RAA 2016 Vol. **16** No. **3**, 52 (6pp) doi: 10.1088/1674–4527/16/3/052
http://www.raa-journal.org *http://iopscience.iop.org/raa*

***R**esearch in*
***A**stronomy and*
***A**strophysics*

Orbital correlation of space objects based on orbital elements

Xiu-Hong Wang[1,2], Jun-Feng Li[1], Xin-Peng Du[2] and Xuan Zhang[2]

[1] School of Aerospace, Tsinghua University, Beijing 100084, China; *wangxiuhong1971@163.com*
[2] State Key Laboratory of Astronautic Dynamics, Xi'an 710043, China

Received 2015 November 12; accepted 2016 January 7

Abstract Orbital correlation of space objects is one of the most important elements in space object identification. Using the orbital elements, we provide correlation criteria to determine if objects are coplanar, co-orbital or the same. We analyze the prediction error of the correlation parameters for different orbital types and propose an orbital correlation method for space objects. The method is validated using two line elements and multisatellite launching data. The experimental results show that the proposed method is effective, especially for space objects in near-circular orbits.

Key words: methods: data analysis — methods: statistical — celestial mechanics

1 INTRODUCTION

After a space object has been detected, it is important to determine whether it has already been cataloged or it is a newly discovered object (Sharma et al. 2002, 2001; Sharma 2000; Stokes et al. 1998). Usually this is performed using the object's orbit or other characteristic data such as radar cross-sections or images (Sharma et al. 2002; Sharma 2000; Stokes et al. 1998; Fujimoto & Scheeres 2010). Because the orbital characteristics of a space object are more distinct than its other characteristics, the first step in space object identification often uses the orbital information. Orbital correlation of objects is often based on properties of the orbital element information or the observation data (Fujimoto & Scheeres 2010; Milani et al. 2005; Früh & Schildknecht 2012). Recently, Milani and Gronchi proposed a primary correlation method of arcs (Milani et al. 2004), and Milani and Tommei developed and tested two different algorithms to solve the correlation problem for objects in geostationary (GEO) orbits (Milani et al. 2011). Milani and his colleagues' work concerning orbital correlation is based on orbital observation data. Compared to orbital observation data, the orbital elements can be applied to orbital correlation more quickly, especially for a cluster of space objects. Therefore, orbital elements are used preferentially when an orbit has been successfully determined.

This paper focuses on orbital correlation based on orbital elements. If there are errors in the parameters used for orbital determination from the observational data (especially for sparse observational data), then the orbital correlation result of the two objects will not be exact (Hoots et al. 2004; Flohrer et al. 2008; Hirose et al. 2010; Vallado et al. 2009). The confidence level is a good measure to judge the orbital correlation results. Therefore, defining a confidence level is a key step in the orbital correlation of space objects.

In this paper, a set of novel correlation criteria is designed according to the physical characteristics of each orbital element to judge whether two space objects are coplanar, co-orbital, or in fact the same. We analyze the error of the correlation parameter for different orbital types and propose a new method to compute the confidence level of these correlation results. Finally, we validate the proposed method using two line elements (TLEs) and multisatellite launching data.

This paper is organized as follows. We study the orbital correlation criteria and the choice of correlation parameters in Section 2. The prediction errors of the correlation parameters are analyzed, and an orbital correlation method is proposed in Section 3. Lastly, we implement several experiments to validate the effectiveness of the method in Section 4.

2 ORBITAL CORRELATION CRITERIA AND THE CHOICE OF CORRELATION PARAMETERS

Orbital correlation is a highly demanding exercise in space object identification, and its main goal is to determine whether two arbitrary tracks are those of the same object. There are a great many space objects, and orbital correlation is complicated (Anz-Meador 2015; Rossi 2005); therefore, in this paper, we chose to study three orbital correlation cases for two space objects. Specifically, the three cases we study are when two space objects are coplanar, co-orbital or the same.

