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Probing model-independent cosmic opacity and its spatial properties

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Abstract Cosmic opacity and its spatial distribution have been constrained with a model independent method. The average opacity of the universe is not zero, but can be zero in the 1σ error range. The best-fit value of the spatial distribution of cosmic opacity is not a constant as the redshift varies, though a homogeneous and transparent universe is favored in the 2σ error range.

Key words: cosmology: observations — cosmology: distance scale — opacity

1 INTRODUCTION

Etherington found the distance duality relation (Etherington 1933)

$$D_{\rm L} = D_{\rm A}(1+z)^2, \quad (1)$$

where $D_{\rm L}$ is the luminosity distance and $D_{\rm A}$ is the angular diameter distance. There are two assumptions about this relation, Lorentz invariance and constant photon number. Equation (1) is very important in cosmic observation (Cunha et al. 2007; Komatsu et al. 2011) because it is independent of any cosmological model. New physics may exist if Equation (1) has been violated (Csáki et al. 2002).

One can test Equation (1) directly using observational data (Avgoustidis et al. 2010; Liao et al. 2015). However, different observational data may give different results. For example, combining the Union2 Type Ia supernova (SNIa) and the elliptical galaxy cluster data (De Filippis et al. 2005) indicates that Equation (1) is satisfied in the 1σ error range. However, Equation (1) is in accord with the 3σ error range (Li et al. 2011; Bonamente et al. 2006).

A very plausible reason for the violation of Equation (1) is that the universe is opaque. Cosmic opacity may account for reduction of an SNIa's (Aguirre 1999) photon number. It is worth testing the opacity of the universe by observations. For this reason, a transparent universe (Avgoustidis et al. 2010; More et al. 2009) has been found by baryon acoustic oscillation data (Percival et al. 2007) and SNIa data (Davis et al. 2007), Hubble parameter ($H(z)$) data (Stern et al. 2010) and Union SNIa data (Kowalski et al. 2008). In our previous work (Chen et al. 2012), using the Union2 SNIa data (Amanullah et al. 2010) and the latest seven baryon acoustic oscillation data, we discussed the transparency of the universe and found that it might be spatially inhomogeneous, though the universe is still transparent in the 1σ error range. However, the ΛCDM model must be assumed in our work, which means that the result is model-dependent. Cosmic transparency is also studied in Nair et al. (2012) and a result similar to ours is obtained.

Recently, Holanda, Carvalho and Alcaniz (Holanda et al. 2013) proposed a model-independent method to get the true $D_{\rm L}$ from Hubble data. Using the 12 Hubble parameter data and the Union2 SNIa data, they studied cosmic transparency and found that a transparent universe is consistent in the 2σ error range. We plan to examine the possible spatial variance of cosmic opacity using this model independent method. Since we use the latest 29 Hubble data points (Zhang et al. 2014; Simon et al. 2005; Stern et al. 2010; Moresco et al. 2012; Gaztañaga et al. 2009; Jimenez & Loeb 2002) and the latest Union2.1 SNIa data (Suzuki et al. 2012), the constraint on cosmic opacity will also be re-visited.

2 COSMIC OPACITY

It is well known that one can obtain observational $D_{\rm L}$ from SNeIa. Of course, the universe is supposed to be transparent. So, the observational $D_{\rm L}$ is equal to the true one and can be used to probe cosmic expansion history (Suzuki et al. 2012; Perlmutter et al. 1999; Riess et al. 1998).

