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Assessing the effects of timing irregularities on radio pulsars anomalous braking indices

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Abstract We investigate the statistical effects of non-discrete timing irregularities on observed radio pulsar braking indices using correlations between the second derivative of the measured anomalous frequency ($\ddot{\nu}_{\rm obs}$) and some parameters that have been widely used to quantify pulsar timing fluctuations (the timing activity parameter (A), the amount of timing fluctuations absorbed by the cubic term ($\sigma_{\rm R23}$) and a measure of pulsar rotational stability ($\sigma_{\rm z}$)) in a large sample of 366 Jodrell Bank Observatory radio pulsars. The result demonstrates that anomalous braking indices are largely artifacts produced by aggregations of fluctuations that occur within or outside the pulsar system. For a subsample of 223 normal radio pulsars whose observed timing activity appeared consistent with instabilities in rotation of the underlying neutron stars (or timing noise) over timescales of $\sim 10 - 40$ yr, $|\ddot{\nu}_{\rm obs}|$ strongly correlates (with correlation coefficient $|r| \sim 0.80 - 0.90$) with the pulsar timing activity parameters and spin-down properties. On the other hand, no meaningful correlations ($r < 0.3$) were found between $\ddot{\nu}_{\rm obs}$ and the timing activity diagnostics and spin-down parameters in the remaining 143 objects, whose timing activity appears significantly dominated by white noise fluctuations. The current result can be better understood if the timing noise in isolated pulsars originates from intrinsic spin-down processes of the underlying neutron stars, but white noise fluctuations largely arise from processes external to the pulsar system.

Key words: methods: statistical — stars: neutron — pulsars: general

1 INTRODUCTION

Radio pulsars (rapidly spinning, highly magnetized neutron stars) are reputed for their astonishing rotational stability, and most pulsars only slow down steadily with time. The theory of radio pulsar spin-down appears well developed and simple (Manchester & Taylor 1977). It is now widely believed that the spin-down of most rotation-powered pulsars is well approximated by a simple power law in the form (e.g. Lorimer & Kramer 2004, and references therein)

$$\dot{\nu} = -K\nu^n, \tag{1}$$

where ν is the pulsar rotation frequency, $\dot{\nu}$ is its first time derivative, n is the braking index and K, which is usually assumed to be an arbitrary positive constant, is a function that absorbs structural factors describing the neutron star (Allen & Horvath 1997). The braking index can be obtained from Equation (1) after differentiation and elimination of the constant K (e.g. Manchester & Taylor 1977) as

$$n = \frac{\nu\ddot{\nu}}{\dot{\nu}^2}, \tag{2}$$

where $\ddot{\nu}$ is the second derivative of pulsar rotation frequency with respect to time. For the simplest case of pure magnetic dipole braking in a vacuum, $n = 3$ (e.g. Manchester & Taylor 1977). Apparently, direct measurements of pulsar braking index can be accomplished through measurements of ν and its first and second time derivatives ($\dot{\nu}$ and $\ddot{\nu}$, respectively).

However, detailed long-term timing observations have revealed that the otherwise smooth spin-down of most radio pulsars is prone to a wide variety of disruptions, the so-called timing activities, which have proven extremely useful in applying pulsars as physical tools. Currently, the observed timing activities are broadly grouped into discrete events (mainly glitches and microglitches) and non-discrete events (timing noise (TN) and white noise (WN)). Glitches are sudden tiny jumps in pulsar spin rate (ν), which are usually accompanied by increases in the magnitude of $\dot{\nu}$ (Yu et al. 2013 and references therein). Large glitches are rare spectacular events which are usually characterized by recovery of the jumps in ν and $|\dot{\nu}|$ on a wide range of timescales (Cordes et al. 1988; Flanagan 1990; Urama 2002). Microglitches ($|\Delta\nu/\nu| < 10^{-9}$) are generally less attractive and relatively poorly understood events that are characterized by variable signatures (Cordes & Downs 1985; Chukwude & Urama 2010). The true status of microglitches, as a scaled-down version of glitches or a special form of TN, is still an issue of intense debate (Okany & Chukwude 2014, in preparation). Currently, it is widely believed that glitches and microglitches originate from some complex dynamical changes within the neutron star interior and their study could provide valuable insight into the internal structure and dynamics of neutron stars (Alpar et al. 1984; Flanagan 1995; Melatos et al. 2008).

