Research in
Astronomy and
Astrophysics

Cosmological parameters for spatially flat dust filled Universe in Brans-Dicke theory

Gopi Kant Goswami

Department of Mathematics, Kalyan P. G. College, Bhilai-490 006, C. G., India; *gk.goswami9@gmail.com*

Abstract We have investigated late time acceleration for a spatially flat dust filled Universe in Brans-Dicke theory in the presence of a positive cosmological constant Λ. Expressions for Hubble's constant, luminosity distance and apparent magnitude have been obtained for our model. The theoretical results are compared with observed values of the latest 287 high redshift ($0.3 \leq z \leq 1.4$) Type Ia supernova data taken from the Union 2.1 compilation to estimate present values of matter and dark energy parameters, $(\Omega_m)_0$ and $(\Omega_\Lambda)_0$. We have also estimated the present value of Hubble's constant H_0 in light of an updated sample of Hubble parameter measurements including 19 independent data points. The results are found to be in good agreement with recent astrophysical observations. We also calculated various physical parameters such as matter and dark energy densities, present age of the Universe and deceleration parameter. The value for Brans-Dicke-coupling constant ω is set to be 40 000 based on accuracy of solar system tests and recent experimental evidence.

Key words: cosmology: cosmological parameters — cosmology: observations — dark energy — Brans-Dicke theory

1 INTRODUCTION

The two independent groups headed by Riess and Perlmutter via Type Ia supernovae (SNe Ia) (Perlmutter et al. 1997; Riess et al. 1998) found that our Universe is accelerating at present. Several theories have been put forward to explain this remarkable discovery (Spergel et al. 2003; Bennett et al. 2003; Tegmark et al. 2004). An exotic bizarre form of energy called dark energy is proposed to understand the accelerating expansion. Dark energy is expected to possess negative pressure, which repels matter from each other and accelerates expansion of the Universe. The simplest candidate for dark energy is the positive cosmological constant Λ which is considered to be a source with equation of state $p_\Lambda = -\rho_\Lambda$. The standard Friedmann-Robertson-Walker (FRW) model of the Universe with cosmological constant as a source of dark energy is often known as the Λ-CDM cosmological model (Copeland et al. 2006; Grøn & Hervik 2007). Basically, the standard FRW model represents a decelerating Universe but the presence of a cosmological con-

stant as a source and its specific value makes the model accelerating. It is found that the Λ-CDM model is in good agreement with the latest observations (Abazajian et al. 2004; Sahni & Starobinsky 2000). Recently Goswami et al. (2015, 2016a,b) have developed Λ-CDM type models for a Bianchi type I anisotropic Universe.

Apart from the Λ-CDM cosmological model, alternative explanations for the accelerated expansion in terms of scalar fields like quintessence (Caldwell et al. 1998), K-essence (Chiba et al. 2000), phantom fields (Caldwell 2002) and Chaplin gas (Kamenshchik et al. 2001) are available. A number of models involving the cosmological term, especially time-varying, have been proposed (Carvalho et al. 1992; Wetterich 1995; Arbab 1997; Padmanabhan 2001; Vishwakarma 2002; Shapiro & Sola 2004; Choudhury & Sil 2006).

It is worth investigating the effect of a cosmological "constant" in the Brans-Dicke (BD) theory of gravity (Brans & Dicke 1961) which describes evolution of the Universe in that it explains the accelerating phase of expansion in the current epoch. In a recent paper, Hrycyna

& Szydłowski (2013) compared dynamical evolution of the standard cosmological model from the perspective of both BD and the general theory of relativity. Singh & Singh (1984) investigated a cosmological model in BD theory by considering the cosmological "constant" as a function of scalar field ϕ. Pimentel (1985) obtained exact cosmological solutions in BD theory with a uniform cosmological "constant." A class of flat FRW cosmological models with cosmological "constant" in BD theory has also been obtained by Ahmadi-Azar & Riazi (1995). The age of the Universe from the perspective nucleosynthesis with term in BD theory was investigated by Etoh et al. (1997). Azad & Islam (2003) extended the idea of Singh & Singh (1984) to study the cosmological constant in Bianchi type I modified BD cosmology. Recently, Qiang et al. (2005) discussed cosmic acceleration in five dimensional BD theory using interacting Higgs and BD fields. Smolyakov (2007) investigated a model which provides the necessary value of effective cosmological "constant" at the classical level. Embedding general relativity with a varying cosmological term in the five dimensional BD theory of gravity in a vacuum has been discussed by Reyes & Madriz Aguilar (2009).

In this paper, we have investigated late time acceleration for a spatially flat dust filled Universe in BD theory in the presence of a positive cosmological constant Λ. The paper is organized as follows: In Section 2, the BD-field equations are developed for a Universe filled with cosmic fluid as the source of matter in spatially homogeneous and isotropic space-time. In Section 3, we have obtained an expression for the gravitational constant in terms of redshift by solving BD-field equations. The value for the coupling constant ω is set to be $40\,000$ on the basis of accuracy of solar system tests and recent experimental evidence. In this section, we have also obtained an expression for Hubble's constant and the relationship between energy parameters Ω_m and Ω_Λ. In Section 4, expressions for luminosity distance and apparent magnitude are obtained. Estimations of energy parameters and Hubble's constant at present are dealt with in Sections 5 and 6 respectively. In Section 7, we obtain various physical parameters such as matter and dark energy densities, present age of the Universe and value of the deceleration parameter based on values of $(\Omega_m)_0$, $(\Omega_\Lambda)_0$ and H_0 obtained by us. The model predicts that acceleration in the Universe began in the past at $z_c = 0.6818 \sim 7.2371 \times 10^9$ yr before the present. Finally, conclusions of the paper are presented in Section 8. The

results of our investigation are consistent with the latest large scale structure measurements by surveys like BOSS, WiggleZ and BAO, and WMAP or Planck results for CMB anisotropies (Planck Collaboration et al. 2016, 2014; Aubourg et al. 2015; Anderson et al. 2014; Delubac et al. 2015; Blake et al. 2012). WMAP quoted the value of dark energy density $\Omega_\Lambda = 0.7184$, whereas the combined WMAP+CMB+BAO+BOSS surveys put $\Omega_\Lambda = 0.7181$. We have obtained $\Omega_\Lambda = 0.704$.

