RAA 2016 Vol. 16 No. 4, 64 (10pp) doi: 10.1088/1674–4527/16/4/064
http://www.raa-journal.org http://iopscience.iop.org/raa

Research in Astronomy and Astrophysics

Interaction of Gravitational field and Brans-Dicke field in R/W universe containing Dark Energy like fluid

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Received 2015 March 11; accepted 2015 September 22

Abstract On studying some new models of Robertson-Walker universes with a Brans-Dicke scalar field, it is found that most of these universes contain a dark energy like fluid which confirms the present scenario of the expansion of the universe. In one of the cases, the exact solution of the field equations gives a universe with a false vacuum, while in another it reduces to that of dust distribution in the Brans-Dicke cosmology when the cosmological constant is not in the picture. In one particular model it is found that the universe may undergo a Big Rip in the future, and thus it will be very interesting to investigate such models further.

Key words: Brans-Dicke scalar field — cosmological constant — dark energy — quintessence — k-essence — big rip

1 INTRODUCTION

Brans and Dicke (B-D) formulated a theory of gravitation (Brans & Dicke 1961), in which besides a gravitational part, a dynamical scalar field is introduced to incorporate a variable gravitational constant and Mach's principle in Einstein's theory. It can be considered as a natural extension of Einstein's general theory of relativity. The simplest case of the scalar tensor theory (Brans 1997) is defined by a constant coupling parameter ω and a scalar field ϕ. In B-D theory, the gravitational constant becomes time-dependent, varying as the inverse of a time-dependent scalar field which couples to gravity with a coupling parameter ω. One important property of this theory is that it gives expanding solutions (Mathiazhagan & Johri 1984; La et al. 1989) for scalar field $\phi(t)$ and scale factor $R(t)$ which are compatible with solar system observations (Perlmutter et al. 1999; Riess et al. 1998; Garnavich et al. 1998). The solar system observations (Bertotti et al. 2003) also impose lower bounds on ω. General relativity is recovered when ω goes to infinity (Weinberg 1972) and from timing experiments using the Viking space probe (Reasenberg et al. 1979), ω must exceed 500. This constraint ruled out many of the extended inflation theories (Weinberg 1989a; La & Steinhardt 1989) and provides a succession of improved models on extended inflation (Holman et al. 1990, 1991; Barrow & Maeda 1990; Steinhardt & Accetta 1990). Furthermore, all important features of the evolution of the universe such as: inflation (Mathiazhagan & Johri 1984), early and late time behavior of the universe (Shogin & Hervik 2014), cosmic acceleration and structure formation (Banerjee & Pavón 2001), quintessence and the coincidence problem (Sen & Seshadri 2003), self-interacting potential and cosmic acceleration (Errahmani & Ouali 2006), and a high energy description of dark energy in an approximate B-D context (Weinberg 1989b) could be explained successfully in the B-D formalism. For a large value of the $\omega-$ parameter, B-D theory gives the correct amount of inflation and early and late time behaviors, while small and negative values explain cosmic acceleration, structure formation and the coincidence problem. Dark energy, identified as being responsible for cosmic acceleration, determines the features related to future evolution of the universe. The nature of this kind of energy may lead to an improvement in our picture of particle physics and gravitation. Investigations into the nature of dark energy have lead to various candidates. Among them, the most popular ones are the cosmological constant Λ (Padmanabhan 2003; Mak et al. 2002; Caldwell et al. 2003a), a dynamical scalar field like quintessence (Bertolami & Martins 2000; Caldwell & Linder 2005; Caldwell 2002; Tsujikawa & Sami 2004; Caldwell et al. 2003b) or a similar phantom field (Cline et al. 2004; Nesseris & Perivolaropoulos 2007; Huang et al. 2007; Bento et al. 1991).

Astronomical observations indicate that the observable universe is undergoing a phase of accelerated expansion. The present day accelerated expansion of the universe is naturally addressed within the B-D theory with evolution described by the inverse of the Hubble scale and power law temporal behavior of a scale factor. The B-D scalar-tensor theory of gravitation is quite important in view of the fact that scalar fields play a vital role in inflationary cosmology. Many cosmological problems (Kolitch & Eardley 1995; Barrow et al. 1993; Bento et al. 1992; Sahoo & Singh 2002, 2003; Lukács 1976) can be successfully explained by using the B-D theory and its extended versions. The solutions of B-D field equations with the Robertson-Walker line element have been obtained by Luke & Szamosi (1972) using a self consistent numerical method. They derived a lower bound on $\frac{\dot{G}}{G}$ by taking $P = 0$ in the field equations of B-D (Dicke 1964) and concluded the presently available data cannot discriminate between different theories. Morganstern (1971) obtained a similar conclusion on the basis of the observed values of matter density, Hubble's constant, the deceleration parameter and the ages of different objects in the universe.

