

Transient acceleration in $f(T)$ gravity

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Abstract Recently an $f(T)$ gravity based on the modification of teleparallel gravity was proposed to explain the accelerated expansion of the universe. We use observational data from type Ia supernovae, baryon acoustic oscillations, and cosmic microwave background to constrain this $f(T)$ theory and reconstruct the effective equation of state and the deceleration parameter. We obtain the best-fit values of parameters and find an interesting result that the constrained $f(T)$ theory allows for the accelerated Hubble expansion to be a transient effect.

Key words: $f(T)$ gravity: cosmology — reconstruction — observation

1 INTRODUCTION

A series of independent cosmological observations including type Ia supernovae (SNIa) (Riess et al. 1998), large scale structure (Tegmark et al. 2004), baryon acoustic oscillation (BAO) peaks (Eisenstein et al. 2005) and cosmic microwave background (CMB) anisotropy (Spergel et al. 2003) have probed the accelerating expansion of the universe. Subsequently, many gravitational theories and cosmological models have been proposed to explain this cosmological phenomenon. Under the assumption of cosmological principles, these theories include the mysterious dark energy with negative pressure in general relativity and modified gravity models based on general relativity. For the former, the acceleration is driven by exotic dark energy, such as the cosmological constant, quintessence or phantom. The cosmological constant model (Λ CDM) is the simplest candidate for dark energy models, and agrees well with current cosmological observations. However, the Λ CDM model is faced with the fine-tuning problem (Weinberg 1989) and coincidence problem (Zlatev et al. 1999). Moreover, the nature of dark energy in the form of other candidates still cannot be revealed. For the latter, the acceleration is realized by modification of general relativity without exotic dark energy, such as the brane-world Dvali-Gabadadze-Porrati model (Dvali et al. 2000), $f(R)$ gravity (Chiba 2003) or Gauss-Bonnet gravity (Nojiri & Odintsov 2005).

Similar to the exotic dark energy and other modified gravity models, it is found that cosmic acceleration can also be successfully obtained from another gravitational scenario described by the $f(T)$ theory (Bengochea & Ferraro 2009). Proposed based on the teleparallel equivalent of general relativity (also known as teleparallel grav-

ity), scalar T is the Lagrangian of teleparallel gravity. Teleparallel gravity is not a new theory of gravity, but rather an alternative geometric formulation of general relativity. In teleparallel gravity, the Levi-Civita connection used in Einstein's general relativity is replaced by the Weitzenböck connection with torsion. However, the torsion vanishes in the dark energy and modified gravity models. Moreover, $f(T)$ theories have several interesting features: they can not only explain the late accelerating expansion, but also have second order differential equations, which are simpler than the $f(R)$ gravity. In addition, when certain conditions are satisfied, the behavior of $f(T)$ will be similar to quintessence (Xu et al. 2012). Although $f(T)$ gravity has attracted wide attention, a disadvantage pointed out in Li et al. (2011a) is that the action and the field equations of $f(T)$ do not respect local Lorentz symmetry. Nonetheless, the $f(T)$ gravity might provide a significant alternative to conventional dark energy in general relativistic cosmology. In addition, Saveliev et al. (2011) indicated that the Lorentz invariance violation is still possible, but $f(T)$ gravity might provide some insights about Lorentz violation. Such $f(T)$ theories are worth further in depth studies.

Up to now, a number of $f(T)$ theories have been proposed (Bengochea & Ferraro 2009; Linder 2010; Yang 2011b; Myrzakulov 2011; Bamba et al. 2011; Wu & Yu 2011). Under these cases, Yang found that $f(T)$ theories are not dynamically equivalent to teleparallel action with an added scalar field (Yang 2011a). Like other gravity theories and models, the $f(T)$ theories also have been investigated using the popular observational data. Investigations show that the $f(T)$ theories are compatible with observations (see e.g. Nesseris et al. 2013; Zheng & Huang 2011

and references therein). We note that the new type of $f(T)$ theory was proposed to explain the accelerating expansion of the universe, and it behaves like a cosmological constant; but because of its dynamic behavior, it is free from the coincidence problem seen in the case of Λ CDM (Yang 2011b). Due to this characteristic, it is impossible to distinguish this type of model from a Λ CDM model. However, observational analysis of this model is still absent. Hence, we would like to perform some further analysis using observational data, such as the SNIa, BAO and CMB.

