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Can cold dark matter paradigm explain the central-surface-densities relation?

Man-Ho Chan

Department of Science and Environmental Studies, The Education University of Hong Kong, Tai Po, New Territories, Hong Kong, China; *chanmh@eduhk.hk*

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Abstract Recently, a very strong correlation between the central surface density of stars and dynamical mass in 135 disk galaxies has been obtained. It has been shown that this central-surface-densities relation agrees very well with Modified Newtonian Dynamics (MOND). In this article, we show that if we assume the baryons have an isothermal distribution and dark matter exists, then it is possible to derive by means of the Jeans equation an analytic central-surface-densities relation connecting dark matter and baryons that agrees with the observed relation. We find that the observed central-surface-densities relation can also be accommodated in the context of dark matter provided the latter is described by an isothermal profile. Therefore, the observed relation is consistent with not only MOND.

Key words: (cosmology:) dark matter

1 INTRODUCTION

Recent empirical fits indicate a very strong correlation between the central surface density of stars Σ_*^0 and dynamical mass $\Sigma_{\rm D}^0$ in 135 disk galaxies (Lelli et al. 2016). The central-surface-densities relation can be described by a double power law (Lelli et al. 2016)

$$\Sigma_{\rm D}^0 = \Sigma_0 \left[1 + \frac{\Sigma_*^0}{\Sigma_{\rm crit}} \right]^{\alpha-\beta} \left[\frac{\Sigma_*^0}{\Sigma_{\rm crit}} \right]^{\beta}, \tag{1}$$

where α, β, $\Sigma_{\rm crit}$ and Σ_0 are fitted parameters. Generally speaking, high surface brightness galaxies give $\Sigma_{\rm D}^0/\Sigma_*^0 \approx 1$ while low surface brightness galaxies systematically deviate from unity. The observed scatter is small overall (~ 0.2 dex) and largely driven by observational uncertainties (Lelli et al. 2016). This result is surprising because there is no obvious reason why the central surface density of stars strongly correlates with the dynamical mass.

Recently, Milgrom (2016) applied Modified Newtonian Dynamics (MOND) to derive this relation analytically. By using quasilinear MOND (QUMOND), we can obtain the expression in Milgrom (2016)

$$\Sigma_{\rm D}^0 = \Sigma_{\rm M} S(\Sigma_{\rm B}^0/\Sigma_{\rm M}), \tag{2}$$

where

$$S(y) = \int_0^y \nu(y')dy',$$

$\nu(y')$ is the interpolating function of the theory, $\Sigma_{\rm B}^0$ is the baryonic central density and $\Sigma_{\rm M} \approx 138\, M_\odot\, {\rm pc}^{-2}$ is a constant. Two different asymptotes give

$$\Sigma_{\rm D}^0 = \Sigma_{\rm B}^0$$

for

$$\Sigma_{\rm B}^0 \gg \Sigma_{\rm M}$$

and

$$\Sigma_{\rm D}^0 = (4\Sigma_{\rm M}\Sigma_{\rm B}^0)^{1/2}$$

for

$$\Sigma_{\rm B}^0 \ll \Sigma_{\rm M}$$

respectively. Milgrom (2016) claims that if $\Sigma_{\rm B}^0$ can fully represent Σ_*^0, the relation in Equation (1) can be explained by this universal MOND relation.

It has been suggested that a wide range of observational data, including the rotation curves of galaxies and the Tully-Fisher relation, are consistent with MOND's predictions but not for the cold dark matter (CDM) model (Milgrom 1983; Sanders 1999; Sanders & McGaugh

2002; McGaugh 2012). However, recent data from gravitational lensing and hot gas in clusters challenge the original idea of MOND and the relativistic version of MOND theory (the tensor vector scalar (TeVeS) model) without any dark matter. For example, Angus (2009); Natarajan & Zhao (2008) show that the missing mass in galaxy clusters cannot be explained by MOND unless $> 10\,\mathrm{eV}$ massive neutrinos exist. Also, based on gravitational lensing data, Ferreras et al. (2012) show that CDM would be required even within the MOND/TeVeS framework. Therefore, generally speaking, MOND is valid only at small scales (such as dwarf and small galaxies). However, MOND is suggested to be a universal theory that can replace the CDM model on all scales. Therefore, it should also work well in galaxy clusters, but not only on the galactic scale. Otherwise, it is reasonable to suspect that MOND is not a universal theory. Previous studies claim that CDM works well on a large scale but not on a small scale (de Blok 2010; Boylan-Kolchin et al. 2011; Burkert 2015; Bull et al. 2016; Del Popolo & Le Delliou 2017). This provides room to invoke other new physics such as MOND to study the missing mass on small scales. Therefore, if CDM also works well on small scales, then we need not invoke new physics to address the missing mass problem. Previously, Kaplinghat & Turner (2002) suggested that the MOND theory may just be a misleading coincidence. Also, Dunkel (2004) and Chan (2013a) show that the generalized MOND equation can be derived from Newtonian dynamics for some specified dark matter contribution. In this article, we show that the observed central-surface-densities relation can also be derived by using the dark matter framework. We conclude that the central-surface-densities relation also supports the dark matter paradigm.

