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Beta decay of nuclides ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni in the crust of magnetars

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Abstract By introducing the Dirac δ-function and Pauli exclusion principle in the presence of superstrong magnetic fields (SMFs), we investigate the influence of SMFs on beta decay and the change rates of electron fraction (CREF) of nuclides ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni in magnetars, which are powered by magnetic field energy. We find that the magnetic fields have a great influence on the beta decay rates, and the beta decay rates can decrease by more than six orders of magnitude in the presence of SMFs. The CREF also decreases by more than seven orders of magnitude in the presence of SMFs.

Key words: physical data and processes: nuclear reactions, nucleosynthesis, abundances — stars: neutron — stars: magnetic fields

1 INTRODUCTION

Beta decay plays a very important role in presupernova evolution, as well as in neutron star (NS) evolution. A strong beta decay rate can contribute to the cooling rate and a much larger value of lepton-to-baryon ratio due to energy loss from antineutrinos. Some authors (e.g., Fuller et al. 1982; Aufderheide et al. 1990, 1994; Langanke & Martinez-Pinedo 1998) have done pioneering works on thermonuclear reactions, such as beta decay and electron capture, and Liu (2012, 2013a,b,c,d,e,f, 2015, 2016) performed detailed studies of the weak interaction and related issues. However, they have lost sight of the influence of superstrong magnetic fields (SMFs) in the relativistic properties of magnetars, which are powered by magnetic field energy. Radio pulsars have typical magnetic fields of $B \sim 10^{12}$ G. Moreover, a high-B pulsar may possess an intense magnetic field of $B \sim 10^{13}$ G (Lai 2001). For some magnetars, their surface dipole magnetic fields can be as high as $10^{14} \sim 10^{15}$ G (Yakovlev et al. 2001; Peng & Tong 2007; Chamel & Haensel 2008; Gao et al. 2012, 2013a,b, 2015).

Recently, the properties and observations of magnetars have been extensively studied. For instance, Olausen & Kaspi (2014) presented a catalog of the 28 currently known magnetars and magnetar candidates and discussed their properties. They also investigated their observed emission properties, particularly the spectral parameters of quiescent X-ray emission. Using the partially screened gap model, Szary et al. (2015) gave an explanation for magnetar radio emission. Based on the estimated ages of their potentially associated supernova remnants (SNRs), Gao et al. (2016) estimated the values of the mean braking indices of eight magnetars with SNRs. Their method provides an effective way to constrain the braking indices of magnetars.

As is well known, NSs may have higher internal magnetic fields. The intensity of the crustal magnetic field of a magnetar can be as high as $B \sim 10^{16}$ G. In such an SMF, the properties of the outer crust of a magnetar will be drastically modified. In particular, the Landau levels of electrons are strongly quantized. By modifying the phase space of relativistic electrons, an SMF can enhance the electron number density $n_{\rm e}$ and decrease the maximum number of Landau levels for electrons, which results in their redistribution. According to the Pauli exclusion principle, degenerate electrons will fill quantum states from the lowest Landau level to the highest Landau level. The enhanced $n_{\rm e}$ in an SMF means an increase in the electron Fermi energy E_F and an increase in the electron degeneracy pressure. In the outer crust of a magnetar, the electron Fermi energy may exceed 30 MeV (Gao et al. 2013a; Li et al. 2016; Zhu et al. 2016). As an extremely important and indispensable physical parameter in the equation of state (EoS) of an NS, the Fermi energy of electrons directly exerts impact on the weak interaction processes, including modified Urca reactions, beta-decay, electron capture, as well as the absorption of neutrinos and antineutrinos. They will in turn influence the intrinsic EoS, internal structure, thermal evolution and even the overall properties of the star. Therefore, it is of great significance to study E_F and the magnetic effects on the weak interaction in an NS.

The weak interaction rates of nuclides ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni are very important and dominant factors during the process of supernova explosions. In pioneering works, the weak interaction rates and some nuclear structure and properties of these nuclides were investigated in detail by some authors (e.g., Fuller et al. 1982; Aufderheide et al. 1990, 1994; Langanke et al. 2003; Domingo-Pardo et al. 2009). Based on works by Peng & Tong (2007) and Gao et al. (2012, 2013a, 2015) in SMFs, we will discuss the beta decay for nuclides ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni due to their importance in the region surrounding a magnetar crust.

The remainder of this paper is organized as follows. In the next section, we address the influence of an SMF on beta decay. In Section 3 we present our results and some discussions. In Section 4 the concluding remarks are given.

2 BETA DECAY IN THE CASE WITHOUT AND WITH SMFS

2.1 Beta Decay in the Case without SMFs

Beta decay rates for the k-th nucleus (Z, A) in thermal equilibrium at temperature T in the case without SMFs is given by a sum over the initial parent states i and the final daughter states f (Fuller et al. 1982; Aufderheide et al. 1990, 1994).

$$\lambda_{bd}^0 = \ln 2 \sum \frac{(2J_i+1)e^{\frac{-E_i}{k_BT}}}{G(Z,A,T)} \sum_f \frac{\xi(\rho, T, Y_{\rm e}, Q_{ij})}{ft_{ij}}, \quad (1)$$

where J_i and E_i are the spin and excitation energies of the parent states and k_B is the Boltzmann factor. ft_{ij} is the comparative half-life connecting states of i and j, and Q_{ij} is the nuclear energy difference between states i and j. $Q_{00} = M_{\rm p}c^2 - M_{\rm d}c^2$, and $M_{\rm p}$ and $M_{\rm d}$ are the masses of the parent nucleus and the daughter nucleus, respectively. E_i and E_j are the excitation energies of the i-th and j-th nuclear state respectively. $G(Z, A, T)$ is the nuclear partition function, which is given by

$$G(Z,A,T) = \sum_i (2J_i+1)\exp\Big(-\frac{E_i}{k_BT}\Big). \quad (2)$$

According to the level density formula (Aufderheide et al. 1994), the nuclear partition function approximately becomes

$$G(Z,A,T) \approx (2J_0+1) + \int_0^\infty dE \int_{J,\pi} dJd\pi(2J_i+1) \times\vartheta(E,J,\pi)\exp\Big(-\frac{E_i}{k_BT}\Big), \quad (3)$$

where the contribution from the excited states are taken into consideration. The level density $\vartheta(E, J, \pi)$ was discussed in detail by Holmes et al. (1976).

