Chaplygin gas of Tachyon Nature Imposed by Noether Symmetry and constrained via $H(z)$ data

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Abstract   An action of general form is proposed for a Universe containing matter, radiation and dark energy. The latter is interpreted as a tachyon field non-minimally coupled to the scalar curvature. The Palatini approach is used when varying the action so the connection is given by a more generic form. Both the self-interaction potential and the non-minimally coupling function are obtained by constraining the system to present invariability under global point transformation of the fields (Noether Symmetry). The only possible solution is shown to be that of minimal coupling and constant potential (Chaplygin gas). The behavior of the dynamical properties of the system is compared to recent observational data, which infers that the tachyon field must indeed be dynamical.

Key words: cosmology: cosmological parameters — cosmology: theory — cosmology: observations — dark energy

1 INTRODUCTION

Tachyons have been extensively studied in M/String theories. Since the realization of their condensation proprieties, researchers have gained interest in their applications in cosmology. At first, there was the problem of describing the string theory tachyon by an effective field theory that would lead to the correct Lagrangian in classical gravity. The first classical description of the tachyon field (Sen 2002a,b) addressed the Lagrangian problem, making way for building the first model within tachyon cosmology (Gibbons 2002).

Being a special kind of a scalar field, it presents negative pressure, making the tachyon a natural candidate to explain dark energy (Padmanabhan 2002; Hao & Li 2002; Bagla et al. 2003; Jassal 2004). The inflationary period could also be explained if one considers the inflaton to behave as a tachyon field. Many different attempts were made under this assumption, testing a wide variety of self-interacting potentials such as power-laws, exponentials and hyperbolic functions of the field (Abramo & Finelli 2003; Liu & Li 2004; Kremer & Alves 2004; Steer & Vernizzi 2004; Campuzano et al. 2006; Herrera et al. 2006; Xiong & Zhu 2007; Balart et al. 2007). The possible scenario where the tachyon plays both roles, inflaton and dark energy, has also been studied in related works (Sami et al. 2002; Cárdenas 2006), where the first constraints on the potential were established so the radiation era could commence.

The studies above introduced a tachyon field which is minimally coupled through the metric, hence providing just another source for the gravitational field. Nevertheless, such fields might also be considered to be non-minimally coupled to the scalar curvature, becoming part of the space-time geometry by generating a new degree of freedom for gravity. In this scenario, the gravitational constant G becomes a variable function of spacetime.

Tachyon fields in the non-minimal coupling context were analyzed for both the inflationary period (Piao et al. 2003) and the current era (Srivastava 2004). In those cases, the coupling functions and the self-interacting potentials were given in an *ad-hoc* manner, as exponentials and power-law forms.

Every time we choose a different coupling or potential function, we create a new cosmological model, or even a new theory of gravity in the non-minimal case. This is a very difficult task since the lack of experiments and observations obligates one to find heuristic arguments to support the choice made. The advantages of searching for symmetries in systems where the Lagrangian is known is widely entertained. This approach not only helps us find exact solutions but might also give us physically meaningful constants of motion. What is less appreciated is the fact that one can constrain a system (one that lacks a closed form of the functional) to present symmetry. In what concerns non-minimally coupled tachyon fields, Noether symmetries were used to establish the coupling and self-interaction functions in the papers (de Souza & Kremer 2009; Collodel & Kremer 2015). The latter makes use of the Palatini approach, in a way to generalize the theory, since the non-minimal coupling can provide a metric-independent connection.

The Chaplygin gas was first introduced by Chaplygin in 1902 (Chaplygin 1902) in the context of aerodynamics. This gas features an exotic equation of state ($p_c \propto -1/\rho_c$), which was originally used to describe the lifting force on a wing of an airplane. Because its pressure is negative, the Chaplygin gas became a good candidate to explain dark energy (Kamenshchik et al. 2001; Fabris et al. 2002; Bilić et al. 2002; Bento et al. 2002; Gorini et al. 2003; Kremer 2003). The attempts to correlate fields and fluids soon showed that the constant potential tachyon field behaves as a Chaplygin gas (Frolov et al. 2002; Gorini et al. 2004; Chimento 2004; Del Campo & Herrera 2008). Its equation of state allows generalizations, giving rise to the so called *Generalized Chaplygin Gas*, or just GCG. This gas exerts a negative pressure proportional in modulus to the inverse of some power of its energy density and was investigated in works such as (Biesiada et al. 2005; Wang et al. 2009; Liao et al. 2013; Xu & Lu 2010; Wang et al. 2013), including its relationship to a - now, not constant potential - tachyon field (Gupta et al. 2012). Originally, the equation of state of a Chaplyigan gas was so simple that even with the exhaustive studies about the GCG there was still plenty of room for further generalizations. Endowing the equation of state (EoS) with a linear barotropic term, which alone would describe an ordinary fluid, enriched the GCG which under this assumption is called the *Modified Chaplygin Gas*, MCG. Its motivations lie precisely on the possible field nature of the gas (Benaoum 2002), and its parameters have been constrained via observational analysis (Paul & Thakur 2013). Further generalizations account for higher order energy density terms in the EoS of a Chaplygin Gas, the *Extended Chaplygin Gas*, ECG (Pourhassan & Kahya 2014; Lu et al. 2014).

