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Light curve inversion of asteroid (585) Bilkis with Lommel-Seeliger ellipsoid method

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Abstract The basic physical parameters of asteroids, such as spin parameters, shape and scattering parameters, can provide us with information on the formation and evolution of both the asteroids themselves and the entire solar system. In a majority of asteroids, the disk-integrated photometry measurement constitutes the primary source of the above knowledge. In the present paper, newly observed photometric data and existing data on (585) Bilkis are analyzed based on a Lommel-Seeliger ellipsoid model. With a Markov chain Monte Carlo (MCMC) method, we have determined the spin parameters (period, pole orientation) and shape (b/a, c/a) of (585) Bilkis and their uncertainties. As a result, we obtained a rotational period of 8.5738209 h with an uncertainty of 9×10^{-7} h, and derived a pole of (136.46°, 29.0°) in the ecliptic frame of J2000.0 with uncertainties of 0.67° and 1.1° in longitude and latitude respectively. We also derived triaxial ratios b/a and c/a of (585) Bilkis as 0.736 and 0.70 with uncertainties of 0.003 and 0.03 respectively.

Key words: asteroids — photometric observation — spin parameter — shape — MCMC method

1 INTRODUCTION

Asteroids are thought to be remnants of planetesimals from the early stage of our solar system. It is very meaningful to study their basic physical properties/parameters, such as spin parameters, shape and scattering parameters, to find clues about the formation and evolution of both the asteroids themselves and the entire solar system. The brightness of an asteroid is due to the reflection of solar light. A disk-resolved image of an asteroid can tell us information about its size, shape and albedo. In practice, the disk-integrated intensities are obtained for most asteroids due to the small size of asteroids and their far distance from observers. The brightness of an asteroid at a certain time is related to the distance of the asteroid from the source of light and observer, its size, shape and albedo of its surface. Its brightness is also related to the position of the asteroid in space if the asteroid is not spherical in shape. In theory, the above physical parameters can be inferred from light curves of an asteroid distributed at different viewing/illumination geometries. This is the reason why disk-integrated photometry data are and will remain a major resource for understanding the physical properties of most asteroids.

The C-type main-belt asteroids are thought to be primitive small bodies in the solar system that preserve information about the early stage of formation and evolution of the solar system (Wang et al. 2015). We, therefore, choose asteroid (585) Bilkis as our target. Asteroid (585), a C-type main-belt asteroid, about 58 km in diameter, was discovered by A. Kopff in February 1906. Several groups have acquired photometric observations of this asteroid (Robinson & Warner 2002; Behrend 2005; Brincat 2012; Behrend 2012). Robinson & Warner (2002) obtained photometric data of (585) Bilkis on six nights from May to August in 2001 and gave a rotation-period of 6.442 h. Behrend (2012) suggested a period of 8.58 h in their website. Brincat (2012) also suggested a period of 8.583 h. Warner (2011) re-analyzed previous light curves of (585) Bilkis and derived a spin period of 8.574 h. Obviously, there are slight differences among the spin periods derived from previous studies. Furthermore, no information on its pole-orientation or shape has been derived until now. To determine the spin parameters and shape

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of (585) Bilkis accurately, new photometric observations were made over seven nights in 2012 and 2014 with the 1.0 m telescope administered by Yunnan Observatories in China and SARA's 0.9 m telescope at Cerro Tololo Inter-American Observatory in Chile.

In the present paper, based on newly observed and previously existing photometric data, we analyzed the spin parameters and shape of (585) Bilkis considering a three-axial ellipsoidal shape with the Lommel-Seeliger scattering law (Muinonen et al. 2015). The best values of parameters and their uncertainties are derived with the Markov chain Monte Carlo (MCMC) method developed by Muinonen et al. (2012). In Section 2, the observations and data reduction for the target are briefly introduced. The light curve inversion method is described in Section 3. The results of the analysis for (585) Bilkis are presented in Section 4. In the last section, a summary and future prospects close the present article.

