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Normal DGP in varying speed of light cosmology

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Abstract Varying speed of light (VSL) has been used in cosmological models in which the physical constants vary over time. On the other hand, the Dvali, Gabadadze and Porrati (DGP) brane world model, especially its normal branch, has been extensively discussed to justify the current cosmic acceleration. In this article we show that the normal branch of DGP in VSL cosmology leads to a self-accelerating behavior and therefore can interpret cosmic acceleration. Applying statefinder diagnostics demonstrates that our result slightly deviates from the ΛCDM model.

Key words: cosmology: theory — cosmology: miscellaneous

1 INTRODUCTION

We know that any physical theory consists of at least one or more free parameters, called fundamental constants. These parameters have been measured in observations and compared with theoretical predictions. Aside from some recent observational results which show the possibility of tiny variations in these constants, one can assume a varying constant theory and deal with its consequences (Chand et al. 2004; Bahcall et al. 2004; Drinkwater et al. 1998; Ubachs & Reinhold 2004; Petitjean et al. 2004; Bertotti et al. 2003).

Varying constant theories have been proposed and studied in literature. For instance, the Brans-Dicke gravity theory (Brans & Dicke 1961), which is an extension of the standard general theory of relativity, considers a varying Newtonian constant G by means of a scalar field. The Barrow-Magueijo theory (Barrow & Magueijo 2005) varies the electron-proton mass ratio $\mu \equiv m_{\rm e}/m_{\rm p}$, via a change in electron mass using a scalar field. The Bekenstein-Sandvik-Barrow-Magueijo scenario (Bekenstein 1982; Sandvik et al. 2002) considers variations in the fine structure constant α, driven again with a scalar field. Also, one model has recently attracted a great deal of attention, the varying speed of light (VSL) theory, which as a cosmological model may be considered as a competitor to inflation, since it can solve some cosmological problems and provides a theory of structure formation. One can regard the VSL theory (Moffat 1993; Magueijo 2000; Magueijo 2003; Barrow & Magueijo 2000) as a result of a varying-α theory, because of the relation between α and c, $\alpha = e^2/\hbar c$. If α varies, e, $\hbar$ or c, or a combination of them, has to be varied.

Although constancy of the speed of light is the foundation of the theory of relativity and apparently it has been verified through many experiments, such as the Michelson-Morely experiment, one can still consider a VSL theory in the sense that the results of such experiments must still hold at the appropriate scale in this part of the Universe and at this time.

On the other hand, a large amount of recent studies investigate the effects of extra dimensions in our Universe (Sami 2003; Farajollahi & Ravanpak 2011; Nojiri & Odintsov 2000; Bouhmadi-López et al. 2010). In the simplest model of higher dimensional gravity, called brane cosmology, we assume our four dimensional (4D) world to be a brane embedded in a five dimensional (5D) spacetime (Randall & Sundrum 1999b,a). The Dvali, Gabadadze and Porrati (DGP) model is a special case of brane cosmology in which the 4D Universe is embedded in a 5D Minkowskian bulk (Dvali et al. 2000). According to how one can embed the 4D brane into the 5D Minkowskian bulk, the DGP model includes two separate branches which are distinguished with a parameter $\epsilon = \pm 1$. The case $\epsilon = +1$ is dubbed a self-accelerating branch, since it can show late time acceleration without any dark energy component (Deffayet 2001; Deffayet et al. 2002). However, the case $\epsilon = -1$, called a normal branch, needs a dark energy component for late time acceleration. The most important feature of the DGP model is its self-accelerating branch which suffers from the ghost problem (Nicolis & Rattazzi 2004; Koyama & Maartens 2006). Thus, it will be very interesting if one can modify the normal branch in such a way that it becomes self-accelerating. Bouhmadi-López (2009), using a $f(R)$-brane in the DGP model, changed the normal branch to a self-accelerating one.

The effects of variation of physical constants, in the context of different higher dimensional theories, have been investigated in recent years. In Brax et al. (2003); Germani & Sopuerta (2003), varying constant theories in brane cosmology and in a string-inspired brane world model have been studied respectively. The varying-G scenario in brane cosmology is the main feature in de Leon (2002); Amarilla & Vucetich (2010). VSL in brane cosmology and in a brane-induced Friedmann-Robertson-Walker Universe has been studied, respectively, in Youm (2001) and Alexander (2000). Also, Steer & Parry (2002) examined VSL in a brane scenario from a different point of view. However, varying constant theories in the context of a DGP brane world model have not been investigated and the results and consequences of such a model are not clear yet.