In this work, we start from a very general Lagrangian for a tachyon field non-minimally coupled to the scalar curvature. Matter and radiation fields are also included in the system as perfect fluids from the beginning. The connection is initially taken to be metric-independent and the action is also varied with respect to it, a process known as the Palatini approach. Since we consider a flat, homogeneous and isotropic Universe, the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric is used to rewrite our functional in the form of a point-like Lagrangian. This presents an extra term than usual, which comes from the independent connection. The system is constrained such that it presents invariance under continuous point transformations, or a Noether symmetry. The coupling and self-interaction potential functions of the tachyon field are then determined. Every new field added to the Lagrangian clearly influences these point transformations. For that matter, it is important to start off from a complete system (including the radiation fields) if one takes symmetry as a first principle. We show that for this system to be Noether symmetrical, the non-minimal coupling must vanish and the self-interaction potential must be constant, hence representing the tachyonic Chaplygin gas. The system is initially composed of five free parameters,

namely the Hubble constant, the three density parameters for recent times and the normalized constant potential. The radiation parameter is then established in an *ad-hoc* way so we are left with four different free parameters. These are determined via the χ^2 analysis for the recent $H(z)$ data from supernovae (SNe) and gamma-ray bursts. We show that although dark energy tends asymptotically to a cosmological constant, any small discrepancies make the tachyon field dynamically active, so the Chaplygin gas exhibits the property of transition from pressureless matter to dark energy.

In order to clarify the typos and notations used here, we remark: the metric signature is $(+, -, -, -)$; the Levi-Civita connection is written with a tilde ($\tilde{\Gamma}^\lambda_{\mu\nu} = \{^\lambda_{\mu\nu}\}$) while the independent connection is given without it ($\Gamma^\lambda_{\mu\nu}$). Natural constants have been rescaled to unity ($8\pi G = c = 1$). Throughout the whole paper, derivatives in equations are presented as follows: dots represent time derivatives, while $\partial_{q^i} \equiv \frac{\partial}{\partial q^i}$ and $\partial_{\dot{q}^i} \equiv \frac{\partial}{\partial \dot{q}^i}$ stand for partial derivatives with respect to the generalized coordinate q^i and velocity $\dot{q}^i$, respectively.

2 ACTION AND POINT LAGRANGIAN

A generalization of the general theory of relativity is proposed through a non-minimal coupling of a function of the tachyon field. The general action for both geometry and source is written

$$S = \int d^4x \sqrt{-g} f(\phi) R - V(\phi)\sqrt{1 - \partial_\mu \phi \partial^\mu \phi} - \mathcal{L}_s, \quad (1)$$

where ϕ is the tachyon field, $f(\phi)$ is the non-minimal coupling function, $V(\phi)$ is the self-interaction potential and $\mathcal{L}_s$ is the Lagrangian density of other sources (matter and radiation).

In order to attain a more general theory we allow the connection to be metric-independent. The variation of the action with respect to the connection $\Gamma^\rho_{\mu\nu}$ results in the well-known form

$$\Gamma^\rho_{\mu\nu} = \tilde{\Gamma}^\rho_{\mu\nu} + \frac{1}{2f}\left(\delta^\rho_\nu \partial_\mu f + \delta^\rho_\mu \partial_\nu f - g_{\mu\nu}\partial^\rho f\right). \quad (2)$$

where $\tilde{\Gamma}^\rho_{\mu\nu}$ is the Levi-Civita connection.

Usually, the self-interaction potential and the coupling function are set in an *ad-hoc* manner. Instead of approaching the problem this way, we would like to constrain the system to that which has a Noether symmetry. This is done by operating a variational vector field on the point-like Lagrangian, and for this, we need to rewrite it in terms of a specific metric. For a flat, homogeneous and isotropic Universe, spacetime is described by the flat FLRW metric. The point-like functional in Equation (1) then becomes

$$\begin{aligned}
L =\ & 6f(\ddot{a}a^2 + \dot{a}^2 a) - \frac{3a^3}{2f}(\partial_\phi f \dot{\phi})^2 \\
& + 3a^3 \partial^2_\phi f \dot{\phi}^2 + 3a^3 \partial_\phi f \ddot{\phi} \\
& + 9\dot{a}a^2 \partial_\phi f \dot{\phi} - a^3 V\sqrt{1 - \dot{\phi}^2} - a^3 \rho_s,
\end{aligned} \quad (3)$$

and ρ_s is a point-like Lagrangian for a perfect fluid (Hawking & Ellis 1973).