2 THE CENTRAL-SURFACE-DENSITIES RELATION

Before a galaxy evolves into a disk galaxy, the baryonic component is self-interacting. Since collision between baryonic matter is vigorous, the baryonic distribution would be close to an isothermal distribution (Evans et al. 2009). The effect of gravity by the baryonic component can be analyzed by using the steady-state Jeans equation (Evans et al. 2009)

$$\frac{d(\rho_\mathrm{B}\sigma^2)}{dr} = -\rho_\mathrm{B}\frac{d\psi}{dr}, \tag{3}$$

where ρ_B is the baryonic mass density, σ is the velocity dispersion of baryonic matter and ψ is the total gravitational potential (including baryonic matter and dark matter). Note that the Jeans equation used here assumes isotropy and spherical symmetry. Nevertheless, it can also be applied in spiral galaxies if we only focus on the cylindrical radial direction in the galactic plane. The velocity dispersion would be directly proportional to the rotational velocity (Croton 2009). Since the isothermal distribution of baryons corresponds to the constant velocity dispersion σ, by Equation (3), we get (Chan 2013a; Evans et al. 2009)

$$\sigma^2\frac{d\rho_\mathrm{B}}{d\psi} + \rho_\mathrm{B} = 0. \tag{4}$$

By substituting the solution of the above equation

$$\psi = \psi_0 - \sigma^2 \ln\rho_\mathrm{B}$$

into the Poisson equation, the total mass density profile is

$$\rho = -\frac{\sigma^2}{4\pi G}\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\ln\rho_\mathrm{B}}{dr}\right)\right]. \tag{5}$$

Let

$$\gamma = -d\ln\rho_\mathrm{B}/d\ln r,$$

and we find

$$\frac{\gamma}{r^2} + \frac{1}{r}\frac{d\gamma}{dr} = \frac{4\pi G\rho}{\sigma^2}. \tag{6}$$

Since the isothermal baryonic component yields $\gamma = 2$, we derive (Chan 2013a)

$$\rho = \frac{\sigma^2}{2\pi G r^2}. \tag{7}$$

This shows that if the baryonic component follows the isothermal distribution, the resulting total matter density also follows the isothermal distribution, which exhibits excellent agreement with observational data $d\ln\rho/d\ln r \approx 2$ (Koopmans et al. 2009; Velander et al. 2011; Grillo 2012). Note that we did not assume any isothermality of dark matter in the first place. We only assume baryons follow an isothermal distribution. Finally, based on the above derivation, the total potential that can yield the isothermal distribution of baryons is also isothermal. However, astrophysicists usually assume that dark matter follows the Navarro-Frenk-White (NFW) profile (Navarro et al. 1997), but not an isothermal profile. Even for some CDM simulations with baryons, no isothermal profile would be generated. Most of the resultant profiles are still an NFW profile, but with a shallower slope near the center (Schaller et al. 2015; Chan et al. 2015). This is because all of the simulations do not assume an isothermal distribution of baryons. Feedback can substantially change the stellar and dark matter dynamics and shape, causing the complete system to depart from a simple isothermal profile. This may be a reason why most CDM simulations do not generate an isothermal spherical distribution. In other words, if the collision between the baryonic matter is vigorous enough that the baryonic distribution can keep the isothermal distribution, the dark matter density and the total density also follow an isothermal profile. This is a result of solving the Jeans equation.

