RAA 2018 Vol. **18** No. **4**, 47(8pp) doi: 10.1088/1674–4527/18/4/47

http://www.raa-journal.org *http://iopscience.iop.org/raa*

Research in Astronomy and Astrophysics

Period variations of Algol-type eclipsing binaries AD And, TW Cas and IV Cas

Štefan Parimucha[1], Pavol Gajdoš[1], Viktor Kudak[1], Miroslav Fedurco[1] and Martin Vaňko[2]

[1] Institute of Physics, Faculty of Science, University of P. J. Šafárik, 044 01 Košice, Slovakia; *stefan.parimucha@upjs.sk*

[2] Astronomical Institute, Slovak Academy of Sciences, 059 60 Tatranská Lomnica, Slovakia

Received 2017 December 4; accepted 2018 January 29

Abstract We present new analyses of variations in $O-C$ diagrams of three Algol-type eclipsing binary stars: AD And, TW Cas and IV Cas. We have used all published minima times (including visual and photographic) as well as newly determined ones from our and SuperWasp observations. We determined orbital parameters of 3$^{\rm rd}$ bodies in the systems with statistically significant errors, using our code based on genetic algorithms and Markov chain Monte Carlo simulations. We confirmed the multiple nature of AD And and the triple-star model of TW Cas, and we proposed a quadruple-star model of IV Cas.

Key words: binaries: close — binaries: eclipsing — techniques: photometric — stars: individual (AD And, TW Cas, IV Cas)

1 INTRODUCTION

Study of $O-C$ diagrams of eclipsing binaries is a powerful tool for an analysis of temporal variations and irregularities in the cyclic phenomena observed in these stars. The most frequently determined quantities in the period studies are minima times of the binary light curve. If we determine minimum time (O – observed), we can calculate the difference between this value and what is predicted by the ephemeris (C – calculated). If changes in the $O-C$ values with time are systematic and if they exceed the experimental errors, we can provide a better model of such a system and reveal other hidden physical phenomena, like mass transfer between both components, angular momentum lost from the system, apsidal motion and/or presence of another body in the system (Sterken 2005).

In this paper we present a new period analysis of three Algol-type eclipsing binaries, which have been overlooked for the few past years.

The light variability of **AD And** was discovered by Guthnick & Preger (1927). They classified the variations as β Lyr type with a photographic amplitude of about 0.9 mag. The first photometric study of AD And was published by Taylor & Alexander (1940). Ruciński (1966) reported the first photoelectric photometry of the object and determined five minima times. Cannon (1934) classified the star as an F-type object, and later classification by Hill et al. (1975) gave a spectral type range from B8 to A0. Giuricin & Mardirossian (1981) published photometric parameters of the system and concluded that both components have almost the same radii, masses, temperatures and luminosities with orbital inclination $i = 81.9^\circ \pm 0.4^\circ$, which was confirmed by Liakos et al. (2012). The period variations of AD And were investigated by several authors (Walker 1957; Ruciński 1966; Frieboes-Conde & Herczeg 1973; Liao & Qian 2009; Liakos et al. 2012). The last mentioned authors determined period of the third body to be 14.3 years and its mass function $f(m) = 0.183\,M_\odot$.

TW Cas was discovered in 1907 by Pickering (1907) and its variability from photographic observations was confirmed by Zinner (1913). Spectroscopic observations of Struve (1950) confirmed B9 spectral type of the primary component and he determined mass function $f(m)$=0.098 $M_\odot$. The most recent photoelectric V observations of TW Cas were obtained by Narita et al. (2001). Their light curve solution led to the conclusion that the secondary component almost fills its Roche lobe. Djurašević et al. (2006) re-analyzed older photoelectric observations from McCook (1971) and determined masses of the primary and secondary components to be

Table 1 Basic Parameters of Studied Objects

Star	TYC	V [mag]	$B-V$ [mag]	T_0 [HJD]	P [d]
AD And	3641–0151–1	11.14	0.20	2452500.3670	0.9862210
TW Cas	4059–0898–1	8.32	0.10	2452500.8245	1.4283346
IV Cas	4001–1104–1	11.34	0.27	2452500.3569	0.9985067

Notes— TYC: number in Tycho-2 catalog, V: magnitude in the V filter, $B-V$: color index, T_0: initial time of minimum and P: period. Linear ephemerides are taken from the on-line database (Kreiner 2004).