2 OBSERVATION AND DATA REDUCTION

For (585) Bilkis, seven new light curves were obtained in 2012 and 2014. Among the newly derived light curves, five were obtained by the 1.0 m telescope administered by Yunnan Observatories in 2012 and 2014 and the other two were obtained by SARA's 0.9 m telescope at Cerro Tololo Inter-American Observatory, Chile in 2014. Additionally, 10 existing light curves (from the Light Curve Database of the Minor Planet Center web page[1]) are involved in our analysis. Detailed information on data used is listed in Table 1. The first column is the date of the observations in UT. The second and third columns present right ascension and declination respectively of the asteroid in J2000.0. Δ and r are the geocentric and heliocentric distances of the asteroid respectively. ph means phase angle of the object and V is the predicted visual magnitude.

The new photometric data obtained at the 1.0 m telescope administered by Yunnan Observatories were acquired with a 2k×2k CCD with a clear filter in 2012 and 2014, while the data from SARA's 0.9 m telescope were obtained in 2×2 bin mode of a 4k×4k CCD with an R filter.

The new images are reduced according to the standard procedure with IRAF[2] software. For all images, the effects of bias, flat fields and dark images are corrected. The images that display cosmic rays are removed properly. The magnitudes of objects are measured by the Apphot task of IRAF with an optimal aperture. Then, some systematic effects in the photometric data are simulated with reference stars in the field of the asteroid and then removed from the target's photometric data (Wang et al. 2013). The time stamps of involved photometric data were corrected by the light time and converted into JD in the TDB system. The light curves of (585) Bilkis used in this study are shown in Figure 1.

[1] *http://www.minorplanetcenter.net*

[2] IRAF means Image Reduction and Analysis Facility package.

3 INVERSION METHOD

As is known in this field, the disk-integrated brightness of an asteroid at any time is the sum of reflected solar light by the visible surface of the asteroid. The observed brightness of an asteroid is related to its albedo and size of its surface area which is both illuminated and visible. Theoretically, we can extract information on spin state, shape and surface properties of an asteroid from the observed light curves. Such a procedure is usually called light curve inversion. The light curve inversion method used here is the Lommel-Seeliger ellipsoid method developed by Muinonen et al. (2015). With this method, the rotation period, longitude and latitude of pole, semimajor axis of the ellipsoid and scattering parameters can be extracted from photometric data. In the present work, only spin parameters and axial ratios of the ellipsoid are estimated because only relative intensities of (585) Bilkis are involved. The relative intensities of (585) Bilkis are calculated from the magnitude differences or reduced magnitudes. Then, they are normalized according to the mean value of each light curve. Photometric data in different filters are processed with the following equation.

$$I_{\rm rel} = 10^{(-(dM-\overline{dM})/2.5)}, \tag{1}$$

where dM means magnitude differences and $\overline{dM}$ is the mean of light curves in each night. The modeled disk-integrated brightness of an asteroid at any time in the Lommel-Seeliger ellipsoid method (Muinonen et al. 2015) is calculated with Equation (2).

$$\begin{aligned} L(\alpha) \;=\; & \pi p F_0 \frac{\Phi_{HG_1G_2}(\alpha)}{\Phi_{\rm LS}(\alpha)} abc \frac{S_\odot S_\oplus}{S} \Big\{ \cos(\lambda' - \alpha') \\ & + \cos\lambda' + \sin\lambda' \sin(\lambda' - \alpha') \\ & \times \ln\left[\cot\frac{1}{2}\lambda' \cot\frac{1}{2}(\alpha' - \lambda') \right] \Big\}, \end{aligned} \tag{2}$$

$$\begin{aligned} \Phi_{HG_1G_2}(\alpha) \;=\; & G_1\Big(1 - \frac{6\alpha}{\pi}\Big) + G_2\Big(1 - \frac{9\alpha}{\pi}\Big) \\ & +(1 - G_1 - G_2) \\ & \times \exp\Big(- 4\pi \tan^{\frac{2}{3}} \frac{1}{2}\alpha \Big), \end{aligned} \tag{3}$$

$$\Phi_{\rm LS}(\alpha) = 1 - \sin\frac{1}{2}\alpha \tan\frac{1}{2}\alpha \ln\Big(\cot\frac{1}{4}\alpha \Big). \tag{4}$$