2.1 Choice of Parameters for Orbital Correlation

A space object travels in an orbital plane, and its orbit at any time can be uniquely determined by six different elements. Two types of orbital elements are used extensively: the position and velocity vectors and the Keplerian elements (Roy 2005; Schutz et al. 2004; Milani & Gronchi 2010; Rossi 2006). The Keplerian elements have specific geometric meanings and are usually computed using NORAD TLEs. Therefore, we use them as parameters for the orbital correlation.

2.2 Orbital Correlation Criteria

Of the six Keplerian elements, the semimajor axis a and the eccentricity e define the shape and size of the orbital ellipse, the inclination i and the longitude of the ascending node Ω define the orientation of the orbital plane, the argument of periapsis ω defines the orientation of the ellipse in the orbital plane, and the mean anomaly M at an epoch defines the momentary position of the space object in the orbital plane (Roy 2005; Schutz et al. 2004; Milani & Gronchi 2010). Because the element ω approaches zero degrees for a near-circular orbit, we use five parameters: a, e, i, Ω and λ ($\lambda = M + \omega$) as the orbit correlation parameters. According to the definitions of the six Keplerian elements, we set the following criteria: Equations (1), (2) and (3) are the orbital correlation criteria to judge if two space objects are coplanar, co-orbital or the same, respectively. In Equations (1), (2) and (3), Δa (Δe, Δi, $\Delta\Omega$, $\Delta\lambda$) denote the difference in the values between the two space orbits for the parameters a (e, i, Ω, λ), where $\{L_i\}_{i=0}^{4}$ and ($L_i > 0$) are the five given threshold values.

The criterion for judging coplanar objects is given by

$$|\Delta i| < L_2, |\Delta\Omega| < L_3\,. \tag{1}$$

The criterion for judging co-orbital objects is given by

$$|\Delta a| < L_0, |\Delta e| < L_1, |\Delta i| < L_2, |\Delta\Omega| < L_3\,. \tag{2}$$

The criterion for judging the same objects is given by

$$|\Delta a| < L_0, |\Delta e| < L_1, |\Delta i| < L_2, |\Delta\Omega| < L_3, |\Delta\lambda| < L_4\,. \tag{3}$$

2.3 Units of the Correlation Parameters

The parameters used for orbital correlation are not all exact. Therefore, the orbital correlation result between two space objects is uncertain, and we used confidence levels to determine whether two space objects are coplanar, co-orbital or the same. The Keplerian elements are measured in the International System (SI) units, and their orders of magnitude are listed in Table 1. Table 1 shows that the orders of magnitude range from 10^{-1} to 10^{6}. To use these five elements with the same order of magnitude, they are normalized before computing the confidence levels. Specifically, we define [L]=L /6 378 140 as the normalized length and use radians for the angular elements a, i, Ω and λ, where L is the length in meters and 6 378 140 m is the usual value used for the equatorial radius (Flohrer et al. 2008; Zeng 1992).

Table 1 Orders of Magnitude for the Keplerian Elements

Keplerian Element	Range of Values	Order of Magnitude
a	6 500 000 m	10^{6}
e	0∼1	10^{-1}
i	0∼180°	10^{-1}∼10^{2}
Ω	0∼360°	10^{-1}∼10^{2}
λ	0∼360°	10^{-1}∼10^{2}

Table 2 Orbital Elements of the Four Objects

ID	NORAD	Hp (km)	Ha (km)	Inclination (°)
1	28254	879	897	99.1
2	25919	678	678	98.1
3	26953	455	448	97.2
4	24680	287	921	97.8

3 THE PROPOSED METHOD FOR ORBITAL CORRELATION

In this section, we will first give an error analysis of the correlation parameters, and then propose an orbital correlation method.

