi t})^{\frac{-6(\omega+1)}{m}}\Big], \end{aligned} \tag{36}$$

$$\sigma^2 = l^2(t^n e^{\xi t})^{\frac{-6}{m}} + \frac{\alpha\rho_0^{(\mathrm{de})^2}}{3\omega^2}(t^n e^{\xi t})^{\frac{-6(\omega+1)}{m}}. \tag{37}$$

Solving Equation (18), we obtain

$$\beta = \zeta(t^n e^{\xi t})^{-\frac{3}{m}}, \tag{38}$$

where ζ is the constant of integration.

In the derived model, we observe that for the generalized hybrid expansion law (19), the expansion scalar (θ) is proportional to mean Hubble parameter (H) and volume is equal to the cube of the scale factor. The anisotropy parameter ($\triangle$) and shear scalar (σ^2) decrease with time, which match the properties of a realistic universe. The behavior of displacement vector (β) versus time is graphed in Figure 1. The displacement vector (β) is a decreasing function of time and finally approaches a very small positive value which co-relates $\beta(t)$ with the nature of cosmological constant $\Lambda(t)$.

The skewness parameters are given by

$$\begin{aligned} \delta(t) &= \alpha_1(H_y + H_z) = \alpha_1\Big[\frac{2}{m}\Big(\frac{n}{t}+\xi\Big) \\ &\quad + \frac{2\alpha\rho_0^{(\mathrm{de})}}{3\omega}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}\Big], \end{aligned} \tag{39}$$

$$\begin{aligned} \gamma(t) &= \eta(t) = -\alpha_1(H_x) = -\alpha_1\Big[\frac{1}{m}\Big(\frac{n}{t}+\xi\Big) \\ &\quad + \frac{2\alpha\rho_0^{(\mathrm{de})}}{3\omega}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}\Big]. \end{aligned} \tag{40}$$

The directional EoS parameters of DE are given by

$$\begin{aligned} \omega_x &= \omega + \delta(t) = \omega + \alpha_1\Big[\frac{2}{m}\Big(\frac{n}{t}+\xi\Big) \\ &\quad + \frac{2\alpha\rho_0^{(\mathrm{de})}}{3\omega}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}\Big], \end{aligned} \tag{41}$$

$$\begin{aligned} \omega_y &= \omega_z = \omega - \alpha_1\Big[\frac{1}{m}\Big(\frac{n}{t}+\xi\Big) \\ &\quad - \frac{2\alpha\rho_0^{(\mathrm{de})}}{3\omega}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}\Big]. \end{aligned} \tag{42}$$

The energy density and pressure of the DE components read as

$$\rho^{(\mathrm{de})} = \rho_0^{(\mathrm{de})}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}, \tag{43}$$

$$p^{(\mathrm{de})} = \omega\rho_0^{(\mathrm{de})}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}. \tag{44}$$

The pressure and energy density of the perfect fluid are obtained as

$$\begin{aligned} p^{(m)} =& \frac{2n}{mt^2} - \frac{3}{m^2t^2}(n+\xi t)^2 - l^2(t^n e^{\xi t})^{\frac{-6}{m}} \\ &- \frac{\alpha^2\rho_0^{(\mathrm{de})^2}}{\omega^2}(t^n e^{\xi t})^{\frac{-6(\omega+1)}{m}} - \frac{3}{4}\zeta(t^n e^{\xi t})^{-\frac{3}{m}}, \end{aligned} \tag{45}$$

$$\begin{aligned} \rho^{(m)} =& \frac{3}{m^2t^2}(n+\xi t)^2 - l^2(t^n e^{\xi t})^{\frac{-6}{m}} \\ &- \rho_0^{(\mathrm{de})}(t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}} \\ &- \frac{\alpha^2\rho_0^{(\mathrm{de})^2}}{3\omega^2}(t^n e^{\xi t})^{\frac{-6(\omega+1)}{m}} + \frac{3}{4}\zeta(t^n e^{\xi t})^{-\frac{3}{m}}. \end{aligned} \tag{46}$$

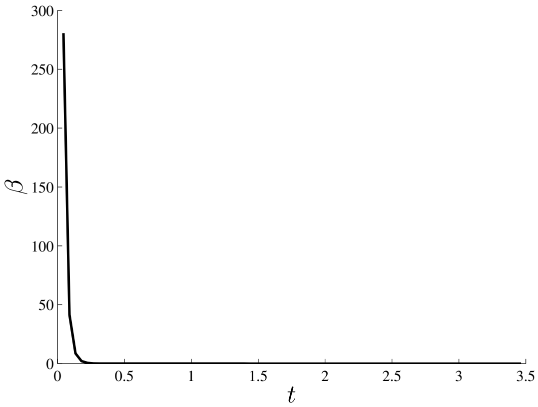

Fig. 1 The displacement vector (β) vs. time for $n = 0.5$, $\xi = 10$ and $\omega = -0.91$.