The beta decay phase space integral $\xi(\rho, T, Y_{\rm e}, Q_{ij})$ is written as

$$\xi(\rho,T,Y_{\rm e},Q_{ij}) = \frac{c^3}{m_{\rm e}c^2}\int_0^{\sqrt{Q_{ij}^2-m_{\rm e}^2c^4}} dpp^2(Q_{ij}-\varepsilon_{\rm n})^2 \frac{F(Z+1,\varepsilon_{\rm n})}{1+\exp[(U_F-\varepsilon_{\rm n})/k_BT]}, \quad (4)$$

where $p, m_{\rm e}, U_F$ and $\varepsilon_{\rm e} = \varepsilon_{\rm n}$ are the electron momentum, mass, chemical potential and energy, respectively. $F(Z+1, \varepsilon_{\rm n})$ is the Coulomb wave correction, which is the ratio of the square of the electron wave function distorted by the Coulomb scattering potential to the wave function of free electrons.

The electron chemical potential depends on the matter density ρ, the electron fraction $Y_{\rm e}$ and the temperature T of the medium. In the case without SMFs in the precollapse phase of a supernova, there is a reasonable approximation of (Bludman & van Riper 1978)

$$U_F^0 = 1.11(\rho_7 Y_{\rm e})^{1/3}\Big[1+\Big(\frac{\pi}{1.11}\Big)\frac{(k_BT)^2}{(\rho_7Y_{\rm e})^{2/3}}\Big]^{-1/3} \text{MeV}. \quad (5)$$

According to Aufderheide et al. (1994), the sum over the total beta decay rate of a parent state can be broken up into two parts, one for the low energy region near the ground state and the other for the resonance region dominated by a Gamow-Teller (GT) resonance transition. Thus, it becomes

$$\lambda_{bd}^0 = \lambda_0^0 + \lambda_{\rm GT}^0, \quad (6)$$

$$\lambda_0^0 = \ln 2\frac{(2J_i+1)}{G(Z,A,T)}\exp(-E_{\rm peak}/k_BT)\frac{\xi(\rho,T,Y_{\rm e},E_{\rm peak}+Q_{00})}{ft_{\rm eff}}, \quad (7)$$

$$\lambda_{\rm GT}^0 = \ln 2\exp(-E_{BGTR(0)}/k_BT)\frac{G(Z+1,A,T)}{G(Z,A,T)}\frac{\xi(\rho,T,Y_{\rm e},E_{BGTR(0)}+Q_{00})}{ft_{0\to BGTR(0)}}, \quad (8)$$

$$ft_{\rm eff} = 6.060 \times 10^4, \qquad ft_{0\rightarrow BGTR(0)} = \frac{10^{3.596}}{|M_{BGTR(0)}|^2}, \tag{9}$$

where J_0 is the initial spin of the parent state and $E_{BGTR(0)}$ is the energy difference between the orbit that the new neutron occupies in the GT resonance and the ground state. The total GT matrix element $|M_{BGTR(0)}|^2$ between the initial parent state $|\psi_i^{\rm P}\rangle$ and the final daughter state $|\psi_f^{\rm D}\rangle$ is given by (Aufderheide et al. 1994)

$$|M_{BGTR(0)}|^2 = |\langle\psi_f^{\rm D} \mid \sum_{\rm n}\sigma_{\rm n}(\tau_{\pm1})|\psi_i^{\rm P}\rangle|^2 = |\langle j_{\rm p}||\sigma_{\rm n}(\tau_{\pm1})||j_{\rm n}\rangle|^2\frac{N_{\rm n}}{2j_{\rm n}+1}\Big(1-\frac{N_{\rm p}}{2j_{\rm p}+1}\Big) \tag{10}$$

where $N_{\rm n}$ and $N_{\rm p}$ are the numbers of neutrons and protons within the $j_{\rm n}$ and $j_{\rm p}$ shell, respectively.

In the case of temperature $\Lambda = k_B T$ (in units of MeV) ranging from roughly 0.3 MeV to 0.8 MeV and electron chemical potentials ranging from 0.5 MeV up to at least 8 MeV, $E_{\rm peak}$ is approximately given by (Aufderheide et al. 1994)

$$E_{\rm peak} = 5\Lambda - Q_{00} + U_F + \delta(\Lambda, U_F), \tag{11}$$

where

$$\delta(\Lambda, U_F) = -0.6604 + 0.9429\Lambda - 0.02119U_F - 0.9432\Lambda U_F - 0.0009524U_F^2 + 0.06224\Lambda U_F^2. \tag{12}$$

If $E_{\rm peak} < 0$, we will consider transitions from the ground state to the low excited state.

2.2 The Beta Decay in the Case with SMFs

For electron Fermi energy in SMFs, here we consider an SMF along the z-axis. By solving the relativistic electron Dirac equation (e.g., Landau & Lifshit's 1991), the positive energy levels of electrons in an SMF are given as (e.g., Peng & Tong 2007; Gao et al. 2012, 2013a)

$$\frac{\varepsilon_{\rm n}}{m_{\rm e}c^2} = \Big[\Big(\frac{p_{\rm z}}{m_{\rm e}c}\Big)^2 + 1 + 2\Big(n+\frac{1}{2}+\sigma\Big)b\Big]^{1/2} = \Big[\Big(\frac{p_{\rm z}}{m_{\rm e}c}\Big)^2 + \Theta\Big]^{1/2}, \tag{13}$$

where $\Theta = 1 + 2(n+\frac{1}{2}+\sigma)b$, $n = 0, 1, 2, 3...$, $b = \frac{B}{B_{\rm cr}} = 0.02266B_{12}$, B_{12} is the magnetic field in units of 10^{12} G, $B_{\rm cr} = \frac{m_{\rm e}^2c^3}{e\hbar} = 4.414 \times 10^{13}$ G is the electron critical magnetic field, p_z is the electron momentum along the field and σ is the spin quantum number of an electron; when $n = 0$, $\sigma = 1/2$, and when $n \geq 1$, $\sigma = \pm 1/2$.