M_1=2.66 $M_\odot$ and M_2=1.15 $M_\odot$, respectively, which are in agreement with values obtained by Narita et al. (2001). Kreiner (1971) used all the available minima times of TW Cas, but could draw no definite conclusions concerning the period variations. Narita et al. (2001) assumed that the orbital period of TW Cas was slowly decreasing, as was confirmed by Lloyd & Guilbault (2002). Khaliullina (2015) noted that the recent minima times demonstrate sinusoidal changes of the orbital period, and thus cyclic variations of the period due to the presence of a third body in the system are observed.

Eclipsing binary **IV Cas** was discovered on Moscow photographic plates by Meshkova (1940). Kim et al. (2005) in their photometric study discovered a short-periodic pulsating component with a frequency of 37.672 cycles per day (period ∼38 minutes). Wolf et al. (2006) and Zasche (2006) reported sinusoidal $O-C$ diagram changes caused by the light-time effect with period about 21 800 days and semi-amplitude 0.03 day. The third component should have a minimal mass of 0.96 $M_\odot$. Detailed analysis of the binary light curve as well as pulsation characteristics of the primary component was studied by Kim et al. (2010). They showed that IV Cas is in a semi-detached configuration with an A3 spectral type primary component and an evolved early-K secondary, which fills its inner Roche lobe. Pulsations correspond to a δ Scuti-type pulsator.

The basic parameters of the studied stars, like their brightness, color indices and linear ephemerides, are given in Table 1.

2 MINIMA TIMES

For our analysis we used minima times collected in the on-line database O–C gateway operated by the Czech Astronomical Society[1]. Almost all published minima times of our objects (including visual and photographic) are accessible in this database.

We also included our two new unpublished minima times for AD And and TW Cas. Moreover, we determined minima times from SuperWasp project observations (Pollacco et al. 2006) available from a public archive[2]. Our observations were obtained using a 508 mm telescope operated by Pavol Jozef Šafárik University (Parimucha & Vaňko 2015). Data reduction and differential photometry were performed by the C-Munipack package[3].

New minima times were calculated by the fitting to the template function of the minimum light curve as proposed by Mikulášek (2015). These minima times are listed in Table 2. Our new light curves of AD And and TW Cas, together with examples of two SuperWasp light curves of IV Cas, are displayed in Figure 1. The best fit with a template function for each minimum is also depicted by a solid line.

Table 2 New unpublished minima times $\mathrm{JD_{min}}$ [HJD - 2400000] of AD And, TW Cas and minima times determined from SuperWASP observations of IV Cas. Errors of minima time determinations are given in parentheses.

	$\mathrm{JD_{min}}$	$\mathrm{JD_{min}}$
AD And	57966.4661(2)	
TW Cas	57948.4948(2)	
IV Cas	54319.6353(3)	54361.5721(2)
	54337.6076(5)	54362.5705(2)
	54348.5919(2)	54363.5691(2)
	54350.5890(4)	54381.5421(3)
	54351.5873(3)	54382.5405(3)
	54352.5853(2)	54396.5202(3)
	54353.5835(3)	54398.5169(4)
	54354.5828(2)	54399.5157(7)
	54357.5784(3)	54402.5113(2)
	54360.5734(3)	54405.5063(3)

3 DATA ANALYSIS

3.1 Theory

Minima times T_C of eclipsing binary stars can be simply calculated by the linear ephemeris

$$T_C = T_0 + P \times E, \tag{1}$$

which predicts minima times of an eclipsing binary with an orbital period P without any other influences. Here E is an epoch of the observation (integer number for a