2 BD-FIELD EQUATION

BD field equations are obtained from the following action

$$\delta \int \sqrt{-g} \left\{ \phi \left(R - 2\Lambda \right) + \omega \frac{\phi_k \phi^k}{\phi} \right\} . \quad (1)$$

The field equations are

$$R_{ij} - \frac{1}{2}R + \Lambda g_{ij} = -\frac{8\pi}{\phi c^4}T_{ij} - \frac{\omega}{\phi^2}\Big(\phi_i \phi_j$$
$$-\frac{1}{2}g_{ij}\phi_k\phi^k\Big) - \frac{1}{\phi}\Big(\phi_{i;j} - g_{ij}\Box\phi\Big), \quad (2)$$

$$(2\omega + 3)\Box\phi = \frac{8\pi T}{c^4} + 2\Lambda\phi . \quad (3)$$

We consider spatially homogeneous and isotropic FRW space-time given by

$$ds^2 = c^2 dt^2 - a(t)^2 \Big[dr^2/(1 + kr^2)$$
$$+ r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big], \quad (4)$$

where $k = -1$ is for a closed Universe, $k = 1$ is for an open Universe and $k = 0$ is for a spatially flat Universe. $a(t)$ is the scale factor.

The energy momentum tensor is taken as that of a perfect fluid. This is given by

$$T_{ij} = (p + \rho)u_i u_j - pg_{ij} , \quad (5)$$

where $g_{ij}u^i u^j = 1$ and u^i is the 4-velocity vector.

In co-moving coordinates

$$u^\alpha = 0, \qquad \alpha = 1, 2, 3 . \quad (6)$$

The BD-field Equations (2) and (3) for line element (4), are obtained as

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + 2\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi}{\phi c^2}p + \frac{kc^2}{a^2} + \Lambda c^2 , \quad (7)$$

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} - \frac{\omega\dot{\phi}^2}{6\phi^2} = \frac{8\pi}{3\phi c^2}\rho + \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} , \quad (8)$$

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} = \frac{8\pi(\rho - 3p)}{(2\omega + 3)c^2\phi} + \frac{2\Lambda c^2}{2\omega + 3} , \quad (9)$$

$$\frac{\dot{\rho}}{\rho} + 3\gamma\frac{(\dot{a})}{a} = 0 \,, \tag{10}$$

$$\Lambda c^2 + 3\frac{kc^2}{a^2} = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} - \omega\frac{\ddot{\phi}}{\phi} - 3\omega\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} + \frac{\omega\dot{\phi}^2}{2\phi} \,. \tag{11}$$

where γ is an equation of state. $\gamma = 1$ is for a dust dominated Universe and $\gamma = 4/3$ is for a radiation filled Universe.

2.1 Dust Model

The Universe is (at present) dust-dominated, so we take $p = 0$. We define density parameters as

$$\Omega_m = \frac{8\pi\rho}{3c^2 H^2 \phi},$$

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2}, \tag{12}$$

$$\Omega_k = \frac{kc^2}{H^2 a^2}.$$

We also define the decelerating parameter for scale factor a as

$$q = -\frac{\ddot{a}}{aH^2}.$$

Equations (7) to (9) and (11) then become

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1 + \xi - \frac{\omega}{6}\xi^2 \,, \tag{13}$$

$$\Omega_\Lambda = \frac{\omega+3}{2\omega}\Omega_m - \frac{2\omega+3}{2\omega}q + \frac{2\omega+3}{6}\xi^2$$
$$- \frac{2\omega+3}{2\omega}\xi, \tag{14}$$

$$\Omega_k = 1 - \frac{3(\omega+1)}{2\omega}\Omega_m + \frac{2\omega+3}{2\omega}q + \frac{4\omega+3}{2\omega}\xi$$
$$- \frac{\omega+1}{2}\xi^2, \tag{15}$$

$$\Omega_m = q - \frac{\omega}{3}q_\phi + (\omega+1)\xi - \frac{\omega}{3}\xi^2 \,, \tag{16}$$

where

$$\xi = \frac{\dot{\phi}}{\phi H} \ \& \ q_\phi = -\frac{\ddot{\phi}}{\phi H^2} \,. \tag{17}$$

Equation (13) is the BD analogue of the density parameter relationship in the CDM relativistic model.

2.2 Spatially Flat Dust Model

We consider spatially flat space ($k = 0, \Omega_k = 0$). Equations (15) and (16) give rise to the following equation.

$$q - (\omega+1)q_\phi + (3\omega+2)\xi = 2 \,. \tag{18}$$

Equation (18) has first integral

$$(\omega+1)\frac{\dot{\phi}}{\phi} - \frac{\dot{a}}{a} = \frac{L}{\phi a^3} \,, \tag{19}$$

where L is the constant of integration.

3 GRAVITATIONAL CONSTANT VERSUS REDSHIFT RELATION

The solution of Equation (19) has a singularity at $a = 0$ and $\phi = 0$, so we take constant $L = 0$. This gives the following power law relation between scalar field ϕ and scale factor a.

$$\xi = \frac{1}{\omega+1}, \ \ \phi = \phi_0\left(\frac{a}{a_0}\right)^{\frac{1}{\omega+1}} \,, \tag{20}$$

where ϕ_0 and a_0 are values of scalar field ϕ and scale factor a at present, respectively. As gravitational constant G is the reciprocal of ϕ i.e. $G = \frac{1}{\phi}$, and $\frac{a_0}{a} = (1+z)$, where z is the redshift,

$$\frac{G}{G_0} = (1+z)^{\frac{1}{\omega+1}} \,.$$

This relationship shows that G grows toward the past and in fact it diverges at cosmological singularity. Radar observations, lunar mean motion and the Viking landers on Mars (Narlikar 2002) suggest that the rate of variation of the gravitational constant must be very slow. Recent experimental evidence shows that $\omega > 40\,000$ (Bertotti et al. 2003; de Felice et al. 2006). Accordingly, we consider a large coupling constant