Since many forms of dark energy are always accompanied and interrelated with a scalar field, we are motivated to see whether the B-D scalar field can manifest some form of dark energy and what roles it can play in causing the accelerated expansion of the universe. We are also motivated to investigate different interesting forms of model universes containing a B-D field interacting with a gravitational field, and especially their interrelation with dark energy in the evolution of our universe. From our study, we find evidence for the existence of dark energy, in one form or another, in almost all model universes obtained by us under different conditions, during the periods of their evolution, which verifies the present accelerated expansion of the universe. One peculiarity of some of the models we obtain is the existence of two forms of dark energy simultaneously in such models, one from the cosmological constant and the other due to the B-D scalar field. In one case there is the possibility of our model universe collapsing and becoming a black hole. Interestingly enough, in yet another case, one of our models is facing the fate of a Big Rip, and one of the model universes we obtain seems to behave like a cyclic model of the universe.

2 SOLUTIONS OF FIELD EQUATIONS

Here, we consider the spherically symmetric Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{1}$$

where k is the curvature index which can take values $-1, 0, 1$.

The B-D theory of gravity is described by the action

$$S = \int d^4x\sqrt{|g|}\left[\frac{1}{16\pi}\left(\phi R - \frac{\omega}{\phi}g^{sl}\phi_{,l}\phi_{,s}\right) + L_m\right], \tag{2}$$

where R represents the curvature scalar associated with the 4D metric g_{ij}; g is the determinant of g_{ij}; ϕ is a scalar field; ω is a dimensionless coupling constant; L_m is the Lagrangian of the ordinary matter component.

The Einstein field equations in their most general form are given by

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{\kappa}{\phi}T_{ij} - \frac{\omega}{\phi^2}\left[\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi^{,s}\phi_{,s}\right] - \frac{1}{\phi}(\phi_{,ij} - g_{ij}\phi_{;s}^{;s}), \tag{3}$$

$$(3+2\omega)\phi_{;s}^{;s} = \kappa T, \tag{4}$$

where $\kappa = 8\pi$, Λ is the cosmological constant, R_{ij} is the Ricci-tensor, g_{ij} is a metric tensor, $\Box\phi = \phi_{;s}^{;s}$, $\Box$ is the Laplace-Beltrami operator and $\phi_{,i}$ is partial differentiation with respect to the x^i coordinate.

The energy-momentum tensor for the perfect fluid distribution is

$$T_{ij} = (P+\rho)U_iU_j - Pg_{ij}, \tag{5}$$

with U_i being a four velocity vector, P the proper pressure and ρ the proper rest mass density. Considering a comoving system, we get $U_1 = U_2 = U_3 = 0; U_4 = 1$ and $g^{ij}U_iU_j = 1$.

A comma $(,)$ or semicolon $(;)$ followed by a subscript denotes partial differentiation or a covariant differentiation respectively. The velocity of light is taken to be unity.

Now using the metric (1), the surviving field equations are

$$\begin{aligned} G_{11} &\equiv \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda \\ &= -\frac{\kappa P}{\phi} - \frac{\omega}{2\phi^2}\left[\frac{(1-kr^2)}{R^2}\phi'^2 + \dot{\phi}^2\right] - \frac{1}{\phi}\left[-\frac{2(1-kr^2)}{R^2 r}\phi' + \frac{2\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right], \end{aligned} \tag{6}$$

$$G_{22} \equiv \frac{k}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} - \Lambda$$
$$= -\frac{\kappa P}{\phi} - \frac{\omega}{2\phi^2}\left[-\frac{(1-kr^2)}{R^2}\phi'^2 + \dot{\phi}^2\right] - \frac{1}{\phi}\left[-\frac{(1-kr^2)}{R^2}\phi'' + \frac{(2kr^2-1)}{R^2 r}\phi' + \frac{2\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right], \tag{7}$$