The perfect fluid density parameter ($\Omega^{(m)}$) and DE density parameter ($\Omega^{(\text{de})}$) are expressed as

$$\begin{aligned}\Omega^{(m)} =& 1 - \frac{m^2t^2}{3(n+\xi t)^2} l^2 (t^n e^{\xi t})^{\frac{-6}{m}} \\ &+ \rho_0^{(\text{de})} (t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}} \\ &+ \frac{\alpha^2 \rho_0^{(\text{de})^2}}{3\omega^2} (t^n e^{\xi t})^{\frac{-6(\omega+1)}{m}} + \zeta (t^n e^{kt})^{-\frac{3}{m}},\end{aligned} \tag{47}$$

$$\Omega^{(\text{de})} = \frac{m^2t^2\rho_0^{(\text{de})}}{3(n+\xi t)^2} (t^n e^{\xi t})^{\frac{-3(\omega+1)}{m}}. \tag{48}$$

Thus, the overall density parameter (Ω) is obtained as

$$\begin{aligned}\Omega \ = \ & \Omega^{(m)} + \Omega^{(\text{de})} = 1 - \frac{m^2t^2}{3(n+\xi t)^2} \\ & \times \left[l^2 (t^n e^{\xi t})^{\frac{-6}{m}} + \frac{\alpha^2 \rho_0^{(\text{de})^2}}{3\omega^2} (t^n e^{\xi t})^{\frac{-6(\omega+1)}{m}} \right].\end{aligned} \tag{49}$$

From Equation (49), it is evident that for $t \to \infty$, the overall density parameter approaches 1, which is supported by astrophysical observations (Perlmutter et al. 1999; Riess et al. 1998).

From Equations (29), (30) and (31), it is clear that the metric functions $X(t)$, $Y(t)$ and $Z(t)$ vanish at $t = 0$. This shows that the derived model has point type singularity at $t = 0$. Further, it can be noted that $n = 0$, $q = -1$ and $\frac{dH}{dt} = 0$ yield the fastest rate for expansion of the universe and generate a singularity free universe, which seem reasonable for understanding the future dynamics of the universe.

From (32), it is evident that when t is in the range $(0, \frac{\sqrt{mn}-n}{\xi})$, the value of q is positive which leads to deceleration. At $t = \frac{\sqrt{mn}-n}{\xi}$, the transition from deceleration to acceleration take place and after $t > \frac{\sqrt{mn}-n}{\xi}$, the universe expands with acceleration.

In Figure 2, the behavior of q is graphed in the accelerating mode of the universe for a particular choice of constants.

4 STABILITY CONDITION

In this section, we check the stability of the corresponding solution with respect to perturbation of the metric (Saha et al. 2012). The perturbations will be considered for all three expansion factors a_i as follows:

$$a_i \to a_{Bi} + \delta a_i = a_{Bi}(1 + \delta b_i). \tag{50}$$

With reference to Equation (50), we obtain the following relations that represent the perturbations of volume scalar, directional Hubble factors and mean Hubble factor

$$\begin{aligned} V &\to V_B + V_B \sum_i \delta b_i, \\ \theta_i &\to \theta_{Bi} + \sum_i \delta b_i, \\ \theta &\to \theta_B + \frac{1}{3} \sum_i \delta b_i. \end{aligned} \tag{51}$$