$$\omega = 40\,000 \,. \tag{21}$$

From Equation (19), the present rate of the gravitational constant is given by

$$\left(\frac{\dot{G}}{G}\right)_0 = -\frac{1}{\omega+1}H_0 = 2.5 \times 10^{-15} \,, \tag{22}$$

where we have taken $H_0 \simeq 10^{-10}\,\mathrm{yr}^{-1}$. Figure 1 demonstrates the fact that G/G_0 varies over ω. For higher values of ω, G/G_0 grows very slowly with redshift, whereas for lower values of ω it grows faster.
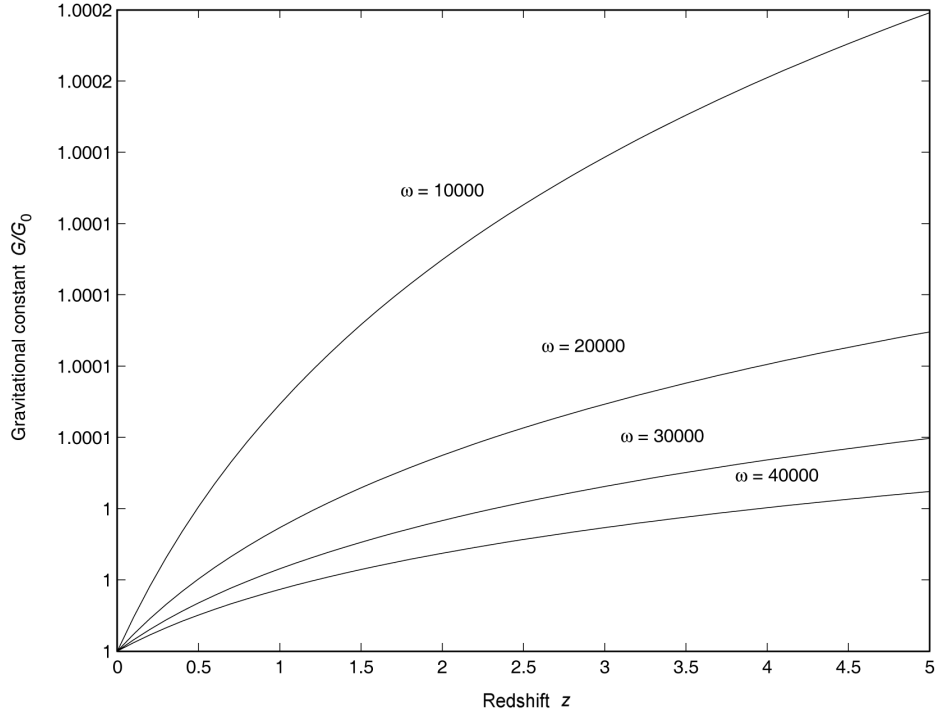
3.1 Density Parameters

Equations (13) and (20) give

$$\Omega_m + \Omega_\Lambda = 1 + \frac{5\omega+6}{6(\omega+1)^2} \,. \tag{23}$$

For $\omega = 40\,000$, Equation (23) becomes

$$\Omega_m + \Omega_\Lambda = 1.0000208 \,. \tag{24}$$

Fig. 1 Variation of gravitational constant over redshift for different ω's.

3.2 Expression for Hubble's Constant

Integration of energy conservation Equation (10) gives

$$\rho = (\rho)_0 \left(\frac{a_0}{a}\right)^3, \qquad (25)$$

where we have taken $\gamma = 1$ for dust matter.

Equations (12), (20), (23) and (25) give the expression for Hubble's constant as

$$H = \frac{H_0}{\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}}} \sqrt{(\Omega_m)_0 \left(\frac{a_0}{a}\right)^{\frac{3\omega+4}{\omega+1}} + (\Omega_\Lambda)_0}, \qquad (26)$$

and

$$\frac{a_0}{a} = (1+z).$$

Hubble's constant in terms of redshift is given by

$$H = \frac{H_0}{\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}}} \sqrt{(\Omega_m)_0(1+z)^{\frac{3\omega+4}{\omega+1}} + (\Omega_\Lambda)_0}. \qquad (27)$$

4 EXPRESSION FOR LUMINOSITY DISTANCE

The luminosity distance in Equation (4) is as follows

$$D_L = a_0 r(1+z).$$

To get the expression for luminosity distance, we consider light traveling along radial direction r. It satisfies a null geodesic given by

$$ds^2 = c^2 dt^2 - a(t)^2 dr^2 = 0.$$

From this, we obtain

$$r = \int_0^{r_s} dr = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_z^0 \frac{dz}{a\dot{z}} = \frac{1}{a_0}\int_0^z \frac{dz}{H(z)},$$

where

$$\dot{z} = \left(\frac{\dot{a}_0}{a}\right) = -H\left(\frac{a_0}{a}\right).$$

So, the luminosity distance D_L is given by

$$D_L = \frac{c(1+z)\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}}}{H_0} \times \int_0^z \frac{dz}{\sqrt{[(\Omega_m)_0(1+z)^{\frac{3\omega+4}{\omega+1}} + (\Omega_\Lambda)_0]}}. \qquad (28)$$

4.1 Expression for Apparent Magnitude

The apparent magnitude of a source of light is related to the luminosity distance via the following expression

$$m = 16.08 + 5\log_{10}\frac{H_0 D_L}{0.026c\,\mathrm{Mpc}}, \qquad (29)$$

$$\log_{10}\left(\frac{H_0 D_L}{c}\right) = (m - 16.08)/5$$
$$+ \log_{10}(0.026 \,\mathrm{Mpc}). \quad (30)$$

Using Equation (28), we get the following expression for apparent magnitude

$$m = 16.08 + 5\log_{10}\left(\frac{(1+z)\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}}}{0.026}\right.$$
$$\left.\int_0^z \frac{dz}{\sqrt{[(\Omega_m)_0(1+z)^{\frac{3\omega+4}{\omega+1}} + (\Omega_\Lambda)_0]}}\right). \quad (31)$$

5 ESTIMATION OF PRESENT VALUES OF ENERGY PARAMETERS

We consider a data set consisting of 287 high redshift ($0.3 \le z \le 1.4$) SNe Ia. They are the observed apparent magnitudes along with their associated errors from the Union 2.1 compilation (Suzuki et al. (2012)). We also obtain a large number of theoretical data corresponding to $(\Omega_m)_0$ in the range ($0 \le (\Omega_m)_0 \le 1$) from Equations (21), (24) and (31).

