RAA 2017 Vol. **17** No. **8**, 88 (6pp) doi: 10.1088/1674–4527/17/8/88

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Research in Astronomy and Astrophysics

Relativistic time transfer for a Mars lander: from Areocentric Coordinate Time to Barycentric Coordinate Time

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Received 2017 April 16; accepted 2017 May 24

Abstract As the second step of relativistic time transfer for a Mars lander, we investigate the transformation between Areocentric Coordinate Time (TCA) and Barycentric Coordinate Time (TCB) in the framework of IAU Resolutions. TCA is a local time scale for Mars, which is analogous to the Geocentric Coordinate Time (TCG) for Earth. This transformation has two parts: contributions associated with gravitational bodies and those depending on the position of the lander. After setting the instability of an onboard clock to 10^{-13} and considering that the uncertainty in time is about 3.2 microseconds after one Earth year, we find that the contributions of the Sun, Mars, Jupiter and Saturn in the leading term associated with these bodies can reach a level exceeding the threshold and must be taken into account. Other terms can be safely ignored in this transformation for a Mars lander.

Key words: reference systems – time – space vehicles

1 INTRODUCTION

Time transfer plays an important role in every deep space mission. Einstein's general relativity (GR) (Iorio 2015; Debono & Smoot 2016) has been an inevitable part of time transfer in both a theoretical sense (Misner et al. 1973; Landau & Lifshitz 1975) and in practice (Petit & Wolf 2005; Nelson 2007, 2011). In the framework of GR, there are two kinds of time: proper time and coordinate time, which are connected by a 4-dimensional spacetime interval, so that clock synchronization and time/frequency transfer for deep space tracking and navigation have been considerably changed (Moyer 2005; Thornton & Border 2005). In the future, time transfer might be used to test GR and alternative theories of gravity (e.g., Wolf et al. 2009; Christophe et al. 2009; Schiller et al. 2009; Deng & Xie 2013; Christophe et al. 2012; Angélil et al. 2014; Delva et al. 2015).

In this work, as a continuation of the time transfer for a Mars lander (Xu et al. 2016), we will focus on the relativistic transformation from Areocentric Coordinate Time (TCA) to Barycentric Coordinate Time (TCB), where TCA is the time coordinate of the Areocentric Celestial Reference System (ACRS). ACRS is introduced to describe events that happen in the vicinity of Mars and its definition is consistent with the International Astronomical Union (IAU) 2000 Resolutions (Soffel et al. 2003) and 2006 Resolution B2 [1] (see Xu et al. 2016, for more details). This transformation is the second step in the whole procedure of time transfer. Although this work is devoted to time transfer for a Mars lander, the major part of the transformation from TCA to TCB only depends on the position and velocity of gravitational bodies in the Solar System and it can also be applied to a Mars orbiter (Deng 2012; Pan & Xie 2013, 2014, 2015). In the practical realization of the time transfer link, we will also consider GR effects on the propagation of electromagnetic waves from Mars to Earth and vice-versa in the post-Newtonian field of the Solar System and the effects of the post-Newtonian field of Mars itself on a Mars probe (Iorio 2010, 2006, 2009; Oberst et al. 2012;

[1] *https://www.iau.org/static/resolutions/IAU2006_Resol2.pdf*

Turyshev et al. 2010), which will be one of our next moves.

Following engineering settings of the previous work (Xu et al. 2016), we also consider a clock onboard the Mars lander with instability at the level of 10^{-13} so that the uncertainty in time is about 3.2 microsecond (μs) after linear drifting for one (Earth) year. Therefore, any effects below these two thresholds will be dropped.

The paper is organized as follows. In Section 2, we describe the relativistic transformation between TCA and TCB based on IAU 2000 Resolutions (Soffel et al. 2003). This transformation has two parts: contributions associated with gravitational bodies in the Solar System and terms directly depending on the position of the lander. Sections 3 and 4 are respectively devoted to examining these two contributions. We summarize the results in Section 5.