According to Saha et al. (2012), for the metric perturbation δb_i to be linear in δb_i, the following equations must be obeyed

$$\sum_i \delta \ddot{b}_i + 2 \sum \theta_{Bi} \delta \dot{b}_i = 0, \tag{52}$$

$$\delta \ddot{b}_i + \frac{\dot{V}_B}{V_B} \delta \dot{b}_i + \sum_j \delta \dot{b}_j \theta_{Bi} = 0, \tag{53}$$

$$\sum \delta \dot{b}_i = 0. \tag{54}$$

Equations (52)−(54) lead to

$$\delta \ddot{b}_i + \frac{\dot{V}_B}{V_B} \delta \dot{b}_i = 0\,, \tag{55}$$

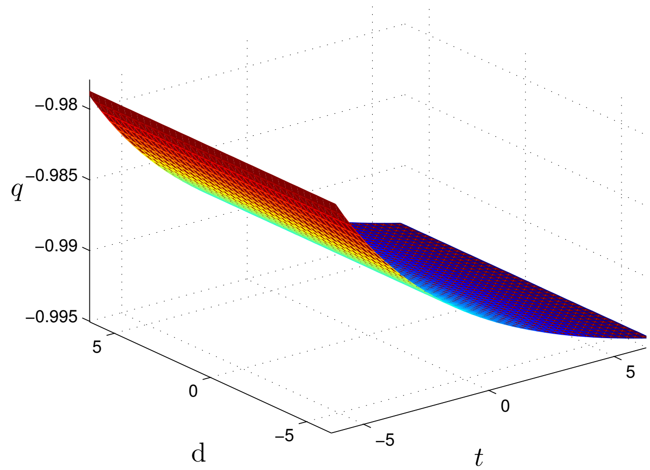

Fig. 2 Variation of DP (q) for $m = d$, $n = 0.5$ and $\xi = 10$.

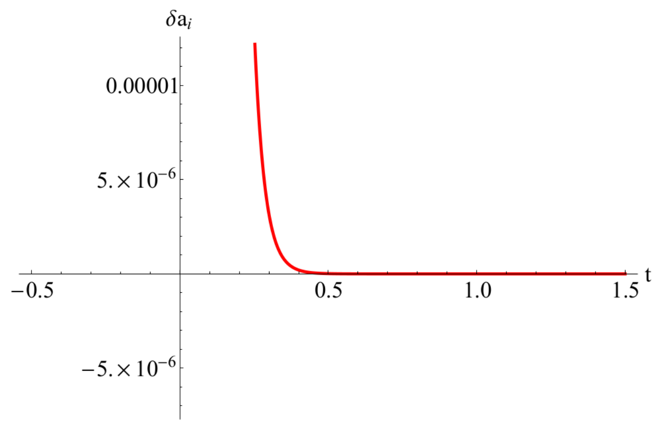

Fig. 3 Plot of a_i versus time for $k = 10$, $n = 0.5$ and $m = 1.25$.

where V_B is the background volume scalar which reads

$$V_B = t^{\frac{3n}{m}} \exp\left(\frac{3kt}{m}\right). \tag{56}$$

Equations (55) and (56) lead to

$$\delta b_i = c_2 - c_1 \left(\frac{3k}{m}\right)^{-1+\frac{3n}{m}} \Gamma\left[1 - \frac{3n}{m}, \frac{3kt}{m}\right], \tag{57}$$

where c_1 and c_2 are the constants of integration.

Thus, the actual fluctuations for each expansion factor $\delta a_i = a_{Bi}\delta b_i$ are expressed as

$$\begin{aligned} \delta a_i \;=\; & c_2 t^{\frac{-3n}{m}} \exp\left(-\frac{3kt}{m}\right) \\ & -c_1\left(\frac{3k}{m}\right)^{-1+\frac{3n}{m}} t^{\frac{-3n}{m}} \exp\left(-\frac{3kt}{m}\right) \\ & \times\Gamma\left[1 - \frac{3n}{m}, \frac{3kt}{m}\right]. \end{aligned} \tag{58}$$

Figure 3 depicts the behavior of actual fluctuations (δa_i) versus time which also shows that δa_i starts with a very small positive value and quickly approaches 0 with evolution of the universe. Thus the background solution is stable against perturbation of the graviton field.