In this manuscript we apply the VSL scenario in the DGP brane world cosmology. Our aim is to study the effect of this modification on the normal branch of the DGP model to find out if the integration of these two could lead the normal branch to be self-accelerating. This paper is outlined as follows. In Section 2, we obtain our model equations in the presence of a varying-c. We should note that variation can be spatial or temporal, or both. Here, we only discuss variation with respect to time. In Section 3, by assuming a widely used function for $c(t)$, we compare the normal DGP model in the presence of a constant c with a time dependent $c(t)$. We constrain our model parameters under which the normal branch will be self-accelerating in a varying-c theory. Section 4 includes conclusions and remarks.

2 DGP VARYING SPEED OF LIGHT THEORY

We start the DGP cosmologies within the framework of VSL theories with the metric

$$ds^2 = -n^2(t,y)c^2(t)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{1}$$

where γ_{ij} is the metric of a three dimensional maximally symmetric space with a constant curvature k, and x^i are the coordinates on the spatial slices. The $a(t,y)$ is the cosmological scale factor on the brane and $b(t,y)$ can be considered to be the scale factor along the extra dimension. Also, we have assumed that the speed of light is only a function of time, $c(t)$.

Since in the VSL theories the Lorentz invariance becomes clearly broken, it is postulated that there exists a preferred Lorentz frame in which the action is similar to a usual Lorentz invariant action with a constant c, replaced by a field $c(x^\mu)$. It is called the principle of minimal coupling. In other words, c varies in the local Lorentzian frames associated with cosmological expansion. This effect is a special relativistic effect and not a gravitational one. So, as proposed in Albrecht & Magueijo (1999), $c(t)$ does not introduce any corrections to the Einstein tensor for the above metric in this preferred frame and then we can derive the non-vanishing components of the 5D Einstein tensor as

$$G_{00} = 3\left[\frac{1}{c^2(t)}\frac{\dot{a}^2}{a^2} - n^2\left(\frac{a'^2}{a^2} + \frac{a''}{a}\right) + k\frac{n^2}{a^2}\right], \tag{2}$$

$$G_{ij} = \left[a^2\left(\frac{a'^2}{a^2} + 2\frac{a''}{a} + \frac{n''}{n} + 2\frac{a'n'}{na}\right) + \frac{a^2}{n^2c^2(t)}\left(-2\frac{\ddot{a}}{a} - \frac{a'^2}{a^2} + \frac{\dot{a}\dot{n}}{an}\right) - k\right]\gamma_{ij}, \tag{3}$$

$$G_{05} = \frac{3}{c(t)}\left(\frac{\dot{a}n'}{an} - \frac{\dot{a}'}{a}\right), \tag{4}$$

$$G_{55} = 3\left(\frac{a'^2}{a^2} + \frac{a'n'}{an}\right) - \frac{3}{n^2c^2(t)}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} - \frac{\dot{a}\dot{n}}{an}\right) - 3\frac{k}{a^2}, \tag{5}$$

where dot and prime respectively mean derivative with respect to time t and y.

In obtaining the above equations, we have assumed that the radius of the extra space is stabilized, i.e., $\dot{b} = 0$. Also, we have considered that the y-coordinate is defined to be proportional to the proper distance along the y-direction with b being the constant of proportionality, i.e., $b' = 0$. According to these assumptions, we have defined the y-coordinate such that $b = 1$.

