

Probing the Lambda-DGP Braneworld model

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Abstract We study cosmic dynamics in the context of the normal branch of the DGP braneworld model. Using current Planck data, we find the best fitting model and associated cosmological parameters in non-flat Λ DGP. With the transition redshift as a basic variable and statefinder parameters, our result shows that the Universe starts its accelerated expansion phase slightly earlier than expected in Λ CDM cosmology. The result also alleviates the coincidence problem of the Λ CDM model.

Key words: (cosmology:) cosmological parameters — observations — theory — dark energy — methods: numerical

1 INTRODUCTION

Nowadays, we know from observations that our Universe is experiencing an accelerated expansion phase according to some unknown mechanism (Riess et al. 1998; Perlmutter et al. 1998, 1999; Kowalski et al. 2008; Amanullah et al. 2010; Sahni & Starobinsky 2000). A mysterious fluid with negative pressure, dubbed dark energy, is the most important candidate which can drive this acceleration (Peebles & Ratra 2003; Padmanabhan 2003; Oliveros & Acero 2015; Brevik et al. 2015; Frieman et al. 2008; Wang & Mukherjee 2007). The cosmological constant, Λ , is the simplest dark energy component despite the related coincidence problem (Weinberg 1989; Zaripov 2014; Basilakos & Lima 2010; Basilakos et al. 2010). Also, we know that the Universe underwent a phase transition during its evolution. In this era, it changed its decelerated expansion to an accelerated phase. Thus, the so called deceleration parameter, $q(z)$, switches from positive to negative values for a specific value of redshift called the transition redshift, z_t .

In addition, we expect the redshift of any cosmological source to change after a time interval because of the evolution and expansion of the Universe. Although this redshift drift is small and could not be measured at low redshifts, it is a unique way to determine the expansion history of the Universe directly and without any ambiguity (Sandage 1962; McVittie 1962). Observationally, we can express this change in redshift as a spectroscopic velocity drift of the source which is on the order of several $\text{cm s}^{-1} \text{yr}^{-1}$.

On the other hand, in the past two decades the theory of extra dimensions has attracted a great deal of attention among researchers (Maia et al. 2004; Rudra et al.

2012; Shahidi & Sepangi 2011). It was derived by using the braneworld scenario of Randall and Sundrum (RS) and developed gradually. Among many extensions of the RS model, the model proposed by Dvali, Gabadadze and Porrati (DGP) is of particular interest, because one branch of this model explains the certain late-time acceleration of the Universe without any dark energy component, irrespective of its own problems. In the DGP braneworld model, our 4D world is a brane embedded in an infinite 5D Minkowskian bulk. Also, all the matter fields are confined to the brane and only gravity can leak into the bulk. According to the two ways that a brane can be embedded in the bulk, the model features two separate branches denoted by $\epsilon = \pm 1$, with distinct characteristics. The $\epsilon = +1$ branch is called the self-accelerating solution in which the Universe experiences a late-time acceleration due to modification of gravity. On the other hand, the $\epsilon = -1$ branch where the Universe is not able to accelerate without a dark energy component is called the normal branch (de Rham et al. 2008; Dvali et al. 2000; Bouhmadi-López 2009; Quiros et al. 2009).

This paper is organized as follows. In Section 2, we study the Λ DGP model, i.e., a spatially non-flat DGP model in the presence of a cosmological constant as a dark energy component. We express the transition redshift in terms of our model parameters via $q(z_t) = 0$ and then estimate it numerically, using a best-fitting procedure. Section 3 deals with the redshift drift and drift velocity. In this section we test our model with observations. Also, in each case we compare our results with the Λ CDM model. In Section 4, a statefinder diagnostic procedure is used to distinguish our model among different dark energy models. In Section 5, the important coincidence problem is inves-

tigated in our model. Section 6 provides conclusions and remarks.

2 TRANSITION REDSHIFT

As we mentioned in the introduction, our Universe has experienced a transition from a decelerated expansion to an accelerated one in its expansion history. The redshift of this transition is called transition redshift, z_t , which is one of the important parameters in cosmology. In this section we are trying to obtain this value directly, using a numer-

ical approach in the non-flat Λ DGP model. With this aim, we start with the Friedmann equation and after calculating the related deceleration parameter, we use the condition $q(z_t) = 0$ to obtain an expression for z_t , in terms of our model parameters. Then, considering z_t as a free parameter, a best-fitting procedure is used to determine the best values of model parameters. Using these values we plot the curves $q(z)$ and (z_t, Ω_{m0}) for the model under consideration and compare them with the Λ CDM model. Also, we can compare the curve (z_t, Ω_{m0}) with the observational constraints from the Planck satellite.

