

## Estimating the mirror seeing for a large optical telescope with a numerical method

En-Peng Zhang<sup>1,2</sup>, Xiang-Qun Cui<sup>3</sup>, Guo-Ping Li<sup>3</sup>, Yong Zhang<sup>3</sup>, Jian-Rong Shi<sup>1,2</sup> and Yong-Heng Zhao<sup>1,2</sup>

<sup>1</sup> National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100101, China; [zhangep@bao.ac.cn](mailto:zhangep@bao.ac.cn)

<sup>2</sup> Key Laboratory of Optical Astronomy, Chinese Academy of Sciences, Beijing 100101, China

<sup>3</sup> Nanjing Institute of Astronomical Optics and Technology, National Astronomical Observatories, Chinese Academy of Sciences, Nanjing 210042, China

Received 2017 December 12; accepted 2018 February 22

**Abstract** It is widely accepted that mirror seeing is caused by turbulent fluctuations in the index of air refraction in the vicinity of a telescope mirror. Computational Fluid Dynamics (CFD) is a useful tool to evaluate the effects of mirror seeing. In this paper, we present a numerical method to estimate the mirror seeing for a large optical telescope ( $\sim 4$  m) in cases of natural convection with the ANSYS ICEPAK software. We get the FWHM of the image for different inclination angles ( $i$ ) of the mirror and different temperature differences ( $\Delta T$ ) between the mirror and ambient air. Our results show that the mirror seeing depends very weakly on  $i$ , which agrees with observational data from the Canada-France-Hawaii Telescope. The numerical model can be used to estimate mirror seeing in the case of natural convection although with some limitations. We can determine  $\Delta T$  for thermal control of the primary mirror according to the simulation, empirical data and site seeing.

**Key words:** astronomical instrumentation — methods: numerical — telescopes

### 1 INTRODUCTION

The primary mirror of a large optical telescope (LOT) has correspondingly large thermal inertia which prevents it from following the temperature variation of ambient air. As a consequence, the temperature of the mirror may often be above that of ambient air during the night (Lowne 1979; Iye et al. 1991). Such a temperature excess (with a typical value of  $\sim 2$  K) can lead to effects from so-called mirror seeing as it causes turbulent fluctuations in the index of refraction of air in the vicinity of the mirror. The image quality (IQ) of a large ground-based optical telescope can be seriously deteriorated by mirror seeing even though the site seeing may be very good. This issue has been the focus of special attention for a long time. However, it is very difficult to measure the mirror seeing of a telescope directly (Racine et al. 1991). Laboratory studies or observational measurements of mirror seeing effects have been done by several researchers. Lowne

(1979) performed the first quantitative measurement of mirror seeing with a Schlieren camera on a 254-mm spherical mirror including the effects of inclination and forced air blowing (Lowne 1979). Iye et al. (1991) conducted a mirror seeing test on a 62-cm telescope with and without forced air convection using a Shack-Hartmann wavefront analyzer. In 1991, Racine et al. studied the dome and mirror seeing of the Canada-France-Hawaii Telescope (CFHT, 3.6-m). Zago (1997) summarized the available data from different authors and methods for estimating mirror seeing and provided empirical relations. For free or natural convection, he derived the expression

$$\theta = 0.38\Delta T^{1.2}, \quad (1)$$

with an uncertainty of 25%. In Equation (1),  $\theta$  is long exposure angular image size (full width at half maximum (FWHM)) in arcsec, and  $\Delta T$  is the temperature difference between the surface of the mirror and the ambient air in K. Equation (1) shows that  $\theta$  is independent

of the mirror size and inclination for natural convection. However, for a 254-mm mirror, it is found that  $\theta$  changes with the inclination angle of the mirror (Lowne 1979).