On the other hand, the non discrete form of pulsar timing activity is generically referred to as TN. Although the prevalence of TN activity (as unmodeled structures in pulsar timing residuals in excess of what is expected from measurement error) among the pulsar population appears fairly well established (Cordes & Downs 1985; D'Alessandro et al. 1995; Hobbs et al. 2010), our understanding of the phenomenon is still largely poor (Lorimer & Kramer 2004). However, it is widely believed that TN activities arise from processes intrinsic to the rotating neutron stars. A popular proposition is that pulsar TN arises from some form of variations in the coupling between neutron star crust and its superfluid core (Jones 1990; D'Alessandro 1996) and/or torque fluctuations in the magnetosphere (Cheng 1987; Kramer et al. 2006; Lyne et al. 2010; Chukwude & Buchner 2012). Lately, analysis of improved timing data has revealed that the spin properties of a sizeable population of pulsars, on a wide range of timescales ($> \sim 10$ years), are strongly dominated by random fluctuations whose statistics are characterized by ensemble average power that is independent of fluctuation frequencies (Cordes & Downs 1985). This class of pulsar timing activity, whose observational manifestation is typified by large amplitude scatters in the observed timing residuals (e.g. D'Alessandro et al. 1993; Chukwude 2007; Hobbs et al. 2010), is generically known as WN. As yet there are no reports that characterize in detail WN fluctuations of pulsars. However, it is widely believed that these fluctuations consist largely of measurement uncertainties and pulse-to-pulse phase jitter (Cordes & Downs 1985; D'Alessandro et al. 1995).

Previous attempts at characterizing the strength of timing irregularities in radio pulsars have led to the introduction of a number of parameters, which are estimated from either the root-mean-square (rms) timing residuals or the second frequency derivative calculated from a cubic fit to times of arrival data (Shannon & Cordes 2010). Cordes & Helfand (1980) quantified the strength of pulsar timing irregularities by defining an activity parameter as

$$A = \log\left[\frac{\sigma_{\rm TN}(2,T)}{\sigma_{\rm TN}(2,T)_{\rm Crab}}\right], \tag{3}$$

where $\sigma_{\rm TN}(2,T)$ is the rms TN from least-square fits of a 2nd-order polynomial over a time interval of length T and $[\sigma_{\rm TN}(2,T)_{\rm Crab}] = 12\,(T/1628)^{3/2}$ is the rms TN of the Crab pulsar in units of milliseconds. The scaling of the Crab pulsar rms TN with T is consistent with the proposition that the observed timing activity is dominated by a random walk in its rotation frequency (Groth 1975; Cordes 1980; Cordes & Helfand 1980). The mean square TN over a timespan length T, $\sigma^2_{\rm TN}(2,T) = \sigma^2_{\rm R}(2,T) - \sigma^2_{\rm W}$, is the quadrature difference between the mean squares of the observed phase residuals and the measurement errors (e.g. Cordes & Downs 1985). Matsakis et al. (1997) introduced a dimensionless Allan variance-like parameter (σ_z) as a quantitative measure of the timing stability of a pulsar clock in the form

$$\sigma_z(T) = \frac{1}{2\sqrt{5}}\left[\frac{\sigma_{\ddot{\nu}}(T)}{\nu}\right]T^2, \tag{4}$$

where $\sigma_{\ddot{\nu}}$ is the rms of the measured $\ddot{\nu}$ over the observed spans of length $T = 10$ yr (Hobbs et al. 2010). Recently, Chukwude (2003) proposed a direct quantification of the 'amount' of contamination introduced in $\ddot{\nu}$ obtained from the standard timing solution, by all forms of timing fluctuations with the parameter $\sigma_{\rm R23}(T)$, defined as

$$\sigma_{\rm R23}(T) = \sqrt{\sigma^2_{\rm R}(2,T) - \sigma^2_{\rm R}(3,T)}, \tag{5}$$

where $\sigma^2_{\rm R}(2,T)$ and $\sigma^2_{\rm R}(3,T)$ are, respectively, the variances of the timing residuals over an observed span of length T after second- and third-order polynomial fits. Trends of TN parameters with various spin-down parameters have been widely used for detailed statistical study of TN in pulsar populations (Urama et al. 2006; Beskin et al. 2006; Hobbs et al. 2010; Biryukov et al. 2012).