In the non-flat Λ DGP model we have the following modified Friedmann equation on the brane (Xu & Wang 2010)

$$H^2 + \frac{K}{a^2} = \left(\sqrt{\frac{\rho}{3M_p^2} + \frac{1}{4r_c^2}} - \frac{1}{2r_c} \right)^2, \quad (1)$$

where $H = \dot{a}/a$, $a = a(t)$, M_p and $r_c = \kappa_{(5)}^2/\kappa_{(4)}^2$ are the Hubble parameter, scale factor, the 4D Planck mass and the so called crossover distance, respectively. Also, $\rho = \rho_m + \rho_\Lambda$ where ρ_m is related to the matter content on the brane and $\rho_\Lambda = M_p^2\Lambda$. In addition, $K = \pm 1$ is the curvature parameter. Using the fractional energy densities of

$$\begin{aligned} \Omega_m &= \frac{\rho_m}{3M_p^2 H^2}, \\ \Omega_\Lambda &= \frac{\Lambda}{3H^2}, \\ \Omega_{r_c} &= \frac{1}{4r_c^2 H^2}, \\ \Omega_K &= -\frac{K}{a^2 H^2}, \end{aligned} \quad (2)$$

one can rewrite the Friedmann equation as below

$$E^2(z) = (\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{r_c0} + \Omega_{\Lambda0}} - \sqrt{\Omega_{r_c0}})^2 + \Omega_{K0}(1+z)^2, \quad (3)$$

where $E(z) = H(z)/H_0$ and the zero index means the present value of any cosmological parameter. The deceleration parameter $q = -\ddot{a}/(aH^2)$, in terms of redshift, is defined as

$$q(z) = \frac{(1+z)}{H(z)} \frac{dH}{dz} - 1. \quad (4)$$

So, it can be rewritten in terms of fractional energy densities as

$$\begin{aligned} q(z) &= \frac{3\Omega_{m0}(1+z)^3(\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda0} + \Omega_{r_c0}} - \sqrt{\Omega_{r_c0}})}{2[(\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda0} + \Omega_{r_c0}} - \sqrt{\Omega_{r_c0}})^2 + \Omega_{K0}(1+z)^2]\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda0} + \Omega_{r_c0}}} \\ &+ \frac{\Omega_{K0}(1+z)^2}{(\sqrt{\Omega_{m0}(1+z)^3 + \Omega_{\Lambda0} + \Omega_{r_c0}} - \sqrt{\Omega_{r_c0}})^2 + \Omega_{K0}(1+z)^2} - 1 \end{aligned} \quad (5)$$

and the transition redshift can be expressed as

$$z_t = \frac{\left\{ \left[4\Omega_{m0}\Omega_{r_c0} + 2\Omega_{m0}\Omega_{\Lambda0} + 2\sqrt{\Omega_{m0}^2\Omega_{r_c0}(4\Omega_{r_c0} + 3\Omega_{\Lambda0})} \right] \Omega_{m0} \right\}^{1/3}}{\Omega_{m0}} - 1. \quad (6)$$

One can check that this expression is exactly the same as the one obtained in a flat Λ DGP model. Now, we use the numerical χ^2 method to obtain the best-fitting values of our model parameters. To this end, we have utilized the observational data from Type Ia Supernovae (SNeIa), Baryon Acoustic Oscillations (BAOs) and Cosmic Microwave Background (CMB) radiation. To constrain our model parameters with respect to SNeIa, we use 557 data points from the Union sample (Amanullah et al. 2010). The related χ^2 value is defined as

$$\chi_{\text{SNeIa}}^2 = \sum_{i=1}^{557} \frac{[\mu_i^{\text{the}}(z_i) - \mu_i^{\text{obs}}(z_i)]^2}{\sigma_i^2}. \quad (7)$$

Here, μ_i^{the} and μ_i^{obs} are the theoretical and observational values of the distance modulus parameter, respectively, and σ_i represents the observationally estimated error. The distance modulus is the difference between the absolute and apparent magnitude of a distant object and is given by $\mu(z) = 5 \log_{10} D_L(z) - \mu_0$ where $\mu_0 = 5 \log_{10} h + 42.38$, $h = (H_0/100) \text{ km s}^{-1} \text{ Mpc}^{-1}$ and

$$D_L(z) \equiv (1+z) \int_0^z \frac{dz'}{E(z')} \quad (8)$$

is called the luminosity distance. Furthermore, we can constrain our model free parameters using the definition of BAO distance

$$D_V(z_{\text{BAO}}) = \left[\frac{z_{\text{BAO}}}{H(z_{\text{BAO}})} \left(\int_0^{z_{\text{BAO}}} \frac{dz}{H(z)} \right)^2 \right]^{1/3}. \quad (9)$$