According to the above studies, the key point for reducing mirror seeing is to control the temperature difference. To make the best of a site with good seeing, thermal control has become more and more important for modern telescopes. The allowable range of  $\Delta T$  for some primary mirrors is very narrow. For example,  $\Delta T$  of the Gemini primary mirror is required to stay in the range  $-0.6 \text{ K} < \Delta T < 0.2 \text{ K}$  (Greenhalgh *et al.* 1994). For the Very Large Telescope (VLT), this range is  $-1 \text{ K} < \Delta T < 0.2 \text{ K}$  (Stanghellini *et al.* 1997). As the passive cooling method cannot realize such precise adjustments, active methods were invented to control the temperature of the mirror. Both Gemini and the VLT have cooling plates behind the primary mirror. The plate enables the mirror to track the average temperature of ambient air. Based on the idea that mirror seeing is only caused by the temperature excess of the front surface of a mirror, surface temperature control was introduced in the Gemini facility. By adjusting the current in the coating, the temperature of the front surface of the mirror can be changed rapidly to track the fluctuation of air temperature (Hansen *et al.* 1997). During the design of the Gemini facility, a numerical method was used to model the temperature distribution through the thickness of the primary mirror to ensure an allowable temperature gradient (Greenhalgh *et al.* 1994).

Numerical studies of mirror seeing have played an important role in telescope design. Computational Fluid Dynamics (CFD) has been applied to evaluate the effects of mirror seeing in addition to laboratory tests or observational measurements. Vogiatzis & Upton (2006) simulated mirror seeing for the Thirty Meter Telescope (TMT) for forced convection cases. Zhang *et al.* (2016) provided a preliminary simulation of mirror seeing for the Chinese Future Giant Telescope (CFGF, 30-m in diameter) with ANSYS ICEPAK software in the case of forced convection. Whether ANSYS ICEPAK can give reasonable simulation results for natural convection still needs testing.

When the mirror is warmer than the ambient air, a boundary layer of heated air will form when the mirror is horizontal. Air streams or bubbles can be seen rising from the boundary layer and image degradation happens until the wind blows them away (Lowne 1979). Evaluating the mirror seeing in the presence of natural convection is helpful for understanding the mechanism of

seeing generation and elimination. In this paper, we will estimate the mirror seeing for an LOT ( $\sim 4$ -m in diameter) with natural convection using ANSYS ICEPAK and test whether the mirror seeing depends on the inclination angle of the mirror. The diameter is chosen as 3.6-m in order to compare the simulation result with observational data from CFHT (Racine *et al.* 1991). We can give the required  $\Delta T$  for thermal control based on the simulation and empirical data when knowing the site seeing. Our method can also be used to give preliminary evaluation of the seeing for a segmented mirror such as the Active Aspherical Correcting Mirror (denoted as Ma) in the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) facility (Cui *et al.* 2012). This can be helpful for improving performance of the telescope. The rest of the paper is organized as follows. In Section 2, we show the theoretical background, model and simulation results. The summary and discussion are given in Section 3.

## 2 THEORETICAL AND NUMERICAL APPROACH

### 2.1 Theoretical Background

Zhang *et al.* (2016) published a detailed introduction on how to relate the image size to the quantities of fluid dynamics and turbulence theory. It can be summarized as follows.

We get the FWHM of the image by

$$\theta = 0.98\lambda/r_0, \quad (2)$$

where  $\lambda$  is the wavelength and  $r_0$  is the Fried parameter.  $r_0$  is given by

$$r_0 = 0.184\lambda^{6/5}(\cos \gamma)^{3/5} \left[ \int_0^z C_N^2(z) dz \right]^{-3/5}, \quad (3)$$

where  $\gamma$  is the zenith angle.