2 TRANSFORMATION FROM TCA TO TCB

According to the IAU Resolutions (Soffel et al. 2003), the post-Newtonian transformation between TCA and TCB can be written in a linear form as

$$\mathrm{TCB} - \mathrm{TCA} = P_1 + L_1 + P_2 + L_2. \tag{1}$$

Here, P_1 and P_2 are contributions associated with gravitational bodies in the Solar System and they are independent of the position and velocity of the Mars lander. $P_{1,2}$ are in the forms of integrals as (Soffel et al. 2003)

$$P_1 = c^{-2}\int_{t_0}^{t}\mathrm{d}t\left[\frac{\boldsymbol{v}_{♂}^2}{2} + \bar{U}_{♂}(\boldsymbol{x}_{♂})\right], \tag{2}$$

$$\begin{aligned} P_2 = c^{-4}\int_{t_0}^{t}\mathrm{d}t\bigg[&\frac{1}{8}\boldsymbol{v}_{♂}^4 + \frac{3}{2}\boldsymbol{v}_{♂}^2\bar{U}_{♂}(\boldsymbol{x}_{♂}) \\ &-4v_{♂}^i\bar{U}_{♂}^i(\boldsymbol{x}_{♂}) - \frac{1}{2}\bar{U}_{♂}^2(\boldsymbol{x}_{♂})\bigg], \end{aligned} \tag{3}$$

where c is the speed of light; t is the coordinate time in the scale of TCB; $\boldsymbol{x}_{♂}$ and $\boldsymbol{v}_{♂}$ are respectively the position and velocity of Mars in the Barycentric Celestial Reference System (BCRS). In Equations (2) and (3), $\bar{U}_{♂}$ and $\bar{U}_{♂}^i$ are respectively the Newtonian gravitational potential and a vector potential, both of which are generated by gravitational bodies except Mars; their expressions read as

$$\bar{U}_{♂}(\boldsymbol{x}_{♂}) = \sum_{\mathrm{A}\neq♂}\frac{GM_{\mathrm{A}}}{r_{♂\mathrm{A}}}, \tag{4}$$

$$\bar{U}_{♂}^i(\boldsymbol{x}_{♂}) = \sum_{\mathrm{A}\neq♂}\frac{GM_{\mathrm{A}}}{r_{♂\mathrm{A}}}v_{\mathrm{A}}^i, \tag{5}$$

where M_{A} is the mass of body A; v_{A}^i is the velocity of body A in the BCRS; and $r_{♂\mathrm{A}}$ is the distance between Mars and body A. Since the distance between bodies is much larger than their characteristic lengths in the Solar System, the non-spherical parts of their gravitational fields can be neglected in this case. It is worth mentioning that all of the variables in the integrands of P_1 and P_2 are functions of TCB, although such a dependence is not shown explicitly.

Unlike P_1 and P_2, the other parts of TCB-TCA, L_1 and L_2, directly depend on the position of the Mars lander as

$$L_1 = c^{-2}\,\boldsymbol{v}_{♂}\cdot\boldsymbol{r}_{\mathrm{L}♂}, \tag{6}$$

$$L_2 = c^{-4}\left[\frac{\boldsymbol{v}_{♂}^2}{2} + 3\bar{U}_{♂}(\boldsymbol{x}_{♂})\right]\boldsymbol{v}_{♂}\cdot\boldsymbol{r}_{\mathrm{L}♂}, \tag{7}$$

where $\boldsymbol{r}_{\mathrm{L}♂} \equiv \boldsymbol{x}_{\mathrm{L}} - \boldsymbol{x}_{♂}$ and $\boldsymbol{x}_{\mathrm{L}}$ is the position of the lander in the BCRS.