$$G_{33} = G_{22}, \tag{8}$$

$$G_{44} \equiv 3(\frac{k}{R^2} + \frac{\dot{R}^2}{R^2}) - \Lambda$$
$$= \frac{\kappa\rho}{\phi} + \frac{\omega}{2\phi^2}\left[\dot{\phi}^2 + \frac{(1-kr^2)}{R^2}\phi'^2\right] + \frac{1}{\phi}\left[\frac{(1-kr^2)}{R^2}\phi'' - \frac{(3kr^2-2)}{R^2 r}\phi' - \frac{3\dot{R}\dot{\phi}}{R}\right], \tag{9}$$

$$G_{14} \equiv \frac{\omega}{\phi^2}\phi'\dot{\phi} + \frac{\dot{\phi}'}{\phi} - \frac{\dot{R}\phi'}{R\phi} = 0. \tag{10}$$

From Equation (4), we get

$$(3+2\omega)\left[-\frac{(1-kr^2)}{R^2}\phi'' + \frac{(3kr^2-2)}{R^2 r}\phi' + \frac{3\dot{R}\dot{\phi}}{R} + \ddot{\phi}\right] = \kappa(\rho - 3P), \tag{11}$$

where a dot and dash denote differentiation with respect to time t and r respectively.

Subtracting Equation (6) from Equation (7), we get

$$0 = \frac{\phi'}{\phi}\left[\frac{1}{r} + \frac{kr}{1-kr^2} - \frac{\phi''}{\phi'} - \omega\frac{\phi'}{\phi}\right]. \tag{12}$$

From Equation (12), we get

$$\frac{\phi''}{\phi'} + \omega\frac{\phi'}{\phi} = \frac{1}{r} + \frac{kr}{1-kr^2}. \tag{13}$$

Integrating Equation (13), we get

$$\frac{1}{\omega+1}\phi^{\omega+1} = -\frac{A\sqrt{1-kr^2}}{k} + B, \tag{14}$$

where A and B are functions of time.

Integrating Equation (10), we get

$$\frac{1}{\omega+1}\phi^{\omega+1} = R(t)g(r) + Q(t). \tag{15}$$

From Equation (12), we get

$$\frac{\phi'}{\phi}\frac{d}{dr}\left[I_n\phi'\phi^{\omega}r^{-1}(1-kr^2)^{\frac{1}{2}}\right] = 0. \tag{16}$$

Using Equation (15) in Equation (16), we get

$$\frac{\phi'}{\phi}\frac{d}{dr}\left[I_n r^{-1}(1-kr^2)^{\frac{1}{2}} + I_n g'(r)\right] = 0, \tag{17}$$

from which it is obvious that ϕ is a function of r only, i.e. $Q(t) = 0$ in Equation (15) gives

$$\frac{1}{\omega+1}\phi^{\omega+1} = R(t)g(r). \tag{18}$$

Comparing Equations (14) and (15), we get $Q(t) = B = 0$. From Equation (14), we get

$$\frac{1}{\omega+1}\phi^{\omega+1} = -\frac{A\sqrt{1-kr^2}}{k}. \tag{19}$$

From Equations (17) and (18), we get

$$\frac{\dot{R}}{R} = \frac{\dot{A}}{A}. \tag{20}$$

Integrating, we get

$$R = NA, \tag{21}$$

where N is a constant of integration.

Using Equations (19) and (20) in Equations (6), (7), (9) and (11), we get

$$\frac{\kappa P}{\phi}=-\frac{k}{R^2}-\frac{2k}{R^2(\omega+1)}-\frac{\omega+3}{\omega+1}\frac{\dot{R}^2}{R^2}-\frac{2\omega+3}{\omega+1}\frac{\ddot{R}}{R}+\Lambda-\frac{\omega}{2}\left[\frac{k^2r^2}{R^2(\omega+1)^2(1-kr^2)}-\frac{\dot{R}^2}{(\omega+1)^2R^2}\right], \tag{22}$$

$$\frac{\kappa\rho}{\phi}=\frac{3k}{R^2}+\frac{3k}{R^2(\omega+1)}+\frac{3\omega+6}{\omega+1}\frac{\dot{R}^2}{R^2}-\Lambda-\frac{\omega}{2}\left[\frac{\dot{R}^2}{(\omega+1)^2R^2}-\frac{k^2r^2}{R^2(\omega+1)^2(1-kr^2)^2}\right], \tag{23}$$

and

$$\frac{\kappa}{\phi}(\rho-3P) \;=\; (3+2\omega)\left[\frac{3k}{R^2(\omega+1)}+\frac{\omega k^2r^2}{R^2(\omega+1)^2(1-kr^2)}+\frac{3\dot{R}^2}{(\omega+1)R^2}+\frac{\ddot{R}}{(\omega+1)R}-\frac{\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2}\right]. \tag{24}$$

