

What do parameterized $Om(z)$ diagnostics tell us in light of recent observations?

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Abstract In this paper, we propose a new parametrization for $Om(z)$ diagnostics and show how the most recent and significantly improved observations concerning the $H(z)$ and SN Ia measurements can be used to probe the consistency or tension between the Λ CDM model and observations. Our results demonstrate that H_0 plays a very important role in the consistency test of Λ CDM with $H(z)$ data. Adopting the Hubble constant priors from Planck 2013 and Riess, one finds considerable tension between the current $H(z)$ data and Λ CDM model and confirms the conclusions obtained previously by others. However, with the Hubble constant prior taken from WMAP9, the discrepancy between $H(z)$ data and Λ CDM disappears, i.e., the current $H(z)$ observations still support the cosmological constant scenario. This conclusion is also supported by the results derived from the Joint Light-curve Analysis (JLA) SN Ia sample. The best-fit Hubble constant from the combination of $H(z)$ +JLA ($H_0 = 68.81^{+1.50}_{-1.49} \text{ km s}^{-1} \text{ Mpc}^{-1}$) is very consistent with results derived both by Planck 2013 and WMAP9, but is significantly different from the recent local measurement by Riess.

Key words: cosmology: theory — cosmology: observations — cosmology: cosmological parameters

1 INTRODUCTION

The fact that our Universe is undergoing an accelerated expansion at the present stage has become one of the most important issues in modern cosmology ever since this aspect was indicated by observations of type Ia supernovae (SNe Ia) (Riess et al. 1998; Perlmutter et al. 1999). This conclusion was also confirmed by other independent astrophysical observations including large scale structure (Tegmark et al. 2004), baryon acoustic oscillation (BAO) peaks (Eisenstein et al. 2005) and cosmic microwave background (CMB) radiation (Spergel et al. 2003). This phenomenon poses a great mystery concerning what component of our Universe could produce a repulsive force to drive this accelerating expansion. In the framework of general relativity, a mysterious substance with negative pressure, dubbed dark energy, was proposed to explain this acceleration. Due to the still

unknown nature of dark energy, the investigation of its equation of state (EoS), $w = p/\rho$, a critical parameter to characterize the dynamical property of dark energy, has also become a significant research theme in modern cosmology. Many cosmologists suspect that dark energy is just the cosmological constant with $w = -1$, due to its simplicity and remarkable consistency with almost all observational data. However, the notable fine-tuning problem (Weinberg 1989) and coincidence problem (Zlatev et al. 1999) still question why Λ CDM is declared to be the concordance cosmological model to describe the overall evolution of the Universe. Thereupon, the possibility that cosmic EoS is variable, depending on time, has been explored in a number of dynamical dark energy models, such as quintessence (Caldwell & Linder 2005; Zlatev et al. 1999), K-essence (Chiba et al. 2000; Armendariz-Picon et al. 2000), phantom (Kahya

& Onemli 2007; Onemli & Woodard 2004; Singh et al. 2003), etc. In the face of so many competing dark energy candidates, it is important to find an effective way to decide whether the EoS of dark energy is time varying, which is significant for us to understand the nature of dark energy.

Following this way, an effective diagnostic named $Om(z)$, which is sensitive to the EoS of dark energy and thus provides a null test of the Λ CDM model, was initially introduced by Sahni et al. (2008) and extensively studied in many subsequent works (Sahni et al. 2014; Ding et al. 2015; Zheng et al. 2016). If the value of $Om(z)$ is a constant at any redshift, dark energy is exactly in the form of a cosmological constant, whereas the evolving $Om(z)$ corresponds to other dynamical dark energy models. On the other hand, the slope of $Om(z)$ could distinguish two different types of dark energy models, i.e., a positive slope indicates a phase of phantom ($w < -1$) while a negative slope represents quintessence ($w > -1$) (Sahni et al. 2008). Based on the above results, many previous works have performed consistency tests of the Λ CDM model, by using reconstructed $Om(z)$ with the combination of Gaussian processes (GP) and observations including SN Ia and Hubble parameter data (Seikel et al. 2012; Qi et al. 2016; Yahya et al. 2014). It was found that Λ CDM is compatible with the Union2.1 SN Ia data set and smaller samples of $H(z)$ measurements. More recently, Shafieloo et al. (2012) developed an improved version of the two-point diagnostic $Om h^2(z_1, z_2)$, which was also extensively used to test Λ CDM with different samples of $H(z)$ data (Sahni et al. 2014; Ding et al. 2015; Zheng et al. 2016). The general conclusion, which revealed the tension between $H(z)$ data and Λ CDM in the framework of *Planck* data (Planck Collaboration et al. 2014), implies that the Λ CDM model may not be the best scenario for our Universe, or dark energy does not exist in the form of the cosmological constant. Considering the significance of this result to understand the nature of dark energy, it is still important to confirm it with alternative techniques.