This paper is organized as follows. In Section 2, the general $f(T)$ gravity and the $f(T)$ model proposed in Yang (2011b) are introduced. In Section 3, we describe the method for constraining the cosmological models and the reconstruction scheme. Subsequently, the parameters of the specific $f(T)$ model are constrained by observational data. Furthermore, through the reconstruction scheme the effective equation of state and the deceleration parameter are reconstructed in Section 4. Finally, we give the summary and conclusions in Section 5.

2 THE $F(T)$ THEORY

The $f(T)$ theory is a modification of teleparallel gravity, which uses the curvatureless Weitzenböck connection instead of the torsionless Levi-Civita connection in Einstein's general relativity. The curvatureless torsion tensor is

$$T_{\mu\nu}^{\lambda} \equiv e_i^{\lambda}(\partial_{\mu}e_{\nu}^i - \partial_{\nu}e_{\mu}^i), \quad (1)$$

where e_i^{μ} ($\mu = 0, 1, 2, 3$) are four linearly independent components of the vierbein field $e_i(x^{\mu})$ ($i = 0, 1, 2, 3$) in a coordinate basis. In particular, the vierbein is an orthonormal basis for the tangent space at each point x^{μ} of the manifold: $e_i \cdot e_j = \eta_{ij}$, where $\eta_{ij} = \text{diag}(1, -1, -1, -1)$. Notice that Latin indices refer to the tangent space, while Greek indices label coordinates on the manifold. The metric tensor is obtained from the dual vierbein as $g_{\mu\nu}(x) = \eta_{ij} e_i^{\mu}(x) e_j^{\nu}(x)$. The torsion scalar is the Lagrangian of teleparallel gravity (Bengochea & Ferraro 2009)

$$T \equiv S_{\rho}^{\mu\nu} T_{\mu\nu}^{\rho}, \quad (2)$$

where

$$S_{\rho}^{\mu\nu} = \frac{1}{2} \left(K^{\mu\nu}_{\rho} + \delta_{\rho}^{\mu} T^{\theta\nu}_{\theta} - \delta_{\rho}^{\nu} T^{\theta\mu}_{\theta} \right), \quad (3)$$

and the contorsion tensor $K^{\mu\nu}_{\rho}$ is given by

$$K^{\mu\nu}_{\rho} = -\frac{1}{2} \left(T^{\mu\nu}_{\rho} - T^{\nu\mu}_{\rho} - T_{\rho}^{\mu\nu} \right). \quad (4)$$

In the $f(T)$ theory, we allow the Lagrangian density to be a function of T (Bengochea & Ferraro 2009; Ferraro & Fiorini 2007; Linder 2010), thus the action reads

$$I = \frac{1}{16\pi G} \int d^4x e f(T), \quad (5)$$

where $e = \det(e_{\mu}^i) = \sqrt{-g}$. The corresponding field equation is

$$\begin{aligned} & [e^{-1} \partial_{\mu} (e S_i^{\mu\nu}) - e_i^{\lambda} T^{\rho}_{\mu\lambda} S_{\rho}^{\nu\mu}] f_T \\ & + S_i^{\mu\nu} \partial_{\mu} T f_{TT} + \frac{1}{4} e_i^{\nu} f(T) = \frac{1}{2} k^2 e_i^{\rho} T_{\rho}^{\nu}, \end{aligned} \quad (6)$$

where $k^2 = 8\pi G$, $f_T \equiv df/dT$, $f_{TT} \equiv d^2f/dT^2$, $S_i^{\mu\nu} \equiv e_i^{\rho} S_{\rho}^{\mu\nu}$, and $T_{\mu\nu}$ is the matter energy-momentum tensor. Obviously, Equation (6) is a second-order equation. Thus, the $f(T)$ theories are simpler than the $f(R)$ theories with fourth-order equations.