To this end, we use the joint analysis of the 2dF Galaxy Redshift Survey at $z = 0.20$ and SDSS data at $z = 0.35$ (Reid et al. 2010; Percival et al. 2010) as the BAO distance ratio which is model independent

$$\frac{D_V(z = 0.35)}{D_V(z = 0.20)} = 1.736 \pm 0.065. \quad (10)$$

The related χ^2 value can be obtained using

$$\chi_{\text{BAO}}^2 = \frac{\left\{ \left[D_V(z = 0.35)/D_V(z = 0.20) \right] - 1.736 \right\}^2}{0.065^2}. \quad (11)$$

Table 1 Best-fitting Values of non-flat Λ DGP Model Parameters

Model parameters	Best-fitting values
Ω_{m0}	$0.291^{+0.001}_{-0.003}$
z_t	$0.638^{+0.001}_{-0.003}$
Ω_{K0}	$0.011^{+0.001}_{-0.002}$
$\Omega_{\Lambda 0}$	$0.671^{+0.004}_{-0.002}$
$\Omega_{r_{c0}}$	$0.0001^{+0.0001}_{-0.0002}$

The CMB shift parameter, R (Wang & Mukherjee 2006; Bond et al. 1997), which is defined as

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_{\text{CMB}}=1091.3} \frac{dz'}{E(z')}, \quad (12)$$

contains the major observational information from CMB. Thus to take into account the contribution of CMB in our analysis we use the χ^2 below

$$\chi_{\text{CMB}}^2 = \frac{[R - R_{\text{obs}}]^2}{\sigma_R^2}, \quad (13)$$

where $R_{\text{obs}} = 1.725 \pm 0.018$ (Komatsu et al. 2011). Now, minimizing $\chi_{\text{SNe}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2$, we obtain the best-fit values of our model parameters. Also, in this procedure we have considered z_t to be a free parameter. The results are shown in Table 1. In comparison with the corresponding transition redshift of the Λ CDM model $z_t = 0.632$ (Planck Collaboration et al. 2014), in the non-flat Λ DGP model the Universe starts its accelerated expansion phase earlier. Note that the value of $\Omega_{r_{c0}}$ has been obtained indirectly, using the best-fitting values of other model parameters, together with Equation (6).

In Figure 1, the deceleration parameter as a function of redshift for the best-fitting values has been drawn with a dashed curve in black. In Figure 2, we have indicated the transition redshift, z_t , as a function of Ω_{m0} . Also, the observational data for z_t and Ω_{m0} , extracted from the Planck satellite, have been shown by horizontal and vertical lines, respectively. It seems that this model fits observations well.

3 REDSHIFT DRIFT

The cosmic redshift parameter is related to the scale factor $a(t)$ and consequently to the cosmic time t indirectly. Then, we conclude that the redshift also changes with time. The variation of redshift with respect to time, called redshift drift, can be used to directly measure the variation of Hubble parameter, $H(z)$, and also the acceleration of the Universe. According to Linder (1997), if we introduce t_0 and t_e , respectively as the time in which a signal is detected in the frame of the observer and the time in which the signal is emitted from the source, then

$$\frac{dz}{dt_0} = \frac{d}{dt_0} \left(\frac{a(t_0)}{a(t_e)} - 1 \right) = \frac{\dot{a}(t_0) - \dot{a}(t_e)}{a(t_e)} \quad (14)$$

and one can derive the McVittie equation

$$\dot{z} = H_0(1+z) - H(z). \quad (15)$$

The appearance of a difference in the value of \dot{a} , in Equation (14), is the reason that we can regard the redshift drift as a measure of acceleration of the Universe. Also, using the best-fitting values in Table 1 and considering the McVittie equation, we can plot the curve $\dot{z}(z)$ and deal with the expansion history of the Universe (see Fig. 3).

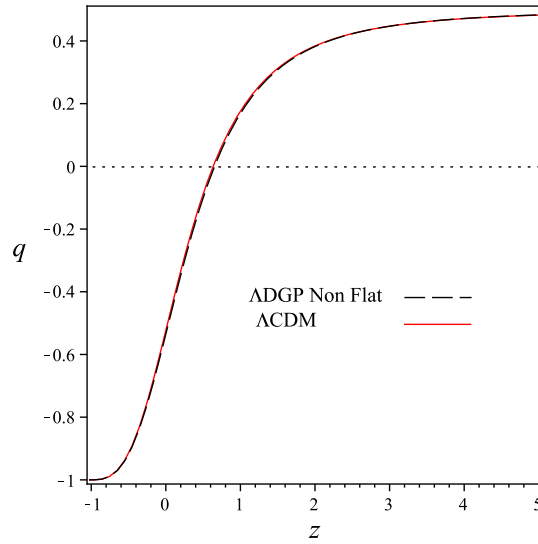


Fig. 1 Deceleration parameter as a function of redshift for the Λ CDM model and non-flat Λ DGP model for the best-fitting values of our model parameters. The curve and the value of z_t for the two models are very similar to each other, but our model switches from deceleration to acceleration a little earlier than the Λ CDM model.