3 CONTRIBUTIONS OF P_1 AND P_2

In order to calculate the contributions of P_1 and P_2, we have to know the positions and velocities of bodies in the Solar System. In this investigation, we consider the Sun, eight planets and the three largest asteroids (Ceres, Pallas and Vesta). We read their positions and velocities with the DE431 ephemeris (Folkner et al. 2014) from the starting time 2023-Jan-01 to the end point 2024-Jan-01. However, the time coordinate of DE431 is the Barycentric Dynamical Time (TDB), instead of TCB, but they have a linear relation described as (Petit & Luzum 2010)

$$\mathrm{d\,TDB} = (1 - L_B)\,\mathrm{d\,TCB}, \tag{8}$$

where $L_B = 1.550519768\times10^{-8}$ is a defining constant. Hence, we can rewrite Equations (2) and (3) as (Soffel et al. 2003)

$$\begin{aligned} P_1 = &\frac{1}{c^2(1-L_B)}\int_{\mathrm{TDB}_0}^{\mathrm{TDB}}\mathrm{dTDB} \\ &\times\left[\frac{\boldsymbol{v}_{♂}^2}{2} + \bar{U}_{♂}(\boldsymbol{x}_{♂})\right], \end{aligned} \tag{9}$$

$$\begin{aligned} P_2 = &\frac{1}{c^4(1-L_B)}\int_{\mathrm{TDB}_0}^{\mathrm{TDB}}\mathrm{dTDB} \\ &\times\bigg[\frac{1}{8}\boldsymbol{v}_{♂}^4 + \frac{3}{2}\boldsymbol{v}_{♂}^2\bar{U}_{♂}(\boldsymbol{x}_{♂}) - 4v_{♂}^i\bar{U}_{♂}^i(\boldsymbol{x}_{♂}) \\ &-\frac{1}{2}\bar{U}_{♂}^2(\boldsymbol{x}_{♂})\bigg], \end{aligned} \tag{10}$$

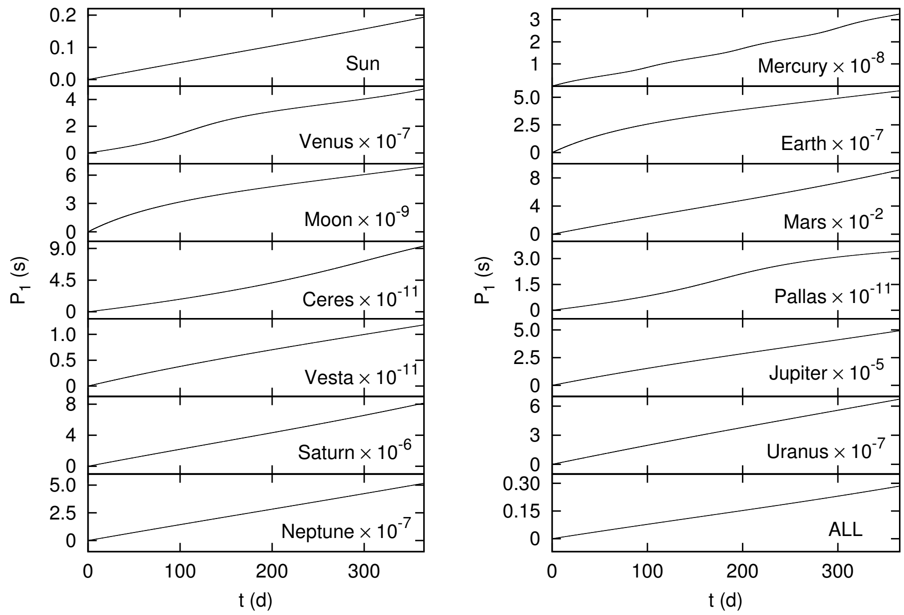

Fig. 1 Components of P_1 contributed by various bodies and their overall values (marked by "ALL").