From Equations (22) and (23), we get

$$\begin{aligned}\frac{\kappa}{\phi}(\rho-3P) \;=\;& \frac{6k}{R^2}+\frac{\rho k}{R^2(\omega+1)}+\frac{6\omega+15}{\omega+1}\frac{\dot{R}^2}{R^2}+\frac{6\omega+9}{\omega+1}\frac{\ddot{R}}{R}\\ &-4\Lambda-\frac{\omega}{2}\left[\frac{4\dot{R}^2}{(\omega+1)^2R^2}-\frac{4k^2r^2}{R^2(\omega+1)^2(1-kr^2)}\right].\end{aligned} \tag{25}$$

From Equations (24) and (25), we get

$$\frac{\rho k}{R^2(\omega+1)}+\frac{6\dot{R}^2}{R^2(\omega+1)}+\frac{4\omega+6}{\omega+1}\frac{\ddot{R}}{R}+\frac{(2\omega+1)\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2}-\frac{(2\omega+1)k^2r^2\omega}{R^2(\omega+1)^2(1-kr^2)}-4\Lambda=0. \tag{26}$$

2.1 Case I: When $\omega = 0$

In this case, Equations (22), (23) and (26) reduce to

$$\frac{\kappa P}{\phi}=-\frac{3k}{R^2}-\frac{3\dot{R}^2}{R^2}-\frac{3\ddot{R}}{R}+\Lambda, \tag{27}$$

$$\frac{\kappa\rho}{\phi}=\frac{6k}{R^2}+\frac{6\dot{R}^2}{R^2}-\Lambda, \tag{28}$$

$$\frac{6k}{R^2}+\frac{6\dot{R}^2}{R^2}+\frac{6\ddot{R}}{R}-4\Lambda=0. \tag{29}$$

Integrating Equation (29), we get

$$R=\sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\},\ \text{when}\quad k=1, \tag{30}$$

$$R=\sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\},\ \text{when}\quad k=-1, \tag{31}$$

$$R=e^{\sqrt{\frac{\Lambda}{3}}(t+D)},\ \text{when}\quad k=0, \tag{32}$$

where D is an arbitrary constant of integration.

Case I(a): When $k=1$, we get

$$R=\sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{33}$$

From Equation (21), we get

$$A=\frac{1}{N}\sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{34}$$

From Equation (19), we get

$$\phi = -\frac{\sqrt{1-r^2}}{N}\sqrt{\frac{3}{\Lambda}}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{35}$$

The gravitational variable is given by

$$G = -\sqrt{\frac{\Lambda}{3}}\frac{4N}{3\sqrt{1-r^2}}\frac{1}{\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}. \tag{36}$$

From Equations (27) and (28), we get

$$P = -\frac{\sqrt{3\Lambda(1-r^2)}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}{\kappa N}, \tag{37}$$

and

$$\rho = -\frac{\sqrt{3\Lambda(1-r^2)}\cosh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}{\kappa N}. \tag{38}$$

Hubble's parameter is given by

$$H = \sqrt{\frac{\Lambda}{3}}\tanh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{39}$$

Scalar expansion is given by

$$\Theta = \sqrt{3\Lambda}\tanh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{40}$$

In this model universe, it is seen that the gravitational variable G has a tendency to increase the pressure and decrease the density of the fluid whereas the B-D scalar field has a tendency to decrease the pressure and increase the density of this universe. This model has a singularity at $r = 1$.

Case I(b): When $k = -1$, we get

$$R = \sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{41}$$

From Equation (21), we get

$$A = \frac{1}{N}\sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{42}$$

From Equation (19), we get

$$\phi = \frac{\sqrt{1+r^2}}{N}\sqrt{\frac{3}{\Lambda}}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}, \tag{43}$$

which is a function of both r and t. When $t \to \infty$, both R and A tend to ∞, and when $r \to \infty$ and $t \to \infty$, the B-D scalar ϕ tends to ∞. Therefore, we conclude that for $k = -1$ the B-D scalar ϕ is an increasing function of both r and t.