The refractive index structure function,  $C_N^2$ , is related to  $C_T^2$  (the temperature structure coefficient) as

$$C_N^2 = C_T^2 \left[ 77.6 \times 10^{-6} \left( 1 + 7.52 \times 10^{-3} \lambda^{-2} \right) \frac{P}{T^2} \right]^2, \quad (4)$$

where  $P$  is the average pressure and  $T$  is average temperature. Tatarskii (1961) related  $C_T^2$  to the dissipation rate of turbulent kinetic energy ( $\varepsilon$ ) and the temperature dissipation rate ( $\varepsilon_\theta$ ) in the inertial domain (Zago 1997)

$$C_T^2 = a^2 \varepsilon_\theta \varepsilon^{-1/3}, \quad (5)$$

where  $a^2 \sim 3$  and

$$\varepsilon_\theta = \kappa_t \left( \frac{\partial T}{\partial x_k} \right)^2, \quad (6)$$

$$\varepsilon = \frac{1}{2} \nu_t \left( \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right)^2. \quad (7)$$

When the flow is parallel to the mirror surface,  $\varepsilon_\theta$  can be reduced to a one-dimensional form

$$\varepsilon_\theta \approx \kappa_t \left( \frac{dT}{dz} \right)^2. \quad (8)$$

In the above expressions,  $\nu_t$  is the turbulent viscosity,  $\kappa_t$  is the equivalent coefficient of turbulent thermal diffusion and  $u_i$  are the components of average wind velocity ( $U$ ) ( $i = 1, 2, 3$ ).

With Equations (3) to (8), we establish the relation between  $\theta$  and CFD parameters. We can get  $\nu_t$  with the output variables  $k$  and  $\varepsilon$  in the simulation results:  $\nu_t = C_\mu \frac{k^2}{\varepsilon}$  where  $C_\mu (= 0.09)$  is an empirical constant in the software. The turbulent thermal diffusion,  $\kappa_t$ , is given by  $Pr_t = \nu_t / \kappa_t$ .  $Pr_t$  is the turbulent Prandtl number which has been set to 0.85 in the software.

## 2.2 The Mirror Model

In this paper, the model of the primary mirror is a cylindrical plate with diameter of  $D = 3.6$  m and thickness of  $H = 0.04$  m. To test the influence of the mirror thickness on mirror seeing, we simulate the case of  $H = 0.5$  m for  $i = 45^\circ$  and  $\Delta T = 2$  K.

Figure 1 shows a representative plot of the model. The ambient air temperature is set to  $T_0 = 273$  K and atmospheric pressure is  $P_0 = 1000$  mb. The gravitational acceleration is set to  $g \sim 9.8$  m s<sup>-2</sup>. The wavelength in Equation (2) is  $\lambda = 0.5$   $\mu$ m and the mirror temperature is  $T_m = 275$  K for  $\Delta T = 2$  K. The residuals of the solution for continuum and energy equations are set to  $10^{-3}$  and  $10^{-7}$ , respectively.

As  $\Delta T (= T_m - T_0)$  is small, we take the reference temperature as  $T_{\text{ref}} = T_0$  and reference ambient pressure  $P_{\text{ref}} = P_0$  for simplification. We also neglect radiation heat transfer due to small  $\Delta T$  and emissivity. From Equation (4), we get

$$C_N^2 = C_T^2 \left[ 7.8 \times 10^{-5} \frac{P_{\text{ref}}}{T_{\text{ref}}^2} \right]^2. \quad (9)$$

We rewrite Equation (2) as

$$\theta = 5.25 \times 2.06 \times 10^5 \lambda^{-1/5} \left[ \int_0^z C_N^2(z) dz \right]^{3/5}, \quad (10)$$

where  $\cos \gamma = 1$ .

## 2.3 Simulation Results and Comparison

Figure 2 shows a representative plot of  $C_N^2$  as a function of perpendicular distance from the plate for  $i = 45^\circ$  and  $\Delta T = 2$  K. The line of sight is perpendicular to the mirror surface. From the distribution of  $C_N^2$ , we can see that most of the mirror seeing occurs in a thin region above the upper surface of the primary mirror.

Racine et al. (1991) gave the relation between  $\theta$  and inclination angle ( $i$ ) of the primary mirror as

$$\theta^{5/3} \propto \Delta T^2 \left[ 1 - \exp \left( \frac{-D \cot i}{l_0} \right) \right], \quad (11)$$

where  $D$  is the mirror diameter and  $l_0$  is size of the convection zone ( $l_0 \sim 50$  cm, Racine et al. 1991; Iye et al. 1991). When  $D \gg l_0$ ,  $\theta$  depends very weakly on  $i$ . To examine the effect of inclination on mirror seeing, we simulate the cases of different  $i$  with the same  $\Delta T$ s,  $T_0$  and  $P_0$ .