In this system, besides dark energy, the Universe is composed of matter (ordinary and dark) and radiation. Both dark matter and ordinary matter are treated as dust, and hence represented by the same entity here. As the Universe expands, the matter density decreases with a^{-3} while the radiation's with a^{-4}. The Lagrangian above contains second-order terms which are more tedious to deal with. Since the action limits are fixed, we can integrate these terms by parts, without loss of generality, so we can work with a first-order Lagrangian, which reads

$$
\begin{aligned}
L =\ & -6f\dot{a}^2 a - 6a^2\dot{a}\partial_\phi f\dot{\phi} - a^3 V\sqrt{1-\dot{\phi}^2} \\
& -\frac{3a^3}{2f}(\partial_\phi F\dot{\phi})^2 - \rho_m^0 - \frac{\rho_r^0}{a},
\end{aligned} \tag{4}
$$

where ρ_m^0 and ρ_r^0 are the recent values of the total density of matter and radiation, respectively, in the Universe.

3 NOETHER SYMMETRIES

Our system may now be constrained to that which is endowed with a Noether symmetry by finding the forms of $f(\phi)$ and $V(\phi)$ that allow symmetrical point transformation. This means that our Lagrangian shall have such a form that a specific continuous transformation of the generalized coordinates $a \to \bar{a}$ and $\phi \to \bar{\phi}$ preserves the general form of the functional,

$$
L(\bar{a}, \bar{\phi}) = L(a, \phi). \tag{5}
$$

In order to find the function forms of $V(\phi)$ and $f(\phi)$ that allow such transformation, we need to apply a certain vector field on the Lagrangian given in Equation (4). This vector field, $\boldsymbol{X}$, is then called a variational symmetry, or *complete lift*, and reads

$$
\boldsymbol{X} \equiv \alpha^i \partial_{q^i} + \dot{\alpha}^i \partial_{\dot{q}^i}, \tag{6}
$$

where the coefficients α^i are functions of the generalized coordinates a, ϕ. The operation of $\boldsymbol{X}$ on the Lagrangian is simply the Lie derivative of L along this vector field ($L_{\boldsymbol{X}}L$). According to the Noether theorem, if this derivative vanishes, there will be a conserved quantity named *Noether charge*. Hence, this will be a variational symmetry if

$$
\boldsymbol{X}L = L_{\boldsymbol{X}}L = 0, \tag{7}
$$

such that

$$
L_\Delta \langle \theta_L, \boldsymbol{X} \rangle = 0, \tag{8}
$$

where $\Delta = d/dt$ is the dynamical vector field and

$$
\theta_L = \frac{\partial L}{\partial \dot{q}^i} dq^i, \tag{9}
$$

is the locally defined Cartan one-form. The brackets represent the scalar product between vector field and one-form, in the Dirac notation. Thus, the Noether charge reads

$$
\Sigma_0 \equiv \langle \theta_L, \boldsymbol{X} \rangle = \alpha^i \frac{\partial L}{\partial \dot{q}^i}. \tag{10}
$$

The condition defined in Equation (7) reads in full form,

$$
\begin{aligned}
0 =\ & XL \\
=\ & \alpha\partial_a L + \beta\partial_\phi L + \left(\dot{a}\partial_a\alpha + \dot{\phi}\partial_\phi\alpha\right)\partial_{\dot{a}}L \\
& + \left(\dot{a}\partial_a\beta + \dot{\phi}\partial_\phi\beta\right)\partial_{\dot{\phi}}L,
\end{aligned} \tag{11}
$$