5 CONCLUDING REMARKS

In this paper, we have searched for a DE model of a transitioning universe with minimal interaction between DE and normal matter in Bianchi-V space-time. The field equations have been solved exactly by taking into account the generalized form of the hybrid expansion law. The main results of this paper are summarized below:

(1) The volume of derived quintessence DE model increases with cosmic time. The parameters H, θ and $\bar{A}$ have extremely large values at $t = 0$ and finally

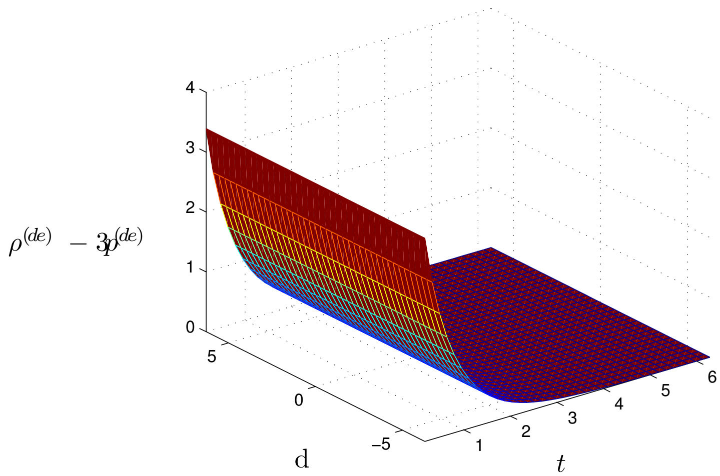

Fig. 4 Validity of SEC, $\rho^{(\mathrm{de})} + 3p^{(\mathrm{de})} \geq 0$, for $m = d$, $n = 0.5$, $\xi = 10$ and $\omega = -0.91$.

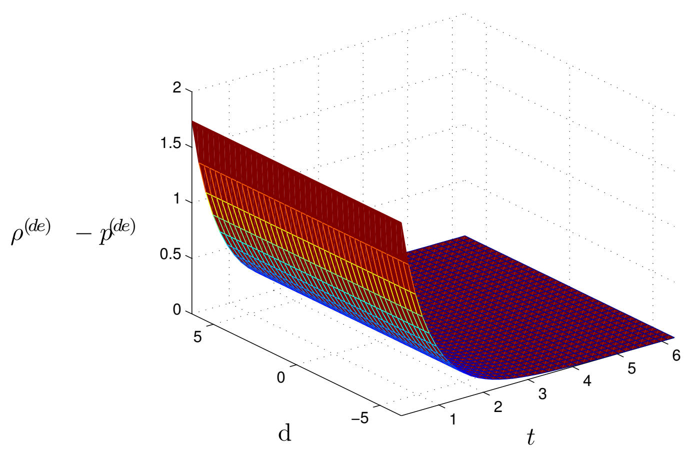

Fig. 5 Validity of DEC, $\rho^{(\mathrm{de})} - p^{(\mathrm{de})} \geq 0$, for $m = d$, $n = 0.5$, $\xi = 10$ and $\omega = -0.91$.

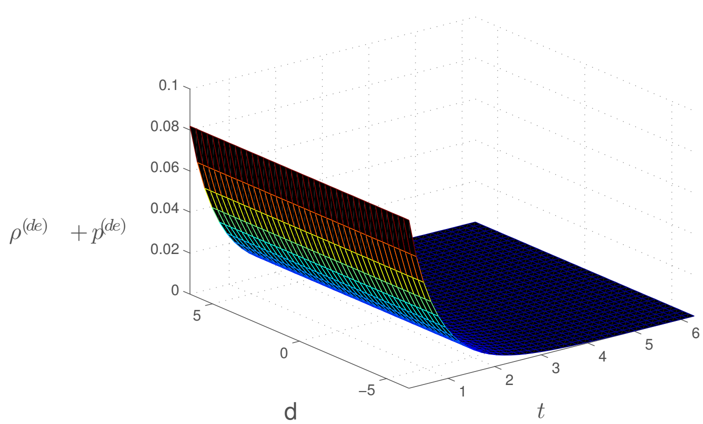

Fig. 6 Validity of NEC, $\rho^{(\mathrm{de})} + p^{(\mathrm{de})} \geq 0$, for $m = d$, $n = 0.5$, $\xi = 10$ and $\omega = -0.91$.

drop to a minimum positive value at $t \to \infty$. Thus the derived model turns out to be a suitable model that exhibits the dynamics of the present universe.

(2) The strong energy condition (SEC), null energy condition (NEC) and dominant energy condition (DEC) are satisfied in the derived model. Plots of energy conditions are displayed in Figures 4, 5, and 6.

