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Linear nonadiabatic pulsation models of the double-mode Cepheid V371 Per

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Abstract A recently found double-mode Cepheid with the shortest known period in our Galaxy and abnormal period ratio, V371 Per, is investigated by linear nonadiabatic pulsation models. V371 Per is likely to be crossing the instability strip for the first time, because the mass derived from pulsation models is larger than the evolution mass for the second or higher crossing objects. This result seems to support the conclusion obtained from spectroscopic observation. We also found that models with observed period and period ratio of V371 Per need to have mass and T_{eff} in a narrow range which shifts as heavy element abundance Z changes. We have checked the agreement between T_{eff} ranges estimated observationally and derived from pulsation models using observational Z. We found that those ranges overlap marginally. We need more spectroscopic estimations of T_{eff} and [Fe/H], and more photometric monitoring to estimate the evolutional period change for confirmation of our result.

Key words: stars: oscillations (including pulsations) — stars: variables: Cepheids — stars: individual (V371 Per)

1 INTRODUCTION

Double-mode Cepheids are a subgroup of classical Cepheids, which pulsate in two radial pulsation modes simultaneously (for a review article, see for example Petersen & Takeuti (2001) and references therein). These stars are checked for the difference from single-mode Cepheids in many aspects, however no difference has been reported up to now. We can interpret that double-mode Cepheids are normal classical Cepheids, which happen to pulsate in two pulsation modes. There are 22 clearly established Galactic members of this subgroup. The period ratios of the two periods are from 0.6967 to 0.7195 with two exceptions with ratios around 0.80 (CO Aur and HD304373), which are considered as first overtone and second overtone pulsators. The periods of the fundamental mode for the other 20 stars are between 2.139 and 6.293 d.

Recently, V371 Per was reported as a new candidate member of this subgroup (Wils et al. 2010). V371 Per was classified as an irregular variable by Weber (1964), but Schmidt et al. (1995) suggested this star to be a double-mode Cepheid from the detected period and excessive scatter in the light curve. Thus, Wils et al. (2010) made new observations and indicated that this star is indeed a double-mode Cepheid with the shortest fundamental mode period of 1.738 d and unusually high period ratio of 0.731. They discussed that the high value of the period ratio may indicate that the metallicity of this star is much lower than the other Galactic double-mode Cepheids from the relation between period and period ratio versus metallicity Z discussed by Buchler & Szabó (2007). They also pointed out that this star is in the Thick Disk or the Halo of our Galaxy. Furthermore, Kovtyukh et al. (2012, 2016) discussed that V371 Per can be considered as a first crossing object from its peculiar abundances, especially from the richness of lithium.

Wils et al. (2010) further discussed some properties of this star, however, the properties they deduced from pulsation models are not convincing because of the lack of computations with appropriate parameters due to its short period and other unusual properties. We would like to present here some results on linear nonadiabatic pulsation models with special interest in this unusual star.

2 MODELS

For calculating the linear non-adiabatic period, we used the same code that was used in Ishida (1995), namely, a Castor (1971) type procedure with OPAL opacity tables (Iglesias et al. 1992; Rogers & Iglesias 1992). The effect of convection does not seem efficient for our purpose, as the derived metallicity for V367 Sct and Y Car using linear non-adiabatic calculations in Ishida (1995) is not far from recent observations (Andrievsky et al. 2002). Therefore, the effect of convective energy transport is not included in all of the models.

The luminosity of the model is estimated from Fouqué et al. (2007)'s period-luminosity relation for Galactic Cepheids and Torres (2010)'s polynomial fits for Flower (1996)'s bolometric correction. The luminosity of V371 Per is estimated to be $460\,L_\odot$. Considering differences between distance derived from the period-luminosity relations in the different photometric bands, $L/L_\odot = 500$ and 420 models are also examined.

The effective temperature of V371 Per is reported to be 6000 K or lower from unreddened $B-V$ by Wils et al. (2010), 6215 K from the average of low-resolution spectra by Schmidt (2011), and 5950∼6213 K from high resolution spectra by Kovtyukh et al. (2012). Taking spectral classification as G0 by Bond (1978) into account, we set a wide parameter range for effective temperatures from 7000 to 5400 K with a difference of 100 K.