which for our system becomes

$$
\begin{aligned}
0 =\ & \alpha\Big(-6f\dot{a}^2 - 12a\dot{a}\partial_\phi f\dot{\phi} - 3a^2 V\sqrt{1-\dot{\phi}^2} \\
& -\frac{9a^2(\partial_\phi f)^2\dot{\phi}^2}{2f} + \frac{\rho_r^0}{a^2}\Big) + \beta\Big(-6\partial_\phi f\dot{a}^2 a \\
& -6a^2\dot{a}\partial_\phi^2 f\dot{\phi} - a^3\partial_\phi V\sqrt{1-\dot{\phi}^2} \\
& +\frac{3a^3(\partial_\phi f)^3\dot{\phi}^2}{2f^2} - \frac{3a^3\partial_\phi f\partial_\phi^2 f\dot{\phi}^2}{f}\Big) \\
& + \left(\partial_a\alpha\dot{a} + \partial_\phi\alpha\dot{\phi}\right)\left(-12f\dot{a}a - 6a^2\partial_\phi f\dot{\phi}\right) \\
& + \left(\partial_a\beta\dot{a} + \partial_\phi\beta\dot{\phi}\right) \times \Big(-6a^2\dot{a}\partial_\phi f \\
& +\frac{a^3 V\dot{\phi}}{\sqrt{1-\dot{\phi}^2}} - 3a^3\frac{(\partial_\phi f)^2\dot{\phi}}{f}\Big),
\end{aligned} \tag{12}
$$

where $\alpha = \alpha^1$ and $\beta = \alpha^2$.

The equation above must hold for any value of $\dot{a}$ and $\dot{\phi}$. If it were a polynomial equation for these dynamical variables one could simply set all coefficients equal to zero, but the different powers of the square roots make the task a little more complicated. We shall differentiate with respect to these quantities and evaluate the resulting equations for different values of $\dot{a}$ and $\dot{\phi}$, then we get the solutions for $\alpha(a,\phi)$, $\beta(a,\phi)$, $V(\phi)$ and $f(\phi)$. Setting $\dot{a} = \dot{\phi} = 0$ in Equation (12) we get

$$
3\alpha a^2 V - \alpha\frac{\rho_r^0}{a^2} + \beta a^3\partial_\phi V = 0. \tag{13}
$$

Differentiating Equation (12) three times with respect to $\dot{\phi}$ and evaluating at $\dot{\phi} = 0$ and $\dot{a} = 1$ gives

$$
3a^3 V\partial_a\beta = 0, \tag{14}
$$

hence $\beta \neq \beta(a)$. Similarly, differentiating Equation (12) once with respect to $\dot{\phi}$, multiplying it by $(1-\dot{\phi}^2)^{3/2}$ and evaluating at $\dot{\phi} = 1$ and $\dot{a} = 0$ we get

$$
V\partial_\phi\beta = 0, \tag{15}
$$

and we conclude that $\beta = \beta_0$ is a constant, since the potential must be non-zero. The fourth derivative of

Equation (12) with respect to $\dot\phi$, evaluated at $\dot\phi = 0$ and taking into account that $\partial_\phi\beta = 0$, leads to

$$9\alpha a^2 V + 3\beta a^3 \partial_\phi V = 0. \tag{16}$$

Since $\rho_r^0 \neq 0$, dividing Equation (16) by 3 and equating with Equation (13) results in $\alpha = 0$ and $V = V_0$, a constant potential. Thus, Equation (12) reduces to

$$-6\partial_\phi f \dot a^2 a - 6a^2 \dot a \partial_\phi^2 f \dot\phi + \frac{3a^3(\partial_\phi f)^3 \dot\phi^2}{2f^2}$$
$$-\frac{3a^3 \partial_\phi f \partial_\phi^2 f \dot\phi^2}{f} = 0, \tag{17}$$

and it is clear that $\partial_\phi f = 0$. The coupling must then be minimal.

4 EQUATIONS OF MOTION

The Lagrangian, Equation (4), for constant self-interaction potential and $f = 1/2$ (to regain Einstein's constant according to the notation adopted), reads

$$L = -3a\dot a^2 - V_0 a^3 \sqrt{1 - \dot\phi^2} - \rho_m^0 - \frac{\rho_r^0}{a}. \tag{18}$$

The Friedmann equation is obtained through the energy equation $E_L = \dot a \frac{\partial L}{\partial \dot a} + \dot\phi \frac{\partial L}{\partial \dot\phi} - L$, which gives

$$H^2 = \frac{1}{3}\rho, \tag{19}$$

where $H = \dot a/a$ is the Hubble parameter and $\rho = \rho_m + \rho_r + \rho_\phi$ is the total energy density of the fields.

$$\rho_\phi = \frac{V_0}{\sqrt{1 - \dot\phi^2}}, \tag{20}$$

is the energy density of the tachyon field; $\rho_m = \rho_m^0/a^3$ and $\rho_r = \rho_r^0/a^4$ are the matter and the relativistic material densities, respectively.