[1] *http://var2.astro.cz/ocgate/*

[2] *http://wasp.cerit-sc.cz/form*

[3] *http://c-munipack.sourceforge.net/*

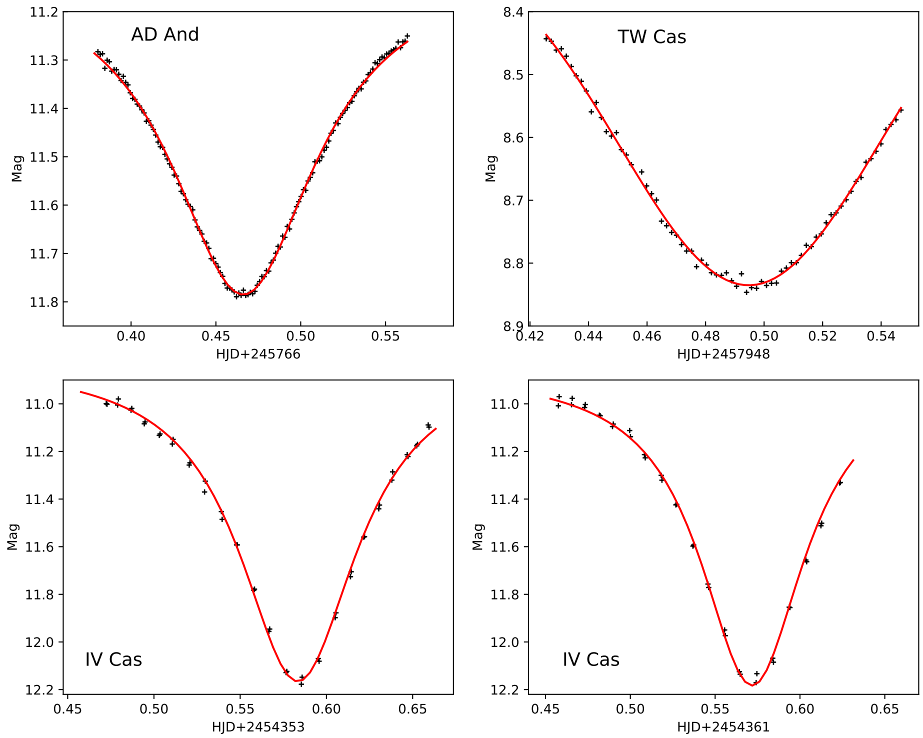

Fig. 1 Our new unpublished light curves of AD And and TW Cas from 2017 and examples of two light curves of IV Cas from the SuperWasp archive. The best fit with a template function for each minimum is displayed by a *solid line*.

primary minimum and/or $E + 0.5$ for a secondary minimum) and it counts how many eclipses elapsed since the zero epoch. T_0 is an initial minimum time (minimum at $E = 0$). To date, the linear ephemerides for our objects are listed in Table 1. The difference between observed T_O and predicted T_C minima times is caused by perturbation δT

$$T_O - T_C \equiv O - C = \delta T. \qquad (2)$$

This perturbation is generally a sum of different effects. For our analysis we consider only mass transfer and presence of another third body in the system (light-time effect). Then we can write

$$\delta T = Q \times E^2 + \frac{a \sin i_3}{c} \times \left[\frac{1 - e_3^2}{1 + e_3 \cos \nu_3} \sin(\nu_3 + \omega_3) + e_3 \sin \omega_3 \right]. \qquad (3)$$

The first term represents period change due to mass transfer (Hilditch 2001). The second one describes period change due to the light-time effect caused by the third component (Irwin 1952). Here $a \sin i_3$ is the projected semi-major axis of the orbit with eccentricity e_3, c is the speed of light, ω_3 is the longitude of the periastron and ν_3 is the true anomaly of the binary orbit around the center of mass of the system. There are no limitations on mass or orbital parameters of the third body. Period of the third body P_3 and time of pericenter passage t_{03} are hidden in the ν_3 calculation, which have to be solved using the Kepler equation. Because we are not able to find the inclination of the orbit i_3 from only $O - C$ analysis, we can only determine the so-called mass function of the third body

$$f(M_3) = \frac{(M_3 \sin i_3)^3}{M^2} = \frac{(a \sin i_3)^3}{P_3^2}, \qquad (4)$$

where $M = M_1 + M_2 + M_3$ is the total mass of the system (M_i - masses of components).

3.2 Fitting Method

To obtain the optimal set of eight parameters (T_0, P, Q, t_{03}, P_3, $a \sin i_3$, e_3, ω_3) there are classical numerical methods based on iterative minimization of the sum of squares, like the Levenberg-Marquardt algorithm or Simplex method (Press et al. 2007). These algorithms can be simply implemented in many programming languages and data analysis packages, and a solution can be found relatively fast. But the convergence to the global minimum (the best solution) is strongly dependent on the initial guess of fitted parameters, so it has to be somewhat close to the final solution.