In order to get the best fit theoretical data set of apparent magnitudes, we calculate χ^2 by using the following statistical formula (Yadav et al. 2012).

$$\chi^2_{\mathrm{SN}} = \frac{A - \frac{B^2}{C} + \log_{10}\left(\frac{C}{2\pi}\right)}{287}, \quad (32)$$

where,

$$A = \sum_{i=1}^{287} \frac{\left[(m)_{\mathrm{ob}} - (m)_{\mathrm{th}}\right]^2}{\sigma_i^2}, \quad (33)$$

$$B = \sum_{i=1}^{287} \frac{\left[(m)_{\mathrm{ob}} - (m)_{\mathrm{th}}\right]}{\sigma_i^2}, \quad (34)$$

and

$$C = \sum_{i=1}^{287} \frac{1}{\sigma_i^2}. \quad (35)$$

Here the sums are taken over data sets of observed and theoretical values of apparent magnitudes for 287 SNe Ia.

Using Equations (32)–(35), we find that for the minimum value of $\chi^2 = 16.6910$, the best fit present values of Ω_m and Ω_Λ are given by $(\Omega_m)_0 = 0.296$ and $(\Omega_\Lambda)_0 = 0.712$.

Now we repeat the above process with luminosity distance. The observed data set of luminosity distances is obtained from those of the apparent magnitude data set given in the Union 2.1 compilation by using

Equation (28). We get a large number of data representing theoretical values of luminosity distances. These correspond to $(\Omega_m)_0$ in the range ($0 \le (\Omega_m)_0 \le 1$) from Equations (21), (24) and (28). We find that for the minimum value of $\chi^2 = 0.6545$, the best fit present values of Ω_m and Ω_Λ are $(\Omega_m)_0 = 0.296$ and $(\Omega_\Lambda)_0 = 0.704$.

Figures 2 and 3 also indicate how well the observed values of apparent magnitudes and luminosity distances agree with the theoretical graphs for $(\Omega_\Lambda)_0 = 0.704$ and $(\Omega_m)_0 = 0.296$.

6 ESTIMATION OF PRESENT VALUES OF HUBBLE'S CONSTANT H_0

We present a data set of observed values for Hubble parameter $H(z)$ versus redshift z with associated error shown in Table 1. These data points were obtained by various researchers over time, by using a differential age approach.

As per our model, the Hubble's constant $H(z)$ versus redshift z relation is given by Equation (27). Hubble Space Telescope (HST) observations of Cepheid variables (Sahni et al. 2014) provide the present value of Hubble's constant in the range $H_0 = 73.8 \pm 2.4 \,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Taking $\omega = 40\,000, (\Omega_m)_0 = 0.296, (\Omega_\Lambda)_0 = 0.704$ and using Equation (27), a large number of data sets representing theoretical values of Hubble's constant $H(z)$ for redshifts as per Table 1 and H_0 in the range ($69 \le H_0 \le 74$) are obtained. It should be noted that each data set will consist of 19 data points and data sets differ in terms of values of H_0.

In order to get the best fit theoretical data set of Hubble's constant $H(z)$ versus z, we calculate χ^2 by using the following statistical formula.

$$\chi^2_{\mathrm{SN}} = \sum_{i=1}^{19} \frac{\left[(H)_{\mathrm{ob}} - (H)_{\mathrm{th}}\right]^2}{\sigma_i^2}. \quad (36)$$

Using Equation (36), we find that for the minimum value of $\chi^2 = 10.2558$, the best fit present value of Hubble's constant H_0 is $72.30 \,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

From Figures 4 and 5 we also observe the dependence of Hubble's constant on redshift and scale factors.

In Figure 4, the observed data points are close to the graph corresponding to $(\Omega_\Lambda)_0 = 0.704, (\Omega_m)_0 = 0.296$ and $H_0 = 72.30 \,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. This validates the proximity of observed and theoretical values.

Table 1 Table of Values for Hubble's Constant

z	$H(z)$	σ_H	Reference	Method
0.07	69	19.6	Moresco et al. 2012	DA
0.1	69	12	Zhang et al. 2014	DA
0.12	68.6	26.2	Moresco et al. 2012	DA
0.17	83	8	Zhang et al. 2014	DA
0.28	88.8	36.6	Moresco et al. 2012	DA
0.4	95	17	Zhang et al. 2014	DA
0.48	97	62	Zhang et al. 2014	DA
0.593	104	13	Moresco 2015	DA
0.781	105	12	Moresco 2015	DA
0.875	125	17	Moresco 2015	DA
0.88	90	40	Zhang et al. 2014	DA
0.9	117	23	Zhang et al. 2014	DA
1.037	154	20	Moresco 2015	DA
1.3	168	17	Zhang et al. 2014	DA
1.363	160	33.6	Moresco 2015	DA
1.43	177	18	Zhang et al. 2014	DA
1.53	140	14	Zhang et al. 2014	DA
1.75	202	40	Zhang et al. 2014	DA
1.965	186.5	50.4	Stern et al. 2010	DA

7 CERTAIN PHYSICAL PROPERTIES OF THE UNIVERSE

7.1 Matter and Dark Energy Densities

The matter and dark energy densities of the Universe are related to the energy parameters through the following equation

$$\Omega_m = \frac{(\rho)_m}{\rho_c}, \ \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}, \tag{37}$$

where

$$\rho_c = \frac{3c^2 H^2}{8\pi G} = \frac{3c^2 \phi H^2}{8\pi}, \tag{38}$$

so

$$(\rho_m)_0 = (\rho_c)_0 (\Omega_m)_0, \ (\rho_\Lambda)_0 = (\rho_c)_0 (\Omega_\Lambda)_0. \tag{39}$$

Now

$$(\rho_c)_0 = \frac{3c^2 H_0^2}{8\pi G} = 1.88 \, h_0^2 \times 10^{-29} \text{ gm cm}^{-3}.$$