Table 1 lists the results. According to Equation (1), the mirror seeing is  $\sim 0.87''$  with a spread of  $0.22''$  for  $\Delta T = 2.0$  K. We can verify that our results agree with Equation (1). Table 1 shows that  $\theta$  nearly does not change with  $i$  for  $D = 3.6$  m. For small mirrors, Racine et al. (1991) suggested an effective diameter ( $D_e$ ) should be applied for Equation (11). Edge effects on the turbulence can reduce  $D_e$  more seriously for small mirrors than large ones. It can be used to explain the larger effect of inclination on mirror seeing found in experiments with a 254-mm mirror (Lowne 1979; Racine et al. 1991). For a much larger mirror ( $D \gg 3.6$  m), Equation (11) should be tested in future studies.

**Table 1** Simulation Results for Different  $i$  with  $\Delta T = 2$  K

Inclination angle ( $i$ )	0°	30°	45°	60°	80°
Mirror seeing ( $\theta$ )	0.82''	0.79''	0.77''	0.76''	0.75''

For  $i < 30^\circ$ , we find that the output of  $T$  and  $U$  keeps fluctuating even if the simulation time is long enough. This may be caused by turbulence in the flow as streams or bubbles can be seen rising intermittently. Thermometers near the mirror show severe fluctuations when the mirror is horizontal (Lowne 1979). The mechanism causing the instability is not very clear. Wang & Zhang (1991) found that random instability of the air flow above a hot plate may be related to the size of the plate and the Rayleigh number. We roughly give the average value of  $\theta \sim 0.82''$  for  $i = 0^\circ$ . Further study should be made for this case in the future.

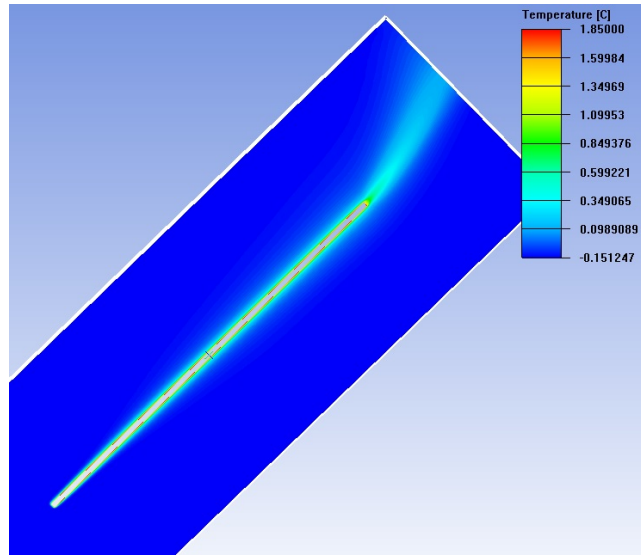


Fig. 1 The simplified CFD model and an example of temperature distribution for the case of inclination angle  $i = 45^\circ$ .