In this paper, we propose a parametrization of $Om(z)$ to provide a null test for Λ CDM, which successfully alleviates the disadvantages of the traditional $Om(z)$ associated with its strong dependence on the smoothing data methodology (Seikel et al. 2012; Qi et al. 2016; Yahya et al. 2014) and the statistical approach used (Sahni et al. 2014; Ding et al. 2015; Zheng et al. 2016). With this new parametrization of $Om(z)$, the purpose of this work is to show how the combination of the most

recent and significantly improved observations regarding the $H(z)$ and SNe Ia can be implemented to probe the consistency or reveal tension between the Λ CDM model and observations. This paper is organized as follows: In Section 2 we briefly introduce the $Om(z)$ and its newly-proposed parametrization. In Section 3, we use the latest $H(z)$ data to constrain the $Om(z)$ parameters and compare with the results obtained from *Planck*, WMAP9 (Hinshaw et al. 2013) and a local determination of H_0 from Riess et al. (2016). Consistency test of Λ CDM with the Joint Light-curve Analysis (JLA) SN Ia sample is also provided in Section 4. Finally, the conclusions are summarized in Section 5.

2 METHODOLOGY AND DATA

Considering flat Friedmann-Lemaître-Robertson-Walker spacetime, the general Friedmann equation for a Universe filled with a perfect fluid with an EoS $w(z)$ (in addition to pressureless matter and now dynamically negligible radiation) can be written as

$$\begin{aligned} E^2(z) &\equiv \frac{H^2(z)}{H_0^2} \\ &= \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0}) \\ &\quad \times \exp\left(3 \int_0^z \frac{1+w(z')}{1+z'} dz'\right), \end{aligned} \quad (1)$$

where Ω_{m0} is the present matter density of the Universe. Inspired by the form of this equation in the Λ CDM case, the $Om(z)$ diagnostic has been defined as (Sahni et al. 2008)

$$Om(z) \equiv \frac{E^2(z) - 1}{(1+z)^3 - 1}. \quad (2)$$

It is obvious that in the flat Λ CDM model, the $Om(z)$ evaluated at any redshift is always equal to the present mass density parameter Ω_{m0} .

Therefore, from observations of the expansion rates at different redshifts, we would be able to differentiate between Λ CDM and other dark energy models including evolving dark energy. For instance, for the simplest phenomenology of dark energy with constant EoS parameter $w = \text{const}$, a positive slope of $Om(z)$ signifies a phase of phantom ($w < -1$) and a negative slope represents the quintessence model ($w > -1$) (Sahni et al. 2008), which is shown in Figure 1. Motivated by the physical indication of $Om(z)$ slope and the well-known Chevalier-Polarski-Linder (CPL) model concerning reconstruction of evolving dark energy EoS, we propose the following

theoretical parametrization for $Om(z)$

$$Om(z) = \alpha \left(\frac{z}{1+z} \right)^n, \quad (3)$$

where α and n are the two constant parameters. From the above expression, it is straightforward to show that Λ CDM is fully recovered when $\alpha = \Omega_{m0}$ and $n = 0$. Moreover, from a simple comparison illustrated in Figure 1, one may easily find a positive slope $n > 0$ indicates a phase of phantom, while a negative slope represents quintessence-like models. Compared with the direct study on the EoS of dark energy in previous works (Cao & Zhu 2014), the introduction of a new parameter n provides a new cosmological-model-independent method to differentiate a wider range of cosmological solutions with effective EoS, which focus on gravitational modifications (i.e., $f(R)$ and $f(T)$ gravity) to account for cosmic acceleration without the inclusion of exotic dark energy (Chiba 2003; Wu & Yu 2011; Qi et al. 2017). We remark that one disadvantage of this $Om(z)$ parameterization is that it would be divergent at $z = 0$ when $n < 0$. However, as is shown in Figure 1, the $Om(z)$ reconstructed by $H(z)$ data (see Table 1) with GP, which is consistent with this parametrization within the 1σ confidence level (CL), exhibits a similar divergence feature at $z \sim 0$. Another disadvantage of this parameterization in this analysis lies in the strong assumption that the slope parameter n is a constant, which only proposes a special candidate to test the possible crossing of the cosmological constant boundary with different value of n . In order to make a comparison with other cosmological models including quintom cosmology (Cai et al. 2010) and other modified gravity models (Chiba 2003; Wu & Yu 2011; Qi et al. 2017), a possible solution is to generalize the slope parameter n as a function of redshift z , which will be considered in our future work concentrating on more cosmological applications. Now, the dimensionless Hubble parameter can be rewritten as

$$\begin{aligned} E^2(z) &= Om(z) [(1+z)^3 - 1] + 1 \\ &= \alpha \left(\frac{z}{1+z} \right)^n [(1+z)^3 - 1] + 1, \end{aligned} \quad (4)$$

and further used to estimate the values of α and n from various observational data by minimizing the respective χ^2 -function. It is noteworthy that we do not aim to pinpoint the right dark energy candidate among many competing models, but our goal is to propose an effective and sensitive probe for testing the validity of the concordance Λ CDM model. One possible controversy here is whether