In addition, the above result can also give a simple explanation to the 'Halo-disk conspiracy problem' (why the transition from disk to halo domination is so smooth) (Battaner & Florido 2000; Remus et al. 2013). Since the total mass density consists of two components, the baryonic matter density $\rho_{\rm B}$ and dark matter density $\rho_{\rm D}$, we can write

$$\rho = \rho_{\rm B} + \rho_{\rm DM} = \frac{\sigma_{\rm B}^2}{2\pi G r^2} + \frac{\sigma_{\rm DM}^2}{2\pi G r^2}. \tag{8}$$

Here, we have used two parameters to represent the velocity dispersion of baryonic matter (i.e. $\sigma^2 = \sigma_{\rm B}^2 + \sigma_{\rm DM}^2$). For a baryon dominated galaxy, we have $\sigma \approx \sigma_{\rm B}$. For a dark matter dominated galaxy, we have $\sigma \approx \sigma_{\rm DM}$. Note that $\sigma_{\rm DM}$ is not the velocity dispersion of dark matter particles. The value of $\sigma_{\rm DM}$ is the limit for the velocity dispersion of baryonic matter in a dark matter dominated galaxy. On the other hand, since the observational data in galaxies strongly support the existence of a core in the dark matter density profile (de Blok 2010), we may slightly modify the dark matter density profile without destroying the isothermal distribution at large r by a cored-isothermal profile (Chan 2013a)

$$\begin{aligned} \rho_{\rm DM} &= \frac{\sigma_{\rm DM}^2}{2\pi G(r^2 + r_{\rm c}^2)} \\ &= \frac{\rho_{\rm c}}{1 + (r/r_{\rm c})^2}, \end{aligned} \tag{9}$$

where $\rho_{\rm c}$ and $r_{\rm c}$ are the central density and core radius of the dark matter profile respectively. The origin of a dark matter core (size $\sim$ kpc) may be due to the self-interaction between dark matter particles (Spergel & Steinhardt 2000; Vogelsberger et al. 2012; Chan 2013b) or some baryonic feedback such as supernovae (de Blok 2010; Governato et al. 2012). In fact, the existence of a dark matter core is still a controversial issue. Some studies claim that cores do not exist in galaxies (Fattahi et al. 2016). However, most observational data seem to favor the existence of dark matter cores (for a review, please see de Blok 2010; Bull et al. 2016; Del Popolo & Le Delliou 2017). Therefore, our model basically follows this assumption. Since the Jeans equation only describes the distribution of matter due to gravitational interaction, the effect of core formation due to other mechanisms at small r is not included. The small modification of the profile here is necessary to match the assumption of the existence of dark matter cores.

Recent studies suggest that the product of the central density and core radius of dark matter is almost a constant for many galaxies ($\rho_{\rm c} r_{\rm c} \equiv \Sigma_1 \sim 100\, M_\odot\,{\rm pc}^{-2}$) (Gentile et al. 2009; Burkert 2015). Therefore, we can write

$$\rho_{\rm c} r_{\rm c} = \sigma_{\rm DM}^2/(2\pi G r_{\rm c}) = \Sigma_1.$$

By integrating the dark matter density in Equation (9) for $r \le r_{\rm c}$, the enclosed 'dark matter core mass' is $M_{\rm c} = 0.429\sigma_{\rm DM}^2 r_{\rm c}/G$. By using this relation, we get

$$\sigma_{\rm DM}^4 = \Sigma_2 G^2 M_{\rm c}, \tag{10}$$

where

$$\Sigma_2 = 2\pi\Sigma_1/0.429 \sim 1500\, M_\odot\,{\rm pc}^{-2}.$$

Let $M_{\rm c} = k_1 M_{\rm DM}$ and $M_{\rm B} = k_2 M_{\rm DM}$, where $M_{\rm DM}$ is the total dark matter mass and $M_{\rm B}$ is the total baryonic mass. Here, we assume that the values of $k_1 \sim 0.1$ (Rocha et al. 2013) and $k_2 \sim 0.2$ (Kassin et al. 2006) are almost constant for all galaxies. The value of k_1 can be approximately calculated by using Equation (9) and the virial radius of a typical galaxy. The value of k_2 is close to the cosmological ratio of baryon to dark matter. Therefore, we have

$$\sigma_{\rm DM}^4 = \Sigma_2 G^2 \left(\frac{k_1}{k_2}\right) M_{\rm B} = \Sigma_3 G^2 M_{\rm B}, \tag{11}$$

where

$$\Sigma_3 = \Sigma_2 (k_1/k_2) \sim 730\, M_\odot\,{\rm pc}^{-2}.$$

The above equation is a traditional form of the Tully-Fisher relation ($M_{\rm B} \propto \sigma^4$) (McGaugh 2005). Therefore, we predict that $M_{\rm B} \propto \sigma^4$ if the galaxy is dark matter dominated since $\sigma \approx \sigma_{\rm DM}$. If the galaxy is baryonic matter dominated, $\sigma \approx \sigma_{\rm B} \propto M_{\rm B}^2$.