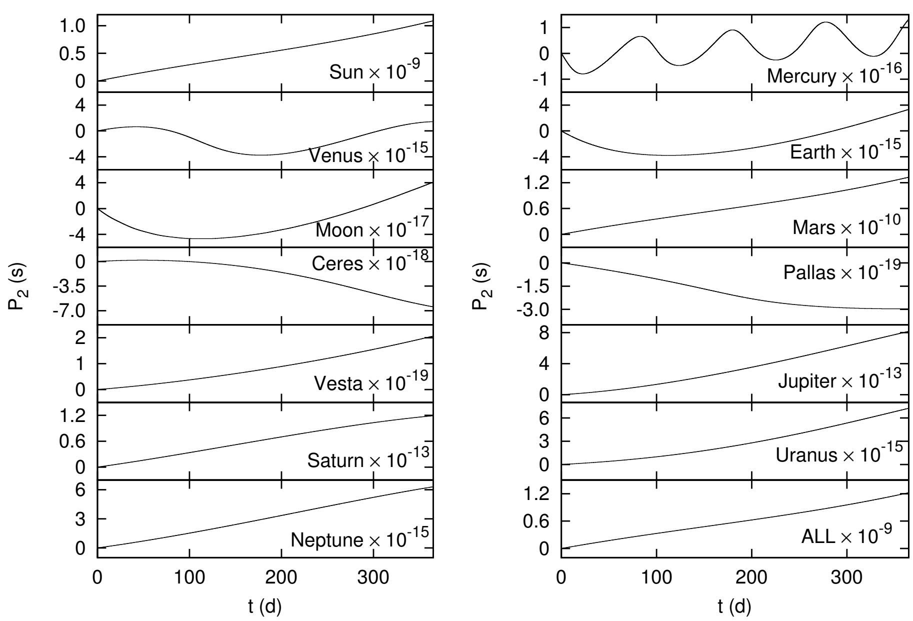

Fig. 2 Components of P_2 contributed by various bodies and their overall values (marked by "ALL").

where all variables in the above integrands are functions of TDB and can be read directly from DE431.

Figure 1 shows components of P_1 contributed by various bodies and their overall values. We find that P_1 can increase to about 0.3 s after one year and contributions of the Sun, Mars, Jupiter and Saturn can reach the level of more than 3 μs.

Figure 2 shows components of P_2 contributed by various bodies and their overall values. Since its overall contribution is about 1 nanosecond (ns), much less than our threshold, we can safely ignore P_2 in this investigation. Like the work of Xu et al. (2016), we also estimate the contribution of the hypothetical Planet Nine or Telisto (Batygin & Brown 2016; Brown & Batygin 2016;

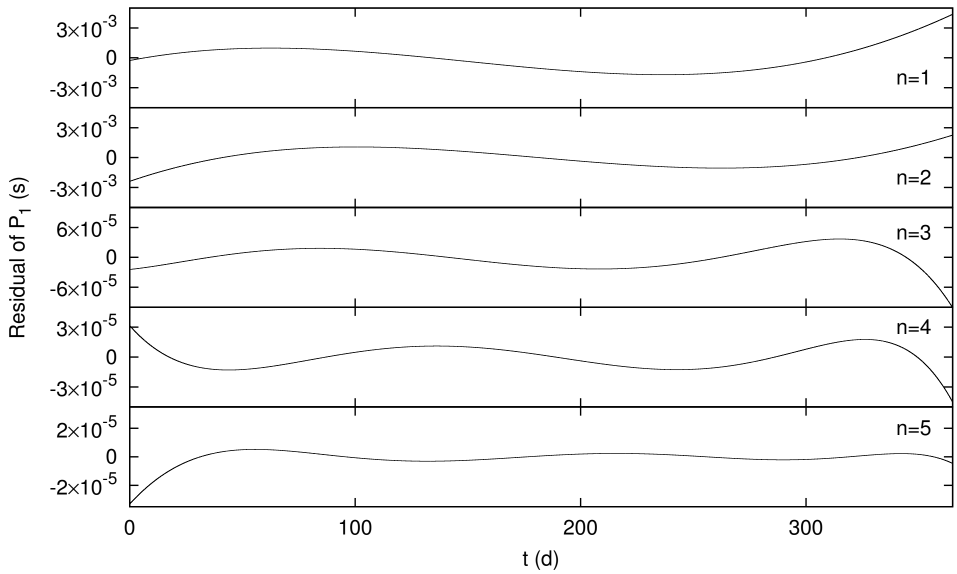

Fig. 3 Post-fit residuals of P_1 for the nth polynomial where $n = 1, \cdots, 5$.