To overcome the problem with initial values of parameters, we have developed our own code[4] based on the use of genetic algorithms and Markov chain Monte Carlo (MCMC) simulation. More details about our code are given in the upcoming paper (Gajdoš & Parimucha 2018). Here we will mention only a brief description of the main principles. Fitting of $O-C$ diagrams with our code is divided into two parts. The first part implements genetic algorithms (e.g Whitley 1994) to determine initial values for the fitting parameters. The second part utilizes these values as input to the MCMC simulation (e.g. Press et al. 2007) which gives as a result the solution with statistically significant error estimates of all parameters. As an input parameter, our code needs only intervals, where the specific parameter can be located. A user can select physically relevant intervals for each fitted parameter. With the previously described approach we can find the best global solution, but its statistical significance strongly depends on number of steps in the MCMC simulation, number of generations and size of population used in the genetic algorithms. A discussion about selection of proper values is given in Gajdoš & Parimucha (2018).

The crucial step in analyzing period changes of eclipsing binaries is setting the weights to individual observations. Minima times are determined from different types of observations, by different instrumentation with various quality. Moreover, authors use unequal methods for minima time determinations. For our solution we choose one weight for the whole group of observations obtained by one technique: visual (vis)- 1, photographic (phot) - 2, photoelectric (phe) - 10, CCD - 10. This weighting scheme is used by many authors (e.g. Zasche et al. 2009; Liakos et al. 2011).

4 RESULTS AND DISCUSSION

4.1 AD And

In Table 3 we list our results from fitting the $O-C$ diagram for AD And together with early published results. Our best fit solution is shown in Figure 2. Parameters of our solution are almost the same as in previous papers, except for the orbital period P_3 of the third body. It is about 2 years shorter (12.1 yr, in contrast to 14.3 yr) than in other solutions. This difference can be explained by the fact that we have used a much longer time interval for an analysis. Liao & Qian (2009) and Liakos et al. (2012) used only minima times from photoelectric and CCD observations and neglected all $O-C$ points obtained before 1990 because of their poorer quality. We also used these older photographic and visual observations even with smaller weight. Moreover, the last CCD minima times cover the full cycle of $O-C$ variations (see Fig. 2).

We also detected a secular period change (parameter Q) not mentioned by other authors. The period change corresponds to increase of the period $dP/dt = 1.06(94) \times 10^{-4}\,\mathrm{s\,yr^{-1}}$ and should be connected with mass transfer from the secondary component to the primary one and/or with the Applegate effect (Applegate 1992), but this is not in agreement with a detached configuration of the system (Liakos et al. 2012). It is necessary to note that the relative statistical error of Q is almost 90%, which degrades its significance. We have also tried a solution with no Q and we surprisingly obtained results with worse statistical significance. We cannot confirm or disprove the secular period changes and only future observations can resolve this problem.

Our solution implies a minimal 3$^{\rm rd}$ body mass (for $i_3 = 90°$) of $\sim 2.33\,M_\odot$, using absolute parameters from Liakos et al. (2012). This would indicate that the third light contributes about 15% to the total luminosity of the system, but the third light resulting from light curve analysis (Liakos et al. 2012) is about 3%. This difference can be explained with the assumption that the 3$^{\rm rd}$ star is actually a binary system with two solar-mass components as mentioned by Liakos et al. (2012).

4.2 TW Cas

Khaliullina (2015) for the first time noted that variations in the $O-C$ diagram of TW Cas can be explained by the presence of another body in the system. This hypothesis is based on the latest CCD minima times which have different trends with respect to the linear ephemeris than older ones. Our analysis of all available minima times of this object confirmed this fact. The results from our study and Khaliullina (2015) are listed in Table 4 and our best fit model is shown in Figure 3.

We confirmed that the 3$^{\rm rd}$ body is on a highly eccentric orbit ($e_3 = 0.71$), although our period P_3 is about 3 years longer and mass function is about twice as small as in the previous solution. However, these values are in the range of statistical errors for these parameters. We did not reveal any secular changes caused by mass transfer and/or magnetic activity. From our model we can find the minimal mass of the 3$^{\rm rd}$ body (for $i_3 = 90°$) to be $\sim 0.48\,M_\odot$, using absolute parameters from Djurašević et al. (2006). If we assume that the third body is a main sequence star, its spectral type should be K6-7, with absolute magnitude in V passband ~ 8 mag. Its contribu-