Therefore, the present values of matter and dark energy densities are given by

$$(\rho_m)_0 = 0.5565 h_0^2 \times 10^{-29} \text{ gm cm}^{-3}, \tag{40}$$

and

$$(\rho_\Lambda)_0 = \rho_c (\Omega_\Lambda)_0$$
$$= 1.3235 h_0^2 \times 10^{-29} \text{ gm cm}^{-3}. \tag{41}$$

Here we have taken

$$(\Omega_m)_0 = 0.296 \text{ and } (\Omega_m)_0 = 0.704.$$

General expressions for matter and dark energy are given by

$$\rho = (\rho)_0 \left(\frac{a_0}{a}\right)^3$$
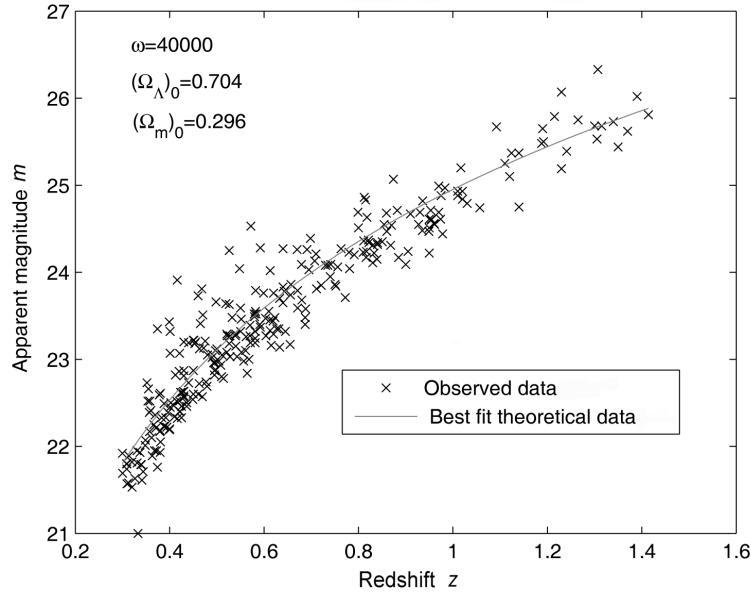$$= (\rho)_0 \left(1 + z\right)^3, \tag{42}$$

and

$$(\rho_\Lambda) = \rho_c \, \Omega_\Lambda. \tag{43}$$

We see that current matter and dark energy densities are very close to the values predicted by various surveys described in the Introduction.
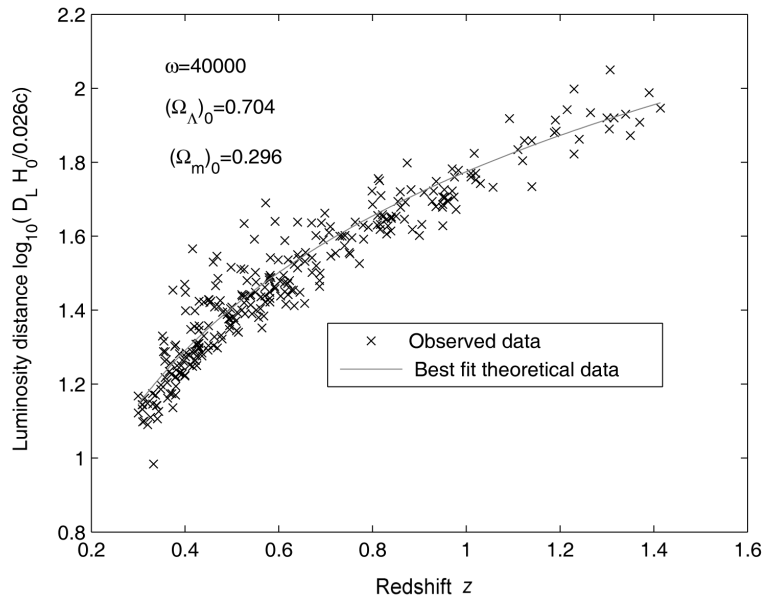
7.2 Age of the Universe

We begin with the integral

$$t = \int\limits_0^t dt = \int\limits_0^a \frac{da}{aH},$$

$$t = \int\limits_0^a \frac{\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}} \, da}{aH_0 \sqrt{(\Omega_m)_0 \left(\frac{a_0}{a}\right)^{\frac{3\omega+4}{\omega+1}} + (\Omega_\Lambda)_0}}. \tag{44}$$

Fig. 2 Best fit curve for apparent magnitude versus redshift.