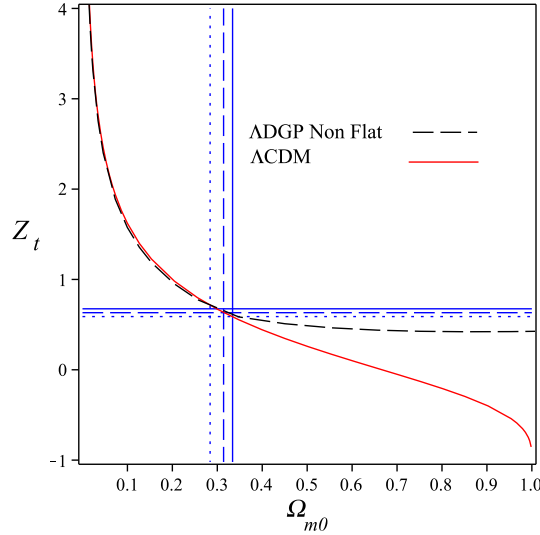


Fig. 2 Transition redshift as a function of Ω_{m0} for the Λ CDM model and non-flat Λ DGP model for the best-fitting values of our model parameters. The horizontal and vertical blue lines are related to observations. The Planck satellite result with a 68% confidence limit on the present matter density parameter is $\Omega_{m0} = 0.314 \pm 0.020$ (Planck Collaboration et al. 2014). The best-fitting value of the transition redshift that corresponds to the 68% confidence limit, with respect to some observational data, has been obtained in Lu et al. (2011) using a deceleration parameter of $z_t = 0.69^{+0.20}_{-0.13}$. Both models are in good agreement with observations.

We should note here that recently, forecasting analysis using the redshift drift has attracted a great deal of attention. It has been shown in a number of works that the mock redshift drift data can significantly improve the constraints on model parameters (Corasaniti et al. 2007; Geng et al. 2014, 2015).

Using some calculations one can reach the relation below

$$q = \frac{1 - \dot{z}'}{1 - \frac{\dot{z}}{1+z}} - 1, \quad (16)$$

where we have represented derivative with respect to z by a prime. Also, from Equation (15), we can obtain

$$\dot{z}' = H_0(1 - E'(z)). \quad (17)$$

With regard to Equation (17), in the diagram $\dot{z}(z)$, the slope of the curve is negative if $E'(z) > 1$, and it is positive if $E'(z) < 1$. Also, the extremum point is related to $E'(z) = 1$. Analyzing Equations (15), (16) and (17) and considering Figure 3, one can find that during decelerated expansion, \dot{z} changes from negative to positive values. Then, starting the accelerating phase it turns and ap-

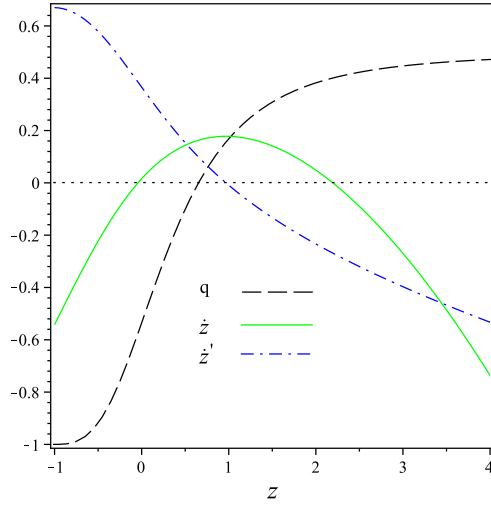


Fig. 3 q , \dot{z} and \dot{z}' as a function of redshift in the non-flat Λ DGP model for the best-fitting values of our model parameters.

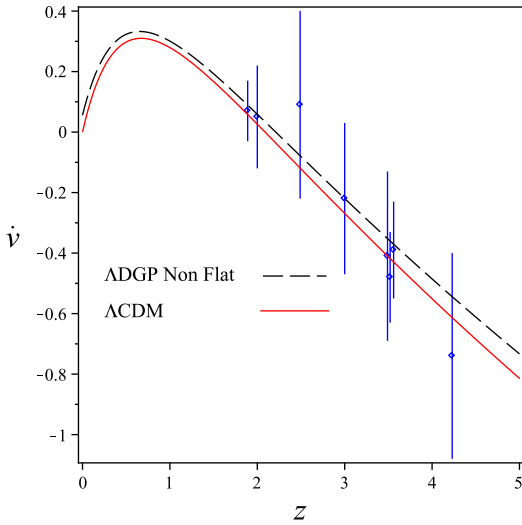


Fig. 4 Drift velocity as a function of redshift for the Λ CDM model and non-flat Λ DGP model for the best-fitting values of our model parameters. Both models fit observations well.

proaches zero at the present and continues to have negative values in the future. Furthermore, as shown in Figure 3, \dot{z}' changes from negative to positive as well in the decelerating phase. But during accelerated expansion, it extends to larger positive values.