3.1 Error Analysis of the Correlation Parameters

We use the TLEs for the orbits of four different space objects to obtain an error analysis for the orbit propagation over 14 days. The orbital elements of the four objects are shown in Table 2. The orbits of the first three objects are nearly circular with different heights, and the fourth is in a low elliptical orbit. To help analyze the errors of the five correlation parameters for the different orbital types, Figure 1 illustrates the position errors and the errors in the radial direction, transverse direction and normal direction for the objects after 14 days of orbital propagation. The position errors after 14 days for the first three objects were approximately 10, 20 and 50 km, respectively, and the errors were primarily distributed in the transverse direction. The position error after 14 days of orbital propagation for the fourth object in an elliptical orbit was approximately 30 km. For the three near-circular orbital objects, we find that the lower orbital heights are related to larger orbital propagation errors (Fig. 1). This conclusion is consistent with Flohrer et al. (2008); Hirose et al. (2010); Legendre et al. (2006).

The correlation parameter errors, with normalized units after 14 days of orbital propagation, for four objects are listed in Table 3 and illustrated in Figures 2–5. We find that the correlation parameter errors of the three near-circular orbits have the same order of magnitude. Compared to the cases with near-circular orbits, the errors in a and e for the elliptical orbit are relatively large.

3.2 The Proposed Method for Computing the Confidence Level

For each type of orbit, the statistical mean values for the propagation errors in the five correlation parameters (a, e, i, Ω and λ) are c_1, c_2, c_3, c_4 and c_5, respectively. We

Table 3 Maximum Error of the Correlation Parameters (normalized units)

NORAD	a	e	i	Ω	λ
28254	1.9×10^{-5}	1.4×10^{-4}	6.8×10^{-5}	3.5×10^{-4}	8.7×10^{-4}
25919	2.2×10^{-5}	1.4×10^{-4}	1.0×10^{-4}	2.1×10^{-4}	2.7×10^{-3}
26953	3.9×10^{-5}	2.0×10^{-4}	6.8×10^{-5}	8.7×10^{-5}	5.2×10^{-3}
24680	1.9×10^{-4}	7.0×10^{-4}	1.7×10^{-4}	2.1×10^{-4}	3.5×10^{-3}

Table 4 Orbits and Orbital Correlation Results for Selected Object from TLEs

Experiment Number	Object Number	Hp (km)	Ha (km)	i (°)	Orbit epoch difference (d)	C_{P}(%)	C_{O}(%)	C_{S}(%)
I	1	879	897	99.1	5	99	99	95
					15	99	99	93
					20	99	99	89
					30	99	99	80
II	2	678	680	98.1	3	99	99	95
					7	99	99	92
					10	99	99	91
					20	99	99	87
					30	99	99	79
III	3	445	448	97.2	5	99	99	95
					11	99	99	89
					20	99	99	86
					29	99	99	78
IV	4	287	921	97.8	1	99	99	95
					4	99	99	88
					8	99	99	83
					15	99	99	76
V	5	1563	38793	64.9	4	99	99	95
					7	99	99	87
					15	99	99	83
					20	99	99	78
VI	5	1563	38793	64.9	5	99	99	95
					10	99	99	93
					15	99	99	88
					20	99	99	84
					30	99	99	78

denote Δa, Δe, Δi, $\Delta\Omega$ and $\Delta\lambda$ to be the actual prediction errors of a, e, i, Ω and λ for one object respectively. C_{P}, C_{O} and C_{S} denote the confidence levels for determining whether two space objects are coplanar, co-orbital or the same, respectively. The corresponding three confidence levels are computed using the following equations:

