

## *C*-field cosmological models: revisited

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**Abstract** We investigate plane symmetric spacetime filled with perfect fluid in the *C*-field cosmology of Hoyle and Narlikar. A new class of exact solutions has been obtained by considering the creation field *C* as a function of time only. To get the deterministic solution, it has been assumed that the rate of creation of matter-energy density is proportional to the strength of the existing *C*-field energy density. Several physical aspects and geometrical properties of the models are discussed in detail, especially showing that some of our solutions of *C*-field cosmology are free from singularity in contrast to the Big Bang cosmology. A comparative study has been carried out between two models, one singular and the other nonsingular, by contrasting the behaviour of the physical parameters. We note that the model in a unique way represents both the features of the accelerating as well as decelerating universe depending on the parameters and thus seems to provide glimpses of the oscillating or cyclic model of the universe without invoking any other agent or theory in allowing cyclicity.

**Key words:** cosmology: miscellaneous — cosmology: theory — cosmology: early universe

### 1 INTRODUCTION

It is generally accepted that spatial anisotropy and the lack of homogeneity would have important consequences in the very early universe. Therefore the study of a creation field cosmological model that relaxes the FRW assumptions is well motivated and is not only a viable alternative to the standard big-bang model but is also theoretically superior to that model (Narlikar & Padmanabhan 1985). As additional support for this superiority, Narlikar & Rana (1983) earlier showed that the theoretical curve of relic radiation in the *G*-varying Hoyle-Narlikar cosmology provides an acceptable fit to the observations at long as well as short wavelengths. A similar problem was also studied by Narlikar et al. (2003) to calculate the expected angular power spectrum of the temperature fluctuations in the microwave background radiation generated in the quasi steady state cosmology and was able to obtain a satisfactory fit to the observational band power estimates of the CMBR temperature fluctuation spectrum. An exhaustive review on the steady state cosmology and *C*-field may be helpful in this research area (Hoyle & Narlikar 1995).

However, alternative theories have been proposed from time to time - the most well known being the steady state theory of cosmology proposed by Bondi & Gold (1948). In this approach the universe neither has any singular beginning nor an end on the cosmic time scale. It has been postulated that the statistical properties of the large scale features of the universe do not change.

Narlikar & Padmanabhan (1985) earlier found a solution of Einstein's equations which admits radiation and a negative-energy based massless scalar creation field as a source. They have shown that the cosmological model connected to this solution satisfies all of the observational tests. The model obtained by them was very important, specifically being free from singularity and it could provide a natural explanation for the flatness problem. Motivated by this fundamental work, in the present work we have studied the Hoyle-Narlikar *C*-field cosmology in plane symmetric spacetime. We have assumed that  $C(x, t) = C(t)$ ; i.e., the creation field *C* is a function of time only. We have extended the method used by Narlikar & Padmanabhan (1985) to the plane symmetric model.

In this regard we note that cosmological models exhibiting plane symmetry have attracted much attention from several scientists. It was Taub (1951, 1956) who first discussed a plane symmetric perfect fluid distribution in which the flow was taken to be isentropic in general relativity. Later on, as a particular case of the plane symmetric models for cosmology, the Bianchi type spacetime was extensively studied by Heckmann et al. (1962), Thorne (1967), Jacobs (1968), Singh & Singh (1992).

More elaborately, in connection with plane symmetric spacetime, Smoot et al. (1992) argued that the earlier predictions of the Friedman-Lemaître-Robertson-Walker (FLRW) type models do not always exactly explain the observed results. Some peculiar outcomes regarding the redshift from extragalactic objects continue to contradict the theoretical explanations given from the FLRW model. It is further known that symmetry plays an important role in understanding the structure of the universe because such distance measurements are usually thought to probe the background metric of the universe. But in reality, the presence of perturbations will lead to deviations from the result expected in an exactly homogeneous and isotropic universe, which suggests considering the cases where perturbations are plane symmetric (Adamek et al. 2014). Although most stars are believed to have spherical symmetry, cylindrical and plane symmetries may be useful to investigate gravitational waves which have been detected very recently. So in literature, many authors consider plane symmetry, which is less restrictive than spherical symmetry and provides an avenue to study inhomogeneities in the early as well as late universe in different physical contexts by da Silva et al. (1998), Anguige (2000), Nouri-Nouri-Zonoz & Tavanfar (2001), Pradhan & Pandey (2003), Pradhan et al. (2007) and Yadav (2010). All these have inspired us to study the model of the universe with plane symmetry.

However, as background of creation field cosmology, we would like to present here some of the relevant works which will provide a thread for our investigation. In their paper on Mach's principle and the creation of matter, Hoyle & Narlikar (1963) used experimental evidence that the local inertial frame is the one with respect to which the distant parts of the universe are non-rotating. They introduced a scalar 'creation field' into the theory of relativity to improve the situation and showed that this explains the observed remarkable degree of homogeneity and isotropy in the universe.

It has also been shown via a  $C$ -field that steady-state cosmology appears as an asymptotic case of the cosmological solutions of Einstein's equations. The source equation has been treated in terms of discrete particles instead of the macroscopic case of a smooth fluid (Hoyle & Narlikar 1964). In this sequel of works on steady-state cosmology, Hoyle & Narlikar (1966) also showed that it is possible to interpret that (i) the expansion rate of fluctuation from the steady-state situation follows the Einstein-de Sitter re-

lations, (ii) the natural scale set by the new steady-state corresponds to the masses of clusters of galaxies  $10^{13} M_{\odot}$  for the 'observable universe', and (iii) it is suggested that elliptical galaxies were formed early in the development of a fluctuation. Some other works on  $C$ -field cosmology are available in the literature (Hoyle & Narlikar 1964b,a; Narlikar 1973) for further study.

Very recently, a study has been carried out (Ghate & Mhaske 2014) in the Hoyle-Narlikar creation field theory of gravitation under plane symmetric and LRS Bianchi type  $V$  cosmological models. The work is on varying gravitational constant  $G$  for the barotropic fluid distribution. The solutions of the field equations have been obtained by assuming that  $G = Bm$ , where  $B$  is a scale factor and  $m$  is a constant. Besides this, Ghate and his collaborators (Ghate & Salve 2014c,a,b) have published a series of works under  $C$ -field cosmology with different physical systems. Some other recent works on  $C$ -field cosmology are also available in the literature (Chatterjee & Banerjee 2004; Singh & Chaubey 2009; Adhav et al. 2010, 2011; Bali & Saraf 2013).

The plan of our study is as follows: In Section 2 we have given an overall view of the creation field theory in cosmology whereas in Section 3 and Section 4 the basic mathematical details of the model and exact solutions of the model have, respectively, been provided. A special section has been added thereafter in Section 5 for the non-singular solution. We have discussed several physical features of the models in Section 6. In Section 7 we have provided some concluding remarks based on comparative studies between two models, one singular and the other nonsingular, by contrasting the behaviour of different physical parameters.

## 2 THE CREATION FIELD THEORY

Einstein's field equations are modified by introducing a massless scalar field called a creation field, viz.  $C$ -field (Hoyle & Narlikar 1963; Hoyle & Narlikar 1964; Hoyle & Narlikar 1964b,a, 1966; Narlikar 1973; Narlikar et al. 2003). The proposed modified field equations have been provided in the form

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi \left( {}^m T_{ij} + {}^c T_{ij} \right), \quad (1)$$

where  ${}^m T_{ij}$  is the matter tensor of the Einstein theory and  ${}^c T_{ij}$  is the matter tensor due to the  $C$ -field which is given by

$${}^c T_{ij} = -f^2 \left( C_i C_j - \frac{1}{2} g_{ij} C^k C_k \right), \quad (2)$$

where  $f^2$  is a coupling constant,  $C_i = \frac{\partial C}{\partial x^i}$  and  $C$  is the creation field function. It is not necessary to take a small value of coupling constant  $f$ . However, it is not large enough and hence one can assume the value of  $f$  in such a way that all the solutions have finite values.

Because of the negative value of  $T^{00}$ , the  $C$ -field has negative energy density producing a repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$${}^m T_{;j}^{ij} = - {}^c T_{;j}^{ij} = f^2 C^i C_{;j}. \quad (3)$$

Here the semicolon (;) denotes covariant differentiation, i.e. matter creation through the non-zero left hand side is possible while conserving the overall energy and momentum.

### 3 THE MODELS: MATHEMATICAL BASICS

The spatially homogeneous and anisotropic plane symmetric spacetime is described by the line element

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2, \quad (4)$$

where  $A$  and  $B$  are the cosmic scale factors and functions of cosmic time  $t$  only (non-static case).