$E(z = 0) = 1$ is still valid for the case of $n < 0$, due to the divergence of the $Om(z)$ parametrization proposed above. In fact, because the term $[(1+z)^3 - 1]$ in Equation (4) approaches zero at $z = 0$, the convergent result of $E(z = 0) = 1$ in this case will be naturally recovered, which can be clearly seen from the enlarged subplot in Figure 2. More importantly, the $Om(z)$ parametrization with different slope parameters also agrees very well with the evolution of $E(z)$ reconstructed by $H(z)$ data with GP.

In this paper, we use the latest $H(z)$ data set including 41 data points to place a constraint on the $Om(z)$ parametrization proposed above. In general, measurement of $H(z)$ could be obtained by two different techniques: galaxy differential age, also known as cosmic chronometer (hereafter CC $H(z)$) and radial BAO size method (hereafter BAO $H(z)$) (Zhang et al. 2010). The latest data set, 41 $H(z)$ data including 31 CC $H(z)$ data and 10 BAO $H(z)$ data, is compiled in Table 1. Moreover, the Hubble function $H(z)$ should be normalized to the dimensionless Hubble parameter $E(z) = H(z)/H_0$, whose uncertainty could be obtained through

$$\sigma_E^2 = \frac{\sigma_H^2}{H_0^2} + \frac{H^2}{H_0^4} \sigma_{H_0}^2, \quad (5)$$

where σ_H and σ_{H_0} are the uncertainty of $H(z)$ and H_0 , respectively. In this work estimate the parameters by minimizing the χ^2 -function defined as

$$\chi_H^2(z, \mathbf{p}) = \sum_{i=1}^{41} \frac{[E_{\text{th}}(z_i, \mathbf{p}) - E_{\text{obs}}(z_i)]^2}{\sigma_E(z_i)^2}, \quad (6)$$

where \mathbf{p} denotes the $Om(z)$ parameters, and E_{th} and E_{obs} respectively stand for the theoretical and observed values of the dimensionless Hubble parameter.

3 RESULTS AND DISCUSSION

We remark here that, as a benchmark for the whole $H(z)$ data set, the influence of the Hubble constant value on the test of the $Om(z)$ parameter should be taken into account. Therefore, three recent measurements of H_0 , $H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with 1.8% uncertainty (Planck Collaboration et al. 2014), $H_0 = 70.0 \pm 2.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with 3.1% uncertainty (Hinshaw et al. 2013) and $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with 2.4% uncertainty (Riess et al. 2016), are respectively used in our analysis. The best-fit parameters (with 1σ uncertainties) for these three priors are presented in Table 2.

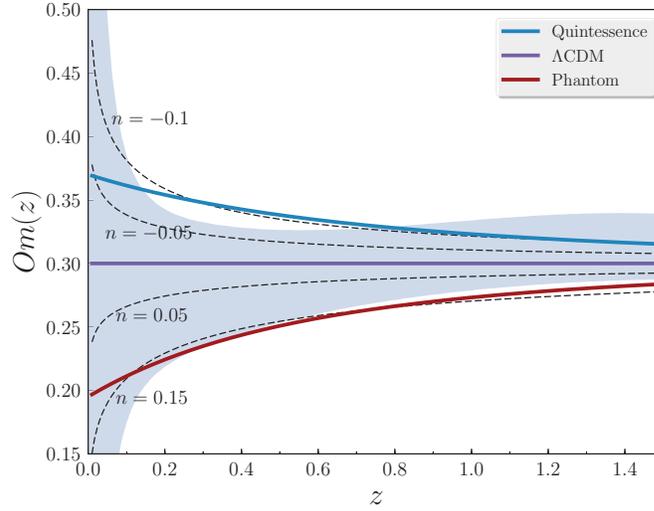


Fig. 1 The evolution of $Om(z)$ versus redshift z (black dashed lines) for the parametrization of $Om(z)$ with $n = -0.1, -0.05, 0.05, 0.15$. Three different cosmologies (Λ CDM, quintessence and phantom) denoted by solid lines are also added for comparison. The light blue shadowed area represents the 1σ confidence region of $Om(z)$ reconstructed by GP applied to $H(z)$ data.