Fig. 3 Best fit curve for luminosity distance versus redshift.
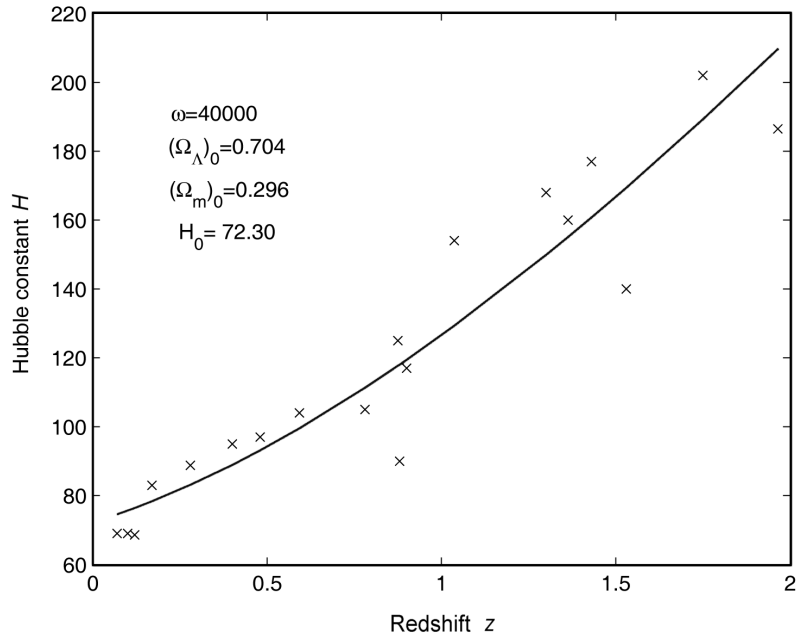
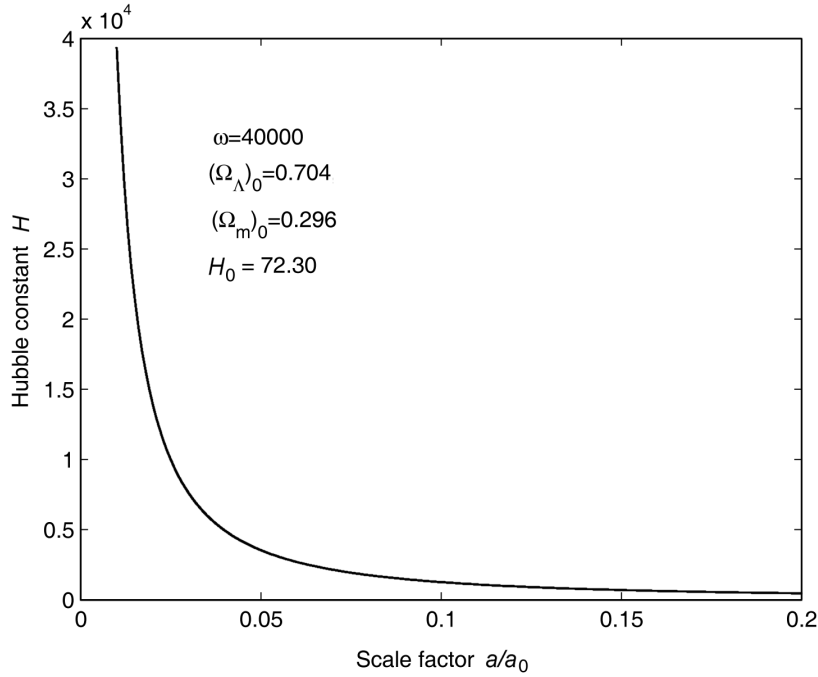Integrating Equation (44), we get the following expression for the age of the Universe.

In terms of redshift, the age is given as

$$H_0 t = \frac{2\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}}}{\frac{3\omega+4}{\omega+1}\sqrt{(\Omega_\Lambda)_0}} \log\left(\sqrt{\frac{(\Omega_\Lambda)_0}{(\Omega_m)_0}\left(\frac{a}{a_0}\right)^{\frac{3\omega+4}{\omega+1}}} + \sqrt{1 + \frac{(\Omega_\Lambda)_0}{(\Omega_m)_0}\left(\frac{a}{a_0}\right)^{\frac{3\omega+4}{\omega+1}}}\right). \tag{45}$$

$$H_0 t = \frac{2\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}}}{\frac{3\omega+4}{\omega+1}\sqrt{(\Omega_\Lambda)_0}} \log\left(\sqrt{\frac{(\Omega_\Lambda)_0}{(\Omega_m)_0}\left(\frac{1}{1+z}\right)^{\frac{3\omega+4}{\omega+1}}} + \sqrt{1 + \frac{(\Omega_\Lambda)_0}{(\Omega_m)_0}\left(\frac{1}{1+z}\right)^{\frac{3\omega+4}{\omega+1}}}\right). \tag{46}$$

Fig. 4 Variation of Hubble's constant with redshift including the corresponding best fit curve.

Fig. 5 Variation of Hubble's constant with scale factor.

The present age of the Universe is given by

$$H_0 t_0 = \frac{2\sqrt{1 + \frac{5\omega+6}{6(\omega+1)^2}}}{\frac{3\omega+4}{\omega+1}\sqrt{(\Omega_\Lambda)_0}} \log\left(\sqrt{\frac{(\Omega_\Lambda)_0}{(\Omega_m)_0}}\right.$$

$$\left. + \sqrt{1 + \frac{(\Omega_\Lambda)_0}{(\Omega_m)_0}}\right). \tag{47}$$

For $\omega = 40\,000$, $(\Omega_\Lambda)_0 = 0.704$ and $(\Omega_m)_0 = 0.296$, $H_0 t_0 = 0.9677$. Since $H_0^{-1} = 9.776\ h^{-1}$ Gyr $= 13.5778$ Gyr when $h = 0.723$, therefore $t_0 = 13.0847$ Gyr. This is consistent with the most recent WMAP data $t_0 = 13.73^{+0.13}_{-0.17}$.