Observational estimates of the iron abundance are $[\mathrm{Fe/H}] = -0.05$ to -0.40 from low-resolution spectra by Schmidt (2011), and $[\mathrm{Fe/H}] = -0.42$ from high-resolution spectra by Kovtyukh et al. (2012). Thus, chemical compositions of the calculated models are $Z = 0.001, 0.004, 0.01, 0.02$ and 0.03.

The mass of the models is changed from 1.5 to $6.0\,M_\odot$ with a difference of 0.5 $M_\odot$. The selected parameter range is summarized in Table 1.

Table 1 Summary of the Model Parameters

Parameter					
Luminosity ($L_\odot$)		420, 460, 500			
T_{eff}		7000 ∼ 5400			
X		0.70			
Z	0.001	0.004	0.01	0.02	0.03
Mass ($M_\odot$)		1.5 ∼ 6.0			

3 RESULTS OF THE LINEAR NONADIABATIC MODELS

As a typical example, the results for $L = 460\,L_\odot$, $Z = 0.004$ are summarized in Figure 1.

The observed period of 1.737 d and period ratio of 0.731 are realized around $M = 3.35\,M_\odot$ and $T_{\mathrm{eff}} = 6020$. In $L = 420\,L_\odot$ models, both of the instability edges move slightly to the higher-temperature direction, the constant period lines move to the lower-temperature direction, and the constant period ratio lines move to the lower-mass direction. Thus, the observed period and period ratio are realized at lower-mass and lower-temperature points than the $L = 460\,L_\odot$ model. In $L = 500\,L_\odot$ models, changes are in opposite directions and the observed period and period ratio are realized at higher-mass and higher-temperature points than the $L = 460\,L_\odot$ models.

The results for the $L = 460\,L_\odot$ and $Z = 0.02$ models are shown in Figure 2. For comparison with Figure 1, we need to remark that the presented mass range is changed from $1.5 \sim 5.0\,M_\odot$ for $Z = 0.004$ models to $4.0 \sim 6.0\,M_\odot$ for $Z = 0.02$ models. In the $Z = 0.02$ models, both of the instability blue edges move to the lower temperature direction, the constant period lines move to the higher-temperature direction, and the constant period ratio lines move to the higher-mass direction, compared to the $Z = 0.004$ models. It is also worth noting that the slope of the constant period ratio lines is steeper in the $Z = 0.02$ models. Thus, the observed period and period ratio is realized at higher-mass and lower-temperature points than the $Z = 0.004$ model, namely, around $M = 5.22\,M_\odot$ and $T_{\mathrm{eff}} = 5600$.

Derived mass and effective temperature that can realize the observed period and period ratio are summarized in Figures 3 and 4. As pointed out in the description of Figure 2, the mass of the satisfactory model increases as Z increases, and T_{eff} of that decreases. We need to remark that both fundamental and first overtone modes are pulsationally stable in the $Z = 0.001$ model.