The Euler-Lagrange equation for the scale factor, together with Equation (19), provides the acceleration equation, which is

$$\frac{\ddot a}{a} = -\frac{1}{6}(\rho + 3p), \tag{21}$$

where $p = p_r + p_\phi$ is the pressure of the fields (as usual, matter behaves as dust so $p_m = 0$), and $p_r = \rho_r/3$. The pressure exerted by the tachyon field is

$$p_\phi = -V_0\sqrt{1 - \dot\phi^2}. \tag{22}$$

The Euler-Lagrange equation for the tachyon field gives the generalized Klein-Gordon equation for the field, which is the same as the fluid equation for dark energy when written in terms of its energy density and pressure

$$\dot\rho_\phi + 3H(\rho_\phi + p_\phi) = 0. \tag{23}$$

An equation of state in the form $p(\rho)$ can now be written for the tachyon field. From Equation (20) we see that $\sqrt{1 - \dot\phi^2} = V_0/\rho_\phi$, which when substituted in Equation (22) yields

$$p_\phi = -\frac{V_0^2}{\rho_\phi}. \tag{24}$$

The Chaplygin gas is a fluid described by an equation of state of the kind

$$p = -\frac{A}{\rho}, \tag{25}$$

where A is a positive defined constant, which is precisely the same as Equation (24) for $A = V_0^2$. Thus, as is widely known from the literature, see e.g. (Frolov et al. 2002; Gorini et al. 2004; Chimento 2004; Del Campo & Herrera 2008), a tachyon field only minimally coupled to the scalar curvature, with constant potential, behaves as a Chaplygin gas.

5 NOETHER CONSTANT

Any Lagrangian system endowed with a Noether symmetry will exhibit a constant of motion, as stated by Noether's theorem. The Noether charge given by Equation (10) here becomes

$$\Sigma_0 = \alpha\frac{\partial L}{\partial \dot a} + \beta\frac{\partial L}{\partial \dot\phi} = \frac{V_0 a^3 \dot\phi}{\sqrt{1 - \dot\phi^2}}, \tag{26}$$

which is simply the first integral of Equation (23).

6 SOLUTIONS

The energy density of the Chaplygin gas, and its pressure, can be rewritten as functions of the scale factor, using Equation (10). These forms are well known from literature and read

$$\rho_\phi = \sqrt{\frac{\Sigma_0^2}{a^6} + V_0^2}; \qquad p_\phi = -\frac{V_0^2}{\sqrt{\frac{\Sigma_0^2}{a^6} + V_0^2}}, \tag{27}$$

where Σ_0 is the Noether constant. From this equation, we see the dual nature of the Chaplygin gas, which behaves as dust matter for $a \leq 1$

$$\rho_\phi \sim \frac{\Sigma_0}{a^3}; \qquad p_\phi \sim 0, \tag{28}$$

and as a cosmological constant for $a \geq 1$

$$\rho_\phi \sim V_0; \qquad p_\phi \sim -V_0. \tag{29}$$

In order to obtain curves showing the variation of our parameters with respect to the redshift, we use the relationship $a = 1/(1 + z)$. The Friedmann equation (Equation (19)) then becomes

$$H^2 = \frac{1}{3}\left(\sqrt{\Sigma_0^2(1 + z)^6 + V_0^2} + \rho_m^0(1 + z)^3 \right.$$
$$\left. + \rho_r^0(1 + z)^4\right). \tag{30}$$

The equation above can be written in a dimensionless form by dividing it by the Hubble constant, $H_0^2 \equiv H(0)^2 = \rho^0/3$, where ρ^0 is the current density of all fluids in the Universe, giving

$$\frac{H^2}{H_0^2} = \left(\sqrt{\bar{\Sigma}_0^2(1+z)^6 + \bar{V}_0^2} + \Omega_m^0(1+z)^3 \right.$$
$$\left. + \Omega_r^0(1+z)^4 \right), \tag{31}$$

where $\Omega_i^0 \equiv \rho_i^0/\rho^0$ is the current density parameter of the i-th component. The bars indicate that the constants have also been divided by the current density, i.e., $\bar{\Sigma}_0 = \Sigma_0/\rho^0$ and $\bar{V}_0 = V_0/\rho^0$, then the density parameter for the Chaplygin gas is simply

$$\Omega_\phi^0 = \sqrt{\bar{\Sigma}_0^2 + \bar{V}_0^2}. \tag{32}$$

This last relationship allows us to investigate the evolution of the Hubble parameter in terms of the dark energy's density parameter, instead of the Noether charge. Finally, we write

$$H = H_0 \left[\sqrt{[(\Omega_\phi^0)^2 - \bar{V}_0^2](1+z)^6 + \bar{V}_0^2} \right.$$
$$\left. + \Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 \right]^{1/2}. \tag{33}$$

Recent observations (Bennett et al. 2013; Planck Collaboration 2014) limit the range of values associated to these parameters. In particular, there is great confidence that $\Omega_r^0 \sim 8.5 \times 10^{-5}$, so we may adopt this result but we will constrain the four remaining parameters (namely $H_0, \Omega_m^0, \Omega_\phi^0$ and $\bar{V}_0$) via $H(z)$ data.