In a Universe filled only with a matter component, the expansion slows because of the effect of gravity. But today, we know from observations that our Universe is at an epoch in its expansion history in which it has started to accelerate. This behavior indicates the existence of a dark energy component with negative pressure. In terms of dimensionless density parameters of dark matter Ω_m and dark energy Ω_d , this epoch is related to the redshift z_{eq} in which $\Omega_m = \Omega_d$. This redshift differs from z_t in that the Universe starts its acceleration. The comparison

between these two values is another interesting quantity in cosmology. The corresponding value of z_{eq} , which can be obtained from Planck Collaboration et al. (2014), shows $z_{\text{eq}} = 0.298$ for the Λ CDM model. In the non-flat Λ DGP model and for simplicity we use new variables $\Omega'_i = E^2 \Omega_i$. Then, $\Omega'_m = \Omega_{m0}(1+z)^3$ and $\Omega'_d = \Omega_{\Lambda 0}$. The condition $\Omega'_m(z_{\text{eq}}) = \Omega'_d(z_{\text{eq}})$ leads to an expression for z_{eq} in terms of our model parameters as

$$z_{\text{eq}} = \left(\frac{\Omega_{\Lambda 0}}{\Omega_{m0}} \right)^{\frac{1}{3}} - 1. \quad (18)$$

Using Table 1, we obtain $z_{\text{eq}} = 0.321$ in our model which is larger than the corresponding value in the Λ CDM model. But, similar to the Λ CDM model, $z_t > z_{\text{eq}}$. Thus, without the need for domination by the dark energy component, the Universe can start its accelerated expansion phase.

In addition, we know from observations that the velocity of a light source changes with respect to t_0 , though this variation is very small. It is just on the order of a few cm s^{-1} , if we consider a time interval of about 30 years. There is a relation between this velocity drift and the redshift drift parameter of

$$\dot{v} = \frac{dv(z)}{dt_0} = \frac{c}{(1+z)} \frac{dz}{dt_0}, \quad (19)$$

which is on the order of several $\text{cm s}^{-1} \text{ yr}^{-1}$.

Figure 4 shows the behavior of $\dot{v}(z)$ for both the Λ CDM and Λ DGP model in comparison with observational data from CODEX, including eight points (Cristiani et al. 2007). Both the curves are in good agreement with observations.

4 STATEFINDER DIAGNOSTIC

To distinguish and classify different dark energy models, a few approaches have been proposed. Among them, the statefinder diagnostic is of particular interest. This approach has been found in terms of two new geometrical

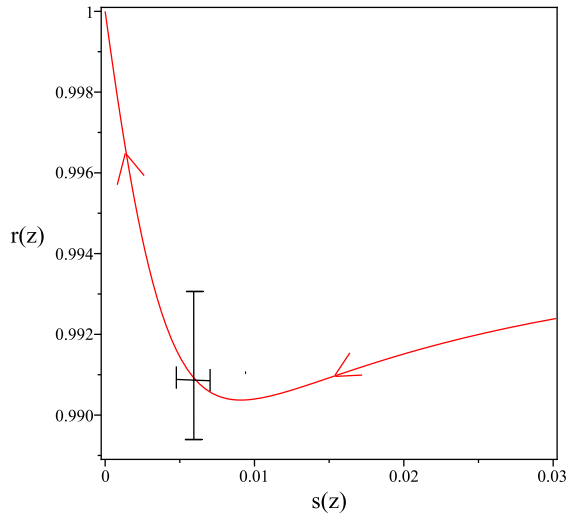


Fig. 5 The evolution of the statefinder parameter r versus s , in the non-flat Λ DGP model for the best-fitting values of our model parameters. There is a very small deviation from the point $(1, 0)$, related to the Λ CDM model. This confirms the analogy and closeness of the two models.