By combining Equation (8) and Equation (11), we have

$$\rho = \rho_{\rm B} + \frac{\sqrt{\Sigma_3 G^2 M_{\rm B}}}{2\pi G(r^2 + r_{\rm c}^2)}. \tag{12}$$

Using the definition of the dynamical central surface density (Milgrom 2016), we get

$$\begin{aligned}\Sigma_{\rm D}^0 &= 2\int_0^\infty \rho^0(z)dz \\ &= \Sigma_{\rm B}^0 + 2\int_0^\infty \frac{\sqrt{\Sigma_3 G^2 M_{\rm B}}}{2\pi G(z^2+z_{\rm c}^2)}dz, \qquad (13)\end{aligned}$$

where $\rho^0(z) = \rho(R = 0, z)$ is the central total mass density. Here, we work in cylindrical coordinates (R, z) with the z-axis along the axisymmetry axis, and $z_{\rm c} = r_{\rm c}(R = 0)$. By integrating the above equation, we get $\Sigma_{\rm D}^0 = \Sigma_{\rm B}^0 + \sqrt{\Sigma_3 G^2 M_{\rm B}}/2Gz_{\rm c}$. Let $z_{\rm c} = az_0$, where z_0 is the characteristic disk length such that the Newtonian gravitational acceleration due to baryons just outside the disk is $g_{\rm B} = GM_{\rm B}/z_0^2$. Finally, we get

$$\Sigma_{\rm D}^0 = \Sigma_{\rm B}^0 + \frac{1}{a}\sqrt{\frac{\Sigma_3\pi}{2}}\sqrt{\Sigma_{\rm B}^0}, \qquad (14)$$

where $\Sigma_{\rm B}^0 = (2\pi G)^{-1}g_{\rm B}$ in Milgrom (2016). Based on the derivation in QUMOND, $\Sigma_{\rm M} = 138\,M_\odot\,{\rm pc}^{-2}$ for $a_0 = 1.2\times10^{-8}\,{\rm cm\,s^{-2}}$ (Milgrom 2016). Taking $\Sigma_3 = 730\,M_\odot\,{\rm pc}^{-2} = 5.29\Sigma_{\rm M}$, we can write the above equation in terms of $\Sigma_{\rm M}$

$$\Sigma_{\rm D}^0 = \Sigma_{\rm B}^0 + \left(\frac{1.45}{a}\right)\sqrt{4\Sigma_{\rm B}^0\Sigma_{\rm M}}. \qquad (15)$$

Since $a \sim 1$ ($z_{\rm c} \sim z_0 \sim 1\,{\rm kpc}$), we have $\Sigma_{\rm D}^0 \approx \Sigma_{\rm B}^0 + \sqrt{4\Sigma_{\rm B}^0\Sigma_{\rm M}}$. For $\Sigma_{\rm B}^0 \gg \Sigma_{\rm M}$ (baryonic matter dominates the galaxy), we have $\Sigma_{\rm D}^0 \approx \Sigma_{\rm B}^0$. The opposite asymptote ($\Sigma_{\rm B}^0 \ll \Sigma_{\rm M}$) gives $\Sigma_{\rm D}^0 \approx (4\Sigma_{\rm M}\Sigma_{\rm B}^0)^{1/2}$. These asymptotic results are identical to the results in (Milgrom 2016). The only difference is the functional form. Milgrom's result gives $\Sigma_{\rm D}^0 = \Sigma_{\rm M}S(\Sigma_{\rm B}^0/\Sigma_{\rm M})$, where $S(y) = \int_0^y \nu(y')dy'$ (Milgrom 2016).

In Figure 1, we fit our result with the observed data obtained from Lelli et al. (2016) and compare with Milgrom's result. Here, we assume that Σ_*^0 is the proxy for $\Sigma_{\rm B}^0$ ($\Sigma_*^0 = \Sigma_{\rm B}^0$) (Milgrom 2016). Both theories can give the same agreement with observational data (our model with $a = 2.9$ gives a better fit). Therefore, we conclude that the observational data also support the dark matter paradigm if dark matter has an isothermal distribution, but not only using MOND theory.