[4] *https://github.com/pavolgaj/OCFit*

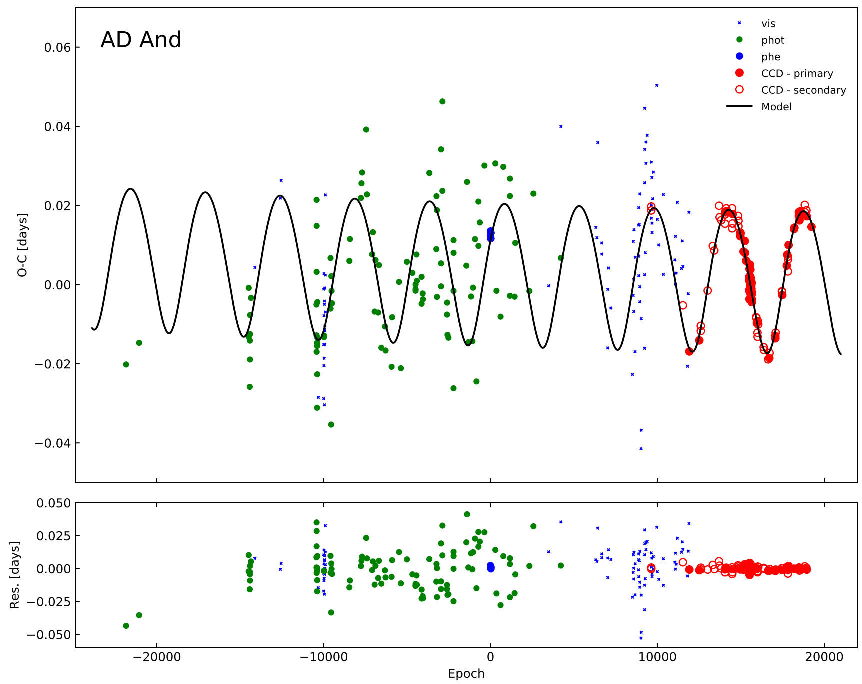

Fig. 2 The $O-C$ diagram of AD And fitted by the light-time effect (*upper*) and the residuals after the subtraction of the best fit (*lower*). Different types of observations are depicted by different points and colors. The best fit (*solid black line*) corresponds to values given in Table 3.

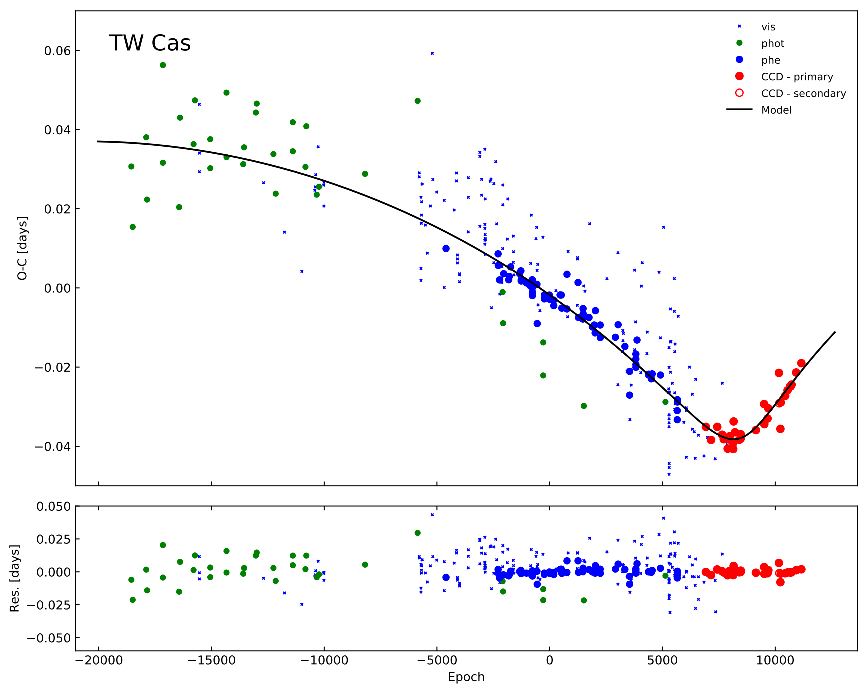

Fig. 3 The $O-C$ diagram of TW Cas fitted by the light-time effect (*upper*) and the residuals after subtraction of the best fit (*lower*). Different types of observations are depicted by different points and colors. The best fit (*solid black line*) corresponds to values given in Table 4.