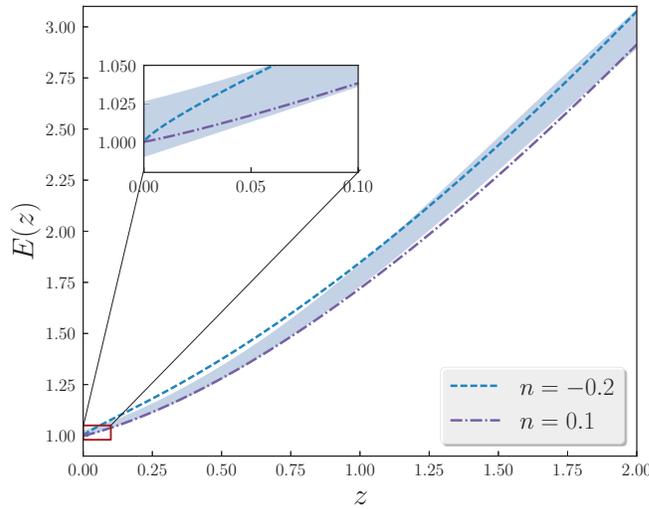


Fig. 2 The evolution of $E(z)$ versus the redshift z for the parametrization of $Om(z)$ with $n = -0.2, 0.1$. The light blue shadowed area represents the 1σ confidence region of $E(z)$ reconstructed by GP applied to $H(z)$ data.

3.1 Comparison with Planck 2013 Results

To start with, we determine the best-fit $Om(z)$ parameters by applying the Markov Chain Monte Carlo method to find the maximum likelihood based on the χ^2 -function. Then we compare the results with cosmological parameters (Planck Collaboration et al. 2014) obtained by Planck 2013. Adopting the Hubble constant prior $H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ in the $H(z)$ data, we obtain the best-fit value of the parameters $\alpha = 0.268^{+0.02}_{-0.02}$ and $n = -0.172^{+0.12}_{-0.114}$ at the 68.3% CL. The value of

slope parameter n is obviously smaller than zero at the 68% CL, which implies that quintessence may be a good candidate for dark energy as suggested by the $Om(z)$ parametrization. The marginalized 2D confidence contours of $\alpha - n$ are shown in Figure 3, in which the Λ CDM model ($n = 0.0$ and $\Omega_{m0} = 0.315$) characterized by Planck 2013 data is also added for comparison. The deviation from Λ CDM at the 2σ confidence region strongly indicates tension between the current $H(z)$ data and Λ CDM, which confirms the conclusion obtained

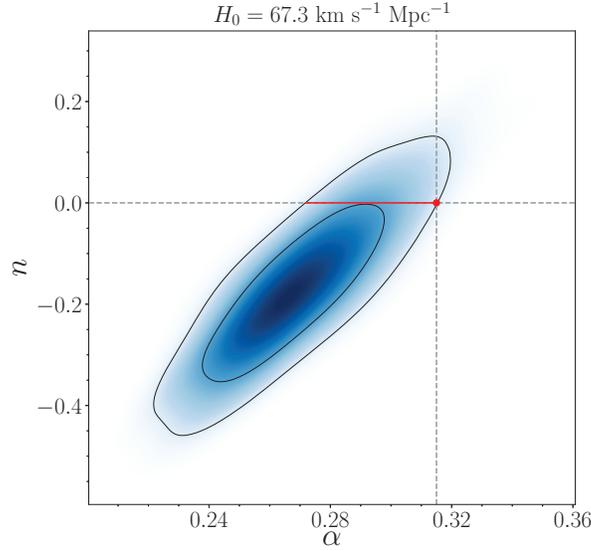


Fig. 3 The 68.3% and 95.4% confidence regions in the $\alpha - n$ parameter space for the $Om(z)$ parameterization constrained from $H(z)$ data (with the prior of $H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from *Planck*). The *red point* represents the Λ CDM model ($n = 0.0$, $\Omega_{m0} = 0.315$).

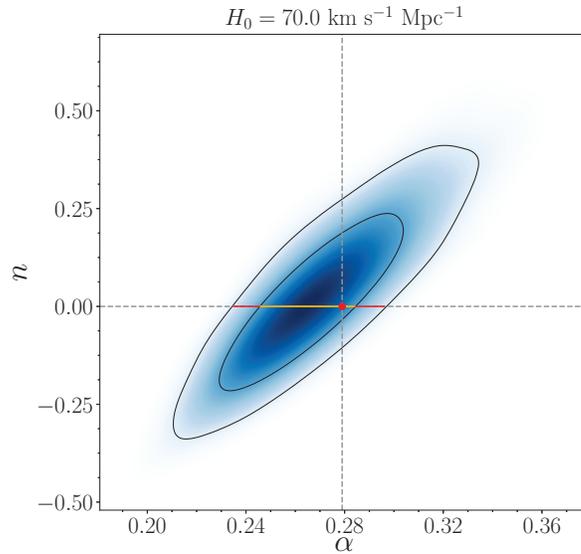


Fig. 4 The 68.3% and 95.4% confidence regions in the $\alpha - n$ parameter space for the $Om(z)$ parameterization of $Om(z)$ constrained from $H(z)$ data (with the prior of $H_0 = 70.0 \pm 2.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from WMAP9). The *red point* represents Λ CDM ($n = 0.0$, $\Omega_{m0} = 0.279$).

in previous works (Sahni et al. 2014; Ding et al. 2015; Zheng et al. 2016).