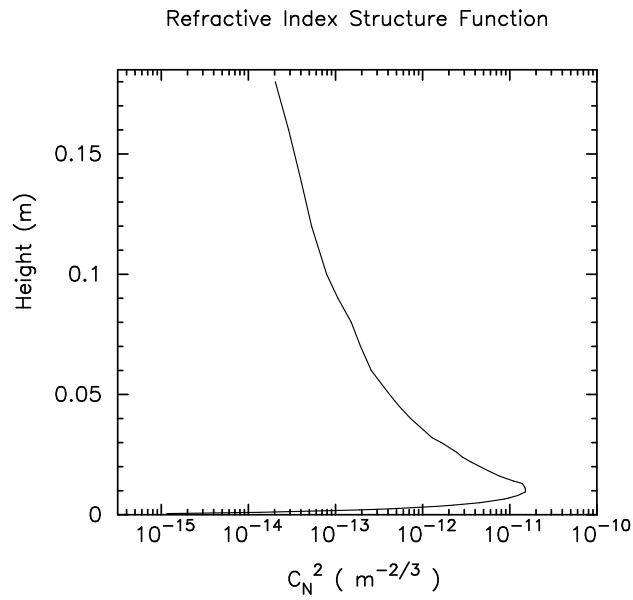
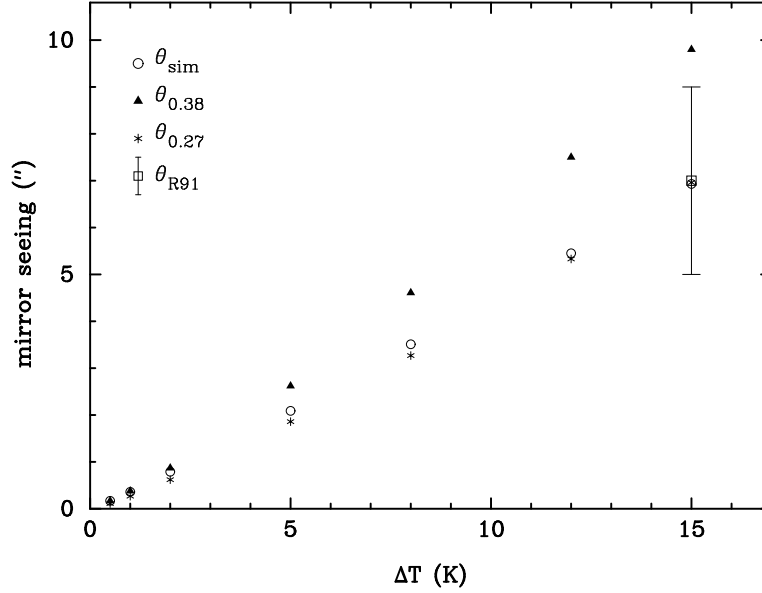


Fig. 2  $C_N^2$  distribution above the center of the mirror surface for  $i = 45^\circ$  and  $\Delta T = 2$  K.

Table 2 Comparison between Simulation Results and Empirical Relations for Different  $\Delta T$  with  $i = 45^\circ$

$\Delta T$	0.5 K	1.0 K	2.0 K	5.0 K	8.0 K	12.0 K	15.0 K
$\theta_{sim}$	0.17''	0.36''	0.79''	2.09''	3.51''	5.45''	6.93''
$\theta_{0.38}$	0.17''	0.38''	0.87''	2.62''	4.61''	7.50''	9.80''
$\theta_{0.27}$	0.11''	0.27''	0.62''	1.86''	3.27''	5.33''	6.96''

Notes:  $\theta_{sim}$  is the simulation results.  $\theta_{0.38}$  and  $\theta_{0.27}$  refer to seeing obtained with  $\alpha_m = 0.38$  and  $\alpha_m = 0.27$ , respectively.



**Fig. 3** Mirror seeing estimated for different  $\Delta T$  with  $i = 45^\circ$ .  $\theta_{\text{sim}}$  is the simulation value.  $\theta_{0.38}$  and  $\theta_{0.27}$  refer to seeing obtained by  $\alpha_m = 0.38$  and  $\alpha_m = 0.27$ , respectively. R91 is the data point taken from Racine et al. (1991).

We also simulate the case of  $H = 0.5$  m for  $i = 45^\circ$  and  $\Delta T = 2$  K to test the influence of mirror thickness. We do not change other conditions as set in the case  $H = 0.04$  m and get  $\theta \sim 0.77''$ . Changing thickness from  $H = 0.04$  m to  $H = 0.5$  m nearly has no influence on mirror seeing. Our result agrees with the idea that only the temperature of the front surface affects mirror seeing.