Table 3 Parameters of the 3$^{\rm rd}$ Body Orbit from $O-C$ Diagram Analysis of AD And and Comparison with Previous Studies

Solution	This paper	(1)	(2)
P [d]	0.98619356(6)	0.98619240(14)	0.9861924(4)
T_0 [HJD]	2439002.9350(9)	2439002.5733(15)	2439002.458(6)
Q [d]	1.67(1.49)$\times 10^{-12}$	–	–
P_3 [d]	4418(16)	5249	5220(37)
t_{03} [HJD]	2442236(537)	2438813(414)	2447012(175)
$a \sin i_3$ [AU]	3.13(7)	3.24(12)	–
e_3	0.15(5)	0.30(24)	0.17(5)
ω_3 [°]	284(43)	270(50)	25(11)
$f(M_3)$ [$M_\odot$]	0.209(14)	0.160(20)	0.183(1)
χ^2	290.596	–	–
χ^2/n	0.723	–	–

Notes– (1): Liao & Qian (2009) and (2): Liakos et al. (2012). P: orbital period of eclipsing pair, T_0: initial minimum, Q: quadratic term, P_3: orbital period of the 3$^{\rm rd}$ body, t_{03}: pericenter passage, $a \sin i_3$: projected semi-major axis of the orbit, e_3: eccentricity, ω_3: the longitude of the periastron, $f(M_3)$: the mass function, χ^2: sum of squares of the best fit and χ^2/n: reduced sum of squares (n: number of data points), errors are given in parenthesis.

Table 4 Parameters of the 3$^{\rm rd}$ Body Orbit from $O-C$ Diagram Analysis of TW Cas and Comparison with Previous Analysis (for description of parameters see Table 3)

Solution	This paper	(1)
P [d]	1.42832665(35)	1.4283273(5)
T_0 [HJD]	2442008.3870(15)	2442008.3560(4)
Q [d]	–	–
P_3 [d]	75300(2900)	74300(400)
t_{03} [HJD]	2454255(388)	2454400(200)
$a \sin i_3$ [AU]	6.49(68)	7.8(1.4)
e_3	0.71(3)	0.74(7)
ω_3 [°]	284(4)	288(6)
$f(M_3)$ [$M_\odot$]	0.006(2)	0.013
χ^2	221.043	–
χ^2/n	0.7569	–

Notes– (1): Khaliullina (2015).

tion to the total luminosity of the system is about 0.1%. Photometric studies of TW Cas did not reveal a significant third light on the light curve, which is in agreement with a low mass 3$^{\rm rd}$ body on a close to edge-on orbit.

4.3 IV Cas

Our results on the analysis of the $O-C$ diagram for IV Cas together with parameter values from the study of Wolf et al. (2006) are listed in Table 5, and our best solution is shown in Figure 4. Unlike the analysis of Wolf et al. (2006), we have obtained different values for three parameters. The first one is a secular period increase of $dP/dt = 4.5(6) \times 10^{-4}$ s yr^{-1}, the second one is higher eccentricity (0.31 vs. 0.09) of the 3$^{\rm rd}$ body orbit and the third one has a larger mass function (0.102 $M_\odot$ vs. 0.056 $M_\odot$). The period increase could be explained by mass transfer from the secondary to primary component. It is in agreement with the semi-detached configuration determined by Kim et al. (2010). Significantly higher eccentricity corresponds to the shape of the $O-C$ diagram with the latest minima times (see Fig. 4).

Our solution gives a minimal mass of the 3$^{\rm rd}$ component (for $i_3 = 90°$) to be $\sim 1.27\,M_\odot$, using masses from Kim et al. (2010). Assuming a main sequence 3$^{\rm rd}$ body, a contribution from the third light of about 10% should be observed. However, Kim et al. (2010) did not report any third light from their light curve solution. The only realistic explanation is that the third body is actually a binary star with less massive and luminous components.