The gravitational variable is given by

$$G = \sqrt{\frac{\Lambda}{3}}\frac{4N}{3\sqrt{1+r^2}}\frac{1}{\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}}, \tag{44}$$

which shows that gravitational variable G decreases as r and t increase and tends to zero when either $r \to \infty$ or $t \to \infty$.

From Equations (27) and (28), we get

$$P = -\frac{\sqrt{3A(1+r^2)}}{\kappa N}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}, \tag{45}$$

and

$$\rho = \frac{\sqrt{3\Lambda(1+r^2)}}{\kappa N}\sinh\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{46}$$

Hubble's parameter is given by

$$H = \sqrt{\frac{\Lambda}{3}}\coth\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{47}$$

Scalar expansion is given by

$$\Theta = \sqrt{3\Lambda}\coth\left\{\sqrt{\frac{\Lambda}{3}}(t+D)\right\}. \tag{48}$$

For this model universe, the scalar field helps in the expansion of the universe. Also, the expansion factor R increases with time thus accurately describing the expansion of the universe. Here in this type of model universe it is seen that pressure is negative and the equation of state $\omega_1 = \frac{P}{\rho} = -1$. Thus this universe seems to be a universe containing dark energy due to cosmological constant Λ. Again, here the scalar field ϕ also contributes to the expansion of this universe. Thus some part of the dark energy contained may be interpreted as quintessence in the form of dark energy which is in agreement with present observations, using equation of state $\omega_1 \simeq -1$.

Case I(c): When $k = 0$, we get

$$R = e^{\sqrt{\frac{\Lambda}{3}}(t+D)}, \tag{49}$$

and

$$A = \frac{1}{N}e^{\sqrt{\frac{\Lambda}{3}}(t+D)}. \tag{50}$$

From Equation (13), we get

$$\phi = \frac{1}{2N}r^2 e^{\sqrt{\frac{\Lambda}{3}}(t+D)}, \tag{51}$$

which is a function of both r and t. When $t \to \infty$, $R \to \infty$, and either $r \to \infty$ or $t \to \infty$, the B-D scalar ϕ tends to infinity.

The gravitational variable is given by

$$G = \frac{8N}{3r^2 e^{\sqrt{\frac{\Lambda}{3}}(t+D)}}, \tag{52}$$

which shows that gravitational variable G decreases as r and t increase and tends to zero as $r \to \infty$ or $t \to \infty$. From Equations (27) and (28), we get

$$P = -\frac{\Lambda r^2}{2\kappa N}e^{\sqrt{\frac{\Lambda}{3}}(t+D)}, \tag{53}$$

and

$$\rho = \frac{\Lambda r^2}{2\kappa N}e^{\sqrt{\frac{\Lambda}{3}}(t+D)}. \tag{54}$$

Hubble's parameter is given by

$$H = \sqrt{\frac{\Lambda}{3}}. \tag{55}$$

Scalar expansion is given by

$$\Theta = \sqrt{3\Lambda}. \tag{56}$$

Again for the solution in this case, it is found that the B-D scalar field ϕ is singular at the origin. However, on the other hand, at the origin, the gravitational force is very strong. As time t increases, the pressure decreases whereas the density increases. Thus there is the possibility that the model universe in this case contracts gradually and at some stage the density will be very high, thereby making it possible for the universe to become a black hole in the course of time. Or, in a different situation, the equation of state is $\omega_1 = \frac{P}{\rho} = -1$ whereas the pressure is negative. This implies that our model universe is expanding and contains dark energy due to the cosmological constant which is in agreement with present observational data, namely, $\frac{p}{\rho} \simeq -1$.

2.2 Case II: When $\omega = 0$ and $\Lambda = 0$

From Equation (29), we get

$$\frac{6k}{R^2} + \frac{6\dot{R}^2}{R^2} + \frac{6\ddot{R}}{R} = 0. \tag{57}$$

Integrating, we get

$$R = \sqrt{-kt^2 + 2at + 2b}, \tag{58}$$

where a and b are constants of integration. From Equation (21), we get

$$A = \frac{1}{N}\sqrt{-kt^2 + 2at + 2b}. \tag{59}$$

Case II(a): When $k = 1$.