In Equation (2), F_0 is incident intensity from the Sun, α is the solar phase angle for an asteroid, and p is the geometric albedo of an asteroid. $\Phi_{HG_1G_2}(\alpha)$ and $\Phi_{\rm LS}(\alpha)$ are the newly adopted phase function and the Lommel-Seeliger phase function respectively. $S_\odot$ and $S_\oplus$ are radii of subsolar and sub-Earth points on the asteroid surface respectively; α', λ' and S are auxiliary quantities computed from vectors describing the light source and observer in the asteroid fixed frame (refer to eqs. (11) and (12) in Muinonen et al. 2015). a, b, c are three semimajor axes of an ellipsoid. H is the absolute magnitude an asteroid and G_1, G_2 are the parameters of the phase function. For

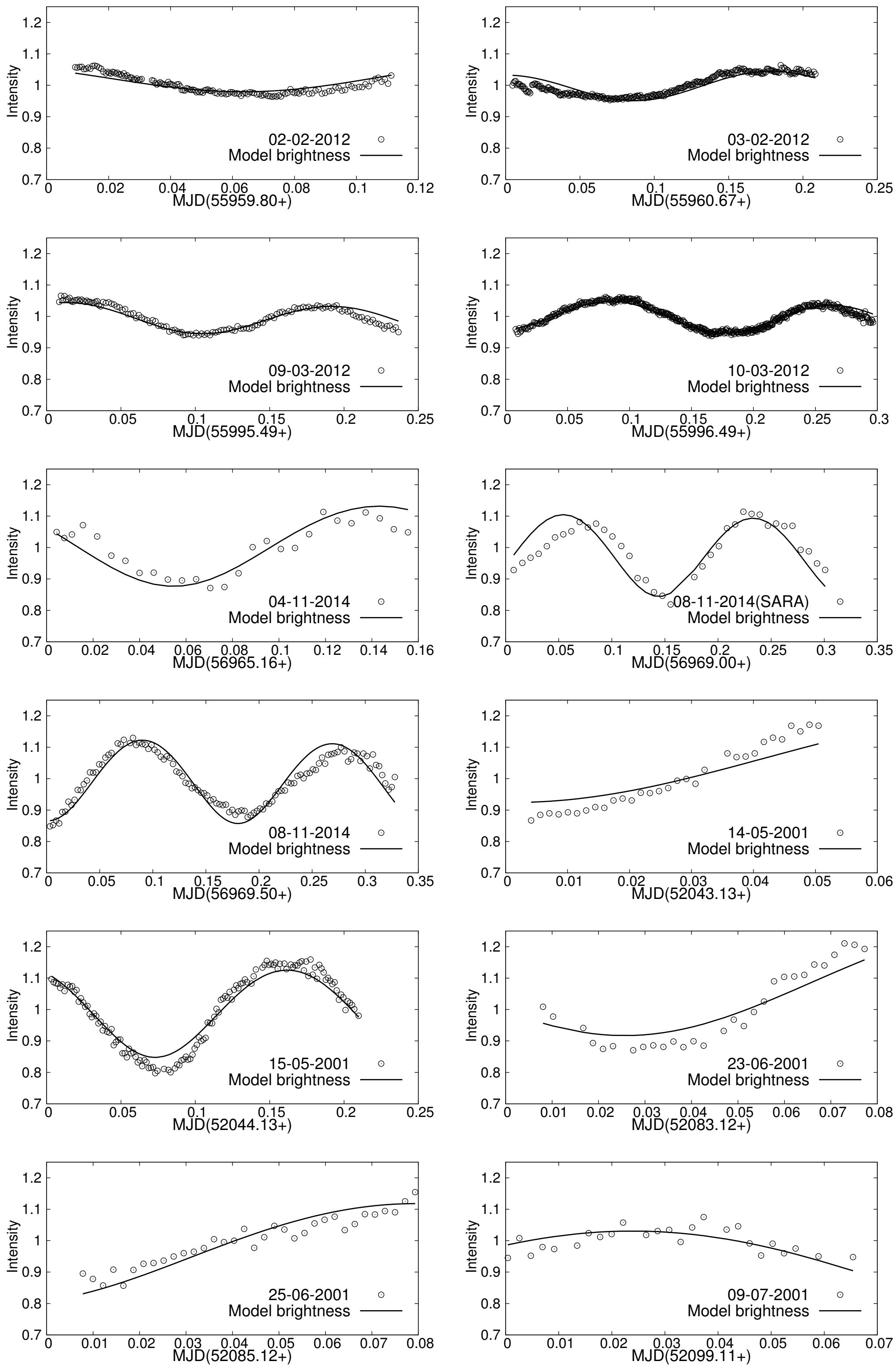

Fig. 1 Light curves of (585) Bilkis. Circles are observed data and lines are modeled light curves using the Lommel-Seeliger ellipsoid model.