The proper volume of the model (4) is given by

$$V = \sqrt{-g} = A^2 B. \quad (5)$$

The matter tensor for perfect fluid is

$${}^m T^{ij} = \text{diag}(\rho, -p, -p, -p), \quad (6)$$

where  $\rho$  is the homogeneous mass density and  $p$  is the isotropic pressure. We have assumed here that the creation field  $C$  is a function of time  $t$  only; i.e.,  $C(x, t) = C(t)$ .

For the line element (4) the Einstein field equation (1) can be written as

$$8\pi\rho = 4\pi\Omega + \frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB}, \quad (7)$$

$$8\pi p = 4\pi\Omega - \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} - \frac{\ddot{A}}{A}, \quad (8)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} = \frac{\dot{A}}{A} \left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right), \quad (9)$$

where dot ( $\dot{\cdot}$ ) indicates the derivative with respect to  $t$  and  $\Omega = f^2 \dot{C}^2$ . From (5), we can write  $B = \frac{V}{A^2}$ . Equation (9) transforms to

$$\frac{\ddot{V}}{3V} - \frac{\ddot{A}}{A} = \frac{\dot{A}}{A} \left( \frac{\dot{V}}{V} - \frac{\dot{A}}{A} \right). \quad (10)$$

The general solution of the above equation is

$$A(t) = a_1 V^{1/3}(t) \exp \left[ a_2 \int \frac{dt}{V(t)} \right], \quad (11)$$

where  $a_1$  and  $a_2$  are constants of integration. Therefore, the coefficient  $B$ , the homogeneous mass density  $\rho$  and the isotropic pressure become

$$B(t) = \frac{V^{1/3}(t)}{a_1^2} \exp \left[ -2a_2 \int \frac{dt}{V(t)} \right], \quad (12)$$

$$8\pi\rho(t) = 4\pi\Omega(t) - \frac{3a_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)}, \quad (13)$$

$$8\pi p(t) = 4\pi\Omega(t) - \frac{3a_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)} - \frac{2\dot{V}(t)}{3V(t)}. \quad (14)$$

In order to obtain a unique solution, one has to specify the rate of creation of matter-energy (at the expense of the negative energy of the  $C$ -field). Without loss of generality, we assume that the rate of creation of matter energy density is proportional to the strength of the existing  $C$ -field energy-density; i.e., the rate of creation of matter energy density per unit proper-volume is given by

$$\frac{d}{dV}(\rho V) + p = f^2 \alpha^2 \dot{C}^2, \quad (15)$$

where  $\alpha$  is a constant of proportionality.

The above equation can be written in the following form

$$V \dot{\rho} + (p + \rho - \alpha^2 \Omega) \dot{V} = 0. \quad (16)$$

By substituting Equations (13) and (14) in Equation (16), we get

$$\frac{\dot{\Omega}}{\Omega} = 2(\alpha^2 - 1) \frac{\dot{V}}{V}. \quad (17)$$

By integrating the above equation we have

$$\Omega(t) = \frac{\Omega_0}{4\pi} V^{2(\alpha^2-1)}, \quad (18)$$

where  $\Omega_0$  is an arbitrary constant of integration. Inserting (18) into (13) and (14) we have

$$8\pi\rho(t) = \Omega_0 V^{2(\alpha^2-1)}(t) - \frac{3k_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)}, \quad (19)$$

$$8\pi p(t) = \Omega_0 V^{2(\alpha^2-1)}(t) - \frac{3k_2^2}{V^2(t)} + \frac{\dot{V}^2(t)}{3V^2(t)} - \frac{2\dot{V}(t)}{3V(t)}. \quad (20)$$

Now, we consider the equation of state of matter as

$$p = \gamma \rho. \quad (21)$$

Here  $\gamma$  varies between the interval  $0 \leq \gamma \leq 1$ , whereas  $\gamma = 0$  describes the dust universe,  $\gamma = 1/3$  represents the radiation universe,  $1/3 \leq \gamma \leq 1$  describes the hard universe and  $\gamma = 1$  corresponds to stiff matter.

By substituting Equations (21) and (18) in Equation (16), we get

$$V \dot{\rho} + \left[ (1 + \gamma)\rho - \Omega_0 \alpha^2 V^{2(\alpha^2-1)} \right] \dot{V} = 0, \quad (22)$$

which yields

$$8\pi\rho(t) = \frac{2\Omega_0 \alpha^2 V^{2(\alpha^2-1)}}{2\alpha^2 + \gamma - 1} + \rho_0 V^{-1-\gamma}, \quad (23)$$

where  $\rho_0$  is an arbitrary constant of integration.

By subtracting Equation (23) from Equation (19), we get

$$(2\alpha^2 + \gamma - 1) \left[ 9a_2^2 + 3\rho_0 V^{1-\gamma} - \dot{V}^2 \right] + 3\Omega_0 (1 - \gamma) V^2 \alpha^2 = 0. \quad (24)$$

The above equation can be written in the following form

$$\int \frac{dV}{\sqrt{9a_2^2 + k_0 V^{2\alpha^2} + 3\rho_0 V^{1-\gamma}}} = t - t_0, \quad (25)$$

where  $k_0 = \frac{3\Omega_0(1-\gamma)}{2\alpha^2 + \gamma - 1}$  and  $t_0$  is an arbitrary constant of integration.

**4 THE MODELS: A CLASS OF EXACT SOLUTIONS**

To obtain the class of exact solutions in terms of cosmic time  $t$ , we consider the following cases and their respective plots. We have used geometrical units, i.e.  $G = c = 1$ . The figures provide information on the natural variation of physical parameters with respect to time only. Usually the units are as follows: energy density  $\rightarrow$  gm cm<sup>-3</sup>, pressure  $\rightarrow$  dyne cm<sup>-2</sup>, creation field  $C =$  density  $\rightarrow$  gm cm<sup>-3</sup>, volume  $\rightarrow$  cm<sup>3</sup>, and time  $\rightarrow$  Gyr.

**4.1  $\rho_0 = 0$**

**4.1.1  $a_2 = 0$**

In this case, we can obtain the following solution:

$$\begin{aligned} V(t) &= \left[ k_1 (1 - \alpha^2) T \right]^{\frac{1}{1-\alpha^2}}, \\ \rho(t) &= \frac{\alpha^2}{12 \pi (1 - \gamma) (1 - \alpha^2)^2 T^2}, \\ p(t) &= \frac{\gamma \alpha^2}{12 \pi (1 - \gamma) (1 - \alpha^2)^2 T^2}, \\ C(t) &= C_0 + \frac{1}{2 f (1 - \alpha^2)} \sqrt{\frac{2 \alpha^2 + \gamma - 1}{3 \pi (1 - \gamma)}} \ln[T], \\ A(t) &= a_1 \left[ (1 - \alpha^2) T \right]^{\frac{1}{3(1-\alpha^2)}}, \\ B(t) &= \frac{1}{a_1^2} \left[ k_1 (1 - \alpha^2) T \right]^{\frac{1}{3(1-\alpha^2)}}, \end{aligned} \tag{26}$$

where  $C_0$  is an arbitrary constant,  $k_0 = k_1^2$  and  $T = t - t_0$ .

**4.1.2  $a_2 \neq 0$**

(i) For the  $\alpha = 0$  case we can obtain the following solution:

$$\begin{aligned} V(t) &= k_2 T, \\ \rho(t) &= p(t) = 0, \\ C(t) &= C_0 + \frac{1}{2 f k_2} \sqrt{\frac{9 a_2^2 - k_2^2}{3 \pi}} \ln[T], \\ A(t) &= a_1 k_2^{1/3} T^{\frac{1}{3} + \frac{a_2}{k_2}}, \\ B(t) &= \frac{k_2^{1/3}}{a_1^2} T^{\frac{1}{3} - \frac{2 a_2}{k_2}}, \end{aligned} \tag{27}$$

where  $C_0$  is an arbitrary constant,  $k_2^2 = 9 a_2^2 - 3 \Omega_0$  and  $T = t - t_0$ .