Considering a flat, homogeneous and isotropic Friedmann-Robertson-Walker universe, we have

$$\begin{aligned} e_i^{\mu} &= \text{diag}(1, a(t), a(t), a(t)), \\ e_i^{\nu} &= \text{diag}\left(1, \frac{1}{a(t)}, \frac{1}{a(t)}, \frac{1}{a(t)}\right), \end{aligned} \quad (7)$$

where $a(t)$ is the cosmological scale factor. By substituting Equations (7), (1), (3) and (4) into Equation (2), we obtain the torsion scalar (Bengochea & Ferraro 2009)

$$T \equiv S^{\rho\mu\nu} T_{\rho\mu\nu} = -6H^2, \quad (8)$$

where H is the Hubble parameter $H = \dot{a}/a$. The dot represents the first derivative with respect to cosmic time. Substituting Equation (7) into (6), one can obtain the corresponding Friedmann equations

$$12H^2 f_T + f = 2k^2 \rho, \quad (9)$$

$$48H^2 \dot{H} f_{TT} - (12H^2 + 4\dot{H}) f_T - f = 2k^2 p, \quad (10)$$

with ρ and p as the total energy density and pressure, respectively. The detailed calculation can be found in Bengochea & Ferraro (2009). The conservation equation reads

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (11)$$

We should note that the only components considered here are matter and radiation, but not dark energy. After a brief simplification of the Friedmann equations ((9) and (10)), we can rewrite them as

$$\frac{3}{k^2} H^2 = \rho + \rho_{\text{eff}}, \quad (12)$$

$$\frac{1}{k^2} (2\dot{H} + 3H^2) = -(p + p_{\text{eff}}), \quad (13)$$

where the effective energy density ρ_{eff} and pressure p_{eff} contributed from torsion are respectively given by (Yang 2011b)

$$\rho_{\text{eff}} = \frac{1}{2k^2} (-12H^2 f_T - f + 6H^2), \quad (14)$$

$$\begin{aligned} p_{\text{eff}} &= -\frac{1}{2k^2} [48\dot{H} H^2 f_{TT} - 4\dot{H} f_T + 4\dot{H}] \\ &\quad - \rho_{\text{eff}}. \end{aligned} \quad (15)$$

We term it ‘‘effective’’ because it is just a geometric effect instead of a specific cosmic component. Therefore, what

we are interested in is the acceleration driven by the torsion, not the exotic dark energy. Using Equations (14) and (15), we can define the total and effective equation of state as (Yang 2011b)

$$w_{\text{tot}} \equiv \frac{p + p_{\text{eff}}}{\rho + \rho_{\text{eff}}} = -1 + \frac{2(1+z)}{3H} \frac{dH}{dz}, \quad (16)$$

$$w_{\text{eff}} \equiv \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = -1 - \frac{48\dot{H}H^2 f_{TT} - 4\dot{H}f_T + 4\ddot{H}}{-12H^2 f_T - f + 6H^2}. \quad (17)$$

The deceleration parameter, as usual, is defined as

$$q(z) \equiv -\frac{\ddot{a}}{aH^2} = -1 + \frac{(1+z)}{H} \frac{dH}{dz}. \quad (18)$$

After reviewing the general formation of $f(T)$ gravity, we now focus on a type of $f(T)$ gravity proposed in Yang (2011b)

$$f(T) = T - \alpha T_0 \left[\left(1 + \frac{T^2}{T_0^2} \right)^{-n} - 1 \right], \quad (19)$$

which is analogous to a type of $f(R)$ theory proposed in Starobinsky (2007), where α and n are positive constants. $T_0 = -6H_0^2$ and H_0 is the current value of the Hubble parameter. This type of $f(T)$ gravity has attracted much attention and has been discussed in detail in Sharif & Azeem (2012). Here we will look into the observational constraints on this type of $f(T)$ gravity. With $f(T)$ taking the form of Equation (19), Equation (9) can be rewritten as