Mustill et al. 2016; Iorio 2012, 2014, 2017) on P_1 to be about 10 ns, which can be neglected.

If the lander has to process the calculation of P_1 for autonomous time transfer, it would be necessary to construct an analytic form to approximate P_1 with sufficient accuracy because the capacity of the onboard computer is expected to be very limited. In principle, we can apply the strategy for constructing time transformations TDB-Terrestrial Time (TT) by using an analytical planetary ephemeris (Fairhead & Bretagnon 1990) and TCB-TCG by Chebyshev polynomial fitting (Fukushima 1995; Irwin & Fukushima 1999; Fukushima 2010) or by non-linear harmonic decomposition (Harada & Fukushima 2003). Although the detailed investigation on this will be left as one of our next moves, we will study the properties of P_1 in order to provide some clues for future works. Following the work of Pan & Xie (2013), we take an nth polynomial to fit P_1 as

$$P_1 \approx \sum_{j=0}^{n} a_j \, t^j, \tag{11}$$

and then perform Fourier analysis on its post-fit residuals. In the future, this preliminary procedure will be replaced with a more sophisticated one, such as the approach in the works (Kudryavtsev 2007, 2016).

Table 1 shows the coefficients a_j of the fitted nth polynomial and Figure 3 represents the post-fit residuals. It is found that, for the cases that $n = 4$ and $n = 5$, the residuals can decrease to about 30 μs and they can even decrease to several μs during t from 100 d to 300 d.

Figure 4 shows frequency spectra of these post-fit residuals. The spectra with $n = 1$ and $n = 3$ have distinct peaks, but these peaks are absent in the others. As demonstrated by the work of Pan & Xie (2015), their spike-like and tip-like shapes do not mean there is any discontinuity at those points, but rather they are caused by the low resolution of the figure.

4 CONTRIBUTIONS OF L_1 AND L_2

The terms L_1 and L_2 are different from P_1 and P_2 by depending on positions of the lander and their values will not accumulate with time. As a straightforward estimation, we can find that

$$L_1 \lesssim c^{-2} v_{♂} r_{\mathrm{L}♂} = 0.9 \ \mu\mathrm{s}, \tag{12}$$

$$L_2 \lesssim c^{-4} \frac{7}{2} v_{♂}^2 v_{♂} r_{\mathrm{L}♂} = 2 \times 10^{-14} \ \mathrm{s}. \tag{13}$$

Therefore, both of them are below the threshold we set and can be safely ignored in this case. In the above estimation, we take $r_{\mathrm{L}♂}$ as the radius of Mars for a lander. When an orbiter of Mars is considered, $r_{\mathrm{L}♂}$ can be larger, which makes L_1 close to or bigger than the threshold so that the term L_1 has to be included in the relativistic transformation between TCA and TCB.