(ii) For the  $\alpha = \frac{1}{\sqrt{2}}$  case we can obtain the following solution:

$$\begin{aligned} V(t) &= \frac{k_0}{4} (T^2 - k_3^2), \\ \rho(t) &= \frac{1}{6 \pi (1 - \gamma) (T^2 - k_3^2)}, \\ p(t) &= \frac{\gamma}{6 \pi (1 - \gamma) (T^2 - k_3^2)}, \\ C(t) &= C_0 + \frac{1}{f} \sqrt{\frac{\gamma}{3 \pi (1 - \gamma)}} \ln \left[ 2 \left( T + \sqrt{T^2 - k_3^2} \right) \right], \\ A(t) &= -a_1 \left( \frac{k_0}{4} \right)^{1/3} (T - k_3)^{2/3}, \\ B(t) &= \frac{1}{a_1^2} \left( \frac{k_0}{4} \right)^{1/3} (T + k_3) (T - k_3)^{-1/3}, \end{aligned} \tag{28}$$

where  $C_0$  is an arbitrary constant,  $k_3^2 = \frac{36 a_2^2}{k_0^2}$  and  $T = t - t_0$ .

(iii) For the  $\alpha = 1$  case we can obtain the following solution:

$$\begin{aligned} V(t) &= k_4^{-1} \sinh [3 a_2 k_4 T], \\ \rho(t) &= \frac{3 a_2^2 k_4^2}{4 \pi (1 - \gamma)}, \\ p(t) &= \frac{3 \gamma a_2^2 k_4^2}{4 \pi (1 - \gamma)}, \\ C(t) &= C_0 + \frac{a_2 k_4 T}{2 f} \sqrt{\frac{3(1+\gamma)}{\pi(1-\gamma)}}, \\ A(t) &= a_1 k_4^{-1/3} \tanh^{1/3} \left[ \frac{3 a_2 k_4 T}{2} \right] \\ &\quad \times \sinh^{1/3} [3 a_2 k_4 T], \\ B(t) &= a_1^{-2} k_4^{-1/3} \coth^{2/3} \left[ \frac{3 a_2 k_4 T}{2} \right] \\ &\quad \times \sinh^{1/3} [3 a_2 k_4 T], \end{aligned} \tag{29}$$

where  $C_0$  is an arbitrary constant,  $k_0 = 2 a_2^2 k_4^2$  and  $T = t - t_0$ .

**4.2  $\rho_0 \neq 0$**

**4.2.1  $a_2 = \Omega_0 = 0$**

We can obtain the following solution:

$$\begin{aligned} V(t) &= \left[ \frac{k_5 (1 + \gamma) T}{2} \right]^{\frac{2}{1+\gamma}}, \\ \rho(t) &= \frac{1}{6 \pi (1 + \gamma)^2 T^2}, \\ p(t) &= \frac{\gamma}{6 \pi (1 + \gamma)^2 T^2}, \\ C(t) &= C_0, \\ A(t) &= a_1 \left[ \frac{k_5 (1 + \gamma) T}{2} \right]^{\frac{2}{3(1+\gamma)}}, \\ B(t) &= \frac{1}{a_1^2} \left[ \frac{k_5 (1 + \gamma) T}{2} \right]^{\frac{2}{3(1+\gamma)}}, \end{aligned} \tag{30}$$

where  $C_0$  is an arbitrary constant,  $k_5^2 = 3\rho_0$  and  $T = t - t_0$ .

#### 4.2.2 $a_2 \neq 0$

When  $\Omega_0 \neq 0$ ,  $\gamma = 0$  and  $p(t) = 0$ .

(i) For the  $\alpha = 0$  case we can obtain the following solution:

$$\begin{aligned} V(t) &= \frac{3\rho_0}{4} (T^2 - k_6^2), \\ \rho(t) &= \frac{1}{6\pi (T^2 - k_6^2)}, \\ C(t) &= C_0 + \frac{1}{f\rho_0 k_6} \sqrt{\frac{4a_2^2 - \rho_0^2 k_6^2}{3\pi}} \ln \left[ \frac{k_6 + T}{k_6 - T} \right], \\ A(t) &= a_1 \left[ \frac{3\rho_0 (T^2 - k_6^2)}{4} \right]^{1/3} \left[ \frac{k_6 - T}{k_6 + T} \right]^{\frac{2a_2}{3\rho_0 k_6}}, \\ B(t) &= \frac{1}{a_1^2} \left[ \frac{3\rho_0 (T^2 - k_6^2)}{4} \right]^{1/3} \left[ \frac{k_6 + T}{k_6 - T} \right]^{\frac{4a_2}{3\rho_0 k_6}}, \end{aligned} \quad (31)$$

where  $C_0$  is an arbitrary constant,  $4\Omega_0 - 12a_2^2 = -3\rho_0 k_6^2$  and  $T = t - t_0$ .

(ii) For the  $\alpha = 1$  case we can obtain the following solution:

$$\begin{aligned} V(t) &= \frac{1}{4k_7^2} e^{-k_7 T} \left[ \left( e^{k_7 T} - 3\rho_0 \right)^2 - 36a_2^2 k_7^2 \right], \\ \rho(t) &= \frac{k_7^2}{12\pi} \left[ 1 - \frac{6\rho_0 e^{k_7 T}}{e^{2k_7 T} + 9\rho_0^2 - 36a_2^2 k_7^2} \right]^{-1}, \\ C(t) &= C_0 + \frac{k_7 T}{2f\sqrt{3\pi}}, \\ A(t) &= -a_1 \left( \frac{1}{2k_7} \right)^{2/3} e^{-\frac{k_7 T}{3}} \left[ e^{k_7 T} - 3\rho_0 - 6a_2 k_7 \right]^{2/3}, \\ B(t) &= \frac{1}{a_1^2} \left( \frac{1}{2k_7} \right)^{2/3} e^{-\frac{k_7 T}{3}} \times \left[ e^{k_7 T} - 3\rho_0 + 6a_2 k_7 \right] \times \left[ e^{k_5 T} - 3k_4 - 6k_2 K_5 \right]^{-1/3}, \end{aligned} \quad (32)$$

where  $C_0$  is an arbitrary constant,  $3\Omega_0 = k_7^2$  and  $T = t - t_0$ .

#### 4.2.3 $a_2 \neq 0$

When  $\Omega_0 > 0$ ,  $3\rho_0 + 9a_2^2 = k_8^2$  and  $\gamma = 1$ . In this case we can obtain the following solution:

$$\begin{aligned} V(t) &= k_8 T, \\ \rho(t) &= p(t) \\ &= \frac{1}{24\pi k_8^2 T^2} \left[ k_8^2 - 9a_2^2 + 3\rho_0 (k_8 T)^{2\alpha^2} \right], \\ C(t) &= C_0 + \frac{\sqrt{\rho_0}}{2f k_8 \alpha^2 \sqrt{\pi}} (k_8 T)^{2\alpha^2}, \\ A(t) &= \frac{k_8^{1/3}}{a_1^2} T^{\frac{1}{3} - \frac{2a_2}{k_8}}, B(t) = a_1 k_8^{1/3} T^{\frac{1}{3} + \frac{2a_2}{k_8}}, \end{aligned} \quad (33)$$

where  $C_0$  is an arbitrary constant and  $T = t - t_0$ .

## 5 NON-SINGULAR SOLUTIONS IN THE C-FIELD COSMOLOGICAL MODELS

Here we assume  $\gamma = 0$ ,  $\alpha^2 = 1$ ,  $a_2 = 0$  and  $3\rho_0 = -k_0 l$ , so that

$$V(t) = l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2, \quad (34)$$

where  $\frac{k_0}{4} = k_1^2$  and  $(t - t_0) = T$ . Also, we obtain the following set of non-singular solutions:

$$\rho(t) = \frac{k_0}{12\pi} \left[ 1 - \frac{l}{2l + \frac{1}{2e^{2k_1 T}} (e^{2k_1 T} - l)^2} \right], \quad (35)$$

$$A(t) = a_1 \left[ l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2 \right]^{\frac{1}{3}}, \quad (36)$$

$$B(t) = \frac{1}{a_1^2} \left[ l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2 \right]^{\frac{1}{3}}, \quad (37)$$

$$C(t) = C_0 + \frac{1}{f\alpha} \sqrt{\frac{k_0}{12\pi}} T. \quad (38)$$

## 6 THE PHYSICAL PROPERTIES OF THE MODELS

The expansion scalar is given by  $\theta = 3H$ ,  $H = \frac{\dot{a}}{a} = \frac{1}{3} \sum_{i=1}^3 H_i$  is the Hubble parameter in our anisotropic models,  $a = V^{1/3}$  is the average scale factor, and  $H_1 = \frac{\dot{A}}{A}$ ,  $H_2 = \frac{\dot{B}}{B}$  and  $H_3 = \frac{\dot{C}}{C}$  are the directional Hubble factors in the directions of  $x$ ,  $y$  and  $z$  respectively. The mean anisotropy parameter is defined by

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i}{H} - 1 \right)^2.$$

The shear scalar is given by

$$\sigma^2 = \frac{1}{2} \sum_{i=1}^3 \left( H_i^2 - 3H^2 \right) = \frac{3}{2} \Delta H^2.$$

The deceleration parameter is defined by  $q = -\left( \frac{\dot{H}}{H^2} + 1 \right)$ .