3 DISCUSSION

In this article, we derive the central-surface-densities relation by using the steady-state Jeans equation in the dark matter framework. If the baryonic density distribution is isothermal, the resultant total mass density also follows an isothermal distribution. This result agrees with the observational data and explains why rotational curves are flat for many galaxies (Sofue & Rubin 2001). We also relate the dark matter density profile with the baryonic matter content and show that $\Sigma_{\rm D}^0 \approx \Sigma_{\rm B}^0 + \sqrt{4\Sigma_{\rm B}^0\Sigma_{\rm M}}$. The asymptotic relations for both regimes are identical to Milgrom's result. Milgrom (2016) claims that there is no reason why $\Sigma_{\rm D}^0$ is so well correlated with local $\Sigma_{\rm B}^0$ in the dark matter paradigm. However, as shown in our derivation, the existence of a dark matter core may give a reason why these quantities are correlated. Therefore, the claim in Milgrom (2016) is wrong. Generally speaking, both MOND and dark matter paradigms can give the same agreement with the observed central-surface-densities relation. In fact, some recent studies also show a similar conclusion by using CDM models (Di Cintio & Lelli 2016; Navarro et al. 2016).

In the derivation, there are a few constants involved: Σ_1, k_1, k_2 and a. Although these values are not universal constants for all galaxies, the ranges of these values are quite narrow (Gentile et al. 2009; Rocha et al. 2013; Kassin et al. 2006). For example, the value of $\Sigma_1 = 141^{+82}_{-52}\,M_\odot\,{\rm pc}^{-2}$ is nearly a constant for a luminosity range of 14 magnitudes and the whole Hubble sequence (Gentile et al. 2009). The variations in these constants among different galaxies would contribute to the scatter in the resulting relation.

However, as mentioned in the introduction, MOND works very poorly in galaxy clusters. Most predictions for galaxy clusters in MOND theory do not match the observational data, including the lensing results (Ferreras et al. 2012). Although the failure of MOND on large scale is not related to our discussion here, MOND, suggested to be a universal theory, should work on all scales. Therefore, it is reasonable to suspect that MOND working well in galaxies is just a coincidence. If the observational data in galaxies support both paradigms but MOND does not work for galaxy clusters, it is reasonable to deny MOND is an effective theory to explain the missing mass problem. Moreover, Chan (2013a) shows that MOND is equivalent to a particular form of dark matter density profile (isothermal distribution) in the dark matter model. It also explains why MOND works in galaxies but not in galaxy clusters (Chan 2013a). Although many studies have shown that the CDM model predictions for small scale structures (e.g. in dwarf galaxies) do not agree with observations (de Blok 2010), recent studies have started to realize that baryonic feedback might be an important mechanism to reconcile the discrepancies

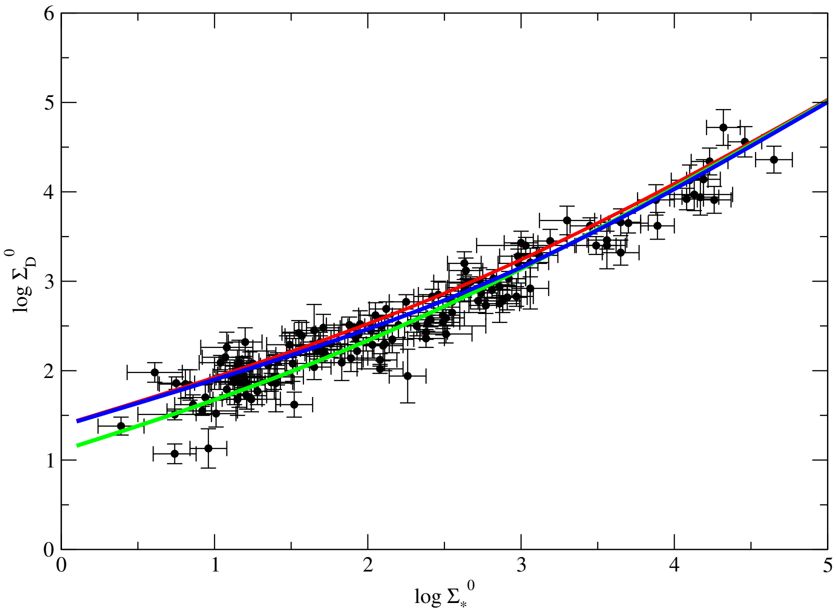

Fig. 1 The resulting central-surface-densities relation (log $\Sigma_{\rm D}^0$ vs. log Σ_*^0) and the observed data with error bars (Lelli et al. 2016). Here, we assume that Σ_*^0 is the proxy for $\Sigma_{\rm B}^0$ ($\Sigma_*^0 = \Sigma_{\rm B}^0$) (Milgrom 2016). Red solid line: the relation in Eq. (15) with $a = 1$. Green solid line: the relation in Eq. (15) with $a = 2.9$. Blue solid line: Milgrom's relation (Milgrom 2016). The units of the central surface densities are in $M_\odot$ pc^{-2}.