As is well known, in a weak magnetic field approximation $B/B_{cr} \ll 1$, for the electron gas in the nondegenerate limit ($T \longrightarrow 0$), the maximum Landau level number $n_{\rm max} \longrightarrow \infty$. However, the maximum Landau level number $n_{\rm max}$ is set by the condition $p_z^F(e) \geq 0$ or $E_F^2(e) \geq m_{\rm e}^2c^4(1+2\nu B/B_{cr})$ for a highly degenerate electron gas in an SMF (e.g., see Lai & Shapiro 1991). According to Gao et al. (2012, 2013a), the maximum Landau level number $n_{\rm max}$ should be re-estimated. According to their discussions, the degeneracy of the n-th Landau level of electrons in a relativistic magnetic field is given by

$$\begin{aligned}\omega_n &= \frac{1}{h^2}g_{n0}\int_0^{2\pi}d\theta\int\delta\Big(\frac{p_\perp}{m_{\rm e}c} - \Big[2\Big(n+\sigma+\frac{1}{2}\Big)b\Big]^{\frac{1}{2}}\Big)p_\perp dp_\perp \\ &= \frac{2\pi}{h^2}g_{n0}\int\delta\Big(\frac{p_\perp}{m_{\rm e}c} - \Big[2\Big(n+\sigma+\frac{1}{2}\Big)b\Big]^{\frac{1}{2}}\Big)p_\perp dp_\perp,\end{aligned} \tag{14}$$

where $p_\perp$ is the electron momentum perpendicular to the magnetic field, $\theta = \arctan p_y/p_x$ and $g_{n0} = 2 - \delta_{n0}$ is the electron spin degeneracy (when $n = 0$, $g_{00} = 1$ and when $n \geq 1$, $g_{n0} = 2$) (Gao et al. 2012, 2013a).

The relationship between the Pauli exclusion principle and electron Fermi energy in SMFs has been discussed in detail by Zhu et al. (2016). According to the Pauli exclusion principle, the electron number density should be equal to its microscopic state density. Thus we have (Peng & Tong 2007, Gao et al. 2013a, 2015, Zhu et al. 2016)

$$\begin{aligned}&N_{\rm phase} = n_{\rm e} = 2\pi\frac{(m_{\rm e}c)^3}{h^3}\int_0^{\frac{E_{\rm F}(e)}{m_{\rm e}c^2}}d\Big(\frac{p_z}{m_{\rm e}c}\Big)\sum_{n=0}^{n_{\rm max}(p_z,\sigma,b)}\sum g_{n0} \\ &\int_0^{\frac{E_{\rm F}(e)}{m_{\rm e}c^2}}\delta\left(\frac{p_\perp}{m_{\rm e}c} - \Big[2\Big(n+\sigma+\frac{1}{2}\Big)b\Big]^{\frac{1}{2}}\right)\frac{p_\perp}{m_{\rm e}c}d\Big(\frac{p_\perp}{m_{\rm e}c}\Big) = N_A\rho Y_{\rm e}\,.\end{aligned} \tag{15}$$

Based on the above discussions and Equations (1)–(3), the Fermi energy of electrons is given by Zhu et al. (2016)

$$E_{\rm F} = U_{\rm F}^B = 59.1\Big(\frac{B}{B_{\rm cr}}\Big)^{1/6}\Big(\frac{\rho Y_{\rm e}}{\rho_0 \times 0.0535}\Big)^{1/3} = 59.1\Big(\frac{B}{B_{\rm cr}}\Big)^{1/6}\Big(\frac{n_{\rm e}}{0.005647 \times \rho_0 N_{\rm A}}\Big)^{1/3}\ {\rm MeV}. \tag{16}$$

2.2.1 The beta becay in SMFs

As discussed above, the total beta decay rate over the sum in SMFs can be broken up into two parts, one for the low energy region near the ground state and the other for the resonance region dominated by the GT resonance transition. Thus it becomes

$$\lambda_{bd}^B = \lambda_0^B + \lambda_{\rm GT}^B, \tag{17}$$

$$\lambda_0^B = \ln 2 \frac{(2J_i+1)}{G(Z,A,T)} \exp(-E_{\rm peak}/k_BT) \frac{\xi^B(\rho, T, Y_{\rm e}, E_{\rm peak}+Q_{00})}{ft_{\rm eff}}, \tag{18}$$

$$\lambda_{\rm GT}^B = \ln 2 \exp(-E_{BGTR(0)}/k_BT) \frac{G(Z+1,A,T)}{G(Z,A,T)} \frac{\xi^B(\rho, T, Y_{\rm e}, E_{BGTR(0)}+Q_{00})}{ft_{0\rightarrow BGTR(0)}}. \tag{19}$$

In SMFs, the beta decay phase space integral $\xi^B(\rho, T, Y_{\rm e}, Q_{ij})$ is written as

$$\xi^B(\rho, T, Y_{\rm e}, Q_{ij}) = \frac{b}{2} \sum_0^\infty \vartheta_{\rm n}, \tag{20}$$

$$\vartheta_{\rm n} = \frac{c^3}{(m_{\rm e}c^2)^5} g_{n0} \int_0^{\sqrt{Q_{ij}^2 - Q_{\rm n}^2}} dp p^2 (Q_{ij} - \varepsilon_{\rm n})^2 \frac{F(Z+1, \varepsilon_{\rm n})}{1+\exp[(U_F^B - \varepsilon_{\rm n})/k_BT]}, \tag{21}$$

where $g_{n0} = 2 - \delta_{no}$ is the electron spin degeneracy and $Q_{\rm n} = (m_{\rm e}^2c^4 - \Theta)^{1/2}$. We assume that an SMF will have no effect on $F(Z, \varepsilon_{\rm n})$, which is only valid under the condition that the electron wave-functions are locally approximated by plane wave functions (Dai et al. 1993). The condition requires that the Fermi wavelength $\lambda_{\rm F} \sim \frac{\hbar}{p_{\rm F}}$ ($p_{\rm F}$ is the Fermi momentum without a magnetic field) is smaller than the radius of the cylinder $\sqrt{2}\psi$ (where $\psi = \frac{\lambda_{\rm e}}{\Theta}$) which corresponds to the lowest Landau level (Baym & Pethick 1975).