It is evident that for all the cases discussed above, the shear scalar is a decreasing function of time and finally diminishes for sufficiently larger time except for the sub-cases (4.1.1) and (4.2.1). For sub-cases (4.1.1) and (4.2.1), the shear scalar is found to be zero which is proposed for the model of a non-shearing universe with an isotropic distribution. However, the decreasing behaviour of a shear scalar corresponds to the isotropisation of the universe with passage of time. It should be noted here that the directional Hubble parameter measures the different rate of expansion along different spatial directions at the same time which governs the anisotropy of the universe (Kristian & Sachs 1966; Collins et al. 1980; Saha & Yadav 2012; Yadav et al. 2012).

### 6.1 The Model (4.1.1)

In this case the solution corresponds to

$$\theta = \frac{1}{(1-\alpha^2)T}, \quad \Delta = \sigma^2 = 0, \quad q = 2 - 3\alpha^2. \quad (39)$$

### 6.2 The Model (i) of (4.1.2)

In this case the solution corresponds to

$$\theta = \frac{1}{T}, \quad \Delta = \frac{18a_2^2}{k_2^2}, \quad \sigma^2 = \frac{3a_2^2}{k_2^2 T^2}, \quad q = 2. \quad (40)$$

### 6.3 The Model (ii) of (4.1.2)

In this case the solution corresponds to

$$\begin{aligned} \theta &= \frac{2T}{T^2 - k_3^2}, \\ \Delta &= \frac{2k_3^2}{T^2}, \\ \sigma^2 &= \frac{4k_3^2}{3(T^2 - k_3^2)^2}, \\ q &= \frac{1}{2} \left( 1 + \frac{3k_3^2}{T^2} \right). \end{aligned} \quad (41)$$

### 6.4 The Model (iii) of (4.1.2)

In this case the solution corresponds to

$$\begin{aligned} \theta &= a a_2 k_4 \coth [3 a_2 k_4 T], \\ \Delta &= 2 \operatorname{sech}^2 [3 a_2 k_4 T], \\ \sigma^2 &= 2 a_2^2 k_4^4 \operatorname{csch}^2 [3 a_2 k_4 T], \\ q &= 3 \operatorname{sech}^2 [3 a_2 k_4 T] - 1. \end{aligned} \quad (42)$$

### 6.5 The Model (4.2.1)

In this case the solution corresponds to

$$\theta = \frac{2}{(1+\gamma)T}, \quad \Delta = \sigma^2 = 0, \quad q = \frac{1+3\gamma}{2}. \quad (43)$$

### 6.6 The Model (i) of (4.2.2)

In this case the solution corresponds to

$$\begin{aligned} \theta &= \frac{2T}{T^2 - k_6^2}, \\ \Delta &= \frac{8a_2^2}{\rho_0^2 T^2}, \\ \sigma^2 &= \frac{16a_2^2}{3\rho_0^2 (T^2 - k_6^2)^2}, \\ q &= \frac{1}{2} \left( 1 + \frac{3k_6^2}{T^2} \right). \end{aligned} \quad (44)$$

### 6.7 The Model (ii) of (4.2.2)

In this case the solution corresponds to

$$\begin{aligned} \theta &= k_7 \left( \frac{e^{2k_7 T} - a_3}{e^{2k_7 T} - 6\rho_0 e^{k_7 T} + a_3} \right), \\ \Delta &= \frac{8(9\rho_0^2 - a_3)e^{2k_7 T}}{(e^{2k_7 T} - a_3)^2}, \\ \sigma^2 &= \frac{4k_7^2(9\rho_0^2 - a_3)e^{2k_7 T}}{3(e^{2k_7 T} - 6\rho_0 e^{k_7 T} + a_3)^2}, \\ q &= -\frac{e^{4k_7 T} - 18\rho_0 e^{3k_7 T} + 2a_3(5e^{2k_7 T} - 9\rho_0 e^{k_7 T}) + a_3^2}{(e^{2k_7 T} - a_3)^2}. \end{aligned} \quad (45)$$

where  $a_3 = 9\rho_0^2 - 36a_2^2 k_7^2$ .

### 6.8 The Model (4.2.3)

In this case the solution corresponds to

$$\theta = \frac{1}{T}, \quad \Delta = \frac{18a_2^2}{k_8^2}, \quad \sigma^2 = \frac{3a_2^2}{k_8^2 T^2}, \quad q = 2. \quad (46)$$

### 6.9 The Model (5)

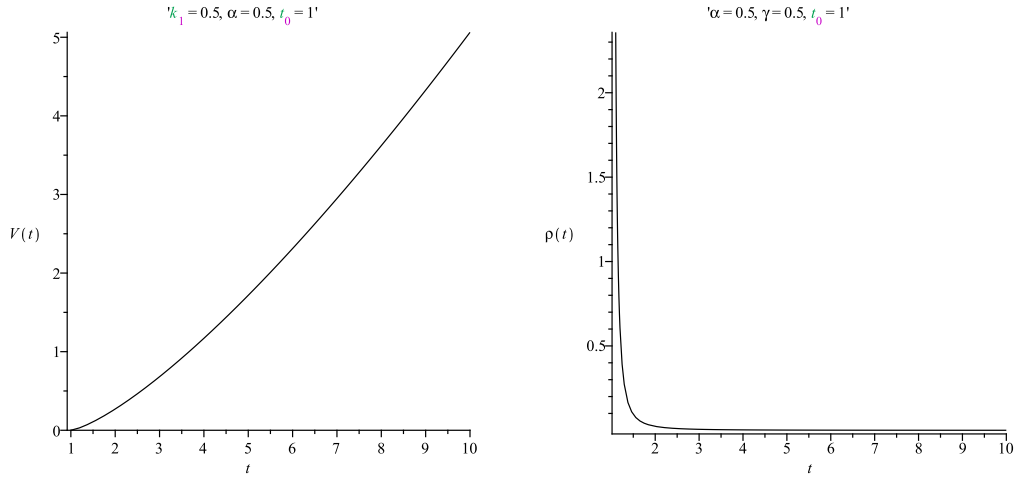
In this case the solution corresponds to

$$\begin{aligned} \theta &= \frac{k_1 (e^{2k_1 T} - l)}{l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2}, \\ \Delta &= \frac{(e^{2k_1 T} - l)^2}{4e^{4k_1 T}}, \\ \sigma^2 &= \frac{k_1^2 (e^{2k_1 T} - l)^4}{24e^{4k_1 T} \left[ l + \frac{1}{4e^{2k_1 T}} (e^{2k_1 T} - l)^2 \right]^2}, \\ q &= \frac{3l (l^2 e^{-2k_1 T} - 3e^{2k_1 T} - 2l)}{2(e^{2k_1 T} - l)^2} - 1. \end{aligned} \quad (47)$$