From Equations (58) and (59), we get

$$R = \sqrt{-t^2 + 2at + 2b}, \tag{60}$$

and

$$A = \frac{1}{N}\sqrt{-t^2 + 2at + 2b}. \tag{61}$$

From Equation (19), we get

$$\phi = -\frac{1}{N}\sqrt{-t^2 + 2at + 2b}\sqrt{1 - r^2}, \tag{62}$$

which is a function of both r and t. The gravitational variable is given by

$$G = -\frac{4N}{3\sqrt{-t^2 + 2at + 2b}\sqrt{1 - r^2}}, \tag{63}$$

when $N < 0$. From Equations (60), (61), (62) and (63), we see that the reality condition for R, A, ϕ and k is $(a^2 + 2b) > (t - a)^2$ and $r^2 < 1$.

From Equations (27) and (28), we get

$$P = 0, \tag{64}$$

and

$$\rho = -\frac{6(a^2 + 2b)\sqrt{(1 - r^2)}}{\kappa N(-t^2 + 2at + 2b)^{\frac{3}{2}}}, \tag{65}$$

which is a function of both r and t. The reality condition is the same as above. Hubble's parameter is given by

$$H = \frac{t-a}{t^2-2at-2b}. \tag{66}$$

Scalar expansion is given by

$$\Theta = \frac{3(t-a)}{t^2-2at-2b}. \tag{67}$$

For this model universe, it is seen that at time t given by $t^2-2at-2b=0$ there may be a gravitational collapse. Since, in this case, the energy density is negative, there is the possibility that this universe contains a phantom form of dark energy. But there is doubt in this case as here the pressure is zero and this universe is closed, since dark energy is assumed to help in the accelerated expansion of the universe. Thus, when $k=1$, $\omega=0$ and $\Lambda=0$, the problem reduces to the case of dust distribution.

Case II(b): When $k=-1$.

From Equations (58) and (59), we get

$$R = \sqrt{t^2+2at+2b}, \tag{68}$$

and

$$A = \frac{1}{N}\sqrt{t^2+2at+2b}. \tag{69}$$

From Equation (19), we get

$$\phi = \frac{1}{N}\sqrt{t^2+2at+2b}\sqrt{1-r^2}, \tag{70}$$

which is a function of both r and t. When $t\to\infty$, the radius of the universe R tends to infinity, and the B-D scalar ϕ tends to infinity either when $r\to\infty$ or $t\to\infty$. The gravitational variable is given by

$$G = \frac{4N}{3\sqrt{t^2+2at+2b}\sqrt{1-r^2}}. \tag{71}$$

From Equation (71), we see that the gravitational variable G decreases when t and r increase and tends to zero when $r\to\infty$ or $t\to\infty$. From Equations (27) and (28), we get

$$P = 0, \tag{72}$$

and

$$\rho = \frac{6(a^2-2b)\sqrt{(1+r^2)}}{\kappa N(t^2+2at+2b)^{\frac{3}{2}}}, \tag{73}$$

which is real where $a^2-2b>0$. Hubble's parameter is given by

$$H = \frac{t+a}{t^2+2at+2b}. \tag{74}$$

Scalar expansion is given by

$$\Theta = \frac{3(t+a)}{t^2+2at+2b}. \tag{75}$$

In the solution for this case, it is obtained that as time t increases, the radius of our (model) universe increases, that is our universe is expanding which is the sign of being a realistic model. But here it is seen that this universe expands initially at a high rate and gradually the expansion slows down until it stops at infinitely large time when preparing for contraction. In this model universe, the B-D field influences the area given by $r=1$, and is inversely proportional to the gravitational potential due to G. Thus, when $k=-1$, $\omega=0$ and $\Lambda=0$, the problem reduces to the case of dust distribution.

Case II(c): When $k=0$.

From Equations (58) and (59), we get

$$R = \sqrt{2at+2b}, \tag{76}$$

and

$$A = \frac{1}{N}\sqrt{2at+2b}. \tag{77}$$

From Equation (76), we know that radius of the universe R tends to infinity when t tends to infinity. From Equation (13), we get

$$\phi = \frac{r^2\sqrt{2at+2b}}{2N}, \tag{78}$$

which is a function of both r and t. When either $r\to\infty$ or $t\to\infty$, the B-D scalar ϕ tends to infinity. The gravitational variable is given by

$$G = \frac{8N}{3r^2\sqrt{2at+2b}}, \tag{79}$$

which shows that the gravitational variable G decreases when r and t increase and tends to zero when either $r\to\infty$ or $t\to\infty$.