$$E^2 + \frac{4n\alpha E^4}{(1+E^4)^{n+1}} + \frac{\alpha}{(1+E^4)^n} - B = \alpha, \quad (20)$$

where $E^2 \equiv H^2/H_0^2$ and $B = \Omega_{m0}(1+z)^3$, with Ω_{m0} being the matter density parameter today. Here we only focus on the evolution of the universe at low redshift, so we neglect the contribution from radiation. For $E(z=0) = 1$, we have $\alpha = (1 - \Omega_{m0}) / (1 - 2^{-n+1}n - 2^{-n})$. This $f(T)$ model has some interesting characteristics: firstly, the cosmological constant is zero in the flat space-time because $f(T=0) = 0$, while the geometrical one contributes as the dark energy; secondly, it can behave like the cosmological constant. Such characteristics indicate that it is possible to accept this type of $f(T)$ model on the basis of observational data, but it is impossible to distinguish it from the Λ CDM. Moreover, though the behavior of this type of $f(T)$ theory is similar to Λ CDM because of its dynamic behavior, it can avoid the coincidence problem suffered by Λ CDM.

3 OBSERVATIONAL DATA AND FITTING METHOD

In this section, we would like to introduce the observational data and constraint method. The corresponding observational data here are distance moduli of SNIa, CMB shift parameter and BAO distance parameter.

3.1 Type Ia Supernovae

As early as 1998, cosmic accelerating expansion was first observed by SNIa acting as ‘‘standard candles’’ which have the same intrinsic luminosity. Therefore, the observable is usually presented in the distance modulus, the difference between the apparent magnitude m and the absolute magnitude M . The latest version is the Union2.1 compilation which includes 580 samples (Suzuki et al. 2012). They were compiled by the Hubble Space Telescope Cluster Supernova Survey over the redshift interval $0.01 < z < 1.42$. The theoretical distance modulus is given by

$$\mu_{th}(z) = m - M = 5 \log_{10} D_L(z) + \mu_0, \quad (21)$$

where $\mu_0 = 42.38 - 5 \log_{10} h$ and h is the Hubble constant H_0 in the units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The corresponding luminosity distance function $D_L(z)$ is

$$D_L(z) = (1+z) \int_0^z \frac{dz'}{E(z'; \mathbf{p})}, \quad (22)$$

where $E(z'; \mathbf{p})$ is the dimensionless Hubble parameter given by Equation (20), and \mathbf{p} stands for the parameter vector of the evaluated model embedded in the expansion rate parameter $E(z)$. We note that parameters in the expansion rate $E(z)$ include the annoying parameter h . In order to exclude the Hubble constant, we should marginalize over the nuisance parameter μ_0 by integrating the probabilities on μ_0 (di Pietro & Claeskens 2003; Nesseris & Perivolaropoulos 2005; Perivolaropoulos 2005). Finally, we can estimate the remaining parameters by minimizing

$$\tilde{\chi}_{\text{SN}}^2(z, \mathbf{p}) = A - \frac{B^2}{C}, \quad (23)$$

where

$$A(\mathbf{p}) = \sum_i \frac{[\mu_{\text{obs}}(z) - \mu_{th}(z; \mu_0 = 0, \mathbf{p})]^2}{\sigma_i^2(z)},$$

$$B(\mathbf{p}) = \sum_i \frac{\mu_{\text{obs}}(z) - \mu_{th}(z; \mu_0 = 0, \mathbf{p})}{\sigma_i^2(z)},$$

$$C = \sum_i \frac{1}{\sigma_i^2(z)},$$

and μ_{obs} is the observational distance modulus. This approach has been used in the reconstruction of dark energy (Wei et al. 2007), parameter constraints (Wei 2010), reconstruction of the energy condition history (Wu et al. 2012), etc.