The correlation between mirror seeing and temperature difference can be expressed as  $\theta = \alpha_m \Delta T^{1.2}$ , and  $\alpha_m$  (in  $''/\text{K}^{1.2}$ ) is called the mirror seeing coefficient. Different experiments yielded different values of  $\alpha_m$ . For example, Racine et al. (1991) obtained  $\alpha_m \sim 0.40$  for  $0 \text{ K} < \Delta T < 3 \text{ K}$  and  $\alpha_m \sim 0.27$  for  $\Delta T \sim 15 \text{ K}$ . Lowne’s test (1979) suggested  $\alpha_m \sim 0.21$  where  $\Delta T$  falls in the interval  $-2 \text{ K} < \Delta T < 8 \text{ K}$ . Generally,  $\Delta T > 5 \text{ K}$  may be found mainly in laboratory study but seldom in astronomical observation. However, it is helpful for understanding the mechanism of mirror seeing as too narrow a range of  $\Delta T$  values may influence the fitting results of  $\alpha_m$ .

To test whether  $\alpha_m$  depends on the temperature difference, we simulate cases with different  $\Delta T$  under the same  $i$ .

Table 2 and Figure 3 display the departure of  $\alpha_m$  between small  $\Delta T$ s and large ones. Our results favor smaller  $\alpha_m$ , especially for larger  $\Delta T$ s. Table 2 also means that an effective way to reduce the effect of mirror seeing is through controlling the mirror temperature.

For the natural convection case of  $\Delta T < 0 \text{ K}$ , Zago (1997) regards it as “quasi-static” image blurring rather than “turbulent” seeing. The particular motion of the image found in the case of  $\Delta T > 0 \text{ K}$  is absent as the light ray is steadily refracted. It can also be effectively reduced by ventilation. The turbulent model in this paper may not be available for this situation. Iye et al. (1991) found that, when  $\Delta T < -2 \text{ K}$ , wind flushing can degrade the Strehl number. Optical aberration produced by the primary mirror and the camera is another factor influencing the IQ. Racine et al. (1991) showed that such optical spread from CFHT is  $\sim 0.38''$ , which is less than the median site seeing of  $\sim 0.43''$ . So, the mirror seeing is not very important until  $\Delta T > 1.0 \text{ K}$  for the delivered IQ.

As astronomers always chase the best IQ or the smallest image size, requirements for the range of temperature control for the primary mirror mainly come from the limit imposed by site seeing. Laboratory studies by Iye et al. (1991) found a reduction in Strehl ratio present when  $\Delta T > 0.2 \text{ K}$ . Racine et al. (1991) suggested that  $\Delta T < 0.5 \text{ K}$  is enough as the best seeing at Mauna Kea is  $\sim 0.2''$ . Our simulation result is also  $\sim 0.2''$  for  $\Delta T = 0.5 \text{ K}$ . Combining the simulation and empirical data, we may give a range of  $\Delta T$  as  $-1 \text{ K} < \Delta T < 0.5 \text{ K}$  in the optical band for good site seeing ( $\sim 0.2''$ ) without mirror ventilation. Practical thermal control of the mirror could include ventilation using natural air flow or a fan. For example, the Gemini

error budget allows only 0.04'' additional contribution at 2.2  $\mu\text{m}$ . To realize such a specification, moderate wind ventilation is considered in addition to temperature control of the cooling plate and surface current (Greenhalgh *et al.* 1994).

### 3 SUMMARY AND DISCUSSION

Mirror seeing is one of the key factors affecting the IQ of a large ground-based optical telescope. It will be an important issue that should be addressed when we design and build an (extremely) large telescope (Su *et al.* 2000, 2016; Li & Yang 2004). CFD provides a useful tool for estimating the mirror seeing so as to define the requirement for controlling the temperature of the mirror.