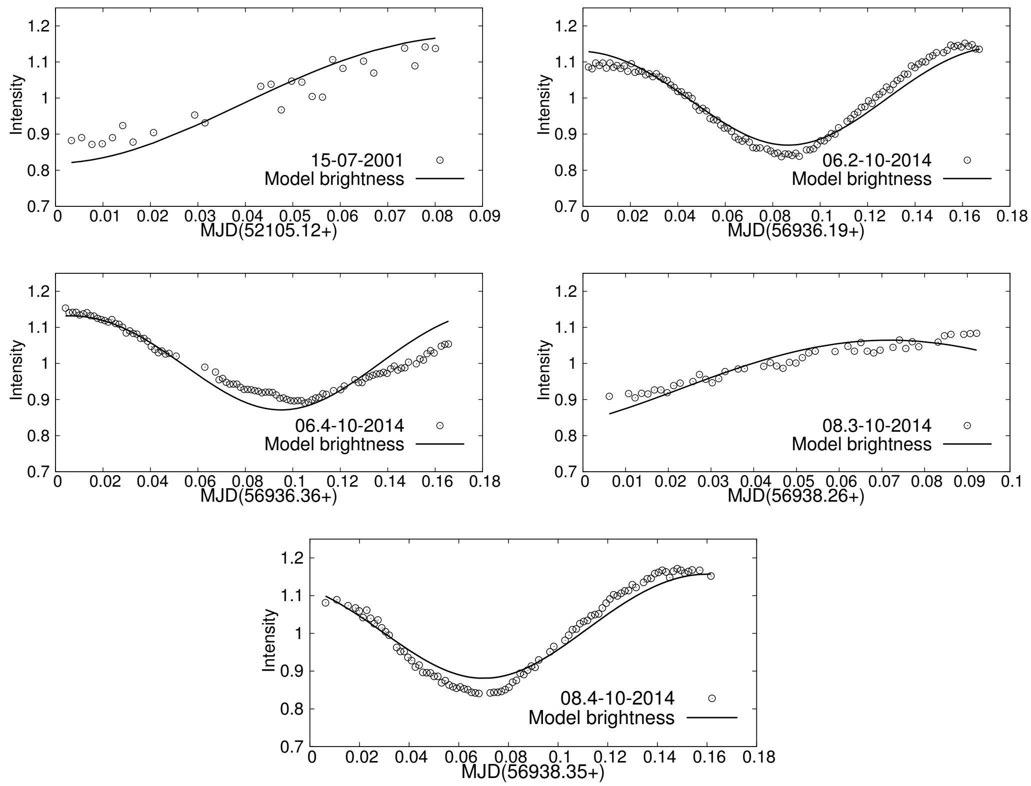

Fig. 1 — *Continued.*

Table 1 Observational Data of Asteroid (585) Bilkis

Date (UT)	R.A. (J2000.0)	Decl. (J2000.0)	Δ (AU)	r (AU)	Ph. (°)	V	Telescope	Filter
2001/05/14.14	13 49 21.3	–02 49 53	1.347	2.288	12.0	13.6	0.25 m, MPC	C
2001/05/15.14	13 48 49.0	–02 45 44	1.353	2.290	12.4	13.6	0.25 m, MPC	C
2001/06/23.14	13 47 26.7	–02 49 52	1.739	2.344	23.3	14.6	0.25 m, MPC	C
2001/06/25.14	13 48 24.1	–02 57 36	1.763	2.346	23.6	14.6	0.25 m, MPC	C
2001/07/09.12	13 57 24.6	–04 05 27	1.941	2.366	24.9	14.9	0.25 m, MPC	C
2001/07/15.14	14 02 25.8	–04 40 51	2.020	2.375	25.1	15.0	0.25 m, MPC	C
2012/02/02.81	09 52 10.7	+03 41 50	1.162	2.122	8.2	12.9	1.0 m, YNAO	C
2012/02/03.67	09 51 25.9	+03 48 21	1.159	2.122	7.8	12.9	1.0 m, YNAO	C
2012/03/09.50	09 25 13.7	+08 49 17	1.196	2.120	13.1	13.2	1.0 m, YNAO	C
2012/03/10.50	09 24 46.9	+08 57 36	1.201	2.120	13.6	13.2	1.0 m, YNAO	C
2014/10/06.19	02 03 30.3	+07 17 40	1.615	2.581	7.3	14.0	0.35 m, MPC	C
2014/10/06.36	02 03 28.4	+07 17 20	1.615	2.581	7.3	14.0	0.35 m, MPC	C
2014/10/08.27	02 01 59.5	+07 02 11	1.606	2.578	6.4	14.0	0.35 m, MPC	C
2014/10/08.36	02 01 55.5	+07 01 32	1.605	2.578	6.4	14.0	0.35 m, MPC	C
2014/11/04.16	01 38 58.4	+03 37 03	1.584	2.546	7.0	13.9	0.9 m, SARA	R
2014/11/08.51	01 35 28.4	+03 09 04	1.600	2.540	8.9	14.0	1.0 m, YNAO	C
2014/11/08.00	01 35 51.8	+03 12 14	1.598	2.541	8.7	14.0	0.9 m, SARA	R