3.2 Comparison with WMAP9 Results

In the second case, we adopt the prior of $H_0 = 70.0 \pm 2.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from WMAP9 results (Hinshaw et al. 2013) to constrain the parametrization of $Om(z)$. By

minimizing the χ^2 , the parameter n implied by our statistical analysis gives $n = -0.021^{+0.151}_{-0.142}$, which indicates that there is no deviation from the Λ CDM scenario. Moreover, the best fit obtained for the dark energy parameter is $\alpha = 0.268^{+0.026}_{-0.024}$ (68.3% CL), which is in excellent agreement with the matter density $\Omega_{m0} = 0.279$ given by WMAP9. As can be seen from Figure 4, the discrepancy between $H(z)$ data and Λ CDM deter-

Table 1 The latest $H(z)$ measurements including 31 data points from the CC $H(z)$ method (I) and 10 data points from the radial BAO size method (II).

z	$H(z)$ (km s ⁻¹ Mpc ⁻¹)	Method	Reference
0.09	69 ± 12	I	Jimenez et al. (2003)
0.17	83 ± 8	I	
0.27	77 ± 14	I	
0.4	95 ± 17	I	
0.9	117 ± 23	I	Simon et al. (2005)
1.3	168 ± 17	I	
1.43	177 ± 18	I	
1.53	140 ± 14	I	
1.75	202 ± 40	I	
0.48	97 ± 62	I	Stern et al. (2010)
0.88	90 ± 40	I	
0.35	82.1 ± 4.9	I	Chuang & Wang (2012)
0.179	75 ± 4	I	
0.199	75 ± 5	I	
0.352	83 ± 14	I	
0.593	104 ± 13	I	Moresco et al. (2012)
0.68	92 ± 8	I	
0.781	105 ± 12	I	
0.875	125 ± 17	I	
1.037	154 ± 20	I	
0.07	69 ± 19.6	I	
0.12	68.6 ± 26.2	I	Zhang et al. (2014)
0.2	72.9 ± 29.6	I	
0.28	88.8 ± 36.6	I	
1.363	160 ± 33.6	I	Moresco (2015)
1.965	186.5 ± 50.4	I	
0.3802	83 ± 13.5	I	
0.4004	77 ± 10.2	I	
0.4247	87.1 ± 11.2	I	Moresco et al. (2016)
0.4497	92.8 ± 12.9	I	
0.4783	80.9 ± 9	I	
0.24	79.69 ± 2.65	II	Gaztañaga et al. (2009)
0.43	86.45 ± 3.68	II	
0.44	82.6 ± 7.8	II	
0.6	87.9 ± 6.1	II	Blake et al. (2012)
0.73	97.3 ± 7	II	
0.35	84.4 ± 7	II	Xu et al. (2013)
0.57	92.4 ± 4.5	II	Samushia et al. (2013)
2.3	224 ± 8	II	Busca et al. (2013)
2.34	222 ± 7	II	Delubac et al. (2015)
2.36	226 ± 8	II	Font-Ribera et al. (2014)

Table 2 The best-fit values of the $Om(z)$ parameters derived from the $H(z)$ data with different H_0 priors. The corresponding values for Λ CDM are also presented for comparison.

H_0 priors	α	n	Λ CDM (α, n)
Planck 2013	$\alpha = 0.268^{+0.02}_{-0.02}$	$n = -0.172^{+0.12}_{-0.114}$	(0.315, 0)
WMAP9	$\alpha = 0.268^{+0.026}_{-0.024}$	$n = -0.021^{+0.151}_{-0.142}$	(0.279, 0)
Riess (2016)	$\alpha = 0.196^{+0.034}_{-0.032}$	$n = 0.162^{+0.163}_{-0.147}$	(-, 0)

mined by Planck 2013 data has gone, i.e., observations of the Hubble parameter still support the existence of a cosmological constant in the framework of this $Om(z)$ parametrization.