However, in the universe there might be some reasons for the photon number to decrease, which may make the universe opaque. So, the true $D_{\rm L}$ should be smaller than the observed one. The relationship between the true $D_{\rm L}$ and the observed one (Chen & Kantowski 2009) is

$$D^2_{L_{\rm true}} = D^2_{L_{\rm obs}} e^{-\tau(z)}. \quad (2)$$

Here, $D_{L_{\rm obs}}$ is obtained from SNeIa data. For a transparent universe, $\tau(z)$ is zero. Apparently, if one can determine $D_{L_{\rm true}}$, then cosmic opacity can be tested by SNeIa. Because baryon acoustic oscillation is not affected by a reduction in photon numbers, baryon acoustic oscillation data have been used to test cosmic opacity and they found that the transparent universe is constrained at the 2σ error range (More et al. 2009). Avgoustidis et al. used the Union SNIa data and the Hubble parameter data to constrain cosmic transparency and found $\Delta\tau < 0.012$ at the 2σ error level in $0.2 < z < 0.35$ (Avgoustidis et al. 2010). But in these works (Avgoustidis et al. 2010; More et al. 2009), a ΛCDM model is required to derive $D_{L_{\rm true}}$.

We can obtain $D_{\rm L}$ from the distance modulus data by

$$D_{\rm L} = e^{(\mu-25)/5}, \tag{3}$$

and the true μ differs from the observed one

$$\mu_{\rm true}(z) = \mu_{\rm obs}(z) - (2.5\log e)\tau(z). \tag{4}$$

Recently, Holanda, Carvalho and Alcaniz (Holanda et al. 2013) proposed a model-independent method to get the true $D_{\rm L}$. Since the comoving distance $D_c(z)$ is a direct integration of $1/H(z')$ with respect to z' from 0 to z, they obtained the $D_c(z)$ data

$$D_c(z) \approx \frac{c}{2}\sum_{i=1}^{N}\left[\frac{1}{H(z_{i+1})} + \frac{1}{H(z_i)}\right], \tag{5}$$

where H_{z_i} is determined by the Hubble data. In Holanda et al. (2013), 12 Hubble data points are used. The error is

$$s_i = \frac{1}{2}(z_{i+1} - z_i)\left(\frac{\sigma^2_{H_{i+1}}}{H^4_{i+1}} + \frac{\sigma^2_{H_i}}{H^4_i}\right)^{1/2}. \tag{6}$$

Thus, the error associated with the integral described by Equation (5) in the interval $0 - z_n$ is $\sum_{i=1}^{n} s_i$. From 12 Hubble data, Holanda, Carvalho and Alcaniz obtained 12 corresponding values of D_c. Then, using $D_{\rm L} = (1+z)D_c$ and the polynomial fit method, they obtained a smoothed curve of $D_{\rm L}$. Since the Hubble data can be characterized by

$$H(z) = -\frac{\frac{dz}{dt}}{1+z}, \tag{7}$$

they are independent of whether the universe is transparent or not. Thus, the obtained $D_{\rm L}$ from the Hubble data represents the true luminosity distance value. From the smoothed curve of $D_{\rm L}$ from the Hubble data, the corresponding value of $\mu_{\rm true}$ at a given SNIa data point can be obtained. Using $\mu_{\rm obs}$ from SNeIa and $\mu_{\rm true}$ from Hubble data, the constraint on ϵ ($\tau(z) = 2\epsilon z$) can be determined. For the Union2 SNIa sample, they find that a perfectly transparent universe ($\epsilon = 0$) is permitted in the 2σ error range, with $\epsilon = 0.03 \pm 0.02$ at the 1σ confidence level.

Recently, 29 $H(z)$ data points have been reported, so we now have many more data points than what were considered in Holanda et al. (2013). With more Hubble data, we hope to obtain a more precise curve of D_c.

In the left panel of Figure 1, we calculate $D_{\rm L}$ from the latest Hubble data, which are represented by red stars. In the analysis, we take $H_0 = 73.8 \pm 2.4\ {\rm Mpc^{-1}\,km\,s^{-1}}$ (Riess et al. 2011). For a comparison, $D_{\rm L}$ from the Union2.1 580 data sample is plotted. The Union2.1 SNIa data set is an updated version of Union2. Using the same method as used in Holanda et al. (2013), we obtain a smoothed curve of $D_{\rm L}$ and then get the corresponding value of $\mu_{\rm true}$ at the redshift of a given Union2.1 SNIa data point. By minimizing the following χ^2 function