3.2 Cosmic Microwave Background

The CMB experiment measures the temperature and polarization anisotropy of cosmic radiation in the early epoch. It generally plays a major role in establishing and sharpening cosmological models. In measurement of the CMB, the shift parameter R is a convenient way to quickly evaluate the likelihood of a cosmological model, and contains

the main information about the CMB observation (Hu & Sugiyama 1996; Hinshaw et al. 2009). It is expressed as

$$R = \sqrt{\Omega_{m0}} \int_0^{z_s} \frac{dz'}{E(z'; \mathbf{p})}, \quad (24)$$

where $z_s = 1090.97$ is the redshift of decoupling. According to the measurement of WMAP-9 (Hinshaw et al. 2013), we estimate the parameters by minimizing the corresponding χ^2 statistic

$$\chi_R^2 = \left(\frac{R - 1.728}{0.016} \right)^2. \quad (25)$$

3.3 Baryon Acoustic Oscillation

The measurement of BAO in large-scale galaxies has rapidly become one of the most important observational pillars in cosmological constraints. This measurement is usually called the standard ruler in cosmology (Eisenstein & Hu 1998). The distance parameter A obtained from the BAO peak in the distribution of SDSS luminous red galaxies (Eisenstein et al. 2005) is a significant parameter and is defined as

$$A_{\text{th}} = \Omega_{m0}^{1/2} E(z_1)^{-1/3} \left[\frac{1}{z_1} \int_0^{z_1} \frac{dz'}{E(z'; \mathbf{p})} \right]^{2/3}. \quad (26)$$

We use the three combined data points in Addison et al. (2013) that cover $0.1 < z < 2.4$ to determine the parameters in evaluated models. The expression of the χ^2 statistic is

$$\chi_A^2 = \sum_i \left(\frac{A_{\text{th}} - A_{\text{obs}}}{\sigma_A} \right)^2, \quad (27)$$

where A_{obs} is the observational distance parameter and σ_A is its corresponding error.

Since the SNIa, CMB and BAO data points are effectively independent measurements, we can simply minimize their total χ^2 values

$$\chi^2(z, \mathbf{p}) = \tilde{\chi}_{\text{SN}}^2 + \chi_R^2 + \chi_A^2,$$

to determine the parameters in the evaluated $f(T)$ model.

3.4 Reconstructing Method

Using the above introduced χ^2 statistic, we can obtain the best-fit values and associated errors of the basic parameters \mathbf{p}' . Further, we can reconstruct the other variable F relative to the known basic parameters \mathbf{p} by error propagation following the method in Lazkoz et al. (2012). For example, estimation from the observational data on the i th parameter p_i is $p_i = p_{0i} \pm \sigma_{i\text{u}} - \sigma_{i\text{l}}$, where p_{0i} is the best-fit value, and $\sigma_{i\text{u}}$ and $\sigma_{i\text{l}}$ are the upper limit and lower limit, respectively. Errors in the reconstructed function F are estimated by

$$\begin{aligned} \delta F_{\text{u}} &= \sqrt{\sum_i \left[\max \left(\frac{\partial F}{\partial p_i} \sigma_{i\text{u}}, -\frac{\partial F}{\partial p_i} \sigma_{i\text{l}} \right) \right]^2}, \\ \delta F_{\text{l}} &= \sqrt{\sum_i \left[\min \left(\frac{\partial F}{\partial p_i} \sigma_{i\text{u}}, -\frac{\partial F}{\partial p_i} \sigma_{i\text{l}} \right) \right]^2}, \end{aligned} \quad (28)$$