Obviously, the same $H(z)$ data set corresponding to different values of H_0 and Ω_{m0} from Planck 2013 and WMAP9 gives very different conclusions. Concerning the previous works (Sahni et al. 2014; Ding et al. 2015; Zheng et al. 2016), their estimates of $Om h^2$ are compared with $\Omega_{m0} h^2$, the combination of Ω_{m0} and H_0 ($h = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$) determined by *Planck* observations. Therefore, it is hard to tell the source of tension between $H(z)$ data and Λ CDM. However, in our method, the impact of H_0 and Ω_{m0} on the final conclusion could be separately discussed. From the above analysis, we may conclude that the value of H_0 is the most influential factor in performing a consistency test of Λ CDM with $H(z)$ data. For instance, in the case of $H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Planck 2013 data, Λ CDM with any value of Ω_{m0} is ruled out at the 68.3% CL. However, with the prior of $H_0 = 70.0 \pm 2.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from WMAP9, the $H(z)$ data exhibit very good consistency with the concordance cosmological constant model.

3.3 Comparison with Riess (2016) Results

Considering the significant influence of H_0 , in the final case a local determination of $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with 2.4% uncertainty from Riess et al. (2016) can be taken to perform a consistency test. We show the contours constrained from the statistical analysis in Figure 5 and the best fit is $\alpha = 0.196^{+0.034}_{-0.032}$ and $n = 0.162^{+0.163}_{-0.147}$ (68.3% CL). Different from the first case based on *Planck* measurements, a positive value for the slope parameter, which corresponds to a phantom cosmology, is favored in the framework of this $Om(z)$ parametrization. Moreover, because this measurement of H_0 is a local determination obtained in a cosmology-independent method, we may comment on the value of matter density in the framework of Λ CDM, i.e., at the 95.4% CL the range of Ω_{m0} is restricted to (0.2118, 0.2504) with the current $H(z)$ data, which is generally lower than the value given by most other types of cosmological observations. Therefore, the measurement of H_0 from Riess et al. (2016) will significantly affect our understanding of Λ CDM and thus the components in the Universe.

4 CONSTRAINTS FROM JLA SN IA SAMPLE

As mentioned above, the parametrization of $Om(z)$ proposed in this paper makes it possible to perform a consistency test for Λ CDM with other astronomical observations. More importantly, the previous literature has examined the role of $H(z)$ and SN Ia data in cosmological constraints, and found that they could play a similar role in constraining the cosmological parameters (Cao et al. 2011a; Cao & Liang 2013; Cao et al. 2015b). Therefore, we turn to the JLA sample of 740 SNe Ia data (Betoule et al. 2014). For the JLA data, the observed distance modulus is given by

$$\mu_{\text{SN}} = m_B^* + \alpha \cdot x_1 - \beta \cdot c - M_B, \quad (7)$$

where m_B^* is the rest frame B -band peak magnitude, and x_1 and c are the time stretch of light curve and the supernova color at maximum brightness respectively, which are the three parameters needed for a light curve fitted by SALT2 (Guy et al. 2007). Moreover, the parameter M_B describes the absolute B -band magnitude, whose value is assumed to be dependent on the host stellar mass (M_{stellar}) by a simple step function (Betoule et al. 2014)

$$M_B = \begin{cases} M_B^1 & \text{for } M_{\text{stellar}} < 10^{10} M_{\odot}, \\ M_B^1 + \Delta_M & \text{otherwise.} \end{cases} \quad (8)$$

Therefore, there are four nuisance parameters (α , β , M_B^1 and Δ_M) to be fitted along with the $Om(z)$ parameters.

On the other hand, the theoretical distance modulus μ_{th} is expressed as $\mu_{\text{th}} \equiv 5 \log(D_L(z)/\text{Mpc}) + 25$, where $D_L(z)$ is the luminosity distance. Thus, χ^2 for the JLA sample is constructed as

$$\chi_{\text{JLA}}^2 = \Delta\mu^T \cdot \mathbf{Cov}^{-1} \cdot \Delta\mu, \quad (9)$$

where $\Delta\mu = \mu_{\text{SN}}(\alpha, \beta, M_B^1, \Delta_M; z) - \mu_{\text{th}}(z)$ and \mathbf{Cov} is the total covariance matrix defined as

$$\mathbf{Cov} = \mathbf{D}_{\text{stat}} + \mathbf{C}_{\text{stat}} + \mathbf{C}_{\text{sys}}. \quad (10)$$

Here \mathbf{D}_{stat} corresponds to the diagonal part of the statistical uncertainty, while \mathbf{C}_{stat} and \mathbf{C}_{sys} denote the statistical and systematic covariance matrices, respectively. The details of the covariance matrix \mathbf{Cov} including its construction can be found in Betoule et al. (2014). Considering the significance of the Hubble constant in testing Λ CDM, in this section we will treat H_0 as a free parameter in the χ^2 -minimization procedure. Thus there are four nuisance parameters plus three parameters (H_0 , α , n) referring to the parametrization of $Om(z)$ that we are interested in.