4 DISCUSSION

As shown in Figures 3 and 4, the observed period and period ratio can be realized with a wide range of chemical composition, however, the mass and effective temperature need to change with chemical composition. Namely, the model with observed period and period ratio should be in a narrow band on the mass and T_{eff} versus Z planes.
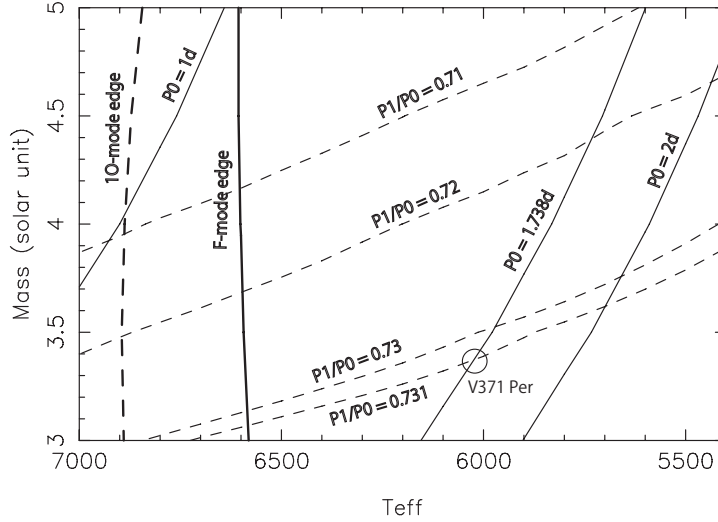
Fig. 1 Contour plot of the fundamental period, period ratio and blue edges on the $T_{\rm eff}$ vs. Mass plane for V371 Per corresponding to $L = 460\ L_\odot$ and $Z = 0.004$ models. Abscissa is the effective temperature and ordinate is the mass in solar units. Solid lines are fundamental period contours with corresponding period values on the side of each line. Dashed lines are period ratio contours. Notifications are the same as the fundamental period. Thick lines are instability blue edges for the fundamental (*solid*) and first overtone (*dashed*). The position of the model parameter satisfies all related criteria, namely, the observed period, the observed period ratio, and both the fundamental and first overtone modes are pulsationally unstable, as indicated by an open circle.
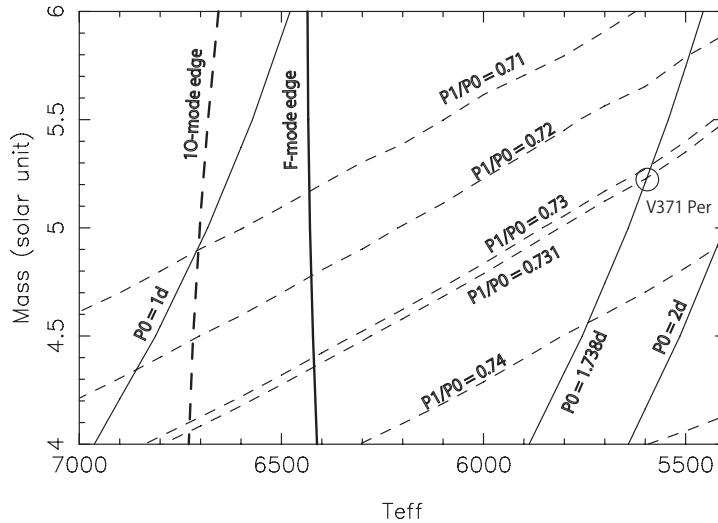

Fig. 2 Contour plot of the fundamental period, period ratio and blue edges on $T_{\rm eff}$ vs. Mass plane of V371 Per corresponding to $L = 460\ L_\odot$ and $Z = 0.02$ models. Notations are the same as in Fig. 1.