5 CONCLUSIONS

We have analyzed period variations of three Algol-type eclipsing binary stars. We used all minima times available in the literature as well as newly determined ones

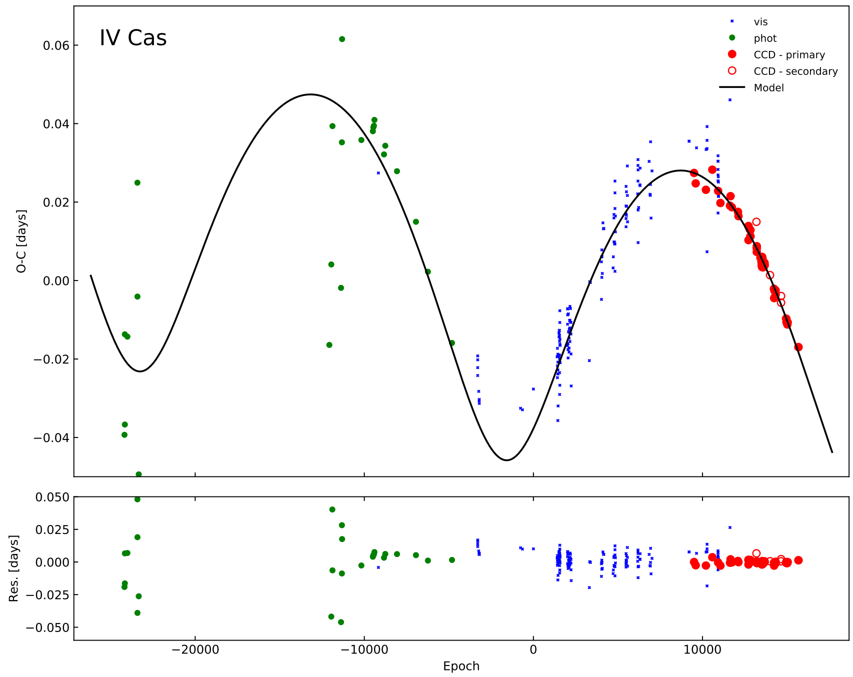

Fig. 4 The $O - C$ diagram of IV Cas fitted by the light-time effect (*upper*) and the residuals after the subtraction of the best fit (*lower*). Different types of observations are depicted by different points and colors. The best fit (*solid black line*) corresponds to values given in Table 5.

Table 5 Parameters of the 3$^{\rm rd}$ Body Orbit from $O - C$ Diagram Analysis of IV Cas and Comparison with a Previous Analysis (for description of parameters see Table 3)

Solution	This paper	(1)
P [d]	0.99851644(18)	0.99851658(12)
T_0 [HJD]	2440854.6186(34)	2440854.6280(5)
Q [d]	7.33(1.69)$\times 10^{-12}$	–
P_3 [d]	21700(444)	21800(500)
t_{03} [HJD]	2439254(858)	2443455(50)
$a \sin i_3$ [AU]	7.13(72)	–
e_3	0.31(10)	0.09
ω_3 [°]	272(14)	341(3)
$f(M_3)$ [$M_\odot$]	0.102(3)	0.056
χ^2	218.658	–
χ^2/n	0.9344	–

Notes– (1): Wolf et al. (2006).

from our observations and from the SuperWasp archive. We applied our code based on genetic algorithms and MCMC simulation. This allows us to determine fitting parameters with statistically significant errors and also measure the quality of the statistical model.

Our new period analysis of all studied Algol-type eclipsing binaries confirmed their multicomponent nature. The third component in the AD And system is most probably also a binary star with two solar-mass components, as shown by the large minimal mass of this component determined from $O - C$ analysis. This is supported also by the solution of the light curve from Liakos et al. (2012). We can speculate that orbital inclination of this binary is much lower than 90°, because we see no other set of eclipses on the light curve. Moreover, absence of ellipsoidal variations on the light curve caused by the second binary system suggests that this pair is a detached binary on the orbit with period in the range of several days.

The detected period increase is disputable and cannot be confirmed or disproved from available data. As a result, we can conclude that AD And is a quadruple-system consisting of two binaries. The first one is an eclipsing pair which we observe and the second one is a binary star with total mass of at least $2.33\,M_\odot$ with orbital inclination and semi-major axis that prevent us from observing the other set of eclipses.

Analysis of period variations for TW Cas confirms the presence of a third body in the system. This body is on a highly eccentric orbit with minimal mass $\sim 0.48\,M_\odot$, which contributes minimally to the total luminosity of the system.

Finally, $O-C$ diagram analysis of IV Cas produced different results than a previous analysis conducted by Wolf et al. (2006). Main discrepancies have been found in secular period increase, eccentricity and mass function. The period increase is caused by mass transfer from the secondary to primary component. The light curve solution of Kim et al. (2010) showed that the secondary component fills its Roche lobe and this supports the above mentioned mass transfer. Due to excessive mass of the third body with respect to no detected third light, we can again conclude that the third body is in fact a binary system with unknown orbital parameters. Therefore, IV Cas could be also considered a quadruple-system consisting of a visible semi-detached binary with pulsating primary component and a second pair composed of cool, low mass and low luminous main sequence stars of K6-7 spectral type.