$$
\begin{aligned}
C_{\mathrm{P}} &= \omega_1 \frac{c_3}{\Delta i} + \omega_2 \frac{c_4}{\Delta\Omega}\,, && (4)\\
C_{\mathrm{O}} &= \omega_3 \frac{c_3}{\Delta i} + \omega_4 \frac{c_4}{\Delta\Omega} + \omega_5 \frac{c_1}{\Delta a} + \omega_6 \frac{c_2}{\Delta e}\,, && (5)\\
C_{\mathrm{S}} &= \omega_7 \frac{c_3}{\Delta i} + \omega_8 \frac{c_4}{\Delta\Omega} + \omega_9 \frac{c_1}{\Delta a} + \omega_{10} \frac{c_2}{\Delta e}\\
&\quad + \omega_{11} \frac{c_5}{\Delta\lambda}\,, && (6)
\end{aligned}
$$

where $0 < w_i < 1$ $(i = 1, 2, 3, \cdots, 11)$ are eleven correction factors that satisfy the conditions that $w_1 + w_2$, $w_3 + w_4 + w_5 + w_6$ and $w_7 + w_8 + w_9 + w_{10} + w_{11}$ are close to 1. Except for these constraining conditions, the values of the correction factors should be related as follows: $C_{\mathrm{P}} > C_{\mathrm{O}} > C_{\mathrm{S}}$ is valid. When Δa, Δe, Δi, $\Delta\Omega$ and $\Delta\lambda$ are close to zero, ${}^{1}\!/_{\Delta a}$, ${}^{1}\!/_{\Delta e}$, ${}^{1}\!/_{\Delta i}$, ${}^{1}\!/_{\Delta\Omega}$ and ${}^{1}\!/_{\Delta\lambda}$ can be set to 1. Since $\{c_i\}_{i=1}^{5}$ are five statistical values, C_{P}, C_{O} and C_{S} may be larger than 1. If C_{P}, C_{O} or C_{S} is larger than 1, its value is set to 99%.

4 EXPERIMENTS AND ANALYSIS OF RESULTS

In this section, we conduct several experiments using TLEs and multisatellite launching data to validate the effectiveness of the proposed orbital correlation method and to analyze the results.

4.1 Experiments Using TLEs

We conducted six experiments using TLEs. In the experiments, we considered five space objects: three traveled in near-circular orbits with orbital heights of 400, 600 and 800 km, respectively; one traveled in an elliptical orbit; and one traveled in a highly elliptical orbit. The five correlation parameters and their errors were obtained from the TLEs. In each experiment, we chose a space object and two sets of orbital TLEs with different orbital epochs. Specifically, in each experiment, we first chose a set of orbital TLEs at the starting epoch for a space object and then chose another set of orbital TLEs for the same object several days later. The experimental results are listed in Table 4.

In the first three experiments, the objects were in near-circular orbits. For objects with near-circular orbits, when the orbital epoch differs by less than 20 days, the confi-