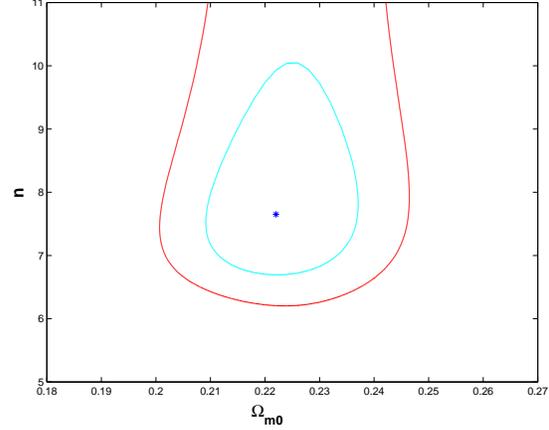


Fig. 1 Constraints on $f(T)$ theory with 68.3% and 95.4% confidence regions in the Ω_{m0} - n plane fitted with a combination of observations from SNIa, BAO and CMB data. The asterisk is the best-fit point.

where δF_{u} and δF_{l} are its upper and lower bound, respectively. In this paper, we will use this method to reconstruct the effective equation of state w_{eff} and deceleration parameter q .

4 CONSTRAINT RESULT

Using the observational data sets, we compute values for the χ^2 statistic and display the associated contour constraints in Figure 1. We find that the combined data provide mild constraints, i.e., $\Omega_{m0} = 0.22_{-0.0094}^{+0.0089}(1\sigma)$ and $n = 7.64_{-0.6700}^{+1.1750}(1\sigma)$ with $\chi_{\text{min}}^2 = 579.4786$. If we consider the degrees of freedom (dof), where $\chi_{\text{min}}^2/\text{dof}=0.9923$, our results indicate that this $f(T)$ model has good consistency with observations. However, we note that the parameter n is worse at the 95.4% confidence level. Namely, n is larger than 6. If the parameter n approaches infinity, we find from Equation (19) that this $f(T)$ model eventually evolves to the standard Λ CDM model.

In terms of Equation (28), we reconstruct the effective equation of state in Figure 2. We find that $w_{\text{eff}}(z)$ is a decreasing function of redshift, and steadily approaches -1 for high redshift $z \gtrsim 1$. That is, the geometric effect behaves like the cosmological constant at the early epoch. However, it generally increases with the decrease in redshift. The present value of the effective equation of state finally reaches $w_{\text{eff}0} = -0.8760$. Moreover, the $w_{\text{eff}}(z)$ crosses through -1 for $z < 1$ within the 1σ confidence level. In Figure 3, we also reconstruct the deceleration parameter $q(z)$. We find that the transition from decelerating to accelerating expansion occurs at $z = 0.95 \pm 0.05$, which is earlier than some phenomenological deceleration parameters (Riess et al. 2004; Cunha & Lima 2008). With the decrease in deceleration parameter, its value today is $q_0 = -0.3750$. In the near future $z = -0.04$, and $q(z)$ will cross zero. That is to say, the accelerating expansion of the

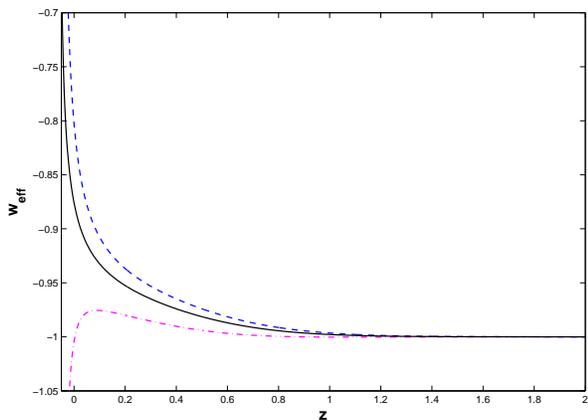


Fig. 2 Reconstruction of the effective equation of state at the 68.3% confidence level for the $f(T)$ gravity considered here.