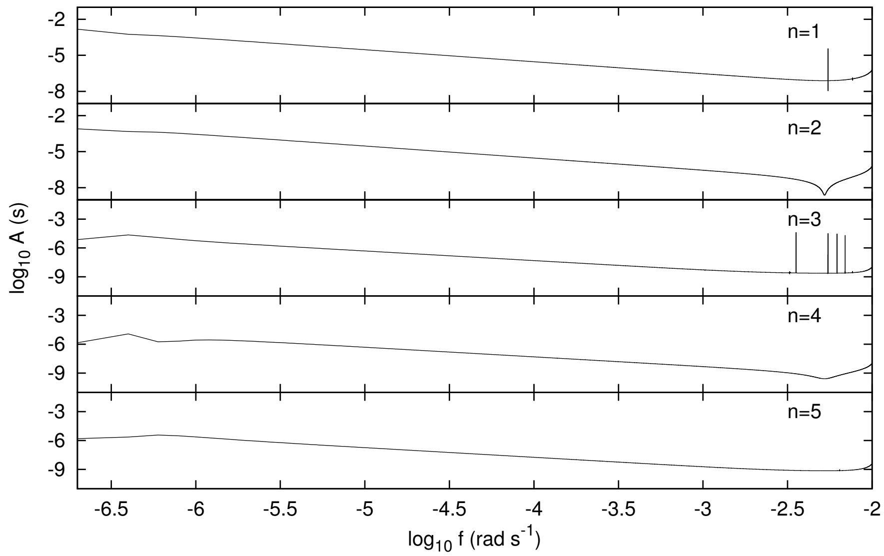

Fig. 4 Frequency spectra of the post-fit residuals of P_1 for the nth polynomial where $n = 1, \cdots, 5$, and where "A" and "f" denote amplitude and frequency, respectively.

Table 1 Coefficients of the fitted nth polynomial

n	a_j	Value
1	a_0	$2.729372422064708 \times 10^{-4}$
	a_1	$8.881818856983953 \times 10^{-9}$
2	a_0	$2.387954153226595 \times 10^{-3}$
	a_1	$8.479410723073467 \times 10^{-9}$
	a_2	$1.276027821887640 \times 10^{-17}$
3	a_0	$2.993148212994718 \times 10^{-5}$
	a_1	$9.376722304005195 \times 10^{-9}$
	a_2	$-5.837430741133446 \times 10^{-17}$
	a_3	$1.503775275456851 \times 10^{-24}$
4	a_0	$-2.586492924316603 \times 10^{-5}$
	a_1	$9.412111185814432 \times 10^{-9}$
	a_2	$-6.342420286347526 \times 10^{-17}$
	a_3	$1.752869730773431 \times 10^{-24}$
	a_4	$-3.949366681198948 \times 10^{-33}$
5	a_0	$3.875654970449868 \times 10^{-5}$
	a_1	$9.362187216272435 \times 10^{-9}$
	a_2	$-5.319681905071191 \times 10^{-17}$
	a_3	$9.241183787895206 \times 10^{-25}$
	a_4	$2.484444940015867 \times 10^{-32}$
	a_5	$-3.587172304388239 \times 10^{-40}$

5 CONCLUSIONS

In this paper, we continue the work on time transfer for a Mars lander (Xu et al. 2016) and investigate its second step that is the relativistic transformation between TCA and TCB. This transformation has two parts: $P_{1,2}$ terms associated with the gravitational bodies [see Eqs. (2) and (3)] and $L_{1,2}$ terms depending on the position of the lander [see Eqs. (6) and (7)].

After setting the instability of the onboard clock as 10^{-13}, we find that the contributions of the Sun, Mars, Jupiter and Saturn in P_1 can reach the level exceeding the threshold and must be taken into account. Other terms, P_2 and $L_{1,2}$, can be safely ignored in this transformation for a Mars lander.

Besides a part of the procedure for time transfer, such a transformation between TCA and TCB might also be used in the model of Doppler tracking of a lander and an orbiter of Mars (Deng & Xie 2014; Zhang et al. 2014; Yang et al. 2014; Xie & Huang 2015), since the time coordinate of their equations of motion is TCA according to the IAU 2000 Resolutions (Soffel et al. 2003). When considering much more precise and accurate measurements, the high-order contributions might be included (Deng & Xie 2012; Deng 2015; Xie & Huang 2015; Deng 2016).

Acknowledgements This work is funded by the National Natural Science Foundation of China (Grant No. 11573015 and No. J1210039).

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