$$\chi^2 = \sum_{z_i}\frac{\left[\mu_{\rm obs}(z_i) - \mu_{\rm true}(z_i) - 2.17\epsilon z\right]^2}{\sigma^2_{\mu_{\rm obs}} + \sigma^2_{\mu_{\rm true}}}, \tag{8}$$

we can estimate the likelihood distribution of ϵ. Here, $\tau(z) = 2\epsilon z$ is considered, and $\sigma^2_{\mu_{\rm obs}}$ and $\sigma^2_{\mu_{\rm true}}$ are the errors associated with the distance modulus from SNIa and distance modulus from the Hubble data, respectively. The result is displayed in the right panel of Figure 1. We find $\epsilon = 0.0097 \pm 0.0262 \pm 0.0426$ at the 1 and 2σ confidence levels. Thus, perfect transparency is allowed at 1σ, which means that the latest data support a transparent universe more strongly.

3 SPATIAL HOMOGENEITY OF COSMIC OPACITY

Although cosmic opacity as a whole is zero in the 1σ error range, a patchy spatial structure for cosmic opacity is still possible. For example, using the seven baryon acoustic oscillations data and the Union2 data, we found that a transparent universe is not preferred in $0.20 - 0.44$ and $0.60 - 0.73$ (Chen et al. 2012), although, at the 1σ confidence level, the result supports a transparent universe. However, the ΛCDM model is assumed since $H(z)$ is unknown when we derive $D_{\rm L}$ from the baryon acoustic oscillation data.

From the above results, one can see that the true value of $D_{\rm L}$ can be calculated in a model-independent way from the $H(z)$ data. Thus, if we use these data to reanalyze the spatial homogeneity of cosmic opacity, a model independent result can be obtained. This is what we are going to do next.

Since the redshifts of some Hubble data are very close to one another, we bin D_c if the redshift difference is less than 0.01, by using the following method

$$D_c(z)^{\rm bin} = \frac{\sum D_c(z_i)/\sigma^2_{D_c(z_i)}}{\sum 1/\sigma^2_{D_c(z_i)}}, \tag{9}$$

with $\sigma^2_{D_c(z)}$ being

$$\sigma^2_{D_c(z)} = \frac{1}{\sum 1/\sigma^2_{D_c(z_i)}}. \tag{10}$$

In addition, Hubble data at redshift $z = 1.43$, 1.53 and 1.75 are discarded in our discussion because there are no corresponding SNIa data at these redshifts. As a result, we obtain 15 D_c data points and show their ratios in Table 1.

Here, the Union2.1 SNIa sample is used. To get the $D_c(z)$ data from $\mu(z)$, we bin all Union2.1 data in the region $[z \pm 0.005]$ and the formula used for calculation is

$$\mu^{\rm bin}_{\rm obs} = \frac{\sum \mu_{\rm obs_i}/\sigma^2_{\mu_{\rm obs_i}}}{\sum 1/\sigma^2_{\mu_{\rm obs_i}}}, \tag{11}$$

Table 1 The Ratio of Comoving Distance $D_c(z_2)/D_c(z_1)$

$\frac{D_c(0.12)}{D_c(0.08)}$	$\frac{D_c(0.175)}{D_c(0.12)}$	$\frac{D_c(0.2)}{D_c(0.175)}$	$\frac{D_c(0.24)}{D_c(0.2)}$	$\frac{D_c(0.275)}{D_c(0.24)}$
1.5183 ± 0.2196	1.4465 ± 0.2755	1.1341 ± 0.2251	1.1853 ± 0.2120	1.1336 ± 0.1870
$\frac{D_c(0.346)}{D_c(0.275)}$	$\frac{D_c(0.423)}{D_c(0.346)}$	$\frac{D_c(0.48)}{D_c(0.423)}$	$\frac{D_c(0.597)}{D_c(0.48)}$	$\frac{D_c(0.68)}{D_c(0.597)}$
1.2333 ± 0.1956	1.1958 ± 0.1738	1.1128 ± 0.1668	1.2018 ± 0.2244	1.1222 ± 0.2243
$\frac{D_c(0.73)}{D_c(0.68)}$	$\frac{D_c(0.781)}{D_c(0.73)}$	$\frac{D_c(0.885)}{D_c(0.781)}$	$\frac{D_c(1.3)}{D_c(0.885)}$	
1.0636 ± 0.2003	1.0571 ± 0.1923	1.0996 ± 0.1915	1.2692 ± 0.2029	