Therefore, when we fix Z to the observationally estimated value, whether the observationally estimated $T_{\rm eff}$ is in the range derived from the pulsation model or not will be a check of the agreement between observation and pulsation theory. From four high-resolution spectra, Kovtyukh et al. (2012) derived [Fe/H] $= -0.42$ and $T_{\rm eff} = 5950 \sim 6213\,{\rm K}$. The simple conversion from [Fe/H] to Z indicates $Z \sim 0.007$. From Figure 4, we can estimate $T_{\rm eff} = 5700 \sim 5900\,{\rm K}$. Considering the above observed $T_{\rm eff}$ range is from only four spectra, we can note that the observations and our estimation from pulsation models marginally overlap. More observational estimations of $T_{\rm eff}$ and chemical composition are needed to confirm whether our results agree with observations.
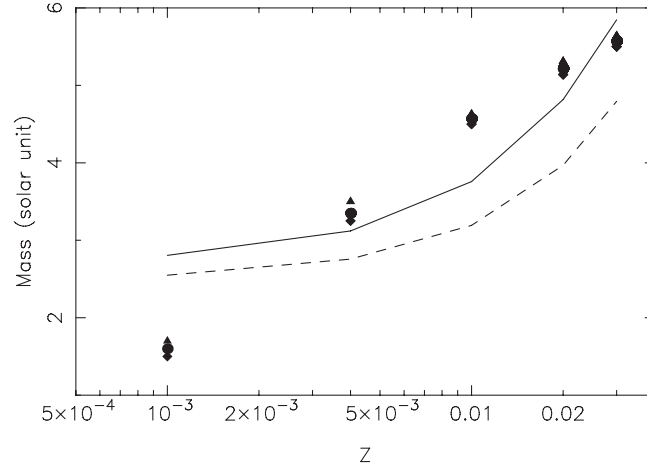
Fig. 3 Derived model mass of V371 Per that can realize the observed period and period ratio. Abscissa is the heavy element abundance Z of the models. Ordinate is the derived mass in solar units. Results from the $L = 460\ L_\odot$ models are plotted by dots. $L = 500\ L_\odot$ and $L = 420\ L_\odot$ models are also plotted by filled triangles and filled diamonds, respectively. Masses estimated from Becker et al. (1977)'s mass-luminosity relations for the first crossing are shown with the solid line, and those for the second crossing are shown with the dashed line.
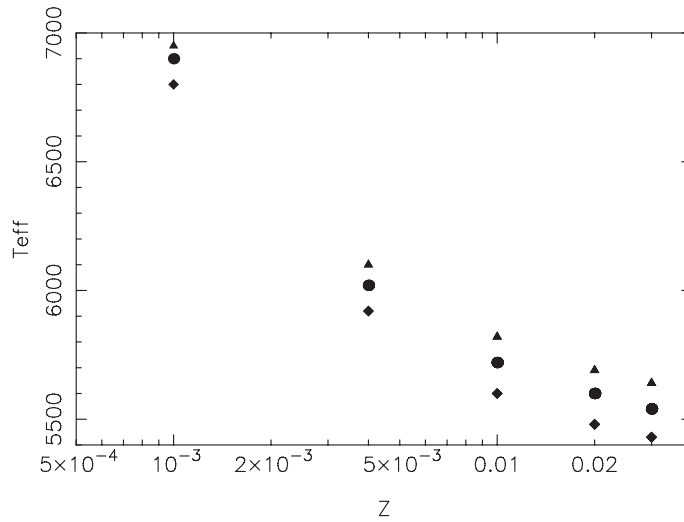

Fig. 4 The derived model's effective temperature of V371 Per that can realize the observed period and period ratio. Abscissa is the same as Fig. 3. Ordinate is the effective temperature. Marks used in this plot are the same as in Fig. 3.

In the low Z models, helium content Y used here is especially high. Therefore, additional calculations for $(X, Z) = (0.80, 0.01)$ models are performed to confirm the effect of high Y values. The process to search for mass and T_{eff} that satisfy the observed properties of V371 Per is the same as that used in the previous section. The obtained mass is $0.2\ M_\odot$ larger than that obtained for $(X, Z) = (0.70, 0.01)$ models. The obtained T_{eff} is almost unchanged. Thus, the result presented in the pre-

vious section will not be largely affected by using high Y values.

The effect of inclusion of convection on the fundamental period and period ratio of the first overtone mode to the fundamental mode is investigated by Saio et al. (1977). Their result indicates that the fundamental period becomes slightly longer and the period ratio becomes smaller as the mixing length l/H_p becomes larger. On Figures 1 and 2, inclusion of the convection will shift the constant period lines slightly to the higher-mass direction

and shift the constant period ratio lines to the lower-mass direction. Because the effect on period ratio is larger than the effect on period, the position of the model with observed period and period ratio will shift to the lower-mass direction, namely the derived mass will be smaller and derived effective temperature will be higher. Therefore, inclusion of convection will produce better agreement between observation and our estimation from pulsation models for T_{eff} discussed in the previous paragraph.

In Figure 3, the mass for the calculated luminosity derived from the evolutional relations by Becker et al. (1977) is also plotted. The masses estimated by the mass-luminosity relation for the second crossing objects are shown by the dashed line and those for the first crossing objects are shown by the solid line. Our results indicate that the pulsationally derived mass is larger than the second crossing mass in almost all ranges of Z. It is even larger than the first crossing mass from about $Z = 0.004$ to 0.02. At the observed chemical composition $Z \sim 0.007$, the mass estimated from our results is $\sim 4.1\, M_\odot$. Taking the effect of convection discussed in the previous paragraph into account, the excess over the first crossing mass will be reduced. Thus, our result from pulsation models seems to indicate that V371 Per is too massive for the second or higher crossing object, and is likely to be the first crossing object as discussed by Kovtyukh et al. (2012, 2016) from spectroscopic observations.