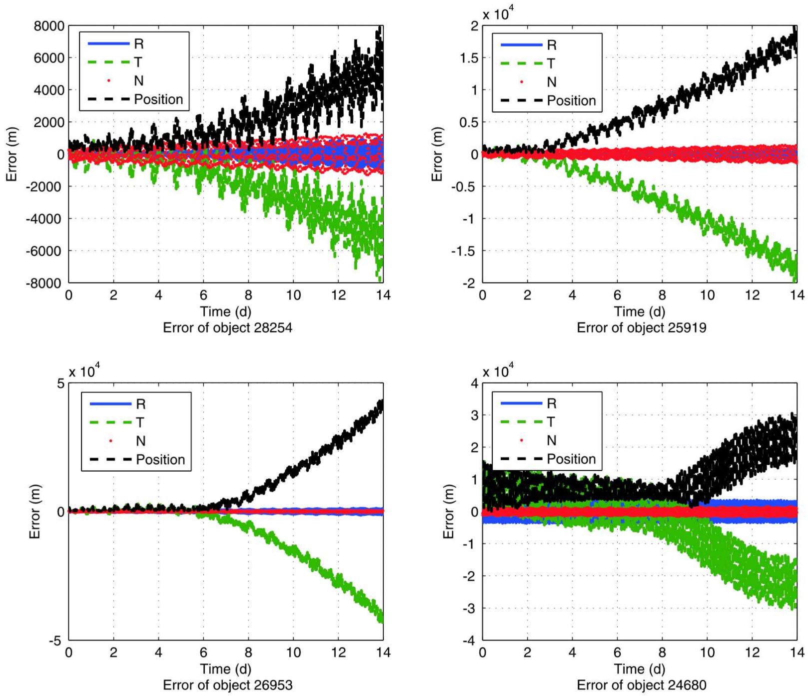

Fig. 1 The position errors and the errors in the radial (R) direction, transverse (T) direction and normal (N) direction for the four objects after 14 days of orbital propagation using TLEs.

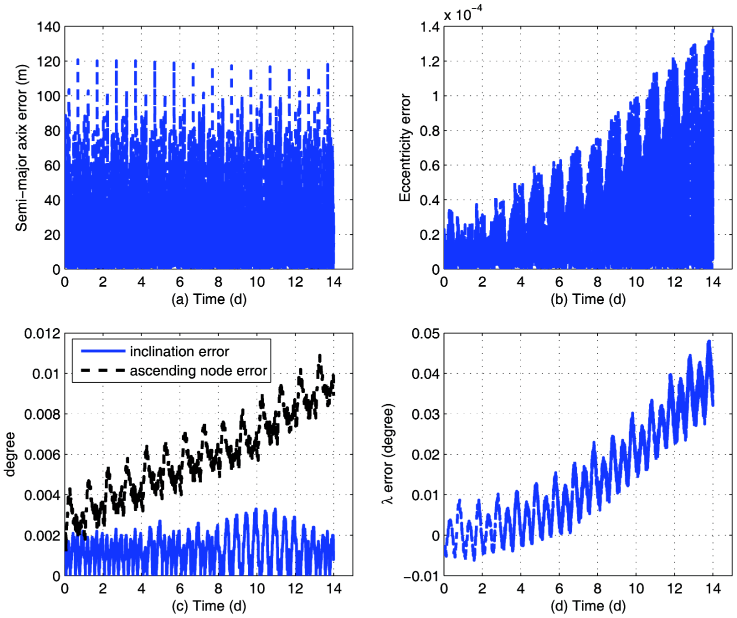

Fig. 2 Errors in correlation parameters of object 28254. (a) error in a; (b) error in e; (c) error in i and Ω; and (d) error in λ.

dence levels for determining coplanar or co-orbital objects are 99%, and the confidence levels for determining if they are the same object are larger than 85%. The results are stable, and the confidence level decreases with the increasing orbital epoch difference. In the fourth experiment, we considered an object with an elliptical orbit. Table 4 shows the confidence level is 83% when the two orbital epochs differ by 8 days, and the confidence level is 76% when the two orbital epochs differ by 15 days. Compared to the results of the near-circular orbital objects, we find that the confidence level decreases with the same epoch difference for elliptical orbits. In the last two experiments, an object with a highly elliptical orbit was considered, and the starting epochs were different. Both results were good, but the results of the sixth experiment were slightly better than the results of the fifth experiment. These experimental results show that the proposed orbital correlation method is very effective (especially for objects in near-circular or-

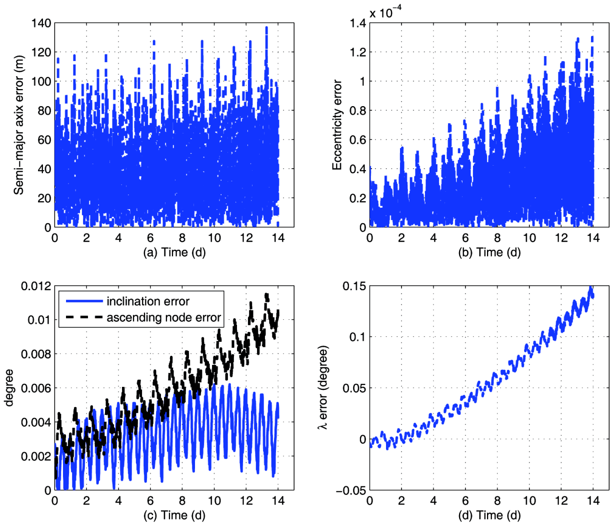

Fig. 3 Errors in Correlation parameters of object 25919.(a) error in a; (b) error in e; (c) error in i and Ω; and (d) error in λ.