Acknowledgements This paper has been supported by a grant from the Slovak Research and Development Agency with number APVV-15-0458. This article was created by realization of the project ITMS (No. 26220120029), based on the supporting operational research and development program financed from the European Regional Development Fund. M.V. would like to thank the project VEGA (2/0143/14).

References

Applegate, J. H. 1992, ApJ, 385, 621
Cannon, A. J. 1934, Harvard College Observatory Bulletin, 897, 12
Djurašević, G., Rovithis-Livaniou, H., Rovithis, P., et al. 2006, A&A, 445, 291
Frieboes-Conde, H., & Herczeg, T. 1973, A&AS, 12, 1
Gajdoš, P., & Parimucha, Š. 2018, submitted to A&C
Giuricin, G., & Mardirossian, F. 1981, A&AS, 45, 499
Guthnick, P., & Preger, R. 1927, Kleine Veroeffentlichungen der Universitaetssternwarte zu Berlin Babelsberg, 1, 4.1
Hilditch, R. W. 2001, An Introduction to Close Binary Stars (Cambridge: Cambridge Univ. Press)
Hill, G., Hilditch, R. W., Younger, F., & Fisher, W. A. 1975, MmRAS, 79, 131
Irwin, J. B. 1952, ApJ, 116, 211
Khaliullina, A. I. 2015, Astronomy Reports, 59, 717
Kim, S.-L., Lee, C.-U., Koo, J.-R., et al. 2005, Information Bulletin on Variable Stars, 5669
Kim, S.-L., Lee, J. W., Lee, C.-U., & Youn, J.-H. 2010, PASP, 122, 1311
Kreiner, J. M. 1971, Acta Astronomica, 21, 365
Kreiner, J. M. 2004, Acta Astronomica, 54, 207
Liakos, A., Zasche, P., & Niarchos, P. 2011, New Astron., 16, 530
Liakos, A., Niarchos, P., & Budding, E. 2012, A&A, 539, A129
Liao, W., & Qian, S. 2009, New Astron., 14, 249
Lloyd, C., & Guilbault, P. 2002, The Observatory, 122, 85
McCook, G. P. 1971, AJ, 76, 449
Meshkova, T. S., 1940, Variable Stars, 5, 304
Mikulášek, Z. 2015, A&A, 584, A8
Narita, E., Schroeder, K.-P., & Smith, R. C. 2001, The Observatory, 121, 308
Parimucha, Š., & Vaňko, M. 2015, in Astronomical Society of the Pacific Conference Series, 496, Living Together: Planets, Host Stars and Binaries, eds. S. M. Rucinski, G. Torres, & M. Zejda, 309
Pickering, E. C. 1907, Astronomische Nachrichten, 175, 91
Pollacco, D. L., Skillen, I., Collier Cameron, A., et al. 2006, PASP, 118, 1407
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 2007, Numerical Recipes: The Art of Scientific Computing (3rd Edition, Cambridge: Cambridge Univ. Press)
Ruciński, S. M. 1966, Acta Astronomica, 16, 307
Sterken, C. 2005, in Astronomical Society of the Pacific Conference Series, 335, The Light-Time Effect in Astrophysics: Causes and Cures of the O-C Diagram, ed. C. Sterken, 3
Struve, O. 1950, ApJ, 112, 184
Taylor, P. H., & Alexander, R. S. 1940, Publications of the University of Pennsylvania Flower Astronomical Observatory, 6, 1
Walker, M. F. 1957, AJ, 62, 371
Whitley, D. 1994, Statistics and Computing, 4, 65
Wolf, M., Zejda, M., Kiyota, S., et al. 2006, Information Bulletin on Variable Stars, 5735
Zasche, P. 2006, in Astronomical Society of the Pacific Conference Series, 349, Astrophysics of Variable Stars, eds. C. Aerts, & C. Sterken, 379
Zasche, P., Liakos, A., Niarchos, P., et al. 2009, New Astron., 14, 121
Zinner, E. 1913, Astronomische Nachrichten, 195, 453