Notes: We select the middle time of each light curve to calculate the position of the asteroid.

more information about the disk-integrated brightness of the Lommel-Seeliger ellipsoid, refer to papers (Muinonen et al. (2015); Muinonen & Lumme (2015)).

In detail, the unknown vector (represented by $\mathbf{P} = (\mathrm{per}, \lambda_\mathrm{p}, \beta_\mathrm{p}, \phi_0, a, b, c, p, G_1, G_2, D)^\mathrm{T}$, where T is transpose) is composed of elements: the rotation period 'per', ecliptic longitude of pole λ_p, ecliptic latitude of pole β_p, rotational phase ϕ_0 at t_0, size of three semimajor axes of an ellipsoid a, b, c ($a \geq b \geq c$), geometric albedo p, parameters of phase function H, G_1, G_2, and equivalent diameter of the asteroid D.

In principle, we can infer these unknown parameters by comparing the observed intensities with the modeled ones. Because only relative intensities of (585) Bilkis are

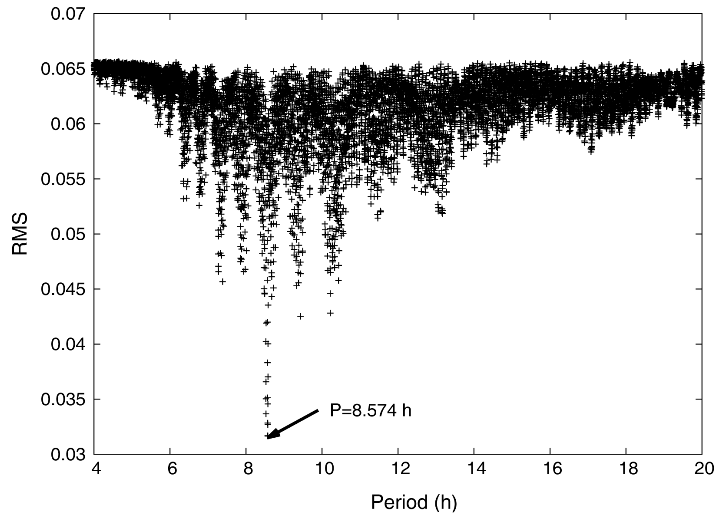

Fig. 2 Period distribution (4 h to 20 h) of (585) Bilkis vs. RMS from the fits.

involved in the present work, only spin parameters (period and orientation of pole) and the ellipsoidal shape of (585) Bilkis (taken with $a = 1$) are analyzed in the present work. The whole analysis procedure contains two parts. First, in order to find the global minimum in root mean square (RMS) of observations, the rotation period and pole of (585) Bilkis are searched by scanning wide ranges. During the process of scanning for the period and pole, a flexible Nelder-Mead downhill method is applied to test the different initial values of period and pole. Second, an MCMC method is applied to find the best solution for the multiple unknown parameters and their uncertainties. During the simulation that utilizes MCMC, the posterior probability density function (PDF) $p_{\rm p}$ corresponding to the unknown parameter vector **P** is derived according to Bayesian inference.