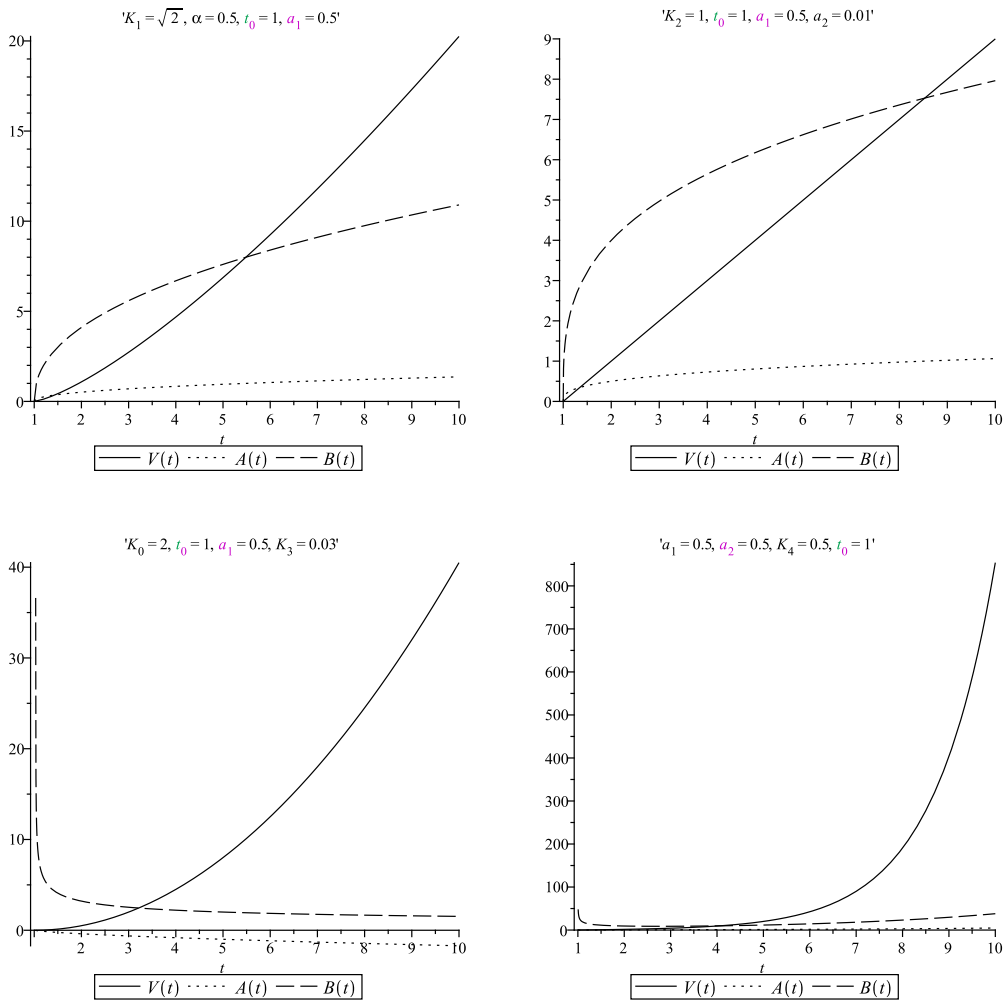
## 7 DISCUSSIONS AND CONCLUSIONS

In the present work, plane symmetric spacetime filled with perfect fluid in the Hoyle-Narlikar  $C$ -field cosmology has been investigated. By considering (i) the creation field is a function of time alone, and (ii) the rate of creation of matter energy-density is proportional to the strength of the existing  $C$ -field energy-density, we have found a new class of exact solutions.

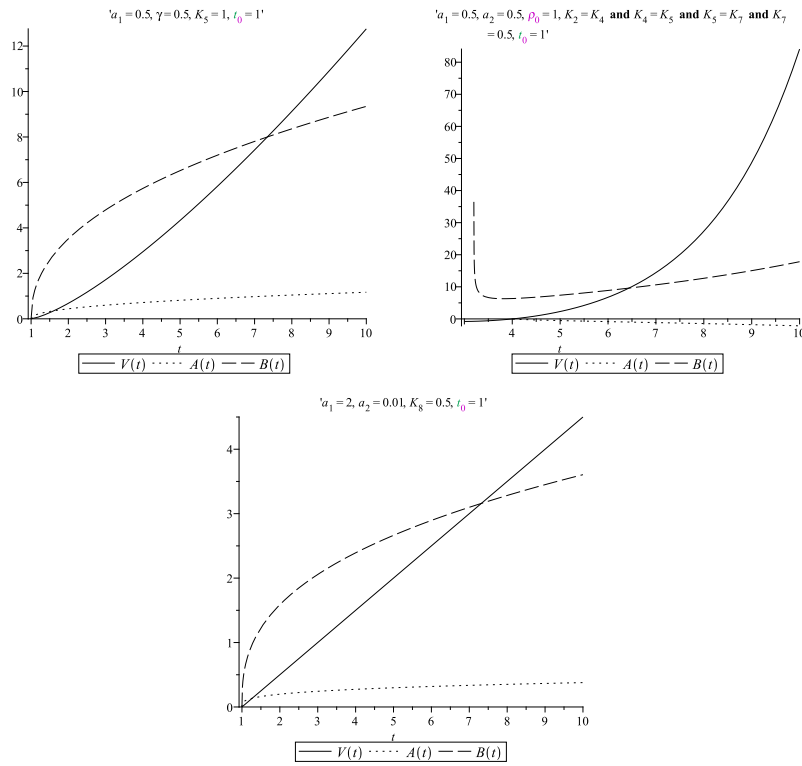
We have, in general, discussed several physical features and geometrical properties of the models. However, as a special case, the most notable aspects of the solution set that have been studied are non-singular in nature. These aspects have been shown through several plots which consist of two kinds: Figures 1–10 for singular cases and Figures 11–12 for the non-singular case. All figures depict interesting features of the present cosmological model in terms of  $C$ -field and other physical parameters.



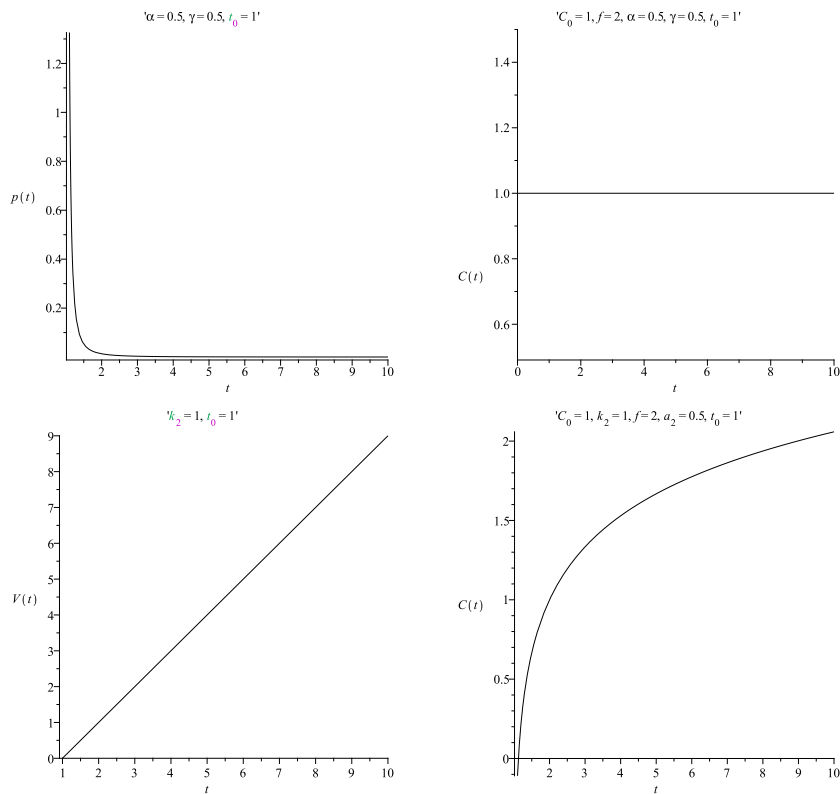
**Fig. 1** Variation of volume (*left panel*) and density (*right panel*) for sub-case 4.1.1.



**Fig. 2** Variation of volume  $V$  and scale factors  $A$  and  $B$ : *upper left panel* for sub-case 4.1.1 when  $\rho_0 = 0$  and  $a_2 = 0$ , *upper right panel* for sub-case 4.1.2 (i) when  $\rho_0 = 0$  and  $a_2 \neq 0, \alpha = 0$ , *lower left panel* for sub-case 4.1.2 (ii) when  $\rho_0 = 0$  and  $a_2 \neq 0, \alpha = 1/\sqrt{2}$ , and *lower right panel* for sub-case 4.1.2 (iii) when  $\rho_0 = 0$  and  $a_2 \neq 0, \alpha = 1$ .