In this paper, we present our numerical model to simulate mirror seeing effects for different inclination angles and temperature differences in cases of natural convection. We obtain the distribution of  $C_N^2$  above the mirror surface and the FWHM of the image. We find that mirror seeing is nearly independent of inclination angle for a large telescope. The simulation results agree with observational data from CFHT and the empirical relation. The thickness of the mirror nearly has no influence on the mirror seeing. Our simulation shows a mirror seeing coefficient ( $\alpha_m$ ) associated with larger  $\Delta T$ s ( $> 5\text{ K}$ ) may be different from that of small ones. This needs further experimental testing. We show that ANSYS ICEPAK can be used to estimate the mirror seeing in the case of natural convection. We can give the allowed  $\Delta T$  for thermal control of the primary mirror according to the simulation, empirical data and site seeing based on the scientific specification.

We note that there are some limitations to the present method for estimating mirror seeing in this paper. Theoretically, some of the uncertainties come from the numerical methodology adopted in the software. These issues are neglected in this study. On the other hand, uncertainties can be introduced by some treatments in the simulation. First of all, the geometry of the mirror model is a plane plate and this is different from the real primary mirror in a telescope. We neglect radiation heat transfer as  $\Delta T$ s are relatively small. The temperature inhomogeneity of the mirror is also neglected. A more realistic model should be used in the future for a more precise simulation. The conclusions in this paper are based on a mirror with diameter  $D = 3.6\text{ m}$ . Whether they are applicable for much larger aperture size telescopes should be tested in the future.

**Acknowledgements** The authors gratefully acknowledge support from the Guo Shou Jing Telescope (the Large Sky Area Multi-Object Fiber Spectroscopic Telescope, LAMOST) and the Large Scientific Equipments Repairing Project of Chinese Academy of Sciences: “Cooling Facility and Monitoring Instruments for LAMOST Dome Seeing Improvement.” This research was supported by National Key Basic Research Program of China Y41J051N01.

### References

- Cui, X.-Q., Zhao, Y.-H., Chu, Y.-Q., *et al.* 2012, RAA (Research in Astronomy and Astrophysics), 12, 1197
- Greenhalgh, R. J. S., Stepp, L. M., & Hansen, E. R. 1994, in Proc. SPIE, 2199, Advanced Technology Optical Telescopes V, ed. L. M. Stepp, 911
- Hansen, E. R., Hagelbarger, D., & Pearson, E. T. 1997, in Proc. SPIE, 2871, Optical Telescopes of Today and Tomorrow, ed. A. L. Ardeberg, 667
- Iye, M., Noguchi, T., Torii, Y., Mikami, Y., & Ando, H. 1991, PASP, 103, 712
- Li, G., & Yang, D. 2004, in Proc. SPIE, 5495, Astronomical Structures and Mechanisms Technology, ed. J. Antebi & D. Lemke, 204
- Lowne, C. M. 1979, MNRAS, 188, 249
- Racine, R., Salmon, D., Cowley, D., & Sovka, J. 1991, PASP, 103, 1020
- Stanghellini, S., Legrand, P., Baty, A., & Hovsepien, T. 1997, in Proc. SPIE, 2871, Optical Telescopes of Today and Tomorrow, ed. A. L. Ardeberg, 314
- Su, D.-Q., Cui, X., Wang, Y.-N., & Wang, S.-G. 2000, in Proc. SPIE, 4004, Telescope Structures, Enclosures, Controls, Assembly/Integration/Validation, and Commissioning, ed. T. A. Sebring & T. Andersen, 340
- Su, D.-Q., Liang, M., Yuan, X., Bai, H., & Cui, X. 2016, MNRAS, 460, 2286
- Tatarskii, V. I. 1961, Wave Propagation in Turbulent Medium (McGraw-Hill)
- Vogiatzis, K., & Upton, R. 2006, in Proc. SPIE, 6271, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 627115
- Wang, Q. J., & Zhang, S. X. 1991, Journal of Engineering Thermophysics, 12, 423
- Zago, L. 1997, in Proc. SPIE, 2871, Optical Telescopes of Today and Tomorrow, ed. A. L. Ardeberg, 726
- Zhang, E.-P., Cui, X.-Q., Li, G.-P., *et al.* 2016, RAA (Research in Astronomy and Astrophysics), 16, 98