Hitherto, statistical studies of the timing activity phenomenon in radio pulsars involving anomalous values of braking indices have failed to examine in detail the effects of the WN component on the observed timing residuals. While some of the analyses assumed that the observed pulsar timing irregularity is strongly dominated by TN (e.g. Chukwude 2003; Chukwude et al. 2010), others were restricted to only pulsars whose observed timing activity appears to show a predominant character of TN (e.g. Urama et al. 2006; Beskin et al. 2006; Biryukov et al. 2007, 2012). However, results of several timing observations (e.g. D'Alessandro et al. 1995; Chukwude 2007; Hobbs et al. 2010) have revealed that WN fluctuations strongly dominate the observed timing properties of a sizeable number of pulsars over long timescales (> 10 yr). Hence, the timing activity parameters and, perhaps, the anomalous $\ddot{\nu}$ of such pulsars could be severely contaminated by WN fluctuations. The basic idea behind the work presented in this paper is to regard estimates of anomalous $\ddot{\nu}$ obtained at a given time for some pulsars as the outcome of either 'intrinsic' noise processes with characteristic red spectra (TN) or 'extrinsic' noise processes with characteristic white spectra (WN). In other words, anomalous values of $\ddot{\nu}$ for the current sample of 366 radio pulsars are regarded as quantitative measures of the strength of either TN or WN fluctuations.

In this paper, we present a statistical assessment of the effects of TN and WN fluctuations on measured braking indices of radio pulsars using the correlations between anomalous values of $\ddot{\nu}$, obtained from a phase-connected coherent timing technique, and timing activity parameters (A, $\sigma_{\rm R23}$ and $\sigma_z(\tau)$, hereafter TAPs) of a large sample of 366 radio pulsars (Hobbs et al. 2010).

2 THEORY OF RELATIONSHIPS

An important diagnostic for assessing the level of timing fluctuations in the otherwise smooth spin-down of a radio pulsar is the rms phase residuals over an observing span of length T after fitting a polynomial of order m whose variance is defined (e.g. Cordes & Downs 1985) as

$$\sigma^2_{\rm R}(m,T) = \frac{1}{N_t}\sum_{i=1}^{N_t} R^2(t_i), \tag{6}$$

where $R(t)$ represents the residuals of the fit and N_t is the number of observations. The variance of the timing residuals can be expressed as a quadrature sum of the TN and WN components in the form (e.g. Cordes & Downs 1985; D'Alessandro et al. 1995)

$$\sigma^2_{\rm R}(m,T) = \sigma^2_{\rm TN}(m,T) + \sigma^2_{\rm WN}, \tag{7}$$

where $\sigma^2_{\rm TN}(m,T)$ and $\sigma^2_{\rm WN}$ are, respectively, the mean square contributions from TN and WN processes (we describe what we mean by TN and WN processes below). The mean square WN fluctuations can be written (e.g. Cordes & Downs 1985; D'Alessandro et al. 1995) as $\sigma^2_{\rm WN} = \sigma^2_{\rm M} + \sigma^2_{\rm J}$, where $\sigma^2_{\rm M}$ and $\sigma^2_{\rm J}$ are the mean square contributions from measurement noise and pulse phase jitter, respectively.

In earlier analyses (Chukwude 2003, 2007; Chukwude et al. 2010), the WN component of Equation (7) was in principle assumed to be zero, a condition which is also sufficiently satisfied for $\sigma^2_{\rm TN}(m,T) \gg \sigma^2_{\rm WN}$. While this assumption might hold reasonably well for the 27 pulsars employed in previous studies, the situation may be noticeably different for larger and more heterogeneous populations of pulsars. Consequently, a more objective approach in analysis of radio pulsar timing activity, which is adopted in this work, is to assume that the observed pulsar timing activity is composed of TN and WN fluctuations. This implies that the calculated pulsar timing properties would be more appropriately considered as the sum of fluctuations produced by two broad types of processes. Consequently, we can express the amount of TN absorbed by the cubic term explicitly as the sum of the TN and WN components in the form

$$\sigma_{\rm R23}(T) = \sigma_{\rm TN}(T) + \sigma_{\rm WN}, \tag{8}$$

where $\sigma_{\rm TN}(T)$ and $\sigma_{\rm WN}$ are, respectively, the rms TN and WN components of $\sigma_{\rm R23}(T)$. Similarly, the calculated TAP A of radio pulsars can be written in the form

$$A(T) = A_{\rm TN}(T) + A_{\rm WN}, \tag{9}$$

where $A_{\rm TN}$ and $A_{\rm WN}$ correspond to the contributions from rotational fluctuations (or TN) and WN fluctuations respectively. Following a similar argument, the timing stability parameter (σ_z) can be decomposed into two parts as