Table 2 The SNIa Distance Modulus Difference

$\mu_{\rm obs}(0.12) - \mu_{\rm obs}(0.08)$	$\mu_{\rm obs}(0.175) - \mu_{\rm obs}(0.12)$	$\mu_{\rm obs}(0.2) - \mu_{\rm obs}(0.175)$
1.0699 ± 0.0668	0.8889 ± 0.0699	0.3003 ± 0.0855
$\mu_{\rm obs}(0.24) - \mu_{\rm obs}(0.2)$	$\mu_{\rm obs}(0.275) - \mu_{\rm obs}(0.24)$	$\mu_{\rm obs}(0.346) - \mu_{\rm obs}(0.275)$
0.2788 ± 0.1060	0.5028 ± 0.1033	0.5931 ± 0.0947
$\mu_{\rm obs}(0.423) - \mu_{\rm obs}(0.346)$	$\mu_{\rm obs}(0.48) - \mu_{\rm obs}(0.423)$	$\mu_{\rm obs}(0.597) - \mu_{\rm obs}(0.48)$
0.5877 ± 0.1213	0.2225 ± 0.1639	0.5420 ± 0.2042
$\mu_{\rm obs}(0.68) - \mu_{\rm obs}(0.597)$	$\mu_{\rm obs}(0.73) - \mu_{\rm obs}(0.68)$	$\mu_{\rm obs}(0.781) - \mu_{\rm obs}(0.73)$
0.6368 ± 0.2254	-0.1082 ± 0.2123	0.3800 ± 0.2031
$\mu_{\rm obs}(0.885) - \mu_{\rm obs}(0.781)$	$\mu_{\rm obs}(1.3) - \mu_{\rm obs}(0.885)$	
0.4679 ± 0.3033	0.7254 ± 0.3097	

with $\sigma^2_{\mu^{\rm bin}_{\rm obs}}$ being

$$\sigma^2_{\mu^{\rm bin}_{\rm obs}} = \frac{1}{\sum 1/\sigma^2_{\mu_{\rm obs_i}}}. \tag{12}$$

Here, $\sigma_{\mu_{\rm obs_i}}$ is the uncertainty in the individual distance modulus.

To find spatial distribution of the cosmic transparency, we utilize the difference in distance modulus between redshifts z_2 and z_1

$$\Delta\mu_{\rm obs} = \mu_{\rm obs}(z_2) - \mu_{\rm obs}(z_1), \tag{13}$$

rather than the distance modulus directly. Using Equations (3) and (4), one has

$$\Delta\mu_{\rm obs} = \Delta\mu_{\rm true} + 2.5\Delta\tau \log e, \tag{14}$$

where

$$\begin{aligned} \Delta\mu_{\rm true} &= 5\log\frac{D_{L_{\rm true}}(z_2)}{D_{L_{\rm true}}(z_1)} \\ &= 5\log\frac{(1+z_2)D_c(z_2)}{(1+z_1)D_c(z_1)}\ , \end{aligned} \tag{15}$$

and $\Delta\tau = \tau(z_2) - \tau(z_1)$. If the transparency of the universe is homogeneous, $\Delta\tau = 0$. From the Union2.1 data we find $\Delta\mu_{\rm obs}$ between different redshifts from the Hubble data and show them in Table 2.