(3) The derived quintessence DE model shows the transition of the universe from its past deceleration phase to current acceleration.

(4) The displacement vector $\beta(t)$ decreases with passage of time and finally approaches a small positive value at large time. The $\beta(t)$ matches the behavior of $\Lambda(t)$.

(5) It is to noted that in the absence of a time varying displacement field, for $m = 2$ and $\xi = 1$, the derived model reproduces the result obtained by Pradhan & Amirhashchi (2011) and Pradhan et al. (2012). Thus, the results of Pradhan & Amirhashchi (2011); Pradhan et al. (2012) are a special case of our result.

(6) The derived model validates the stability condition which confirms that the solution demonstrated in this paper is stable and may be useful for better understanding dynamics of the accelerating universe.

(7) As a final comment, we note that the derived model shows the possibility of incorporating both features of the universe, the decelerating as well as accelerating phases, depending upon the values of parameters under consideration. It can also be noted that for some certain values of problem parameters, the derived model describes an accelerating universe with non-negative pressure of its matter/energy constituent, which needs to be tested by other theories.

Acknowledgements The authors wish to express their sincere thanks to the referee whose valuable comments helped in improving the quality of this paper.

References

Akarsu, Ö., & Kılınç, C. B. 2010, General Relativity and Gravitation, 42, 119

Akarsu, Ö., Kumar, S., Myrzakulov, R., Sami, M., & Xu, L. 2014, J. Cosmol. Astropart. Phys., 1, 022

Amirhashchi, H. 2017a, arXiv:1712.02072

Amirhashchi, H. 2017b, Phys. Rev. D, 96, 123507

Amirhashchi, H., & Amirhashchi, S. 2018, arXiv:1802.04251

Bhamra, K. S. 1974, Australian Journal of Physics, 27, 541

Caldwell, R. R., Komp, W., Parker, L., & Vanzella, D. A. T. 2006, Phys. Rev. D, 73, 023513

Carroll, S. M., Hoffman, M., & Trodden, M. 2003, Phys. Rev. D, 68, 023509

Halford, W. D. 1970, Australian Journal of Physics, 23, 863

Halford, W. D. 1972, Journal of Mathematical Physics, 13, 1699

Hinshaw, G., Weiland, J. L., Hill, R. S., et al. 2009, ApJS, 180, 225

Kumar, S., & Singh, C. P. 2011, General Relativity and Gravitation, 43, 1427

Kumar, S., & Yadav, A. K. 2011, Modern Physics Letters A, 26, 647

Lyra, G. 1951, Mathematische Zeitschrift, 54, 52

Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565

Pradhan, A., Yadav, L., & Yadav, A. K. 2005, Ap&SS, 299, 31

Pradhan, A., & Amirhashchi, H. 2011, Modern Physics Letters A, 26, 2261

Pradhan, A., Amirhashchi, H., & Zainuddin, H. 2012, Electronic Journal of Theoretical Physics, 9, 159

Rahaman, F., Bhui, B. C., & Bag, G. 2005, Ap&SS, 295, 507

Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009

Riess, A. G., Nugent, P. E., Gilliland, R. L., et al. 2001, ApJ, 560, 49

Rodrigues, D. C. 2008, Phys. Rev. D, 77, 023534

Saha, B., Amirhashchi, H., & Pradhan, A. 2012, Ap&SS, 342, 257

Sen, D., & Vanstone, J. 1972, Journal of Mathematical Physics, 13, 990

Singh, T., & Singh, G. 1992, International Journal of Theoretical Physics, 31, 1433

Yadav, A. K. 2010, FIZIKA B, 19, 53

Yadav, A. K. 2011, Ap&SS, 335, 565

Yadav, A. K. 2012, RAA (Research in Astronomy and Astrophysics), 12, 1467

Yadav, A. K. 2016, Ap&SS, 361, 276

Yadav, A. K., & Haque, A. 2011, International Journal of Theoretical Physics, 50, 2850

Yadav, A. K., & Sharma, A. 2013, RAA (Research in Astronomy and Astrophysics), 13, 501

Yadav, A. K., Srivastava, P., & Yadav, L. 2015, International Journal of Theoretical Physics, 54, 1671

Yadav, A. K., & Yadav, L. 2011, International Journal of Theoretical Physics, 50, 218