$$\begin{aligned} p_{\rm p} &\propto p_{\rm pr}(\mathbf{P}) p_\epsilon(\Delta L(\mathbf{P})), \\ \Delta L(\mathbf{P}) &= L_{\rm obs} - L(\mathbf{P}), \end{aligned} \tag{5}$$

where '$p_{\rm pr}$', the prior PDF, is assumed to be constant. p_ϵ, the PDF of observational errors, is calculated by the residuals $(O - C)$. The final posterior PDF is

$$\begin{aligned} p_{\rm p}(\mathbf{P}) &\propto \exp[-\tfrac{1}{2}\chi^2(\mathbf{P})] \\ \chi^2(\mathbf{P}) &= \Delta L(\mathbf{P})^{\rm T} \wedge \Delta L(\mathbf{P}), \end{aligned} \tag{6}$$

where '$\wedge$' means the covariance matrix of the errors. The RMS is

$$\mathrm{RMS} = \sigma\sqrt{\chi^2(\mathbf{P})/N_{\rm obs}}, \tag{7}$$

where σ is observational error and $N_{\rm obs}$ is the number of observations.

The sampling of the MCMC simulation is carried out using the Metropolis-Hastings algorithm. To efficiently derive a regular Gaussian shape for the distribution of parameters, a proposal PDF of parameters is generated by the virtual observation MCMC method. In detail, the virtual observations are generated by adding Gaussian noise into the original photometric data. Then, the corresponding virtual least-square solutions of parameters are derived with an optimization method, the Nelder-Mead downhill simplex method.

4 RESULTS

4.1 Downhill Simplex Solution

To find the most probable value of period, we scan the period from 4 h to 20 h with a resolution of per$^2/2\Delta T$ (ΔT denotes the time span of photometric data used) and find that the most probable period is near 8.574 h (see Fig. 2). Then, a range between 8.55 h and 8.6 h is scanned with a high resolution of per$^2/10\Delta T$. Finally, the most significant value of period is found around 8.5738 h (see Fig. 3). This value is very close to Warner's result (Warner 2011).

Every period scanning step is carried out roughly for hundreds of different pole orientations distributed as uniformly as possible on the unit sphere. For each rotation period, the pole orientation, rotational phase and axial ratios are optimized with the Nelder-Mead downhill simplex method. Optionally, the scattering parameters can also be optimized if the photometric data have been converted to the absolute magnitude system. With the optimal period obtained, the most probable pole ($\lambda_{\rm p}$,$\beta_{\rm p}$) is scanned systematically over the full solid angle with a high resolution of about a few degrees on a unit sphere.

Figure 4 shows the tested poles. It is worth noting that the tested poles with the same pole latitude and 180° differences in pole longitude will have a similar effect on disk-integrated brightness. By comparison, we find a pole at (135°, 36°) (the white asterisk in Fig. 4) gives a significant minimum RMS for the observations among tested poles. Taking the scanned spin parameters as initial values, unknown parameters are re-estimated with the Nelder-Mead

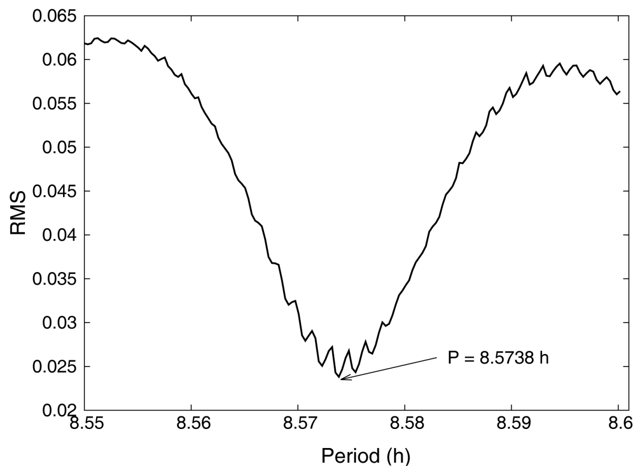

Fig. 3 Period distribution (8.55 h to 8.60 h) of (585) Bilkis vs. RMS from the fits.