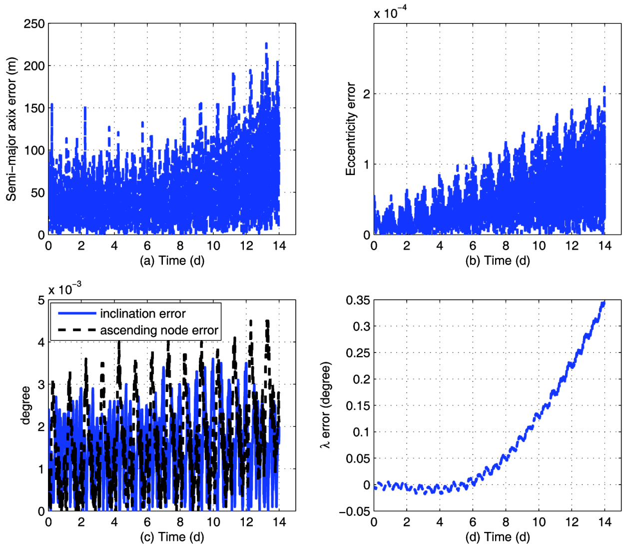

Fig. 4 Errors in correlation parameters of object 26953. (a) error in a; (b) error in e; (c) error in i and Ω; and (d) error in λ.

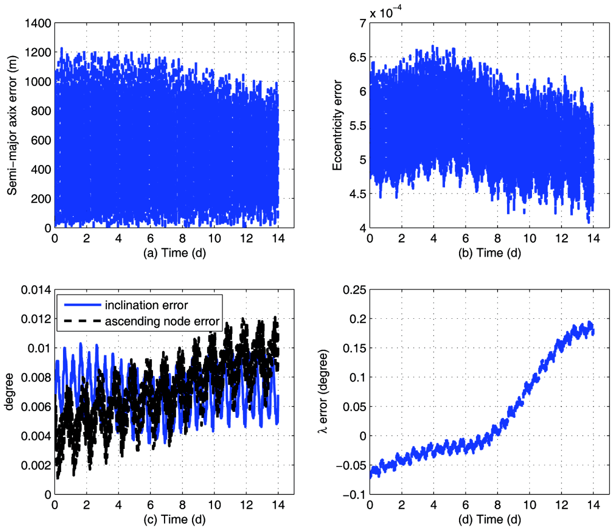

Fig. 5 Errors in correlation parameters of object 24680. (a) error in a; (b) error in e; (c) error in i and Ω; and (d) error in λ.

Table 5 Experimental Results Using the Multisatellite Launching Data Set

Space Object	Hp (km)	Ha (km)	i (°)	$C_{\rm P}$ (%)	$C_{\rm O}$ (%)	$C_{\rm S}$ (%)
S-A	1076	1105	63.41	99	95	56
S-B	1075	1105	63.41			
S-A	1076	1105	63.41	99	88	44
S-C	1076	1105	63.41			
S-B	1075	1105	63.41	99	94	65
D-A	1080	1105	63.40			

bits). The last two experiments demonstrate that the confidence levels differ with different starting epochs, even for the same object, because the precisions of the TLEs are different at different epochs.