Now, we estimate the best-fit value for $\Delta\tau$ by using $L \propto e^{-\chi^2/2}$, with

$$\chi^2 = (\Delta\mu_{\rm obs} - \Delta\mu_{\rm true})^2/(\sigma^2_{\rm obs} + \sigma^2_{\rm true}). \tag{16}$$

The obtained results are shown in Figure 2 and Table 3, which demonstrate that in redshift regions $0.08 - 0.12$, $0.24 - 0.423$, $0.597 - 0.68$ and $0.73 - 0.885$, the best fit value of $\Delta\tau$ is larger than zero, while in other regions $\Delta\tau = 0$. Thus, the best-fit values of cosmic opacity are not constant at different redshift, which is similar to our previous result. In the 68% error range, $\Delta\tau = 0$ is in accord with the observation except for the redshift region $0.597 - 0.68$ where $\Delta\tau = 0$ is only permitted in the 95% error range. Therefore, a homogeneous universe is still favored by observations.

Table 3 The Obtained $\Delta\tau$ in Different Redshift Regions

	Best fit value	1σ	2σ	3σ
$\Delta\tau_{0.08-0.12}$	0.077	0.207	0.495	0.785
$\Delta\tau_{0.12-0.175}$	0	0.326	0.653	0.918
$\Delta\tau_{0.175-0.2}$	0	0.346	0.694	0.917
$\Delta\tau_{0.2-0.24}$	0	0.264	0.572	0.857
$\Delta\tau_{0.24-0.275}$	0.157	0.230	0.537	0.762
$\Delta\tau_{0.275-0.346}$	0.018	0.266	0.575	0.839
$\Delta\tau_{0.346-0.423}$	0.072	0.234	0.521	0.805
$\Delta\tau_{0.423-0.48}$	0	0.254	0.555	0.836
$\Delta\tau_{0.48-0.597}$	0	0.346	0.713	0.937
$\Delta\tau_{0.597-0.68}$	0.255	0.254	0.602	0.734
$\Delta\tau_{0.68-0.73}$	0	0.265	0.593	0.898
$\Delta\tau_{0.73-0.781}$	0.181	0.268	0.634	0.798
$\Delta\tau_{0.781-0.885}$	0.128	0.321	0.687	0.835
$\Delta\tau_{0.885-1.3}$	0	0.306	0.653	0.919

Now we give a comparison with our previous results obtained using the baryon acoustic oscillation data (Chen et al. 2012) by finding $\Delta\tau$ between redshift regions of the baryon acoustic oscillation data, that is, $0.106 - 0.2, 0.2 - 0.35, 0.35 - 0.44, 0.44 - 0.57, 0.57 - 0.6$ and $0.6 - 0.73$. The true values of $D_{\rm L}$ and their corresponding errors at