Due to energy conservation, the electron, proton and neutron energies are related to the neutrino energy and Q-value for the capture reaction (Cooperstein & Wambach 1984)

$$Q_{i,f} = \varepsilon_{\rm e} - \varepsilon_\nu = \varepsilon_n - \varepsilon_\nu = \varepsilon_f^n - \varepsilon_i^p, \tag{22}$$

and we have

$$\varepsilon_f^n - \varepsilon_i^p = \varepsilon_{if}^* + \hat{\mu} + \Delta_{np}, \tag{23}$$

where $\hat{\mu} = \mu_n - \mu_p$ is the difference between the chemical potentials of neutrons and protons in the nucleus and $\Delta_{np} = M_nc^2 - M_pc^2 = 1.293\,{\rm MeV}$ is the mass difference between a neutron and a proton. $Q_{00} = M_fc^2 - M_ic^2 = \hat{\mu} + \Delta_{np}$, with M_i and M_f being the masses of the parent nucleus and the daughter nucleus, respectively; ε_{if}^* corresponds to the excitation energies in the daughter nucleus at the states with zero temperature.

In order to compare the results (λ_{bd}^B) in SMFs with those of the rates (λ_{ec}^0) in the case without SMFs, we define an enhancement factor C, which is given by

$$C = \frac{\lambda_{bd}^B}{\lambda_{bd}^0}. \tag{24}$$

On the other hand, the change rate of electron fraction (CREF) is also an important parameter, which is determined by each nucleus in the weak interaction reaction in the process of stellar evolution. The CREF due to beta decay of the k-th nucleus in SMFs is given by

$$\dot{Y}_{\rm e}(k) = \frac{dY_{\rm e}}{dt}(k) = \frac{X_k}{A_k}\lambda_{bd}^B \tag{25}$$

where X_k and A_k are the mass fraction and the mass number of the k-th nucleus, respectively.

The k-th nucleus has charge and mass number Z and A, respectively, and $N = A - Z$. The mass fraction of the k-th nucleus is given by X_k. The distribution of the nuclei must conserve mass of $\sum_k X_k = 1$ and the charge must satisfy $\sum_k Z_kX_k/A_k = Y_{\rm e} = (1-\eta)/2$, where η is the neutron excess. The abundance for nuclei, which is related to the neutrons and protons, is expressed by

$$\begin{aligned} X_k \; = \; & \frac{G(Z,A,T)}{2}\Big(\frac{\rho N_{\rm A}\lambda^3}{2}\Big)^{A-1} \\ & \times A^{5/2} X_{\rm n}^N X_{\rm p}^Z \exp(Q_k/k_{\rm B}T), \end{aligned} \tag{26}$$

where $\lambda = (h^2/2\pi M_{\rm H}k_{\rm B}T)^{1/2}$ is the thermal wavelength, and $Q_k = (Zm_{\rm H} + Nm_{\rm n} - M_k)^2$ is binding energy, where all masses are atomic. $X_{\rm n}$ and $X_{\rm p}$ are the abundance for the neutrons and protons, respectively.

3 NUMERICAL RESULTS AND DISCUSSIONS

Here we will focus on the outer crust of magnetars and discuss the beta decay process. The outer crust extends from the bottom of the atmosphere (the density can be

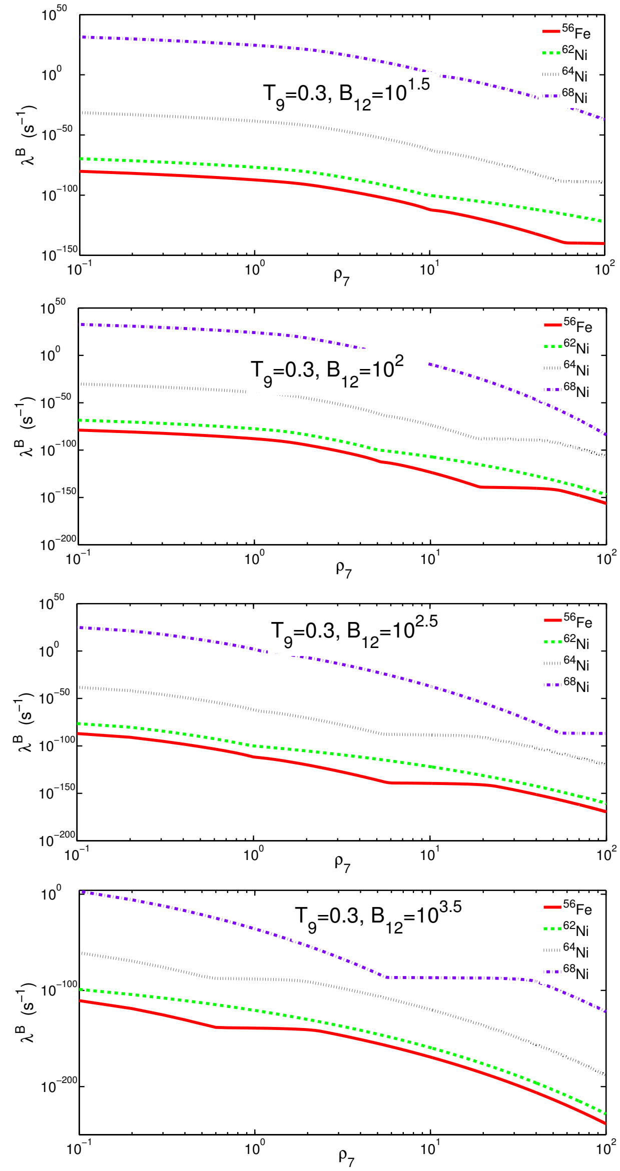

Fig. 1 The beta decay rates of ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni as a function of electron density ρ_7 when $B_{12} = 10^{1.5}, 10^2, 10^{2.5}$ and $10^{3.5}$ at $T_9 = 0.3$.