Table 1 presents 25 measurements of the Hubble parameter from SNe and gamma-ray bursts (Simon et al. 2005; Stern et al. 2010; Moresco et al. 2012; Gaztañaga et al. 2009; Zhang et al. 2014). The constructed function that describes the Hubble parameter, Equation (33), depends on the redshift, plus four different parameters. Hence, $H = H(z, H_0, \Omega_m^0, \Omega_\phi^0, \bar{V}_0)$. The values assumed by these parameters that best fit the observational data are the ones that minimize the function

$$\chi^2 = \sum_{i=1}^{25} \left[\frac{H_{\mathrm{obs}}(z_i) - H(z_i, H_0, \Omega_m^0, \Omega_\phi^0, \bar{V}_0)}{\sigma_i} \right]^2. \tag{34}$$

A primary condition for a good fit is that $\chi^2/\mathrm{dof} \leq 1$, where 'dof' stands for degrees of freedom and in this case is given by the number of data points, dof= 25. Our minimized χ^2 is given by $H_0 = 69.6524, \Omega_m^0 = 0.288261, \Omega_\phi^0 = 0.711654$ and $\bar{V}_0 = 0.709957$ resulting in $\chi^2 = 12.8676$ and $\chi^2/\mathrm{dof} = 0.507504$. Marginalizing over two parameters allows us to analyze the correlation by plotting the contours of their distributions within some confidence interval.

The correlation between dark energy and matter density parameter is strong. The $H(z)$ does not seem to impose very strict constraints for our current matter density,

Table 1 Observational values for $H(z)$ and their respective errors (Simon et al. 2005; Stern et al. 2010; Moresco et al. 2012; Gaztañaga et al. 2009; Zhang et al. 2014).

z	H_{obs}	σ
0.07	69	19.6
0.09	69	12
0.12	68.6	26.2
0.17	83	8
0.179	75	4
0.199	75	5
0.2	72.9	29.6
0.24	79.69	3.32
0.27	77	14
0.28	88.8	36.6
0.352	83	14
0.4	95	17
0.43	86.45	3.27
0.48	97	62
0.593	104	13
0.68	92	8
0.781	105	12
0.875	125	17
0.88	90	40
0.9	117	23
1.037	154	20
1.3	168	17
1.43	177	18
1.53	140	14
1.75	202	40
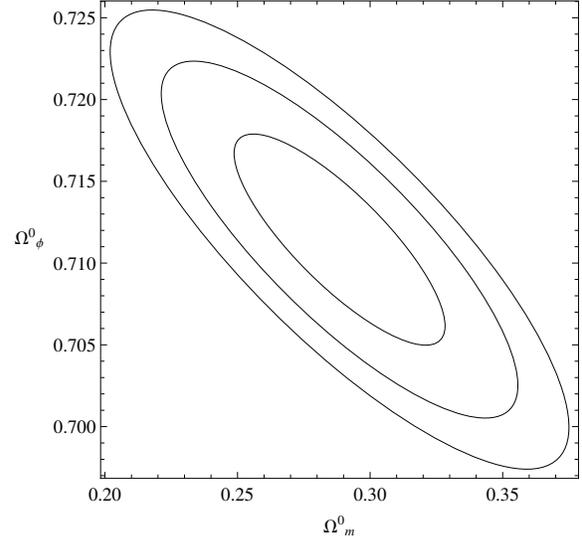
Fig. 1 Confidence intervals for 1-, 2- and 3-σ for the density parameters Ω_m^0 and Ω_ϕ^0.

but for dark energy we see that within 3-σ all points lie in the range $0.697 \leq \Omega_\phi^0 \leq 0.726$, as shown in Figure 1. The correlation between Ω_ϕ^0 and $\bar{V}_0$ is much stronger, as one would expect since the potential defines the energy density. Nevertheless, it is quite interesting to see the form these ellipses take in Figure 2; the current density parameter for dark energy is given by Equation (32). The case $\Omega_\phi^0 = \bar{V}_0$ is just the cosmological constant scenario. From the figure, we see very thin ellipses with a slope close to unity.
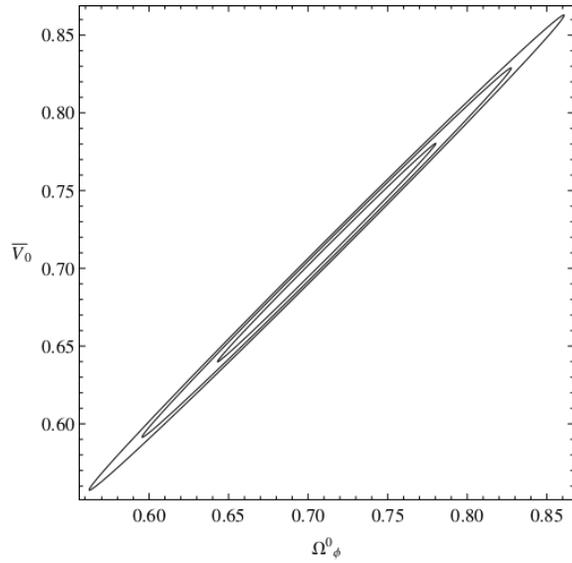
Fig. 2 Confidence intervals for 1-, 2- and 3-σ for the density parameter Ω_ϕ^0 and the constant potential $\bar{V}_0$.