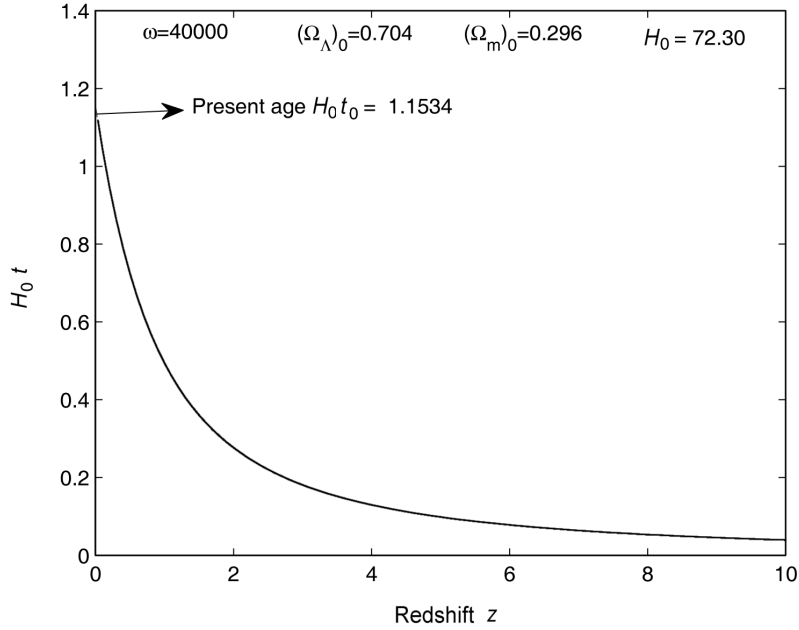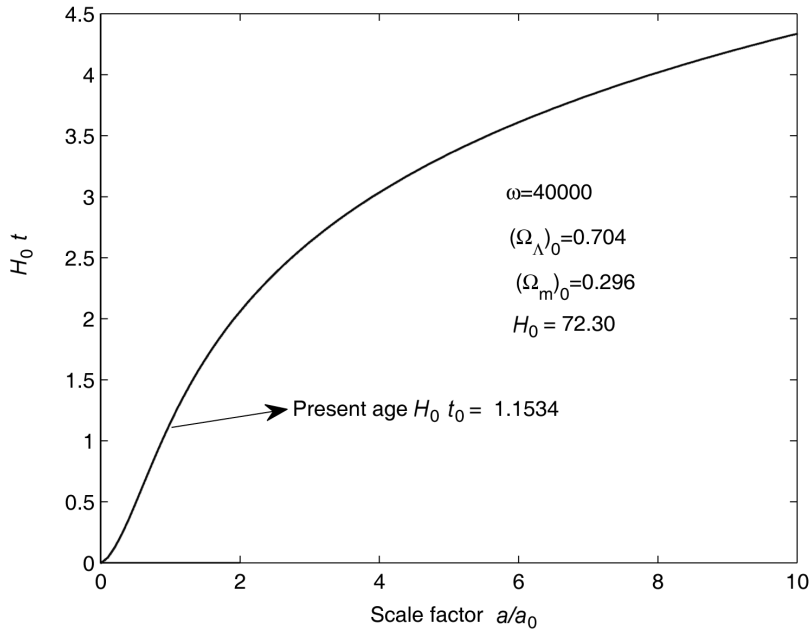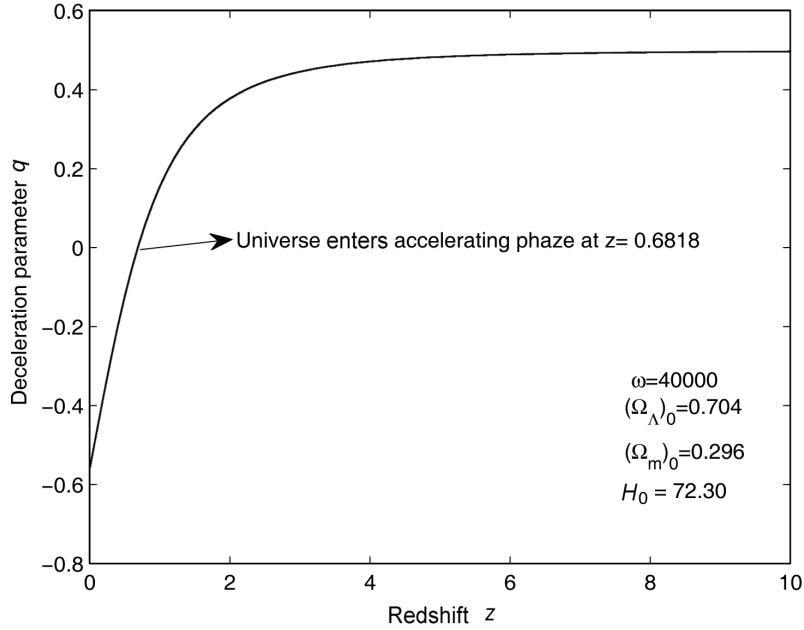**Fig. 6** Variation of redshift over time.

Fig. 7 Variation of scale factor over time.

In Figures 6 and 7, we have shown the variation of scale factor and redshift over time. They are also consistent with recent observations.

7.3 Deceleration Parameter

Equations (14), (20) and (23) give

$$q = \frac{\omega + 2}{2(\omega + 1)} - \frac{3(\omega + 1)}{2\omega + 3} \, \Omega_\Lambda \, . \tag{48}$$

Fig. 8 Variation of deceleration parameter over redshift. It represents an accelerating Universe at present.

Table 2 Cosmological Parameters at Present

Cosmological Parameter	Value at Present
BD coupling constant ω	40 000
$\left(\frac{\dot{G}}{G}\right)_0$	2.5×10^{-15}
Dark energy parameter $(\Omega_\Lambda)_0$	0.704
Dust energy parameter $(\Omega_m)_0$	0.296
Hubble's constant H_0	72.30
Deceleration parameter $(q)_0$	–0.5560
Dust energy density $(\rho_m)_0$	$0.5565 h_0^2 \times 10^{-29}$ gm cm^{-3}
Dark energy density $(\rho_\Lambda)_0$	$1.3235 h_0^2 \times 10^{-29}$ gm cm^{-3}
Age of the Universe t_0	13.0847 Gyr

Using Equations (26) and (27), we get the following expression for the deceleration parameter

$$q = \frac{\omega + 2}{2(\omega + 1)} - \frac{3(\omega + 1)}{2\omega + 3}\ (\Omega_\Lambda)_0$$
$$\times \frac{\left(1 + \frac{5\omega + 6}{6(\omega + 1)^2}\right)}{\left((\Omega_m)_0 \left(\frac{a_0}{a}\right)^{\frac{3\omega + 4}{\omega + 1}} + (\Omega_\Lambda)_0\right)}. \tag{49}$$

In terms of redshift it is given by

$$q = \frac{\omega + 2}{2(\omega + 1)} - \frac{3(\omega + 1)}{2\omega + 3}\ (\Omega_\Lambda)_0$$
$$\times \frac{\left(1 + \frac{5\omega + 6}{6(\omega + 1)^2}\right)}{\left((\Omega_m)_0 (1 + z)^{\frac{3\omega + 4}{\omega + 1}} + (\Omega_\Lambda)_0\right)}. \tag{50}$$

As the present phase $(z = 0)$ of the Universe is accelerating $q \leq 0$ i.e. $\frac{\ddot{a}}{a} \geq 0$, we must have

$$(\Omega_\Lambda)_0 \geq \frac{(2\omega + 3)(\omega + 2)}{6(\omega + 1)^2}. \tag{51}$$

For $\omega = 40\,000$, the limit is as follows

$$(\Omega_\Lambda)_0 \geq 0.3333,$$

which is consistent with the present observed value of $(\Omega_\Lambda)_0 = 0.704$. Putting $\omega = 40\,000$, $(\Omega_\Lambda)_0 = 0.704$, $(\Omega_m)_0 = 0.296$ and $z = 0$ in Equation (50), we get the present value of the deceleration parameter

$$q_0 = -0.5560. \tag{52}$$

The Universe reaches its accelerating phase when $z < z_c$ where $z = z_c$ at $q = 0$. Equation (50) provides

$$z_c \cong 0.6818 \,. \tag{53}$$

Thus, the acceleration must have begun in the past at $z_c = 0.6818 \sim 0.5180 H_0^{-1}\text{yr} \sim 7.2371 \times 10^9$ yr before from the present. We have calculated redshift with time using Equation (46).