universe may be slowing down again until a decelerating expansion takes place in the future. It is possible, however, to have an eternally accelerating phase at the 68.3% confidence level as shown in Figure 3. The feature of transient acceleration makes this $f(T)$ gravity compatible with the S-matrix description of string theory (Banks & Dine 2001; Hellerman et al. 2001). Most of the dark energy models including the current standard Λ CDM scenario predict an eternally accelerating universe. But, the consequent cosmological event horizon does not allow the construction of a conventional S-matrix to describe particle interactions (Guimarães & Lima 2011). However, from the standpoint of string theory, the existence of the conventional S-matrix is absolutely essential for an asymptotically large space at infinity (Cui et al. 2013). Therefore, the S-matrix is ill-defined in an eternally accelerating universe. In order to alleviate the conflict between dark energy and string theory, several dynamic dark energy models have been proposed to achieve the possibility of transient acceleration occurring (Cui et al. 2013; Russo 2004; Carvalho et al. 2006). In addition, recently it was argued that the SNIa data favor a transient acceleration (Shafieloo et al. 2009) which is not excluded by current observations (Guimarães & Lima 2011; Zuñiga Vargas et al. 2012; Bassett et al. 2002). Our result indicates that this type of $f(T)$ gravity serves as an alternative to modified gravity and dynamic dark energy models.

5 SUMMARY AND CONCLUSIONS

The $f(T)$ gravity based on modification of teleparallel gravity was proposed to explain the accelerating expansion of the universe without the need for dark energy. A brief overview of a specific $f(T)$ gravity proposed in Yang (2011b) was also given. We also introduced the method used to constrain cosmological models with observational data including SNIa, BAO and CMB. After constraining the $f(T)$ gravity proposed in Yang (2011b), we find that the best-fit values of the parameters at the 68.3% confidence level are: $\Omega_{m0} = 0.22^{+0.0089}_{-0.0094}$ and $n = 7.64^{+1.175}_{-0.67}$

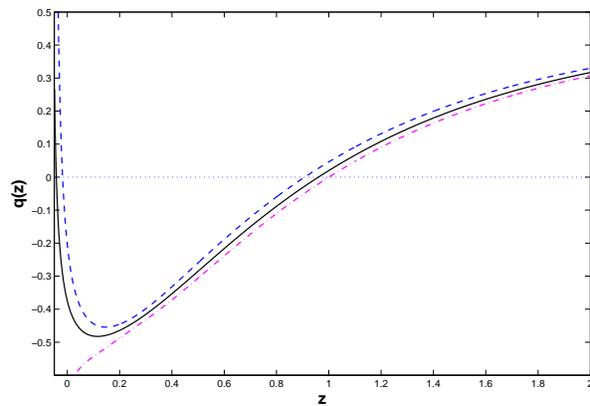


Fig. 3 Reconstruction of the deceleration parameter at the 68.3% confidence level for the $f(T)$ gravity considered here.

with $\chi^2_{\min} = 579.4786$ ($\chi^2_{\min}/\text{dof}=0.9923$). The parameters Ω_{m0} and n can be constrained well at the 68.3% confidence level by these observational data.

We also reconstructed the effective equation of state and the deceleration parameter from observational data. We found that the transition from deceleration to acceleration occurs at $z = 0.95 \pm 0.05$. The present value of the deceleration parameter was found to be $q_0 = -0.3750$, meaning that the cosmic expansion has passed a maximum value (at about $z \sim 0.1$) and is now slowing down again. This is a theoretically interesting result because an eternally accelerating universe (like Λ CDM) is endowed with a cosmological event horizon which prevents the construction of a conventional S-matrix describing particle interactions. Such a difficulty has been pointed out as a severe theoretical problem for any eternally accelerating universe (Hellerman et al. 2001; Cline 2001; Carvalho et al. 2006). Some research also indicated that a transient phase of accelerated expansion is not excluded by current observations (Guimarães & Lima 2011; Zuñiga Vargas et al. 2012; Bassett et al. 2002). We note, however, it is possible to have an eternally accelerating phase and an effective equation of state crossing through -1 at the 68.3% confidence level, according to the reconstruction of the effective equation of state and the deceleration parameter. We look forward to a more comprehensive investigation including the observations of structure growth which is widely used to study $f(T)$ gravity (Izumi & Ong 2013; Chen et al. 2011; Geng & Wu 2013; Zheng & Huang 2011; Li et al. 2011b), to reduce errors in the effective equation of state and the deceleration parameter at $z \sim 0$.

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