The best fit parameters listed above show a very small difference between the two of them, and as we will see this difference grows bigger in the past, but there is a high tendency for the cosmological constant.

The evolution of the density parameters for different redshift scales is shown below. In Figure 3 radiation is neglected for its energy density is too small to be observed. As the redshift increases dark energy falls but ever more slowly, and for values $z \geq 2$ its density decreases so smoothly that it almost appears to be constant. This is due to the small difference between Ω_ϕ^0 and $\bar{V}_0$ that makes the tachyon field dynamical and the Chaplygin gas property thrive. From Equations (27) and (28) it becomes clear that dark energy decays into matter fields as the redshift grows and the term $\bar{\Sigma}_0$ outpaces $\bar{V}_0$. Furthermore, we are now looking at the matter era, hence the almost constant behavior. In Figure 4 we see the evolution of the density parameters for radiation and the combination of the Chaplygin gas and matter, since the first behaves as the latter. In this scenario, the equality of the densities happens a bit earlier in our history than expected from the cosmological constant case. For instance, we have $z_{\text{eq}} = 3968.15$ (approximately 37.5 thousand years since the beginning of the Universe) whereas for a non-dynamical field description we would expect $z_{\text{eq}} \sim 3600$ (about 47 thousand years old). Although a transition happening at higher redshifts does not influence the time at which recombination occurs (once it depends on the temperature), the earlier increase in dark matter energy density allows it to combine and form potential wells earlier, giving room for structure formation, some of which we have recently discovered and which turned out to be quite old (Andreon et al. 2009; Andreon & Huertas-Company 2011).

The ratio $\omega_\phi = p_\phi/\rho_\phi$, between the pressure and energy density for dark energy, is shown in Figure 5. As expected from Equation (27), the ratio tends asymptotically
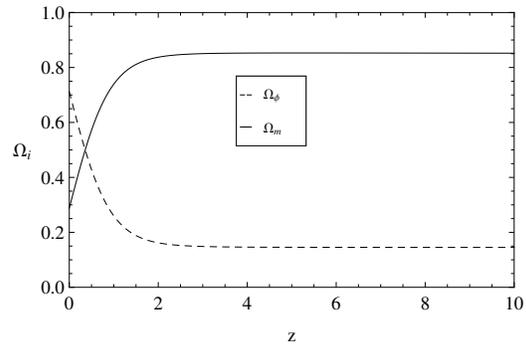
Fig. 3 The dashed line stands for dark energy while the solid line represents dark matter. As we enter the matter dominated era, dark energy decays into matter fields contributing even more to its dominance, with its energy density falling ever more slowly, assuming an almost constant behavior.
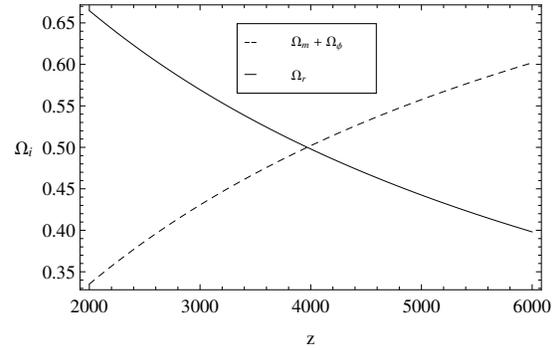
Fig. 4 Density parameters plotted for high redshift values. The dashed line represents the matter fields, where the Chaplygin gas is included once it behaves as dust at this point. The solid line stands for radiation's energy density parameter. Equality in densities happens at $z = 3968.15$.
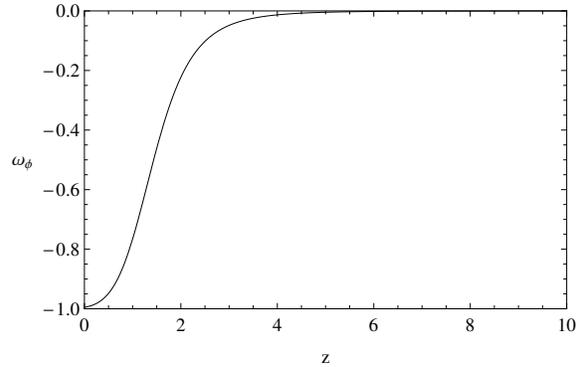
Fig. 5 Ratio between pressure and energy density for the Chaplygin gas. Any small difference between Ω_ϕ^0 and $\bar{V}_0$ grows considerably with the redshift and dark energy eventually becomes a pressureless field, and hence matter.