Table 3 The best-fit values (with 1σ uncertainties) of the Hubble constant and $Om(z)$ parameters with different data combinations ($H(z)$, JLA and $H(z)$ +JLA).

	H_0	α	n
JLA	$67.63^{+4.06}_{-2.18}$	$0.292^{+0.097}_{-0.075}$	$-0.014^{+0.240}_{-0.250}$
$H(z)$	$67.799^{+5.67}_{-13.29}$	$0.276^{+0.070}_{-0.026}$	$-0.164^{+0.434}_{-0.545}$
$H(z)$ +JLA	$68.81^{+1.50}_{-1.49}$	$0.262^{+0.020}_{-0.018}$	$-0.096^{+0.105}_{-0.098}$

In order to break the strong degeneracy between parameters, we also perform a joint statistical analysis by using the JLA data and the Hubble parameter measurements to constrain the parametrization of $Om(z)$. The total χ^2 with the combined data set of JLA and $H(z)$ can be given by

$$\chi_{\text{tot}}^2 = \chi_{\text{H}}^2 + \chi_{\text{JLA}}^2. \quad (11)$$

The best-fit parameters (with 1σ uncertainties) for different data sets are presented in Table 3.

The marginalized 2D confidence contours of parameters (α and n , α and H_0 , and n and H_0) are shown in Figure 6. It is apparent that the principal axes of confidence regions obtained with $H(z)$ data and JLA data intersect, which implies that the joint analysis with $H(z)$ and JLA could effectively break the strong degeneracy between parameters and thus provide a more stringent constraint on the three parameters. On the one hand, although the best-fit $Om(z)$ slope parameter is slightly smaller than zero, which suggests that the current observational data tend to support a quintessence cosmology, the Λ CDM model ($n = 0$) is still included within the 1σ confidence region. On the other hand, the best-fit Hubble constant from the combination of $H(z)$ +JLA ($H_0 = 68.81^{+1.50}_{-1.49} \text{ km s}^{-1} \text{ Mpc}^{-1}$) is consistent with the results derived by both Planck 2013 and WMAP9, but is significantly different from the recent local measurement by Riess et al. (2016).

5 CONCLUSIONS AND DISCUSSIONS

An important issue in modern cosmology is whether the EoS of dark energy is a constant or varying with time. Based on an effective diagnostic $Om(z)$ and its improved versions $Om(z_1, z_2)$ and $Om h^2(z_1, z_2)$, many recent works have performed a null test of Λ CDM determined by *Planck* observations, which implies that the Λ CDM model may not be the best scenario for our Universe, or dark energy does not exist in the form of a cosmological constant. In this paper, we have proposed a parametrization of $Om(z)$ to investigate the validity of Λ CDM,

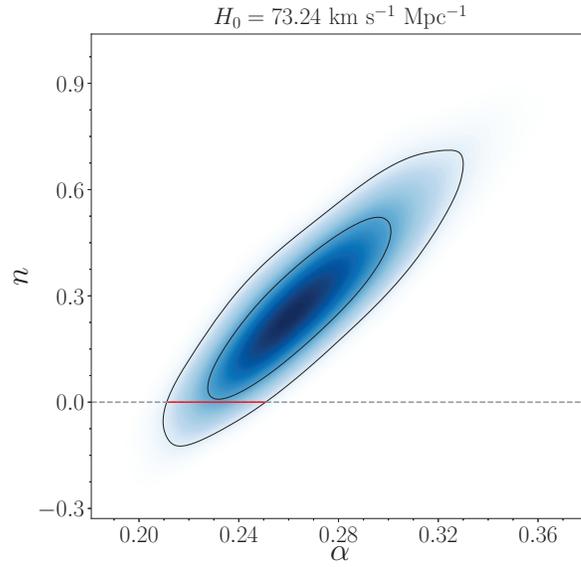


Fig. 5 The 68.3% and 95.4% confidence regions in $\alpha - n$ parameter space for the $Om(z)$ parameterization of $Om(z)$ constrained from $H(z)$ data (with the prior of $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Riess et al. (2016)). The horizontal black line delineates Λ CDM ($n = 0.0$).