More modern evolution calculation with OPAL opacity is presented by Bono et al. (2000) and they obtained a mass-luminosity relation for classical Cepheids. Although their relation is for the second and third crossing objects, and for objects in the mass range 4–$15\, M_\odot$, the estimated mass for $L = 460\, L_\odot$ with $(Y, Z) = (0.293, 0.007)$ is about $3.34\, M_\odot$ from their relation. This estimation is less massive than the mass estimated from our results in the previous paragraph, and seems to support that V371 Per is a first crossing object. It is also worth noting that the $4.0\, M_\odot$ model in Bono et al. (2000)'s table 7 with $(Y, Z) = (0.23, 0.004)$ has similar first crossing luminosity $\log L/L_\odot = 2.620\,(416\, L_\odot)$ to the present model $460\, L_\odot$ with shorter period $\log P = 0.0171\,(1.040\,\mathrm{d})$ at higher temperature $\log T = 3.808$ (6426 K). Our consequence that V371 Per is likely to be a first crossing object also seems valid with state-of-the-art evolution calculation.

Wils et al. (2010) reported that both the fundamental and first overtone mode frequencies are decreased. Frequency decrease means period increase. Period increase is realized when the Cepheid is the 1st, 3rd or 5th crossing of the instability strip. Effects of the chemical composition and periods on the evolutionary period change of Cepheids was already investigated by Saitou (1989) using Becker et al. (1977)'s evolution calculation. His results indicate that evolutionary period changes for the 1st, 4th and 5th crossings are larger than those for the 2nd and 3rd crossings, especially in the short period region. He also derived abundance dependent period change-period relations for each crossing. We can estimate the period change for V371 Per to be about $\log[\Delta P/P]_{100} = -3.11$ from Wils et al. (2010)'s figure 5. On the other hand, we can estimate from Saitou (1989)'s fittings that $\log[\Delta P/P]_{100} \sim -2.88$ for the first crossing and ~ -4.67 for the 3rd crossing with $Z = 0.007$. Consequently, the observed period change seems to fit the first crossing. Recently, evolutionary period change was also discussed by Pietrukowicz (2003) and Turner et al. (2006). We can convert the above period change to $\log \dot{P} \sim 0.063$ and $\dot{P}/P^2 \sim 12.14$. Both values put V371 Per in the first crossing region of figure 2 of Pietrukowicz (2003) and figure 3 of Turner et al. (2006). Therefore, the period change reported by Wils et al. (2010) seems to support our result that V371 Per is a first crossing object derived from linear pulsation models. However, Wils et al. (2010) reported that the period change of V371 Per should be treated with caution, because this change highly depends on photographic data. Thus, we need more photometric monitoring of this interesting and unusual Cepheid and more precise estimation of its period change.

5 CONCLUSIONS

We have investigated the recently found double-mode Cepheid V371 Per by linear nonadiabatic pulsation models. Our result indicates that V371 Per is likely to be a first crossing object, because the mass derived from linear pulsation results is larger than the evolutionary mass for the second crossing object. Thus, our result obtained from pulsation models seems to support the conclusion from the spectroscopic observation by Kovtyukh et al. (2012, 2016). Decreases of the frequencies reported by Wils et al. (2010) seem to support this consequence. We also found that models with observed period and period ratio of V371 Per need to have mass and T_{eff} in a narrow range which shifts as heavy element abundance Z changes. We have checked whether observational T_{eff} is in the range derived from pulsation models using obser-

vational Z or not. Two T_{eff} ranges are marginally overlapping, and agreement between observation and our result seems to be marginal. We need more spectroscopic estimations of T_{eff} and [Fe/H] to check our result, and more photometric monitoring to estimate the evolutional period change and thus confirm the evolutional status of V371 Per.

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