$10^4\,\mathrm{g\,cm^{-3}}$) to the layer with a density of $4 \times 10^{11}\,\mathrm{g\,cm^{-3}}$ and the depth of the outer crust can be a few hundred meters (Shapiro & Teukolsky 1983, Yakovlev et al. 2001). From observations, the surface temperatures of magnetars cluster in the range of $10^6 \sim 10^7$ K. Their internal temperature $T_{\rm in}$ (including the crust temperature) will be higher and the maximum of $T_{\rm in}$ can be estimated as $(1 \sim 9) \times 10^8$ K (e.g., Mereghetti 2008; Olausen & Kaspi 2014; Li et al. 2016). It has been shown that in the magnetar outer crust, the matter density in which nuclides ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni exist is in the range of about $8 \times 10^6 - 1.5 \times 10^9\,\mathrm{g\,cm^{-3}}$ (e.g., Yakovlev et al. 2001), corresponding to magnetic field $B_{12} = 5 \sim 10^4$. Thus, for convenience, we select several typical parameter sets as follows: The density range is $0.1 < \rho_7 < 10^3$ (e.g., $\rho_7 = 4.17, 8.17, 14.17$ and 34.17), and two typical temperature points are $T_9 = 0.3$ and 0.7. We also select the parameters of the magnetic field strength, whose range is $5 < B_{12} < 10^4$ (e.g., $B_{12} = 10^{1.5}, 10^2, 10^{2.5}$ and $10^{3.5}$); here ρ_7 is the density in units of $10^7\,\mathrm{g\,cm^{-3}}$, and T_9 is the temperature in units of 10^9 K.

Figures 1 and 2 show that the beta decay rates of ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni are functions of ρ_7 in different astronomical conditions. One can find that the beta decay rates decrease greatly as their matter density increases, and the maximum beta decay rate can exceed five orders of magnitude. We also find that a stronger magnetic field has a larger influence on the decay rates. For example, when $T_9 = 0.3$ and $\rho_7 = 19.7$, as B_{12} increases from $10^{1.5}$ to 10^2, the beta decay rates for ^{68}Ni decrease from $1.225 \times 10^{-7}\ \mathrm{s^{-1}}$ to $5.307 \times 10^{-26}\ \mathrm{s^{-1}}$. On the other hand, the lower the temperature is, the larger the influence of SMFs on the beta

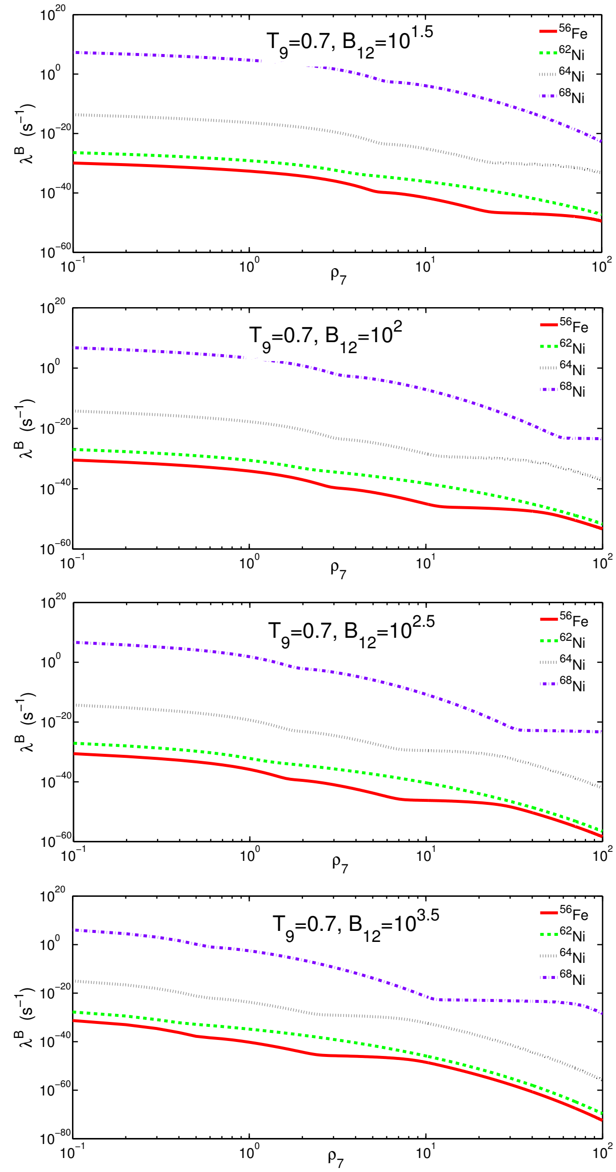

Fig. 2 The beta decay rates of ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni as a function of electron density ρ_7 when $B_{12} = 10^{1.5}, 10^2, 10^{2.5}$ and $10^{3.5}$ at $T_9 = 0.7$.

decay becomes. This happens because the electron energy and electron chemical potential are so low at relatively low temperatures (e.g., $T_9 = 0.3$) that the SMFs can strongly affect the rates. For example, the beta decay rates of ^{68}Ni are 8.206×10^{-11} s^{-1} and 4.177×10^{-8} s^{-1}, corresponding to $T_9 = 0.3$ and 0.7, respectively, when $B_{12} = 10^2$.