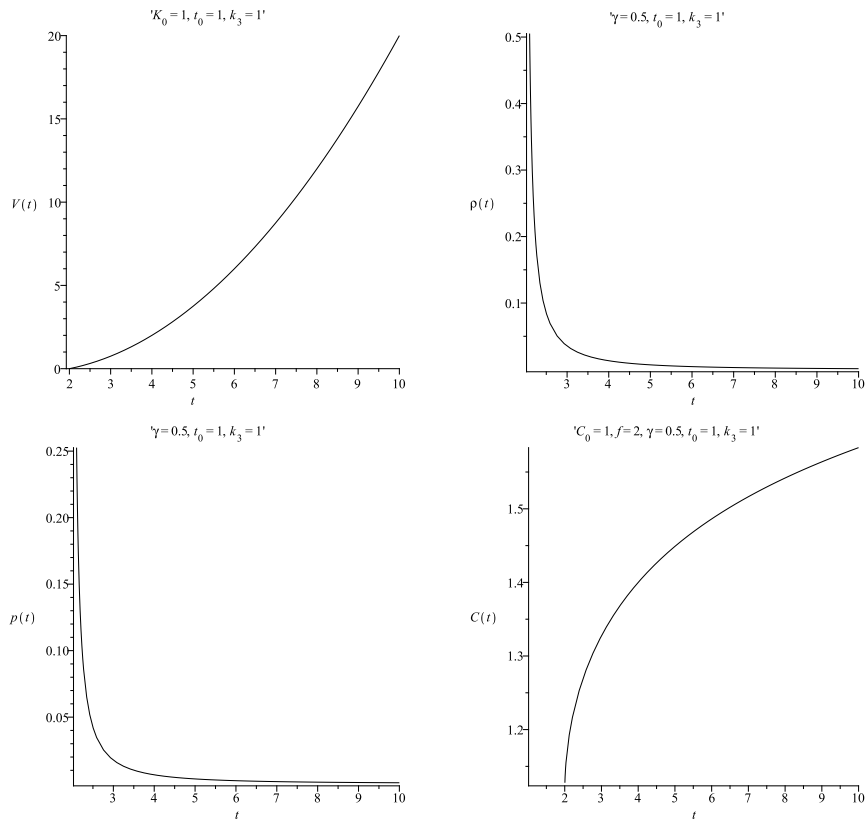


**Fig. 3** Variation of volume  $V$  and scale factors  $A$  and  $B$ : upper left panel for sub-case 4.2.1 when  $\rho_0 \neq 0$  and  $a_2 = 0$ , upper right panel for sub-case 4.2.2(ii) when  $\rho_0 \neq 0, a_2 \neq 0, \gamma = 0, p(t) = 0$  and  $\alpha = 1$ , and lower panel for sub-case 4.2.3 when  $\rho_0 \neq 0, a_2 \neq 0, \gamma = 1$ .

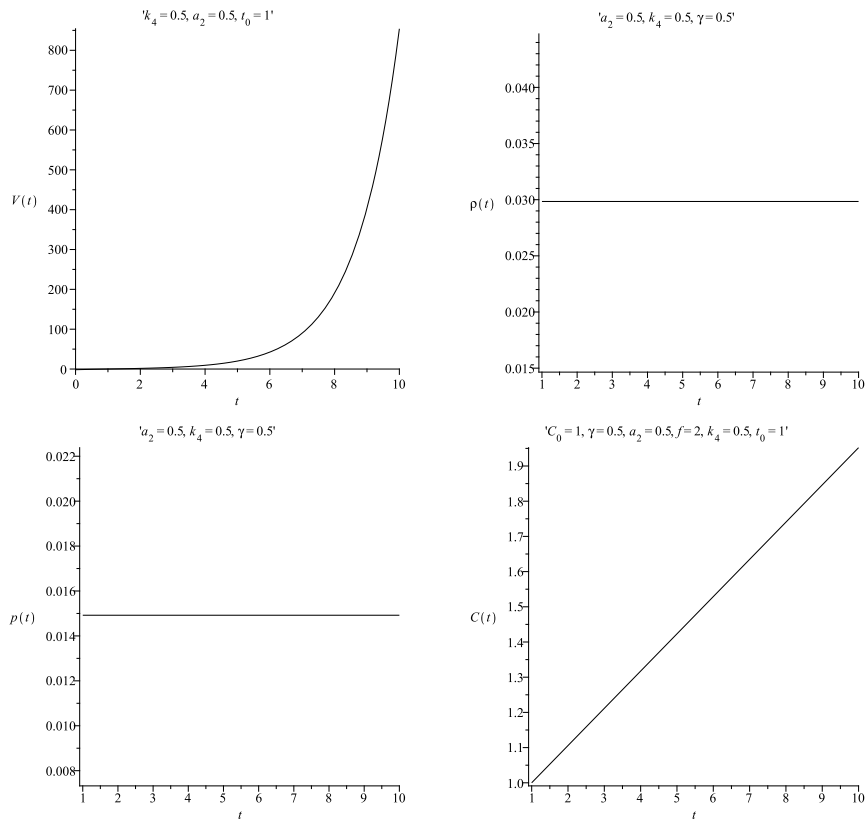


**Fig. 4** Variation of pressure (upper left panel) and creation field (upper right panel) for sub-case 4.1.1 when  $\rho_0 = 0$  and  $a_2 = 0$  whereas variation of volume (lower left panel) and creation field (lower right panel) for sub-case 4.1.2 (i) when  $\rho_0 = 0$  and  $a_2 \neq 0, \alpha = 0$ .

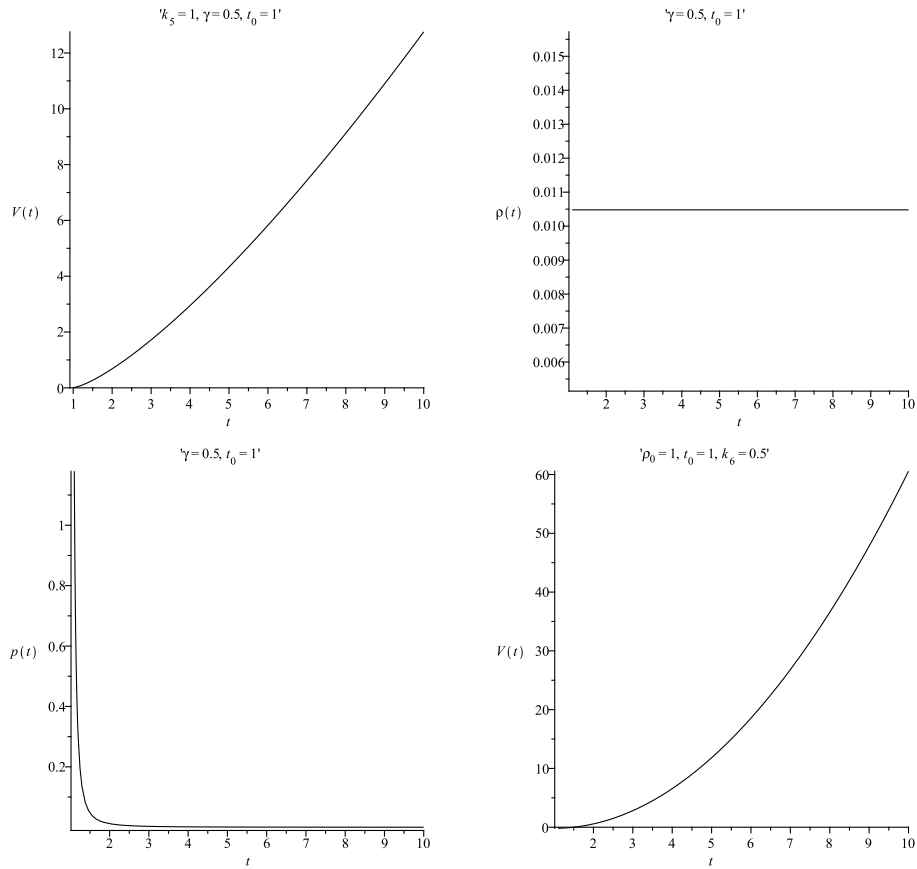




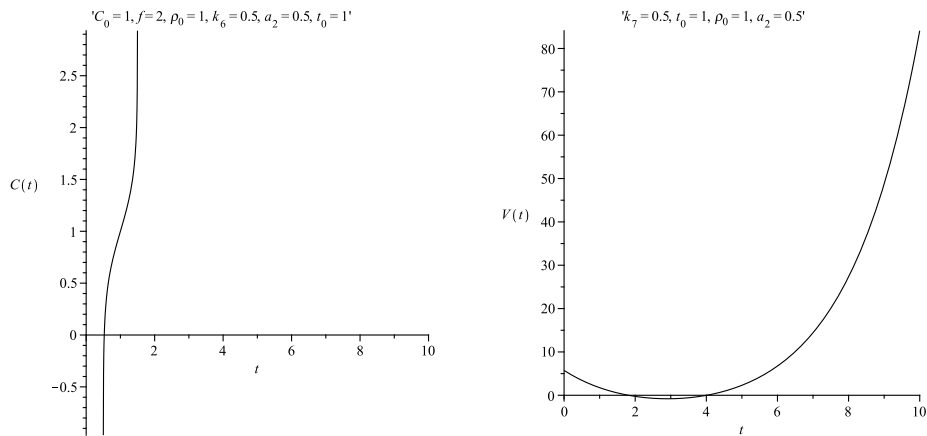
**Fig. 5** Variation of volume, density, pressure and creation field for sub-case 4.1.2 (ii) when  $\rho_0 = 0$ ,  $a_2 \neq 0$ ,  $\alpha = 1/\sqrt{2}$ .