variables which are related to the third derivative of scale factor with respect to time (Sahni et al. 2003). In a non-flat Universe these two new variables are defined as

$$r = \frac{\ddot{a}}{aH^3},$$

$$s = \frac{r - 1 + \Omega_K}{3(q - 1/2 + \Omega_K/2)}. \quad (20)$$

Also, they can be rewritten in terms of the equation of state parameter w and its first derivative with respect to time (Sahni et al. 2003) as

$$r = 1 - \Omega_K + \frac{9}{2}w_d(1 + w_d)\Omega_d - \frac{3}{2}\frac{\dot{w}_d}{H}\Omega_d,$$

$$s = 1 + w_d - \frac{1}{3}\frac{\dot{w}_d}{w_d H}. \quad (21)$$

So, for the Λ CDM model with $w_d = -1$, we have $(r, s) = (1, 0)$. The pair (r, s) has been utilized frequently in the literature to discriminate a wide variety of dark energy models (Alam et al. 2003; Visser 2004; Zhang et al. 2008, 2010; Setare & Jamil 2011; Sami et al. 2012; Cui & Zhang 2014). For this aim, one can compare the corresponding trajectories in the r - s plane. Moreover, deviation from the fixed point $(1, 0)$ related to the Λ CDM model can be studied using these curves.

Figure 5 illustrates the trajectories belonging to the Λ DGP model. The range of change of statefinder parameters, especially r , is very small, as can be seen from Figure 6, which means that our model has a tiny departure from the Λ CDM model. Also, the curve $r(s)$ approaches the fixed point $(1, 0)$ at late times.

5 COINCIDENCE PROBLEM

One of the most important problems in the Λ CDM model is the coincidence problem, namely why the energy densities of dark matter and dark energy are of precisely the same order today? In another words if we introduce

$$R = \frac{\rho_m}{\rho_d}, \quad (22)$$

then the coincidence problem asks why R is of order unity now?

Many scenarios have been proposed to solve or at least alleviate this problem. For instance, some dynamical dark energy models have been put forward to replace the cosmological constant (Peebles & Ratra 2003; Padmanabhan 2003). Also, the coupling and interaction between dark sectors of the Universe have been used for this goal (Chimento et al. 2003; Olivares et al. 2006; Amendola et al. 2006; Del Campo et al. 2006; Olivares et al. 2008; Karwan 2008; Egan & Lineweaver 2008; Lee et al. 2008; Zhang et al. 2009). In these articles the authors investigated different approaches to resolve the coincidence problem. Some of them show that R is independent of initial conditions and study attractor solutions. Some others argue that R does not change much during the whole history of the Universe. Also, in many of them the authors introduce a mechanism in which R tends to a constant value at late times or varies more slowly than the scale factor today.

But here, we do not replace Λ with a dynamical dark energy term. Also, we do not consider any interaction between dark sectors. We only try to investigate the effect of the extra dimensions. We can show that in a Λ DGP model, though the coincidence problem is not solved in full, it can at least be ameliorated. With this aim we can introduce an effective dark energy term in our model if we rewrite the Friedmann equation in standard general relativistic form as

$$\Omega_m + \Omega_{\text{eff}} + \Omega_K = 1.$$

Thus we obtain

$$\Omega_{\text{eff}} = \Omega_\Lambda - 2\sqrt{\Omega_{r_c}}\sqrt{1 - \Omega_K} \quad (23)$$

and we can interpret the ratio defined in Equation (22) in our model as $R = \rho_m/\rho_{\text{eff}}$.

Figure 7 illustrates the behavior of R in the whole history of the Universe until now for both Λ CDM and Λ DGP models. It is obvious that in our model the coincidence problem has been slightly alleviated and this is only because of considering the effect of extra dimensions. Also, Figure 8 shows the behavior of Ω_m , Ω_{eff} and Ω_K versus redshift in our model.

6 CONCLUSIONS

In this paper we used a non-flat Λ DGP model and obtained best-fitting values of transition redshift z_t and other model parameters using SNe+BAO+CMB data. We found that the

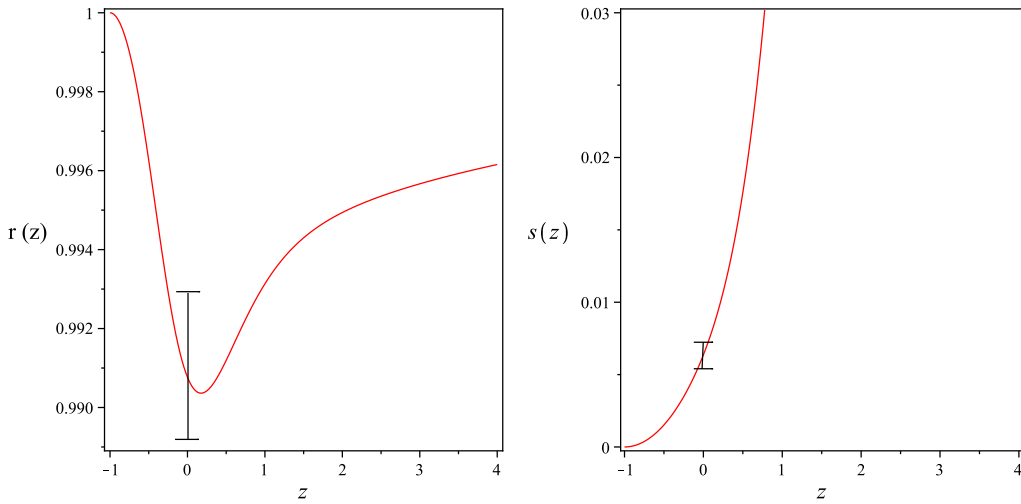


Fig. 6 The evolution of the statefinder parameters r and s versus redshift, in the non-flat Λ DGP model for the best-fitting values of our model parameters.