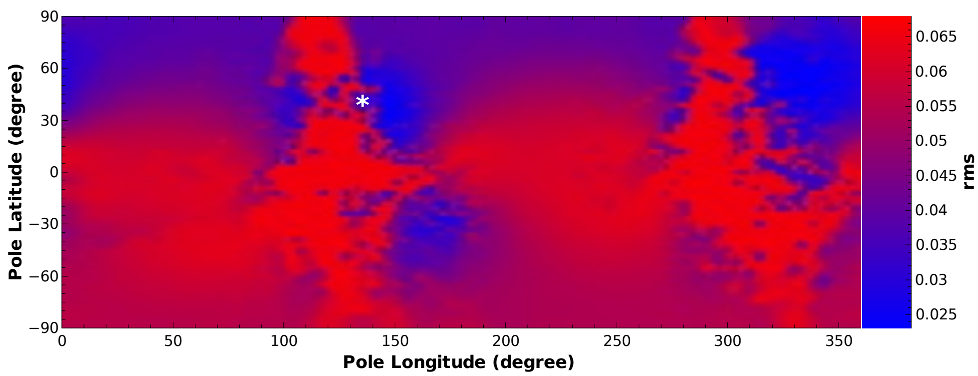

Fig. 4 Tested poles ($\lambda_{\rm p}$,$\beta_{\rm p}$); the white asterisk corresponds to the significant minimum of RMS.

downhill simplex method, and a pole at (135°, 36°) with a spin period of 8.5738 h is derived and an ellipsoidal shape with axial ratios of $b/a = 0.75$ and $c/a = 0.64$ is obtained as well.

4.2 Solution Derived from the MCMC Method

In order to understand the uncertainties in the estimated parameters, an MCMC simulation is carried out for photometric data of (585) Bilkis. In detail, proposal distributions of the parameters are derived with the virtual observation MCMC method, and then a random-walk Metropolis-Hastings algorithm is run based on those proposal distributions.

The proposal distributions of parameters represent least-square solutions of virtual observations, which are generated by adding Gaussian noise into the original observations. The standard deviation of a Gaussian added in one night's observation is the scatter of the light curve compared to a certain model. Such a proposal distribution actually suggests high probability regions in the parameter space that correspond to the observations and, therefore, it speeds the MCMC sampling procedure. Usually, 10 000 virtual least-square solutions are incorporated in the proposal distributions. Detailed information on the MCMC sampling can be referred to in Muinonen's work (Muinonen et al. 2012) on asteroid orbital inversion.

Finally, a joint distribution of parameters is derived with the MCMC simulation, in which more than 20 000 samples are contained. The best values and uncertainties in model parameters are estimated from the statistical features of the marginal posterior distribution of an individual parameter (see Fig. 5). We have obtained the peak value of Gaussian fitting for the distribution as the best value of the corresponding parameter, and taken the $1-\sigma$ limit as the uncertainty in an estimated parameter.

With the method mentioned above, the best period of 8.5738209 h and its uncertainty of 9×10^{-7} h are derived. In the same way, the best values and corresponding uncertainties of pole orientation and the three axes representing ellipsoid parameters (b/a and c/a) are derived. We obtain the most likely pole of (136.46°, 29.0°), and the uncertainties in longitude and latitude are 0.67° and 1.1°, respectively. For the shape of (585) Bilkis, axial ratios b/a and