**Fig. 6** Variation of volume, density, pressure and creation field for sub-case 4.1.2 (iii) when  $\rho_0 = 0$ ,  $a_2 \neq 0$ ,  $\alpha = 1$ .



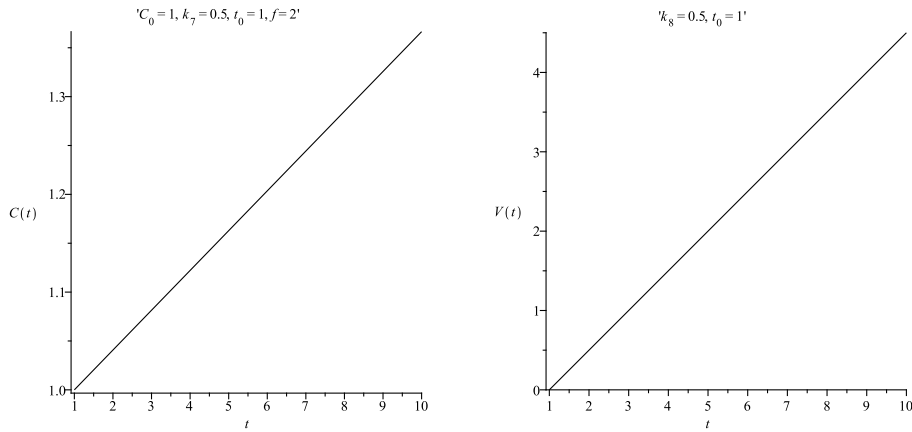
**Fig. 7** Variation of volume (*upper left panel*), density (*upper right panel*) and pressure (*lower left panel*) for sub-case 4.2.1 when  $\rho_0 \neq 0$ ,  $a_2 = 0$ ,  $\Omega_0 = 0$ , and variation of volume (*lower right panel*) for sub-case 4.2.2. (i) when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 \neq 0$ ,  $\gamma = 0$ ,  $p(t) = 0$  and  $\alpha = 0$ .



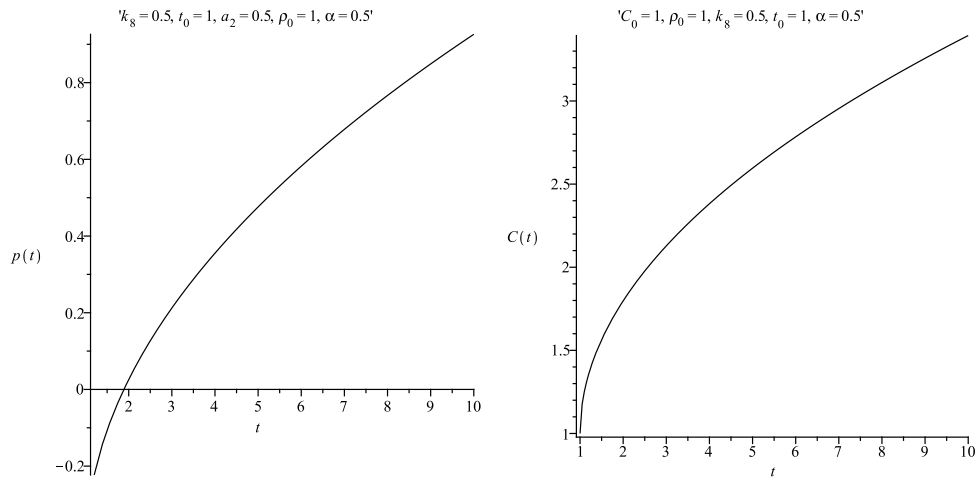
**Fig. 8** Variation of creation field (*left panel*) for sub-case 4.2.2 (i) when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 \neq 0$ ,  $\gamma = 0$ ,  $p(t) = 0$  and  $\alpha = 0$  whereas variation of volume (*right panel*) for sub-case 4.2.2 (ii) when  $\rho_0 \neq 0$ ,  $a_2 \neq 0$ ,  $\Omega_0 \neq 0$ ,  $\gamma = 0$ ,  $p(t) = 0$  and  $\alpha = 1$ .

However, as one possible improvement to the present investigation we would like to perform a comparative study between the singular and non-singular solutions of the two models. In this regard we draw a few specific plots to show variation of  $C$ ,  $\theta$ ,  $\Delta$ ,  $\sigma^2$  and  $q$  for singular and non-singular cases in Figure 13. Here, we are basically doing a comparison of the singular case 4.1.2 (ii) with non-singular

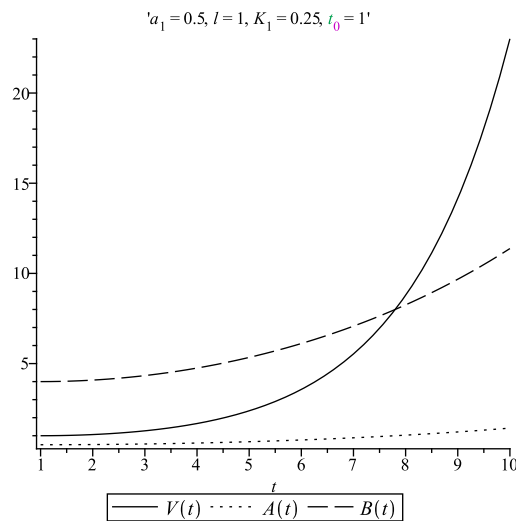
case 5. One can observe that in the model of the singular case the deceleration parameter  $q$  gets a positive value whereas a non-singular model gives an accelerating universe. Comparing  $C$  in both the cases we note that initially the creation field in the singular case assumes a higher value than the non-singular one; however, after a certain time has lapsed, the creation field in the non-singular case



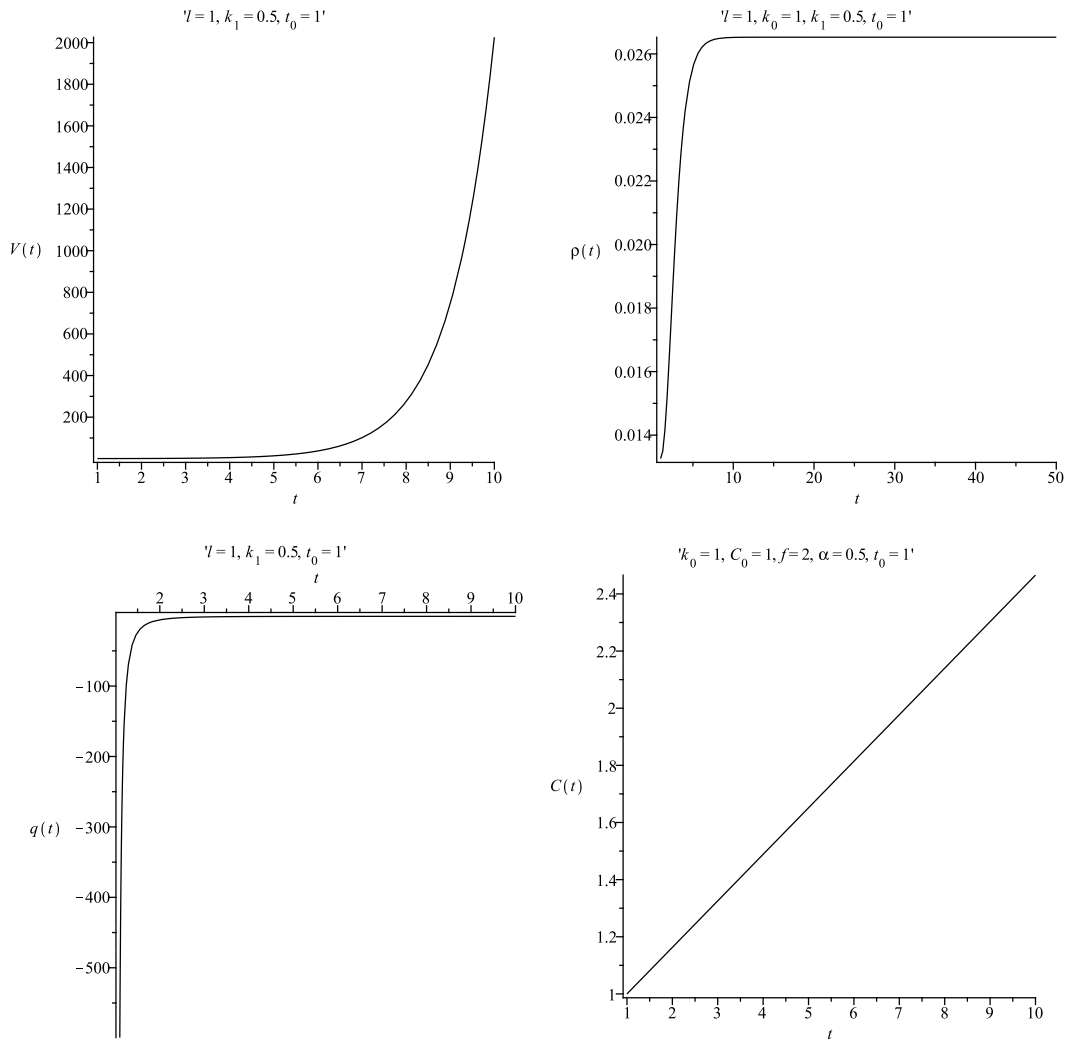
**Fig. 9** Variation of creation field (left panel) for sub-case 4.2.2 (ii) when  $\rho_0 \neq 0, a_2 \neq 0, \Omega_0 \neq 0, \gamma = 0, p(t) = 0$  and  $\alpha = 1$  whereas variation of volume (right panel) for sub-case 4.2.3 when  $\rho_0 \neq 0, a_2 \neq 0, \Omega_0 > 0, 3\rho_0 + 9a_2^2 = k_8^2$  and  $\gamma = 1$ .