From Equations (27) and (28), we get

$$P = 0, \tag{80}$$

and

$$\rho = \frac{3a^2r^2}{\kappa N(2at+2b)^{\frac{3}{2}}}. \tag{81}$$

Hubble's parameter is given by

$$H = \frac{t}{2(at+b)}. \tag{82}$$

Scalar expansion is given by

$$\Theta = \frac{3t}{2(at+b)}. \tag{83}$$

From Equation (81), we see that ρ decreases when r is fixed and t increases and ρ increases when r increases and t decreases.

Regarding our model universe in this case, we have seen, from the expressions of R and ϕ, that the scalar field has a tendency to increase the radius of the universe, thereby helping in the expansion of the universe. The density of this universe is also seen to decrease with time which is a sign of a realistic universe. The expansion factor here is found to increase with time, thereby implying that our universe is expanding, which accurately describes the present universe. Thus, when $k = 0, \omega = 0$ and $\Lambda = 0$, the problem reduces to the case of dust distribution.

2.3 Case III: When $\omega \neq 0$ and $\Lambda = 0$

Since R is a function of t, we only consider the case $k = 0$. Then, Equation (26) reduces to

$$\frac{6\dot{R}^2}{R^2(\omega+1)} + \frac{4\omega+6}{\omega+1}\frac{\ddot{R}}{R} + \frac{(2\omega+1)\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2} = 0. \tag{84}$$

Integrating, we get

$$R = \left[\frac{(4+3\omega)(at+b)}{(2+2\omega)}\right]^{\frac{2+2\omega}{4+3\omega}}, \tag{85}$$

where a and b are arbitrary constants of integration. From Equation (21), we get

$$A = \frac{1}{N}\left[\frac{(4+3\omega)(at+b)}{(2+2\omega)}\right]^{\frac{2+2\omega}{4+3\omega}}. \tag{86}$$

If $\omega > 0$, the radius of the universe increases as t increases and tends to infinity as t tends to infinity. From Equation (13), we get

$$\phi = \left\{\frac{(\omega+1)r^2}{2N}\right\}^{\frac{1}{\omega+1}} \left\{\frac{(4+3\omega)(at+b)}{2+2\omega}\right\}^{\frac{2}{4+3\omega}}, \tag{87}$$

which is a function of both r and t. If $\omega > 0$, the B-D scalar ϕ tends to infinity either when $r \to \infty$ or $t \to \infty$. The gravitational variable is given by

$$G = \frac{4+2\omega}{3+2\omega}\left\{\frac{2N}{(\omega+1)r^2}\right\}^{\frac{1}{\omega+1}} \left\{\frac{2+2\omega}{(4+3\omega)(at+b)}\right\}^{\frac{2}{4+2\omega}}. \tag{88}$$

If $\omega > 0$, G decreases as r and t increase and tends to zero when either $r \to \infty$ or $t \to \infty$. From Equations (22) and (23), we get

$$P = -\frac{4a^2(2\omega+3)^2}{\kappa(4+3\omega)^2(at+b)^2}\left\{\frac{(\omega+1)r^2}{2N}\right\}^{\frac{1}{\omega+1}} \left\{\frac{(4+3\omega)(at+b)}{2+2\omega}\right\}^{\frac{2}{4+3\omega}}, \tag{89}$$

and

$$\rho = \frac{2a^2(2\omega+3)}{\kappa(3\omega+4)(at+b)^2}\left\{\frac{(\omega+1)r^2}{2N}\right\}^{\frac{1}{\omega+1}} \left\{\frac{(4+3\omega)(at+b)}{2+2\omega}\right\}^{\frac{2}{4+3\omega}}. \tag{90}$$

Hubble's parameter is given by

$$H = \frac{2a(\omega+1)}{(4+3\omega)(at+b)}. \tag{91}$$

Scalar expansion is given by

$$\Theta = \frac{6a(\omega+1)}{(4+3\omega)(at+b)}. \tag{92}$$

Considering the solution obtained in this case, the gravitational variable G is found to vary inversely with the scalar field ϕ. Thus in this case, the B-D scalar field has a tendency to decrease the gravitational potential. For this universe it is seen that the equation of state $\omega_1 < -1$, namely, $\omega_1 = \frac{P}{\rho} = -\frac{2(2\omega+3)}{4+3\omega} = -1 - \frac{\omega+2}{4+3\omega} < -1$. Thus the dark energy contained in this universe may be taken as the k-essence form of energy. Here we see that for the k-essence energy, with $\omega_1 < -1$, the scalar field grows in the future. Since the k-essence fields are similarly uniform on a small scale, the abundance of k-essence energy within a bound object actually grows with time, thereby increasing its influence on the internal dynamics. Ultimately, there is the possibility that the repulsive k-essence energy will overcome the forces holding this model together and pulls this universe apart in a Big Rip. Thus, when $k = 0, \omega \neq 0$ and $\Lambda = 0$, the problem reduces to the case of dust distribution.