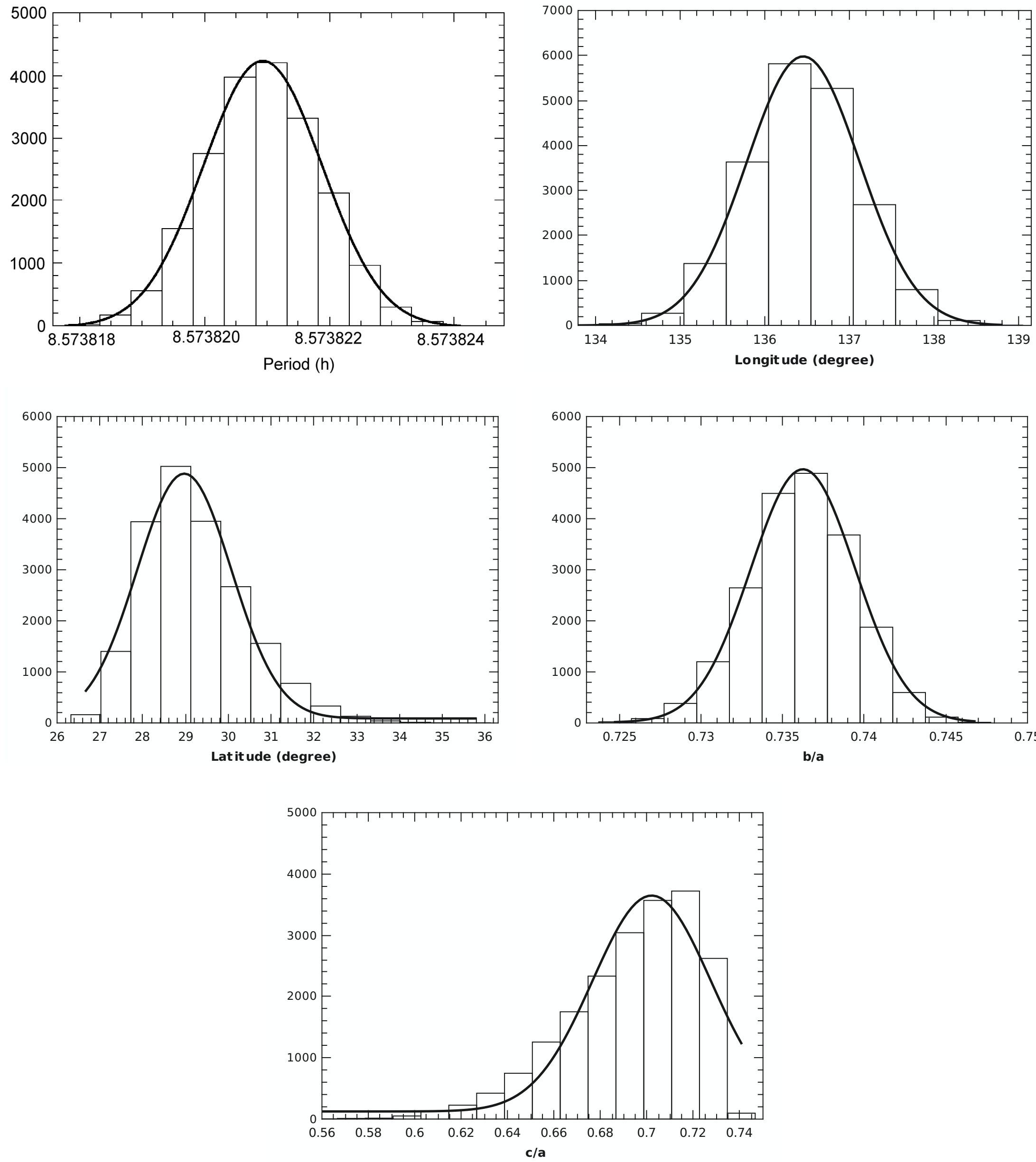

Fig. 5 Histogram distributions of spin period, pole orientation and axial ratios.

c/a are 0.736 and 0.70 with uncertainties of 0.003 and 0.03 respectively.

It is easy to see that the uncertainty in pole latitude is larger than that of pole longitude and the uncertainty in axial ratio c/a is significantly larger than that of b/a. This happens because the latitude and shape parameter c/a are coupled together to some extent in our model.

5 SUMMARY

Using the Lommel-Seeliger ellipsoid model and the MCMC method, we derived the spin parameters (rotation period, pole orientation) and the shape of (585) Bilkis from new and existing photometric data. Actually, such an analysis procedure will be applied to near Earth asteroids (NEAs), especially potentially hazardous asteroids because fewer parameters are involved compared to the convex inversion method. It is useful to analyze physical properties of these newly discovered NEAs with the Lommel-Seeliger ellipsoid method, because of the limited photometric data on the newly discovered NEAs.

The derived period of 8.5738209 h is consistent with previous results. For the first time, its pole and ellipsoidal shape are determined. The ecliptic longitude of the pole is 136.46° with a standard deviation of 0.67° and the ecliptic latitude is 29.0° with a standard deviation of 1.1°. Regarding its shape, the relative triaxial dimensions are $b/a = 0.736$ and $c/a = 0.70$ with standard deviations of 0.003 and 0.03 respectively.

At present, it is still difficult to easily decouple the effects of ecliptic latitude and the shape parameter c/a using disk-integrated brightness. However, we can still estimate the spin parameters and shape with reasonable errors using a relatively small amount of photometric data. In the near future, we will have better limits on the spin and shape parameters by the convex inversion method when more photometric observations and/or occultation data of (585) Bilkis are obtained.

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