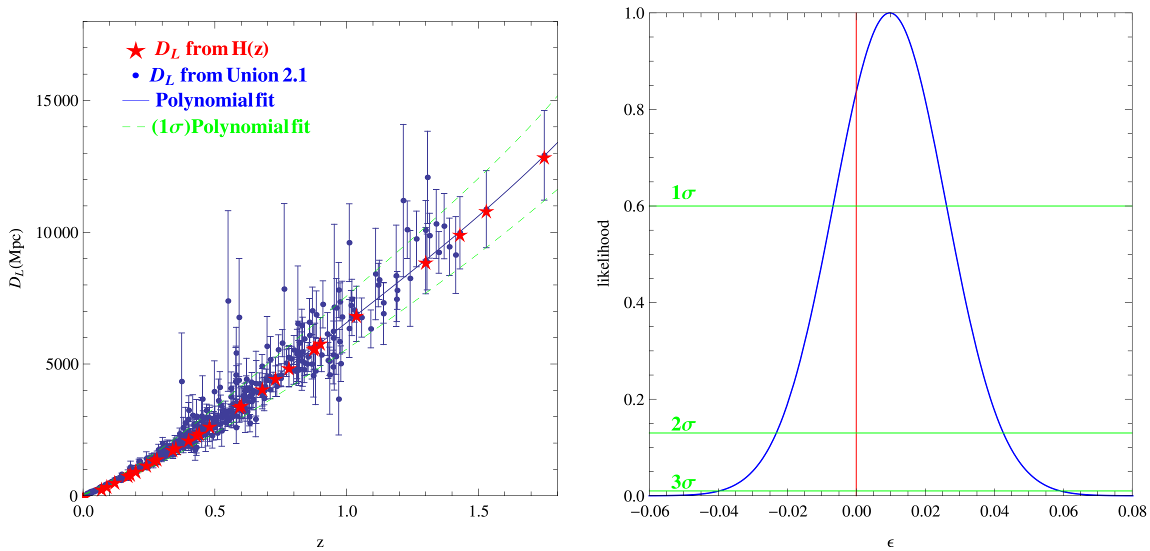

Fig. 1 *Left:* red stars show $D_{\rm L}$ obtained from Hubble data. For a comparison, values of $D_{\rm L}$ from the Union2.1 SNIa sample are plotted as blue dots. The solid curve represents the second degree polynomial fit of red star points and the dashed curves are the corresponding 1σ errors. *Right:* likelihood function for ϵ.

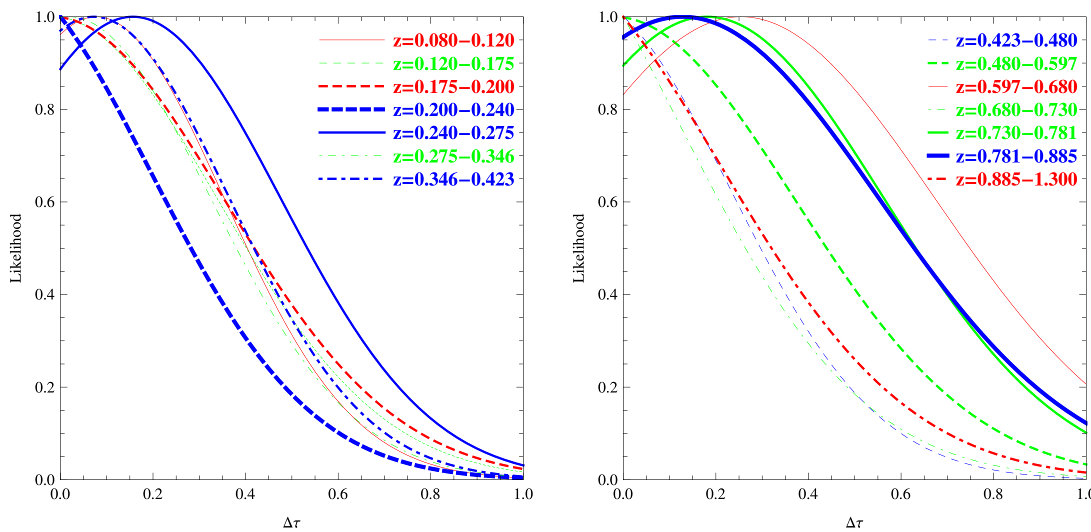

Fig. 2 The posterior probabilities of $\Delta\tau$.

these redshifts are obtained from the smoothed curves in the left panel of Figure 1.

The results are given in Figure 3 and Table 4. We find that a homogeneous universe is favored between redshift ranges $0.106-0.2$ and $0.44-0.6$, while an inhomogeneous one is favored at redshift ranges $0.2-0.44$ and $0.6-0.73$, though in the 68% error range $\Delta\tau = 0$ is still permitted. So, this conclusion is similar with that drawn from the baryon acoustic oscillation data (Chen et al. 2012) and is consistent with that given in Table 3.