2.4 Case IV: When $\omega \neq 0$ and $\Lambda \neq 0$

Since R is only a function of t, we just consider the case $k = 0$.

Then, Equation (26) reduces to

$$\frac{6\dot{R}^2}{R^2(\omega+1)} + \frac{4\omega+6}{\omega+1}\frac{\ddot{R}}{R} + \frac{(2\omega+1)\omega}{(\omega+1)^2}\frac{\dot{R}^2}{R^2} - 4\Lambda = 0. \tag{93}$$

Integrating, we get

$$R = e^{\frac{2(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}, \tag{94}$$

and

$$A = \frac{1}{N} e^{\frac{2(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}. \tag{95}$$

If $\omega > 0$, the radius of the universe R tends to infinity as t tends to infinity.

From Equation (13), we get

$$\phi = \left\{ \frac{(\omega+1)r^2}{2N} \right\}^{\frac{1}{\omega+1}} e^{\frac{2\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}, \tag{96}$$

which is a function of both r and t. When either $r \to \infty$ or $t \to \infty$, the B-D scalar ϕ tends to infinity. The gravitational variable is given by

$$G = \frac{4+2\omega}{3+2\omega} \left\{ \frac{2N}{(\omega+1)r^2} \right\}^{\frac{1}{\omega+1}} e^{-\frac{2\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}, \tag{97}$$

which is a function of both r and t. From Equation (97), we see that the gravitational variable G decreases when r and t increase and tends to zero when either $r \to \infty$ or $t \to \infty$. From Equations (22) and (23), we get

$$P = \frac{\Lambda}{\kappa(4+3\omega)} \tag{98}$$

and

$$\rho = \frac{\Lambda}{\kappa} \left\{ \frac{(\omega+1)r^2}{2N} \right\}^{\frac{1}{\omega+1}} e^{\frac{2\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}t}. \tag{99}$$

Hubble's parameter is given by

$$H = \frac{2(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}. \tag{100}$$

Scalar expansion is given by

$$\Theta = \frac{6(\omega+1)\sqrt{\Lambda}}{\sqrt{(2\omega+3)(3\omega+4)}}. \tag{101}$$

In this model universe, the scalar field is seen to have a tendency to increase the expansion of the universe, thereby flattening the universe. Here, also the B-D field has a tendency to decrease the gravitational potential, and the gravitational variable G tends to decrease the pressure and density of the universe. Since here, as $t \to \infty$, it is found that $R \to \infty$ as well as $\rho \to \infty$, there is the possibility of a bounce at some point in time, thereby indicating that this universe shows cyclic behavior. If $8\sqrt{\Lambda} > \frac{\omega\sqrt{(2\omega+3)(3\omega+4)}}{(\omega+1)^2}$, then this model universe will have an accelerated expansion instigated by the negative pressure. Also, in this model the vacuum energy due to the cosmological constant may be taken as the dark energy part causing the accelerated expansion of the universe.

3 CONCLUSIONS

The universes we have investigated are found to behave in different ways and to show different manifestations under different conditions. Some of them show signs of containing a cosmological constant form and quintessence form of dark energy, whereas others seem to contain fluids behaving like phantom and k-essence forms of dark energy, which can explain the present accelerated expansion of the universe. Thus the model universes we obtain in these cases may be taken as realistic models of our universe, and many more unknown properties of the universe and of dark energy may be realized and known from further studies of these models, which we will perform and report elsewhere afterwards. Furthermore, one model of ours seems to undergo a gravitational collapse leading to a black hole; whereas another model surprisingly seems to face the fate of a Big Rip. Another new finding in some of our models is that they simultaneously contain two forms of dark energy, one due to a cosmological constant and another due to a B-D scalar field. Also, interestingly enough, one of our models seems to behave like a universe obeying the newly proposed cyclic theory of the universe.

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