4.2 Experiments Using the Multisatellite Launching Data

We conducted several experiments using a multisatellite launching data set, which measured three satellites (denoting them respectively by S-A, S-B, and S-C for simplicity) and a piece of debris (denoting it by D-A). We used our proposed orbital correlation method to determine whether two of these four objects were coplanar, co-orbital or the same. The experiment results are shown in Table 5. We found that the coplanar confidence levels were all 99%, the co-orbital confidence levels were high, and the confidence levels for being the same object were low. These results are consistent with common sense. Considering the multisatellite launching case, the five objects are coplanar. After satellite and rocket separation, the three satellites and the debris are traveling nearly in the same plane, but their tracks are slightly different. They are different objects, and the confidence levels for them being the same are low.

5 CONCLUSIONS

Orbital correlation is important for cataloging space objects, and the confidence level is a key factor to judge their classification. In this paper, we first give the correlation criteria for determining whether two space objects are coplanar, co-orbital, or the same object using the physical meanings of the orbital elements. We then analyze the propagation errors of the correlation parameters for different orbital types and present a method for computing the confidence level. Lastly, we conduct several experiments to validate the method using TLEs and multisatellite launching data. From the experimental results, we find that the confidence level is related to several factors, such as orbital type, orbital precision, and orbital epoch difference. Orbital correlation is a complicated subject and therefore deserves further study.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant Nos. 11572166 and 61401515).

References

Anz-Meador, P. 2015, Orbital Debris Quarterly News, 19

Flohrer, T., Krag, H., & Klinkrad, H. 2008, in Advanced Maui Optical and Space Surveillance Technologies Conference, E53

Früh, C., & Schildknecht, T. 2012, Journal of Guidance, Control, and Dynamics, 35, 1483

Fujimoto, K., & Scheeres, D. 2010, in AIAA/AAS Astrodynamics Specialist Conference, Toronto, Canada, AIAA-2010-7975

Hirose, C., Kudo, N., Matsuda, I., et al. 2010, in Proceedings of 61st International Astromautical Congress, Prague, Czech

Hoots, F. R., Schumacher, Jr., P. W., & Glover, R. A. 2004, Journal of Guidance Control Dynamics, 27, 174

Legendre, P., Deguine, B., Garmier, R., & Revelin, B. 2006, in Astrodynamics Specialist Conference and Exhibit, Keystone, Colorado

Milani, A., Gronchi, G. F., Vitturi, M. D., & Knežević, Z. 2004, Celestial Mechanics and Dynamical Astronomy, 90, 57

Milani, A., Gronchi, G. F., Knežević, Z., Sansaturio, M. E., & Arratia, O. 2005, Icarus, 179, 350

Milani, A., & Gronchi, G. F. 2010, Theory of Orbital Determination (Cambridge University Press)

Milani, A., Tommei, G., Farnocchia, D., et al. 2011, MNRAS, 417, 2094

Rossi, A. 2005, Serbian Astronomical Journal, 170, 1

Rossi, A. 2006, in 24th IADC Meeting Tsukuba

Roy, A. E. 2005, Orbital motion (Bristol (UK): Institute of Physics Publishing)

Schutz, B., Tapley, B., & Born, G. H. 2004, Statistical Orbit Determination (Academic Press)

Sharma, J. 2000, Journal of Guidance Control Dynamics, 23, 153

Sharma, J., Wiseman, A., & Zollinger, G. 2001, Improving Space Surveillance with Space-based Visible Sensor, Tech. rep., DTIC Document

Sharma, J., Stokes, G. H., von Braun, C., Zollinger, G., & Wiseman, A. J. 2002, Lincoln Laboratory Journal, 13, 309

Stokes, G. H., von Braun, C., Sridharan, R., Harrison, D., & Sharma, J. 1998, Lincoln Laboratory Journal, 11, 205

Vallado, D. A., Kelso, T., Agapov, V., & Molotov, I. 2009, in 5th European Conference on Space Debris. Darmstadt, Germany

Zeng, Y.-C. 1992, Flight Dynamics of Spacecraft (Xian, China: Northwestern Polytechnical University Press), 31