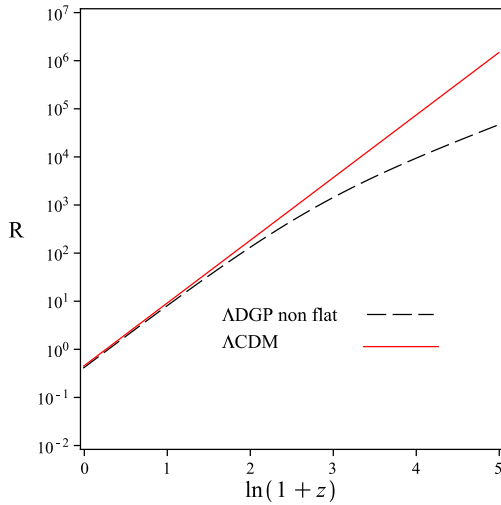


Fig. 7 The trajectory of R versus $\ln(1+z)$ for the Λ CDM model and non-flat Λ DGP model for the best-fitting values of our model parameters. The coincidence problem in our model has been alleviated.

transition from decelerating expansion to the accelerating phase in our model happens earlier than in the Λ CDM model. The cosmic redshift drift was studied exactly and the correlations between q , \dot{z} and \dot{z}' were investigated in this model. We obtained z_{eq} in our model and found that, like the Λ CDM model, before domination by the dark energy component, the Universe starts its accelerated expansion.

With regard to Figures 2 and 4, we concluded that our model is in a good agreement with observational data released by both Planck and CODEX. We see that our model is marginally consistent with the transition redshift derived indirectly from observations, better than the Λ CDM model. We applied a statefinder diagnostic scenario in our model and found that this model shows a tiny deviation from the

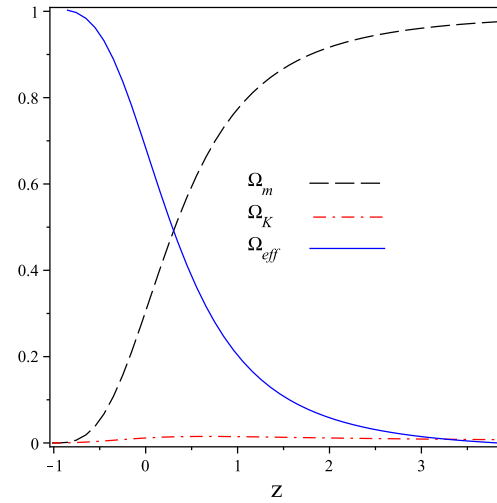


Fig. 8 The trajectory of Ω_m , Ω_{eff} and Ω_K versus redshift in the non-flat Λ DGP model for the best-fitting values of our model parameters. It is obvious that $\Omega_m + \Omega_{\text{eff}} + \Omega_K = 1$.

Λ CDM model. Also, from Figure 7, we found that our model improves the coincidence problem.

References

- Alam, U., Sahni, V., Deep Saini, T., & Starobinsky, A. A. 2003, MNRAS, 344, 1057
- Amanullah, R., Lidman, C., Rubin, D., et al. 2010, ApJ, 716, 712
- Amendola, L., Tsujikawa, S., & Sami, M. 2006, Physics Letters B, 632, 155
- Basilakos, S., & Lima, J. A. S. 2010, Phys. Rev. D, 82, 023504
- Basilakos, S., Plionis, M., & Lima, J. A. S. 2010, Phys. Rev. D, 82, 083517
- Bond, J. R., Efstathiou, G., & Tegmark, M. 1997, MNRAS, 291, L33