Figure 8 shows how the deceleration parameter increases from negative to positive with changing redhift, which means that in the past the Universe was decelerating, then at the instant $z_c \cong 0.6797$ it became stationary, and thereafter it started accelerating.

8 CONCLUSIONS

We summarize our results by presenting Table 2 which displays the values of cosmological parameters at present obtained by us.

We have found that the acceleration would have begun in the past at $z_c = 0.6818 \sim 7.2371 \times 10^9$ yr before the present. These results are in good agreement with various surveys described in the Introduction.

References

Abazajian, K., Adelman-McCarthy, J. K., Agüeros, M. A., et al. 2004, AJ, 128, 502

Ahmadi-Azar, E., & Riazi, N. 1995, Astrophysics and Space Science, 226, 1

Anderson, L., Aubourg, É., Bailey, S., et al. 2014, MNRAS, 441, 24

Arbab, A. I. 1997, General Relativity and Gravitation, 29, 61

Aubourg, É., Bailey, S., Bautista, J. E., et al. 2015, Phys. Rev. D, 92, 123516

Azad, A., & Islam, J. 2003, Pramana, 60, 21

Bennett, C. L., Halpern, M., Hinshaw, G., et al. 2003, ApJS, 148, 1

Bertotti, B., Iess, L., & Tortora, P. 2003, Nature, 425, 374

Blake, C., Brough, S., Colless, M., et al. 2012, MNRAS, 425, 405

Brans, C., & Dicke, R. H. 1961, Physical Review, 124, 925

Caldwell, R. R., Dave, R., & Steinhardt, P. J. 1998, Physical Review Letters, 80, 1582

Caldwell, R. R. 2002, Physics Letters B, 545, 23

Carvalho, J. C., Lima, J. A. S., & Waga, I. 1992, Phys. Rev. D, 46, 2404

Chiba, T., Okabe, T., & Yamaguchi, M. 2000, Phys. Rev. D, 62, 023511

Choudhury, S. B. D., & Sil, A. 2006, Ap&SS, 301, 61

Copeland, E. J., Sami, M., & Tsujikawa, S. 2006, International Journal of Modern Physics D, 15, 1753

de Felice, A., Mangano, G., Serpico, P. D., & Trodden, M. 2006, Phys. Rev. D, 74, 103005

Delubac, T., Bautista, J. E., Busca, N. G., et al. 2015, A&A, 574, A59

Etoh, T., Hashimoto, M., Arai, K., & Fujimoto, S. 1997, A&A, 325, 893

Goswami, G., Yadav, A. K., & Mishra, M. 2015, International Journal of Theoretical Physics, 54, 315

Goswami, G. K., Dewangan, R. N., Yadav, A. K., & Pradhan, A. 2016a, Ap&SS, 361, 47

Goswami, G. K., Dewangan, R. N., & Yadav, A. K. 2016b, Ap&SS, 361, 119

Grøn, Ø., & Hervik, S. 2007, Einstein's General Theory of Relativity with Modern Applications in Cosmology (Springer Science+Business Media)

Hrycyna, O., & Szydłowski, M. 2013, Physical Review D, 88, 064018

Kamenshchik, A., Moschella, U., & Pasquier, V. 2001, Physics Letters B, 511, 265

Moresco, M., Cimatti, A., Jimenez, R., et al. 2012, J. Cosmol. Astropart. Phys., 8, 006

Moresco, M. 2015, MNRAS, 450, L16

Narlikar, J. V. 2002, An Introduction to Cosmology (Cambridge: Cambridge Univ. Press)

Padmanabhan, T. 2001, gr-qc/0112068

Perlmutter, S., Aldering, G., Deustua, S., et al. 1997, in Bulletin of the American Astronomical Society, 29, American Astronomical Society Meeting Abstracts, 1351

Pimentel, L. O. 1985, Astrophysics and Space Science, 112, 175

Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2014, A&A, 571, A16

Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, A&A, 594, A26

Qiang, L.-E., Ma, Y., Han, M., & Yu, D. 2005, Phys. Rev. D, 71, 061501

Reyes, L. M., & Madriz Aguilar, J. E. 2009, arXiv:0902.4736

Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009

Sahni, V., Shafieloo, A., & Starobinsky, A. A. 2014, ApJ, 793, L40

Sahni, V., & Starobinsky, A. 2000, International Journal of Modern Physics D, 9, 373

Shapiro, I. L., & Sola, J. 2004, astro-ph/0401015

Singh, T., & Singh, T. 1984, Journal of Mathematical Physics, 25, 2800

Smolyakov, M. N. 2007, arXiv:0711.3811

Spergel, D. N., Verde, L., Peiris, H. V., et al. 2003, ApJS, 148, 175

Stern, D., Jimenez, R., Verde, L., Kamionkowski, M., & Stanford, S. A. 2010, J. Cosmol. Astropart. Phys., 2, 008

Suzuki, N., Rubin, D., Lidman, C., et al. 2012, ApJ, 746, 85

Tegmark, M., Strauss, M. A., Blanton, M. R., et al. 2004, Phys. Rev. D, 69, 103501

Vishwakarma, R. G. 2002, Classical and Quantum Gravity, 19, 4747

Wetterich, C. 1995, A&A, 301, 321

Yadav, A. K., Rahaman, F., Ray, S., & Goswami, G. 2012, The European Physical Journal Plus, 127, 1

Zhang, C., Zhang, H., Yuan, S., et al. 2014, RAA (Research in Astronomy and Astrophysics), 14, 1221