By assuming all the matter fields are confined on the brane and using junction conditions, after some calculations we reach

$$H^2 + \frac{kc^2(t)}{a^2(t)} = \left(\sqrt{\frac{8\pi G}{3}\rho + \frac{1}{4r_\mathrm{c}^2}} + \frac{\epsilon}{2r_\mathrm{c}}\right)^2 \tag{6}$$

and

$$2\dot{H} + 3H^2 + \frac{kc(t)^2}{a^2} = -\frac{3H^2 + \frac{3kc(t)^2}{a^2} - 2\epsilon r_\mathrm{c}\sqrt{3H^2 + \frac{kc(t)^2}{a^2}}8\pi G}{1 - 2\epsilon r_\mathrm{c}\sqrt{3H^2 + \frac{kc(t)^2}{a^2}}} \tag{7}$$

as the effective Friedmann equations on the 4D brane. Here, ρ and p are energy density and pressure of the matter fields respectively, G is the gravitational constant and r_c is the crossover length scale which separates 4D and 5D regimes of the model.

The violation of energy conservation is a general feature of the VSL theory. It can be seen via combining the above two Friedmann equations that

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2(t)}\right) = \frac{3kc(t)\dot{c}(t)}{4\pi G a^2(t)}. \tag{8}$$

For $\dot{c}(t) \neq 0$, the conservation of energy is destroyed. So, any change in the speed of light may be considered as a source of matter creation. To solve this problem, the following two solutions have been proposed. (1) We can modify the energy momentum $T_{\mu\nu}$ (Shojaie & Farhoudi 2006) by including other physical terms or varying gravitational constant $G(t)$, such that $G(t)c(t)^{-4} = \mathrm{const}$ (Barrow & Magueijo 2000). Thus, the energy-momentum remains conserved. (2) We can neglect the energy-momentum conservation, and regard variation in the speed of light as a source of matter creation (Shojaie & Farhoudi 2006). In this paper, we adopt the latter and in the next section discuss the consequences.

3 THE NORMAL DGP BRANCH IN VSL

Let us investigate the effect of VSL in the normal branch of the DGP model. We start with the Friedmann equation of the normal branch in the original DGP, in which the speed of light c is a constant,

$$H^2 + \frac{kc^2}{a^2(t)} = \left(\sqrt{\frac{8\pi G}{3}\rho + \frac{1}{4r_\mathrm{c}^2}} - \frac{1}{2r_\mathrm{c}}\right)^2, \tag{9}$$

where ρ is ordinary matter. Therefore, in the limit of late time, we can neglect it and the equation reduces to

$$H^2 + \frac{kc^2}{a^2(t)} = 0, \tag{10}$$

or, in terms of the new variable $\Omega_k = -k/(a^2H^2)$, to

$$H^2 = \frac{\Omega_{k0}H_0^2c^2}{a^2(t)}, \tag{11}$$

where the subscript, ‘zero’, represents the present value of parameters. Integrating this equation gives us the behavior of the scale factor at late time as

$$a(t) = (\sqrt{\Omega_{k0}}cH_0)t. \tag{12}$$

Regardless of the values of Ω_{k0} and H_0, this relation shows no acceleration at late time.

Now, we apply the same procedure in the presence of a varying $c(t)$. With regard to Equation (9), we obtain at late time

$$H^2 + \frac{kc^2(t)}{a^2(t)} = 0, \tag{13}$$

or

$$H^2 = \frac{\Omega_{k0}H_0^2c^2(t)}{a^2(t)}. \tag{14}$$

In the following we assume the widely used expression for $c(t)$ to be (Barrow & Magueijo 2000)

$$c(t) = c_0a^n(t) = c_0(1+z)^{-n}, \tag{15}$$

where c_0 is the current value of the speed of light and n is a constant where for $n \to 0$; $c(t)$ approaches the constant speed of light limit. This is called the Machian scenario which has significant advantages compared to the phase transition scenario in which the speed of light varies abruptly at a critical temperature (Moffat 1993; Albrecht & Magueijo 1999). Also, since $\dot{c}/c = n\dot{a}/a$, the speed of light decreases with time for $n < 0$, and grows for $n > 0$. Inserting Equation (15) in Equation (14), one obtains