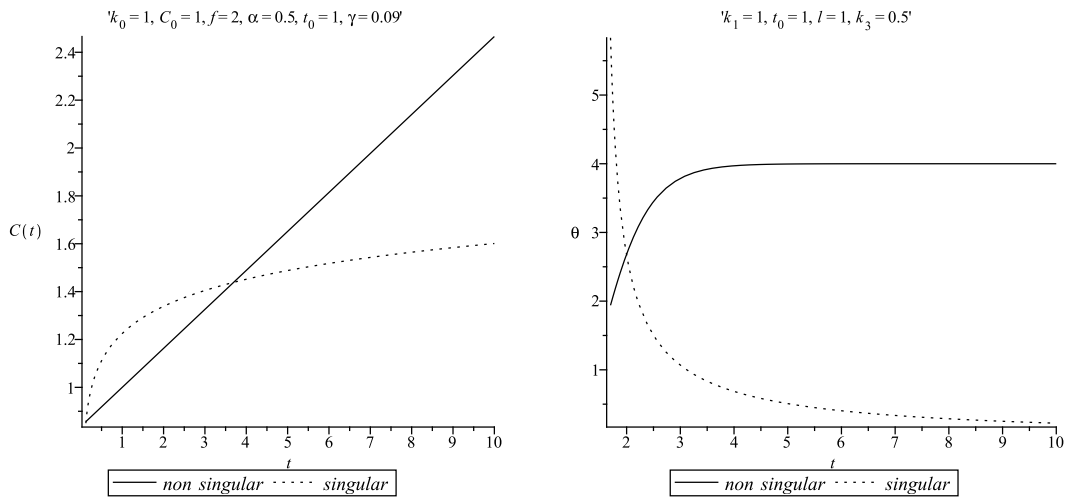
**Fig. 10** Variation of pressure and creation field for sub-case 4.2.3 when  $\rho_0 \neq 0, a_2 \neq 0, \Omega_0 > 0, 3\rho_0 + 9a_2^2 = k_8^2$  and  $\gamma = 1$ .



**Fig. 11** Variation of volume  $V$  and scale factors  $A$  and  $B$  for non-singular case 5 when  $\gamma = 0, a_2 = 0, \alpha^2 = 1$  and  $3\rho_0 = -k_0 l$ .



**Fig. 12** Variation of volume, density, deceleration parameter and creation field for non-singular case 5 when  $\gamma = 0, a_2 = 0, \alpha^2 = 1$  and  $3\rho_0 = -k_0l$ .



**Fig. 13** Variation of  $C, \theta, \Delta, \sigma^2$  and  $q$  for singular and non-singular cases as a comparative study.

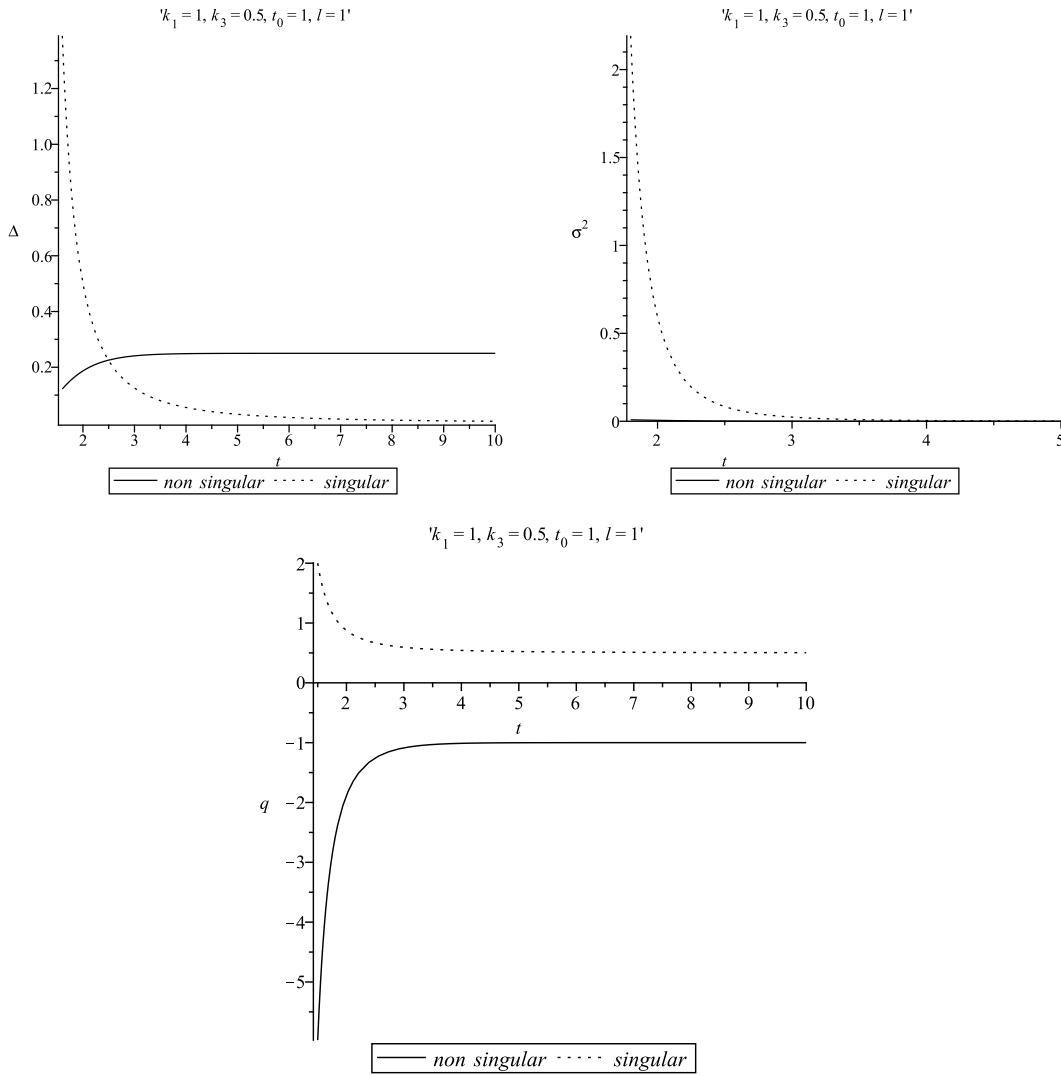


Fig. 13 — Continued.

acquires a higher value than the singular one. In a similar way one can continue comparison for other parameters, which is also quite evident from the contrasting behaviour of other parameters in Figure 13.

As a final comment, we note from the above comparative study that the present model in a unique way represents both the features of the accelerating as well as decelerating universe depending on the parameters and thus seems to provide glimpses of the oscillating or cyclic model of the universe (see Frampton 2006 and refs. therein). However, it can be noted that our model is based on Hoyle-Narlikar type  $C$ -field cosmological theory and does not invoke any other agent or theory, e.g. dark energy (Khoury et al. 2001; Steinhardt & Turok 2002a,b; Boyle et al. 2004; Steinhardt & Turok 2006), branes (Randall & Sundrum 1999b,a; Csáki et al. 2000; Binetruy & Langlois 2000; Brown et al. 2008), modified gravity (Frampton & Takahashi 2003, 2004), etc. in allowing cyclicity.

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