The beta decay rates as functions of B_{12} are shown in Figures 3 and 4. One can see that an SMF has a great influence on the beta decay rates at different densities and temperatures. As an SMF increases, the rates decrease by more than six orders of magnitude. For a given density and SMF, the higher the temperature is, the larger the beta decay rate becomes. A significant reason for this may be that at a relativity higher temperature, the electron chemical potentials are so high that the rates would be increased. For example, when $B_{12} = 10^3$ and $\rho_7 = 4.17$, the rate of ^{68}Ni is about 3.499×10^{-15} s^{-1} for $T_9 = 0.3$, whereas the rate is 1.017×10^{-9} s^{-1} for $T_9 = 0.7$. On the other hand, for a given temperature and SMF, the higher the density is, the smaller the beta decay rate becomes. This is due to the fact that at a relativity higher density, the influence of an SMF on beta decay can be strongly weakened by density and beta decay rates can be suppressed. For example, when $B_{12} = 10^2$ and $T_9 = 0.7$, the rate of ^{68}Ni is about 3.885×10^{-3} s^{-1} for $\rho_7 = 4.17$, while the rate is 4.215×10^{-17} s^{-1} for $\rho_7 = 34.17$. We also find that the curve representing the rates shows some erratic fluctuation. This is caused by the positive threshold energy (Q_0) in the decay process and contributions from the partial decay rates due to different selected states in the parent nucleus.

On the other hand, according to previous work of Lai & Shapiro (1991), the beta decay rate decreases due to the

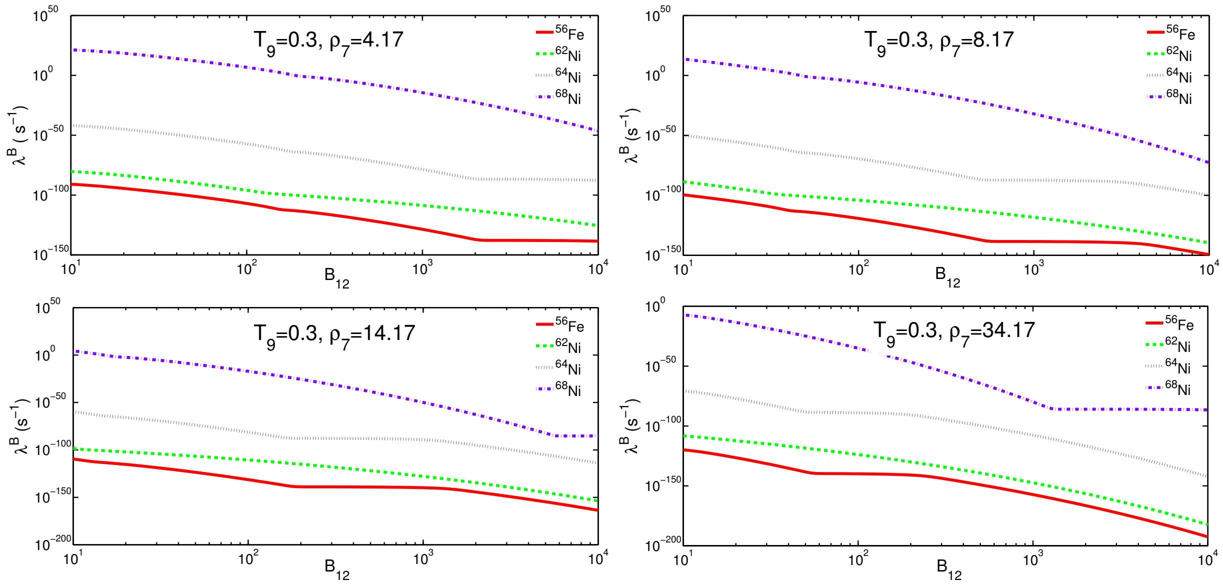

Fig. 3 The beta decay rates of ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni as a function of B_{12} when $\rho_7 = 4.17, 8.17, 14.17$ and 34.17 at $T_9 = 0.3$.

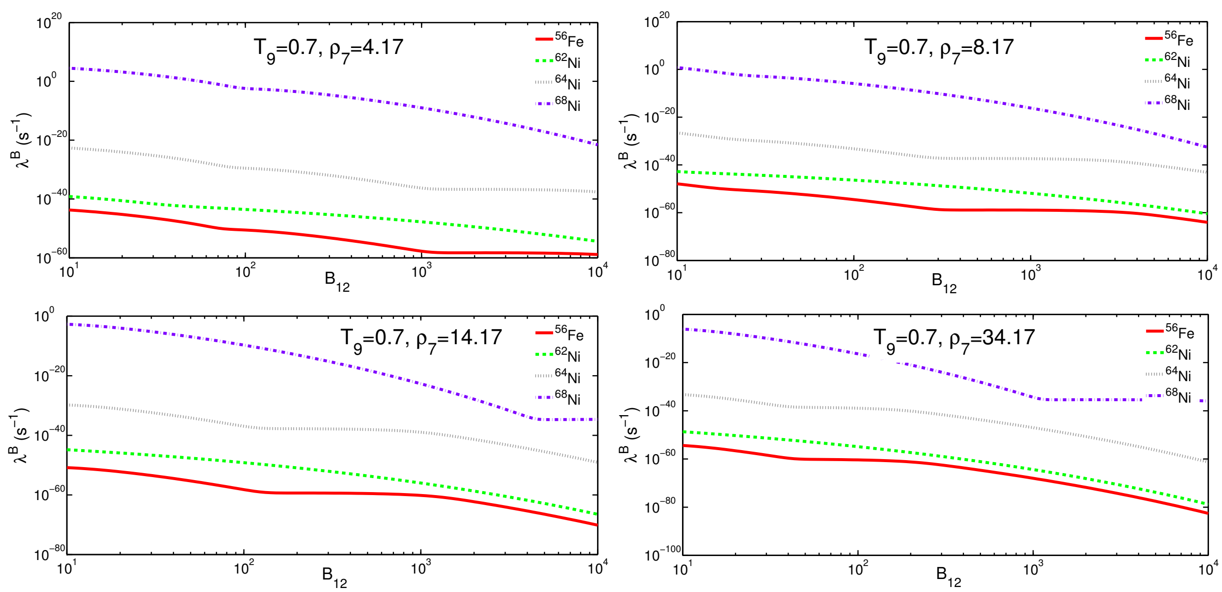

Fig. 4 The beta decay rates of ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni as a function of B_{12} when $\rho_7 = 4.17, 8.17, 14.17$ and 34.17 at $T_9 = 0.7$.

decrease in electron chemical potential (i.e. electron Fermi energy) with an increase in the magnetic field. However, in our paper, based on the model of Gao et al. (2013a, 2015) and Peng & Tong (2007), according to the Pauli exclusion principle, degenerate electrons will fill quantum states from the lowest Landau level to the highest Landau level. The enhanced n_e in an SMF means that there is an increase in the electron Fermi energy E_F and an increase in the electron degeneracy pressure. For a given temperature and density, the stronger the SMF is, the higher the electron chemical potential becomes. Thus, the beta decay rate will greatly increase.