$$H^2 = \frac{\Omega_{k0}H_0^2c_0^2a^{2n}(t)}{a^2(t)}. \tag{16}$$

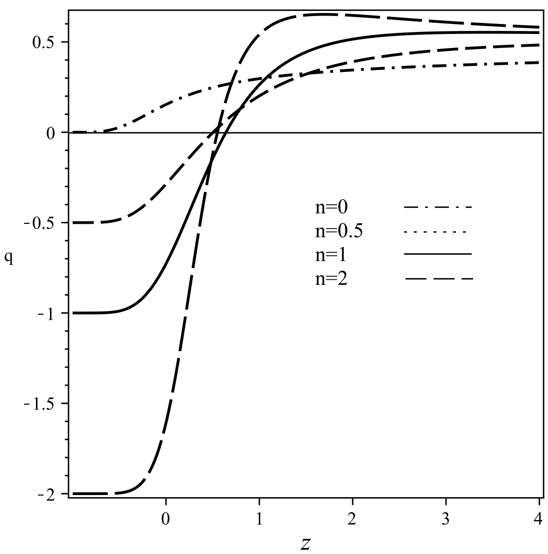

Fig. 1 The behavior of deceleration parameter versus redshift for different values of VSL-DGP parameter n. The case $n = 0$ is related to an ordinary DGP model with constant speed of light. For $n > 0$, the late time acceleration is obvious.

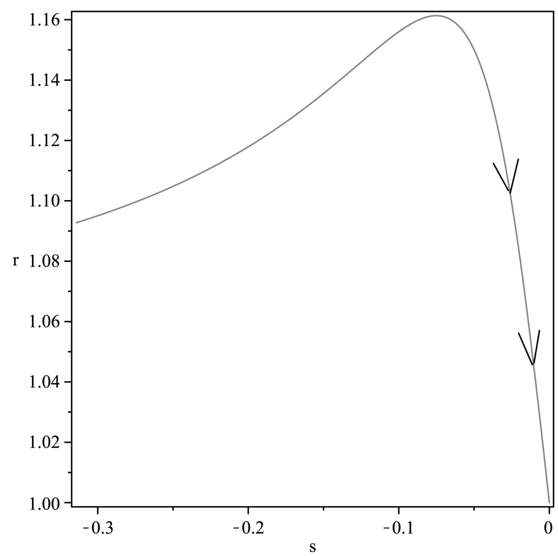

Fig. 2 The evolution of the statefinder parameter r versus s, in the non-flat VSL-DGP model with $n = 1$. There is a very small deviation from point $(1, 0)$, related to the ΛCDM model. This confirms the closeness and analogous nature of the two models.

Integration leads to

$$a(t) = \left(\left[\sqrt{\Omega_{k0}}c_0 H_0(1-n)\right]t\right)^{\frac{1}{1-n}}, \tag{17}$$

where regardless of the values of Ω_{k0}, H_0 and c_0, one can find the deceleration parameter as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -n. \tag{18}$$

According to Akarsu & Dereli (2012), the Universe would display power-law accelerating expansion for $-1 < q < 0$, exponential or de Sitter expansion for $q = -1$ and super-exponential expansion for $q < -1$. We know that our Universe is experiencing an accelerated expansion phase, so with attention to Equation (18), the normal DGP branch with a time VSL described by Equation (15) can naturally lead to late time acceleration for $n > 0$. It approaches a power-law, de Sitter or super-exponential acceleration for $0 < n < 1$, $n = 1$ and $n > 1$, respectively. The latter is related to the case when the Universe ends with a Big Rip (Caldwell et al. 2003). The result of an ordinary normal DGP model with a constant speed of light is covered when $n = 0$ (see Fig.1).

4 STATEFINDER DIAGNOSTIC

The statefinder diagnostic is an approach that can distinguish different dark energy models. In this approach, two new geometrical variables related to the third derivative of the scale factor with respect to time play a crucial role (Sahni et al. 2003). In a non-flat Universe these variables are defined as

$$\begin{aligned} r &= \frac{\dddot{a}}{aH^3} = q + 2q^2 - \frac{\dot{q}}{H}, \\ s &= \frac{r - \Omega_t}{3(q - 1/2)}, \end{aligned} \tag{19}$$

where $\Omega_t = 1 - \Omega_k$. We can rewrite the above equation in terms of the equation of state parameter of dark energy, w_d, and its first time derivative as

$$\begin{aligned} r &= \Omega_t + \frac{9}{2}w_d(1 + w_d)\Omega_d - \frac{3}{2}\frac{\dot{w}_d}{H}\Omega_d, \\ s &= 1 + w_d - \frac{1}{3}\frac{\dot{w}_d}{w_d H}. \end{aligned} \tag{20}$$

Thus, for the flat ΛCDM model, in which $w_d = -1$, we have $(r, s) = (1, 0)$. As mentioned, the pair (r, s) is usually used to discriminate different dark energy models. Also, one can compare the (r, s) trajectories of these models with each other and study their deviation from the ΛCDM model.