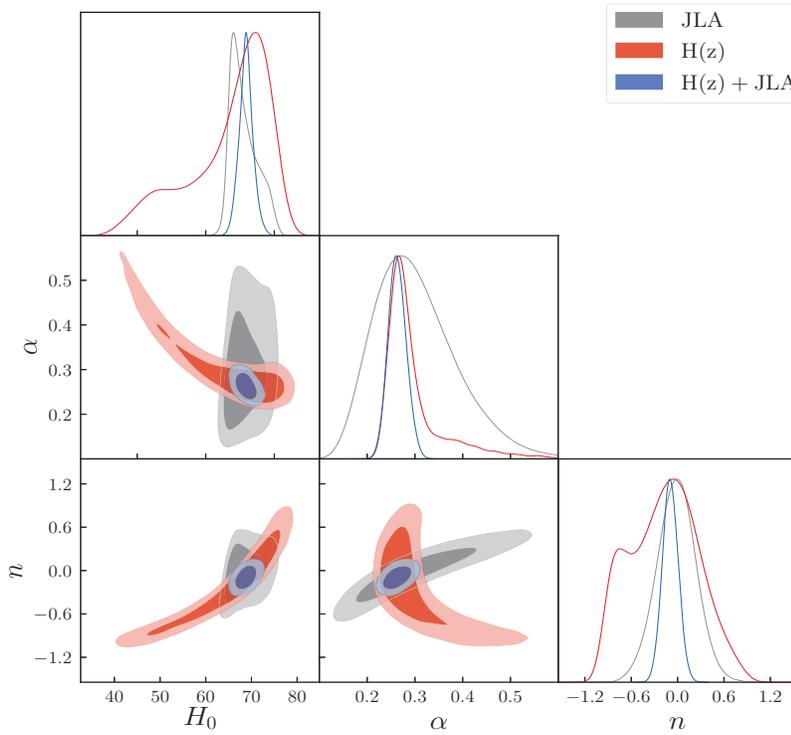


Fig. 6 The 68.3% and 95.4% confidence regions in the $H_0 - \alpha - n$ parameter space derived from JLA, $H(z)$ and $H(z)$ +JLA.

which successfully clarifies the impact of H_0 and Ω_{m0} on the final conclusion. With three different priors of the Hubble constant H_0 , the latest $H(z)$ data are used to set a constraint on the $Om(z)$ parameters of interest. Our re-

sults showed that the value of H_0 plays a very important role in the consistency test of Λ CDM. Here we summarize our main conclusions in more detail:

- Adopting the Hubble constant prior $H_0 = 67.3 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Planck Collaboration et al. 2014) for the $H(z)$ data, we find the value of the slope parameter n to be smaller than zero at the 68% CL, which implies that quintessence may be a good candidate for dark energy according to this $Om(z)$ parametrization. The deviation from Λ CDM at the 2σ confidence region strongly indicates tension between the current $H(z)$ data and Λ CDM, which confirms the conclusion obtained in the previous works.
- With the prior of $H_0 = 70.0 \pm 2.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from WMAP9 results, the discrepancy between $H(z)$ data and Λ CDM disappeared, i.e., the data analyzed in the framework of this $Om(z)$ parametrization still support the cosmological constant scenario.
- In the third case with the local determination of $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Riess et al. (2016), a positive value for the slope parameter, which corresponds to a phantom cosmology, is strongly favored by the current $H(z)$ data. Moreover, at the 95.4% CL the range of matter density is restricted to $\Omega_{m0} = (0.2118, 0.2504)$, which is generally lower than the value given by most other types of cosmological observations.

Moreover, the parametrization of $Om(z)$ makes it possible to perform a consistency test of Λ CDM with other astronomical observations. We studied the constraining power of the JLA sample that has 740 SNe Ia (Betoule et al. 2014) and its combination with the Hubble parameter measurements on the parametrization of $Om(z)$. Here we summarize our main conclusions in more detail:

- Although the best-fit $Om(z)$ slope parameter is slightly smaller than zero, which suggests that the current observational data tend to support a quintessence cosmology, the Λ CDM model ($n = 0$) is still included within the 1σ confidence region.
- The best-fit Hubble constant from the combination of $H(z)$ +JLA ($H_0 = 68.81^{+1.50}_{-1.49} \text{ km s}^{-1} \text{ Mpc}^{-1}$) is very consistent with the results derived both by Planck 2013 and WMAP9, but is significantly different from the recent local measurement by Riess et al. (2016).

As a final remark, the parametrization of $Om(z)$ proposed in this paper has opened a robust window for testing the validity of the concordance Λ CDM cosmology and suggesting other possible dynamical dark energy models. However, more precise model selection

still remains a difficult task with current accuracy of the data and the important role played by the Hubble constant. We hope that future data related to strong gravitational lensing observations (Cao et al. 2011b; Cao & Zhu 2012; Cao et al. 2012b,a, 2015a), high-redshift SNe Ia from SDSS-II and the Supernova Legacy Survey (SNLS) collaboration (Betoule et al. 2014), ultra-compact structure in high-redshift radio quasars (Cao et al. 2017) and weak lensing surveys combined with CMB measurements (Planck Collaboration et al. 2016) will lead to substantial progress in this respect.

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