The enhancement factor C of beta decay rates is a function of B_{12}, as shown in Figures 5 and 6. The results demonstrate that the enhancement factor C decreases by more than six orders of magnitude as an SMF increases. For example, when $\rho_7 = 8.17$ and $T_9 = 0.3$, the factor C of ^{62}Ni decreases from 2.892×10^{-3} s^{-1} to 2.094×10^{-9} s^{-1} as B_{12} increases from 10^2 to 10^3. This happens because the beta decay rates greatly decrease as the SMF increases.

CREF is one of the key parameters for describing the evolution of magnetars. CREF strongly influences the electron degenerate pressure because a tremendous amount of electrons will be emitted by beta decay. CREF for each of the nuclides also influences the change in the equation of state and composition in the evolution process of the magnetar due to the beta decay reaction.

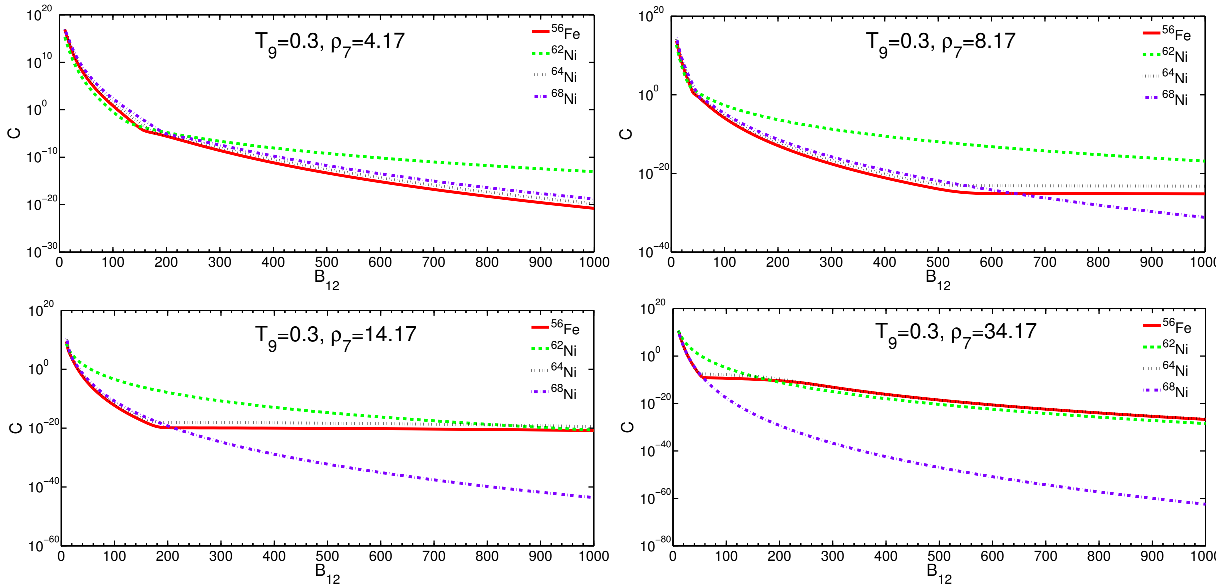

Fig. 5 The factor C for ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni as a function of B_{12} when $\rho_7 = 4.17, 8.17, 14.17$ and 34.17 at $T_9 = 0.3$.

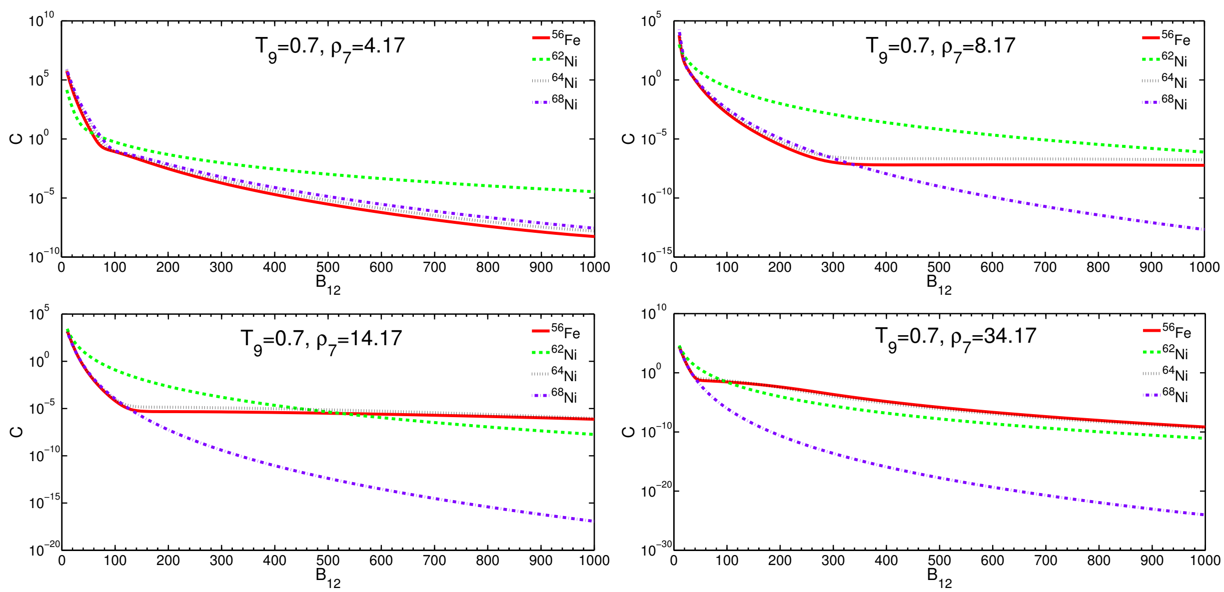

Fig. 6 The factor C for ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni as a function of B_{12} when $\rho_7 = 4.17, 8.17, 14.17$ and 34.17 at $T_9 = 0.7$.