between theory and observations (Macciò et al. 2012; Peñarrubia et al. 2012; Pontzen & Governato 2014). Our result basically supports this argument. Provided the dark matter and baryon distributions are described by an isothermal sphere profile, the dark matter can also accommodate the missing mass problem. It is therefore not essential to invoke new physics (MOND) to address the current missing mass problem.

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References

Angus, G. W. 2009, MNRAS, 394, 527
Battaner, E., & Florido, E. 2000, Fund. Cosmic Phys., 21, 1
Boylan-Kolchin, M., Bullock, J. S., & Kaplinghat, M. 2011, MNRAS, 415, L40
Bull, P., Akrami, Y., Adamek, J., et al. 2016, Physics of the Dark Universe, 12, 56
Burkert, A. 2015, ApJ, 808, 158
Chan, M. H. 2013a, Phys. Rev. D, 88, 103501
Chan, M. H. 2013b, MNRAS, 433, 2310
Chan, T. K., Kereš, D., Oñorbe, J., et al. 2015, MNRAS, 454, 2981
Croton, D. J. 2009, MNRAS, 394, 1109
de Blok, W. J. G. 2010, Advances in Astronomy, 2010, 789293
Del Popolo, A., & Le Delliou, M. 2017, Galaxies, 5, 17
Di Cintio, A., & Lelli, F. 2016, MNRAS, 456, L127
Dunkel, J. 2004, ApJ, 604, L37
Evans, N. W., An, J., & Walker, M. G. 2009, MNRAS, 393, L50
Fattahi, A., Navarro, J. F., Sawala, T., et al. 2016, arXiv:1607.06479
Ferreras, I., Mavromatos, N. E., Sakellariadou, M., & Yusaf, M. F. 2012, Phys. Rev. D, 86, 083507
Gentile, G., Famaey, B., Zhao, H., & Salucci, P. 2009, Nature, 461, 627
Governato, F., Zolotov, A., Pontzen, A., et al. 2012, MNRAS, 422, 1231
Grillo, C. 2012, ApJ, 747, L15
Kaplinghat, M., & Turner, M. 2002, ApJ, 569, L19
Kassin, S. A., de Jong, R. S., & Weiner, B. J. 2006, ApJ, 643, 804
Koopmans, L. V. E., Bolton, A., Treu, T., et al. 2009, ApJ, 703, L51
Lelli, F., McGaugh, S. S., Schombert, J. M., & Pawlowski, M. S. 2016, ApJ, 827, L19
Macciò, A. V., Stinson, G., Brook, C. B., et al. 2012, ApJ, 744, L9

McGaugh, S. S. 2005, ApJ, 632, 859
McGaugh, S. S. 2012, AJ, 143, 40
Milgrom, M. 1983, ApJ, 270, 365
Milgrom, M. 2016, Physical Review Letters, 117, 141101
Natarajan, P., & Zhao, H. 2008, MNRAS, 389, 250
Navarro, J. F., Benítez-Llambay, A., Fattahi, A., et al. 2016, arXiv:1612.06329
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Peñarrubia, J., Pontzen, A., Walker, M. G., & Koposov, S. E. 2012, ApJ, 759, L42
Pontzen, A., & Governato, F. 2014, Nature, 506, 171
Remus, R.-S., Burkert, A., Dolag, K., et al. 2013, ApJ, 766, 71
Rocha, M., Peter, A. H. G., Bullock, J. S., et al. 2013, MNRAS, 430, 81
Sanders, R. H. 1999, ApJ, 512, L23
Sanders, R. H., & McGaugh, S. S. 2002, ARA&A, 40, 263
Schaller, M., Frenk, C. S., Bower, R. G., et al. 2015, MNRAS, 451, 1247
Sofue, Y., & Rubin, V. 2001, ARA&A, 39, 137
Spergel, D. N., & Steinhardt, P. J. 2000, Physical Review Letters, 84, 3760
Velander, M., Kuijken, K., & Schrabback, T. 2011, MNRAS, 412, 2665
Vogelsberger, M., Zavala, J., & Loeb, A. 2012, MNRAS, 423, 3740