to -1 as the Universe expands but approaches zero quickly as the redshift increases, when dark energy finally becomes a pressureless field. The role of the Chaplygin gas becomes clearer even though we are approaching the cosmological constant in recent times.
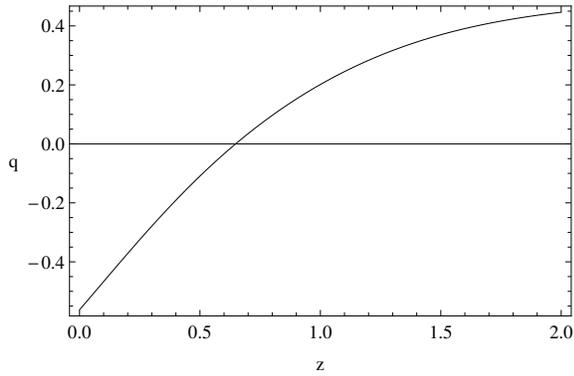
Fig. 6 Deceleration parameter. Expansion becomes accelerated at $z_t = 0.65$ and the current value of this parameters stands at $q_0 = -0.56$.
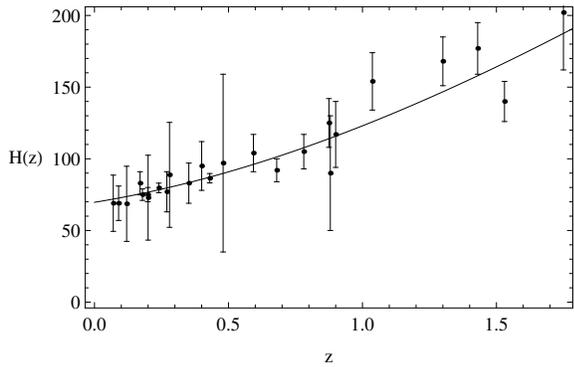
Fig. 7 The Hubble parameter curve together with its observational data (Simon et al. 2005; Stern et al. 2010; Moresco et al. 2012; Gaztañaga et al. 2009; Zhang et al. 2014).

The deceleration parameter q is plotted in Figure 6. The transition from a decelerated to an accelerated expansion happens at $z_t = 0.65$, while for our current time $q_0 = -0.56$, both of which are in agreement with observations (del Campo et al. 2012).

Although the Chaplygin gas has been extensively studied before, new observational data provide great motivation to revisit the model and set new constraints. The evolution of the Hubble parameter described by this model, together with the data we used to define our parameters, is shown in Figure 7. Unfortunately, there are not many satisfactory measurements to compute solid statistics for this parameter as there are for the distance modulus, for instance. Also, the errors associated with the data from gamma-ray bursts are much bigger than one would desire them to be. Nevertheless, these sources provide information from a much younger Universe compared to SNe, making it worthwhile for testing models and constraining parameters.

7 CONCLUSIONS

In this work, we started from a general action where a tachyon field represents the nature of dark energy. We allowed it to be non-minimally coupled to the scalar curvature and we considered the connection to be independent through the Palatini approach. Dark matter, baryonic matter and relativistic material were included in source fields, as our intention was to build a more complete model. Instead of establishing the self-interaction potential and the coupling function in an *ad-hoc* manner, we stated that symmetry should play a more primary role and only functions capable of composing a continuous and symmetric point transformation on the generalized coordinates would be considered. This led to the simpler case where the tachyon field is only minimally coupled and its potential is constant, behaving as a Chaplygin gas.

The theoretical framework of the Chaplygin gas has been thoroughly investigated and is widely found in the literature.? For this reason, we focused on more recent observational data to constrain the dynamics of the system. The Hubble parameter suggests that, if the Chaplygin gas is the underlying nature of dark energy, it should be slightly dynamical, as opposed to the particular case of a cosmological constant since, as we see, any small difference between its constant potential and current density parameter grows considerably with redshift. As this component exhibits a dual behavior, acting as dark energy for small redshifts and decaying into matter fields later on, the matter era begins earlier in the history of the Universe, which could help explain older structures.

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