Table 4 The $\Delta\tau$ in Different Redshift Ranges of Seven Baryon Acoustic Oscillation data

	Best fit value	1σ	2σ	3σ
$\Delta\tau_{0.106-0.2}$	0	0.356	0.713	0.937
$\Delta\tau_{0.20-0.35}$	0.030	0.378	0.765	0.948
$\Delta\tau_{0.35-0.44}$	0.142	0.388	0.735	0.848
$\Delta\tau_{0.44-0.57}$	0	0.367	0.734	0.937
$\Delta\tau_{0.57-0.60}$	0	0.394	0.779	0.978
$\Delta\tau_{0.60-0.73}$	0.055	0.374	0.760	0.926

4 CONCLUSIONS

In previous work (Chen et al. 2012), using the latest baryon acoustic oscillation data and Union2 Ia SNIa data, we placed constraints on cosmic opacity between different redshift regions. We find that the best-fit values of cosmic transparency are not constant with redshift, although in the

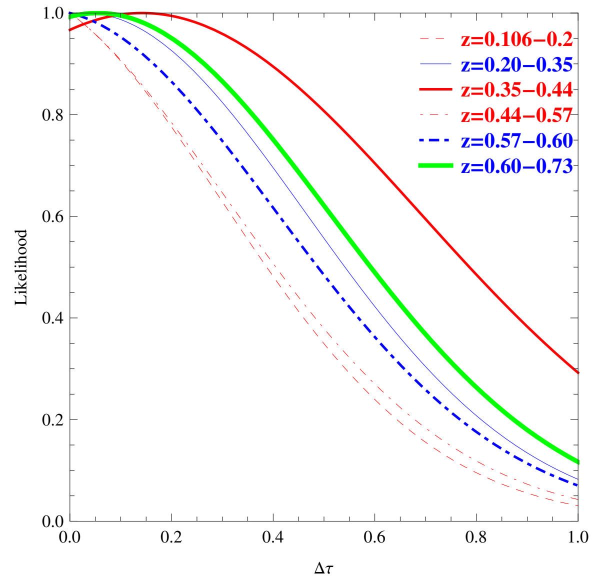

Fig. 3 The $\Delta\tau$ in different redshift ranges of seven baryon acoustic oscillation data.

68% error range a transparent universe is still permitted. However, a ΛCDM cosmological model is assumed, so the results are model-dependent. Recently, Holanda, Carvalho and Alcaniz (Holanda et al. 2013) proposed a model independent method to obtain the true comoving distance from 12 Hubble data. Then they studied the constraint on cosmic opacity by assuming $\tau = 2\epsilon z$ using the Union2 SNIa data and found that a transparent universe is allowed in the 95% error range.

We use the same model independent way as Holanda et al. (2013) to obtain the true $D_{\rm L}$ in this paper. Since we now consider the latest 26 Hubble data and the Union2.1 Ia supernova in our calculation, we first discuss the constraint on cosmic opacity and find that $\epsilon = 0.0097 \pm 0.0262\,(1\sigma)$, which means that the universe is transparent according to observations in the 68% error range. Then we study the spatial distribution of cosmic opacity. By binning the obtained μ at the redshift of Hubble data within the redshift ranges $\Delta z < 0.01$ and discarding three high redshift comoving distance data points since the corresponding SNIa data are absent, 14 different redshift regions are obtained. The results in these redshift regions show that the cosmic transparency's best fit value also is not constant at different redshift and $\Delta\tau = 0$ is still permitted in the 68% error range except for the redshift region $0.597 - 0.68$.

We then give a comparison with our previous results obtained using the baryon acoustic oscillation data (Chen et al. 2012) by finding $\Delta\tau$ between redshift regions of the baryon acoustic oscillation data. We first find the comoving distance at the redshifts of baryon acoustic oscillation data and then study cosmic transparency. A similar result to that from the baryon acoustic oscillation data is found. The cosmic transparency's best fit value is not zero in the redshift range $0.106-0.20$ and $0.44-0.60$, but it is zero in $0.20-0.44$ and $0.60-0.73$. In the 68% error range, the universe is homogeneous according to observational results. Since the results obtained in this paper and our previous work show that the best fit values for cosmic transparency are not constant at different redshift, a patchy spatial structure for cosmic opacity seems to be allowed by current observations, although a homogeneous and transparent universe remains in accord with observational data in the 68% error range.

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