- Bouhmadi-López, M. 2009, *J. Cosmol. Astropart. Phys.*, 11, 011
- Brevik, I., Obukhov, V. V., & Timoshkin, A. V. 2015, *Ap&SS*, 355, 399
- Chimento, L. P., Jakubi, A. S., Pavón, D., & Zimdahl, W. 2003, *Phys. Rev. D*, 67, 083513
- Corasaniti, P.-S., Huterer, D., & Melchiorri, A. 2007, *Phys. Rev. D*, 75, 062001
- Cristiani, S., Avila, G., Bonifacio, P., et al. 2007, *Nuovo Cimento B Serie*, 122, 1165
- Cui, J.-L., & Zhang, J.-F. 2014, *European Physical Journal C*, 74, 2849
- de Rham, C., Dvali, G., Hofmann, S., et al. 2008, *Physical Review Letters*, 100, 251603
- Del Campo, S., Herrera, R., Olivares, G., & Pavón, D. 2006, *Phys. Rev. D*, 74, 023501
- Dvali, G., Gabadadze, G., & Porrati, M. 2000, *Physics Letters B*, 485, 208
- Egan, C. A., & Lineweaver, C. H. 2008, *Phys. Rev. D*, 78, 083528
- Frieman, J. A., Turner, M. S., & Huterer, D. 2008, *ARA&A*, 46, 385
- Geng, J.-J., Zhang, J.-F., & Zhang, X. 2014, *J. Cosmol. Astropart. Phys.*, 12, 018
- Geng, J.-J., Li, Y.-H., Zhang, J.-F., & Zhang, X. 2015, *European Physical Journal C*, 75, 356
- Karwan, K. 2008, *J. Cosmol. Astropart. Phys.*, 5, 011
- Komatsu, E., Smith, K. M., Dunkley, J., et al. 2011, *ApJS*, 192, 18
- Kowalski, M., Rubin, D., Aldering, G., et al. 2008, *ApJ*, 686, 749
- Lee, J., Kim, H.-C., & Lee, J.-W. 2008, *Physics Letters B*, 661, 67
- Linder, E. V. 1997, *First Principles of Cosmology* (Prentice Hall)
- Lu, J., Xu, L., & Liu, M. 2011, *Physics Letters B*, 699, 246
- Maia, M. D., Monte, E. M., & Maia, J. M. F. 2004, *Physics Letters B*, 585, 11
- McVittie, G. C. 1962, *ApJ*, 136, 334
- Olivares, G., Atrio-Barandela, F., & Pavón, D. 2006, *Phys. Rev. D*, 74, 043521
- Olivares, G., Atrio-Barandela, F., & Pavón, D. 2008, *Phys. Rev. D*, 77, 063513
- Oliveros, A., & Acero, M. A. 2015, *Ap&SS*, 357, 12
- Padmanabhan, T. 2003, *Phys. Rep.*, 380, 235
- Peebles, P. J., & Ratra, B. 2003, *Reviews of Modern Physics*, 75, 559
- Percival, W. J., Reid, B. A., Eisenstein, D. J., et al. 2010, *MNRAS*, 401, 2148
- Perlmutter, S., Aldering, G., della Valle, M., et al. 1998, *Nature*, 391, 51
- Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, *ApJ*, 517, 565
- Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2014, *A&A*, 571, A20
- Quiros, I., García-Salcedo, R., Matos, T., & Moreno, C. 2009, *Physics Letters B*, 670, 259
- Reid, B. A., Percival, W. J., Eisenstein, D. J., et al. 2010, *MNRAS*, 404, 60
- Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, *AJ*, 116, 1009
- Rudra, P., Debnath, U., & Biswas, R. 2012, *Ap&SS*, 339, 53
- Sahni, V., & Starobinsky, A. 2000, *International Journal of Modern Physics D*, 9, 373
- Sahni, V., Saini, T. D., Starobinsky, A. A., & Alam, U. 2003, *Soviet Journal of Experimental and Theoretical Physics Letters*, 77, 201
- Sami, M., Shahalam, M., Skugoreva, M., & Toporensky, A. 2012, *Phys. Rev. D*, 86, 103532
- Sandage, A. 1962, *ApJ*, 136, 319
- Setare, M. R., & Jamil, M. 2011, *General Relativity and Gravitation*, 43, 293
- Shahidi, S., & Sepangi, H. R. 2011, *International Journal of Modern Physics D*, 20, 77
- Visser, M. 2004, *Classical and Quantum Gravity*, 21, 2603
- Wang, Y., & Mukherjee, P. 2006, *ApJ*, 650, 1
- Wang, Y., & Mukherjee, P. 2007, *Phys. Rev. D*, 76, 103533
- Weinberg, S. 1989, *Reviews of Modern Physics*, 61, 1
- Xu, L., & Wang, Y. 2010, *Phys. Rev. D*, 82, 043503
- Zaripov, F. 2014, *Ap&SS*, 352, 289
- Zhang, J., Zhang, X., & Liu, H. 2008, *Physics Letters B*, 659, 26
- Zhang, H., Yu, H., Zhu, Z.-H., & Gong, Y. 2009, *Physics Letters B*, 678, 331
- Zhang, L., Cui, J., Zhang, J., & Zhang, X. 2010, *International Journal of Modern Physics D*, 19, 21