In our model, for late time we have

$$r = -n + 2n^2, \tag{21}$$

where we have used Equation (18). So, we conclude that only for the case $n = 1$ does our model approach the ΛCDM model and in a power-law acceleration $0 < n < 1$, or in a super-exponential acceleration $n > 1$, the

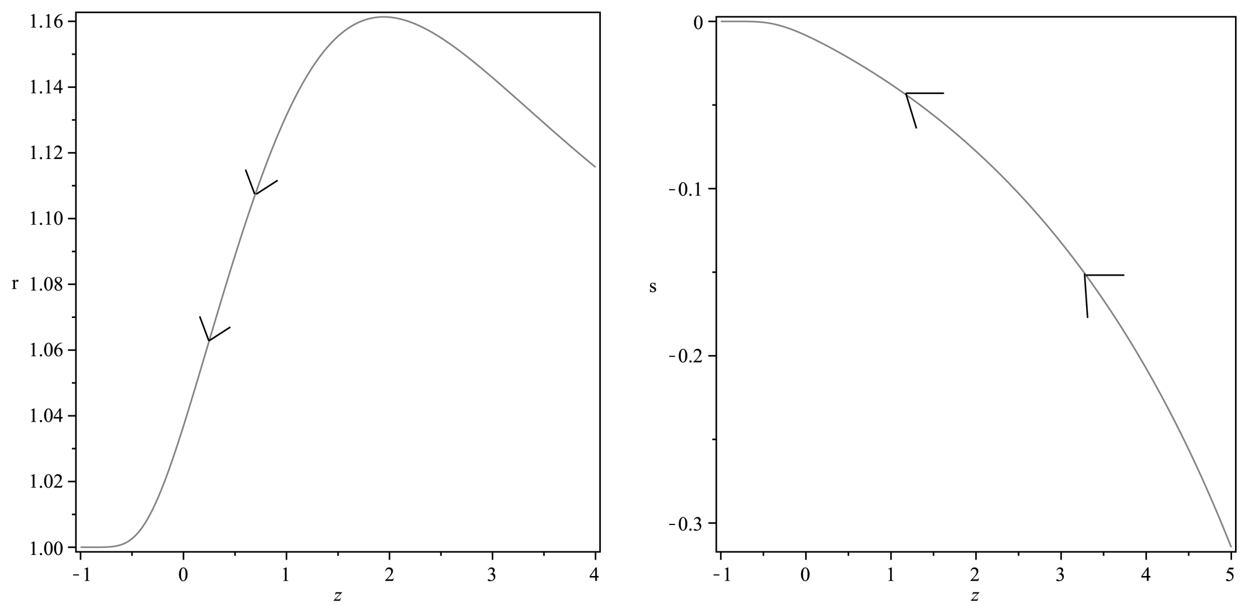

Fig. 3 The evolution of the statefinder parameters r and s versus redshift, in the non-flat VSL-DGP model with $n = 1$.

model deviates from the ΛCDM model. Figure 2 illustrates the trajectories belonging to the VSL-DGP model with $n = 1$. The range of change for the statefinder parameters, especially r, is small, as can be seen from Figure 3. This means that our model has a tiny departure from the ΛCDM model. Also, the curve $r(s)$ approaches the fixed point $(1, 0)$ at late time.

5 CONCLUSIONS

In this article we investigated VSL theory in the context of the normal branch of DGP brane cosmology. With this aim, we considered a time dependent speed of light described by $c_0 a^n(t)$. We derived the modified Friedmann equations of the model. In comparison with the ordinary DGP model and in late time approximation, we concluded that our model can experience a late time acceleration for $n > 0$. We found that our model may lead to a power-law acceleration for $0 < n < 1$, an exponential acceleration for $n = 1$ and also may end up with a Big Rip for $n > 1$. Using the statefinder diagnostic, we found that only the exponential or de Sitter expansion approaches the ΛCDM model.

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