Figure 7 demonstrates that CREF is a function of magnetic field at different sets of temperature and matter densities. The results show that CREF decreases by more than seven orders of magnitude. For example, when $\rho_7 = 4.17$ and $T_9 = 0.7$, the CREF of ^{68}Ni decreases from 2.834×10^{-17} s^{-1} to 3.758×10^{-22} s^{-1} as B_{12} increases from 30.54 to 394.4. In contrast, when $\rho_7 = 34.17$ and $T_9 = 0.7$, the CREF of ^{68}Ni decreases from 3.097×10^{-25} s^{-1} to 2.902×10^{-37} s^{-1} as B_{12} increases from 30.54 to 394.4.

According to Zhu et al. (2016), the electron number density $n_{\rm e}$ will greatly increase due to the fact that an SMF strongly modifies the phase space of relativistic electrons and decreases the maximum of electron Landau level number. Thus, it will cause a redistribution of electrons. According to the Pauli exclusion principle, the electrons are strongly degenerate, and more and more electrons will occupy quantum states from the lowest Landau level (the ground level) to the highest Landau level. However, as the magnetic field strength increases, more and more electrons will occupy the lowest Landau levels. The enhancement of $n_{\rm e}$ in an SMF means an increase in the electron Fermi energy and an increase in electron degeneracy pressure.

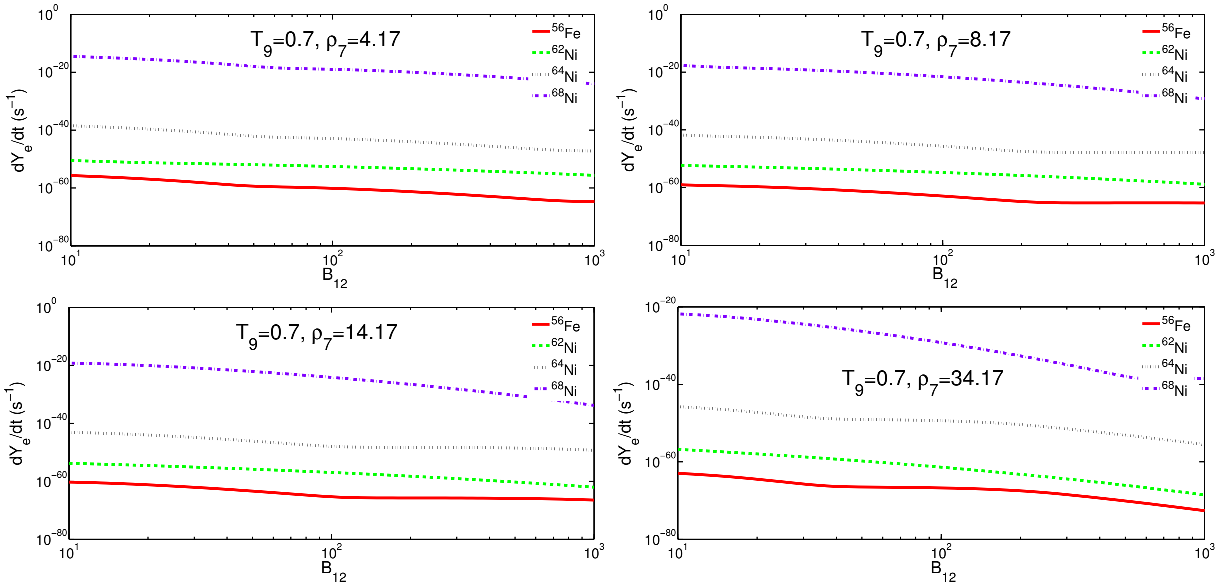

Fig. 7 The CREF for ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni as a function of B_{12} when $\rho_7 = 4.17, 8.17, 14.17$ and 34.17 at $T_9 = 0.7$.

Therefore, these are bound to lead to an increase in the beta decay rate in SMFs.

In summary, by analyzing the effect of an SMF on the beta decay rates for nuclides ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni, one can see that an SMF will exert different effects on the beta decay rates for different density and temperature in the surrounding region. Based on the Dirac δ-function and the Pauli exclusion principle, we have derived new results on the rates and discussed in detail the electron Fermi energy in the magnetar surface. Our results show that an SMF can decrease beta reaction rates by more than six orders magnitude when $10 \leq B_{12} \leq 10^4$.

4 CONCLUDING REMARK

By introducing the Dirac δ-function and Pauli exclusion principle in the presence of SMFs, we have carried out an estimation on the influence of an SMF on electron Fermi energy in magnetars. Based on the model of relativistic SMF theory, we investigate the beta decay processes of ^{56}Fe, ^{62}Ni, ^{64}Ni and ^{68}Ni in magnetars. The results show that the beta rates can decrease by more than six orders of magnitude in the presence of SMFs. The CREF will also decrease by more than seven orders of magnitude when $1.0 \leq B_{12} \leq 10^4$.

As is well known, the beta decay rates in an SMF are quite relevant for numerical simulations of thermal evolution and magnetic field evolution for magnetars. The antineutrino energy loss by beta decay reaction also plays an important role in the process of magneto-thermal evolution of magnetars. Our conclusions may be helpful for investigation of the associated thermal evolution, the nucleosyntheses of heavy elements and numerical calculations and simulation of NSs and magnetars.

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