$$\sigma_{\rm z}(T) = \sigma_{\rm z[TN]}(T) + \sigma_{\rm z[WN]}, \tag{10}$$

where $\sigma_{\rm z[TN]}(T)$ and $\sigma_{\rm z[WN]}$ are the components of the parameter from TN and WN, respectively. In this paper, TN is used to label all processes producing fluctuations that are characterized by excess power at lower fluctuation frequencies. Models of TN include random walks, microglitches, long-term systematic excursions of pulsar spin properties and/or some mixtures of two or more of these activities (e.g. Boynton et al. 1972; Cordes & Downs 1985; D'Alessandro et al. 1995; Chukwude 2002; Lyne et al. 2010; Chukwude & Buchner 2012). On the other hand, WN denotes all fluctuations that have white spectra: equal levels of power at all fluctuation frequencies (e.g. Cordes & Downs 1985; Shannon & Cordes 2010). Following Hobbs et al. (2010), the analysis presented in this paper is based on the assumption that the observed WN fluctuations are largely measurement noise (assumed to be largely thermal random noise in the observing hardware). The dominant presence of TN and WN in radio pulsar timing history has been widely observed, respectively, as unmodeled structures with different shapes and scatters with uncorrelated amplitudes in the timing residuals (e.g. Cordes & Downs 1985; D'Alessandro et al. 1995; Chukwude 2002; Hobbs et al. 2010). The dependence of the timing activity statistics $\sigma_{\rm R23}$, A and $\sigma_{\rm z}$ on dataspan length T stems from the non-stationarity of TN fluctuations (e.g. Cordes & Downs 1985). It follows that $\sigma_{\rm R23}(T)$, $A(T)$ and $\sigma_{\rm z}(T)$ only apply to the case where the contribution from TN strongly dominates over that of WN (i.e. $\sigma_{\rm TN} \gg \sigma_{\rm WN}$, $A_{\rm TN} \gg A_{\rm WN}$ and $\sigma_{\rm z[TN]} \gg \sigma_{\rm z[WN]}$).

In principle, the observed second derivative of pulsar spin frequency ($\ddot{\nu}_{\rm obs}$) can be modeled as a superposition of systematic spin-down and irregular components in the form (e.g. Chukwude 2003; Urama et al. 2006)

$$\ddot{\nu}_{\rm obs} = \ddot{\nu}_{\rm dip} + \delta\ddot{\nu}, \tag{11}$$

where $\ddot{\nu}_{\rm dip}$ is the component of the estimated parameter from the deterministic pulsar spin-down and $\delta\ddot{\nu}$ denotes the aggregation of contributions from all forms of timing fluctuations. The exact nature of fluctuations that contribute to $\delta\ddot{\nu}$ is still not well understood (Lorimer & Kramer 2004). However, for the purposes of this work, we assume that they can be classified as two broad types (TN and WN). Hence, the random component of Equation (11) can be written explicitly as

$$\delta\ddot{\nu} = \ddot{\nu}_{\rm tno} + \ddot{\nu}_{\rm wno}, \tag{12}$$

where $\ddot{\nu}_{\rm tno}$ and $\ddot{\nu}_{\rm wno}$ denote contributions from TN and WN fluctuations, respectively. The prevailing scenario, in which most pulsars are associated with anomalous values of $\ddot{\nu}$, can plausibly be attributed to cases where the random component of $\ddot{\nu}$ strongly dominates over the deterministic part (i.e. $\delta\ddot{\nu} \gg \ddot{\nu}_{\rm dip}$, which implies that $\delta\ddot{\nu} \approx \ddot{\nu}_{\rm obs}$).

Most previous TN analyses involving the second derivative of radio pulsar rotation frequency (Chukwude et al. 2010 and references therein) were predicated on the assumption that $\ddot{\nu}_{\rm obs} \approx \ddot{\nu}_{\rm tno}$ (which is also true for $\ddot{\nu}_{\rm wno} = 0$ or more realistically for $\ddot{\nu}_{\rm wno} \ll \ddot{\nu}_{\rm tno}$), $A = A_{\rm TN}(T)$ and $\sigma_{\rm R23}(T) = \sigma_{\rm TN}(T)$. Consequently, the observed strong $|\ddot{\nu}_{\rm obs}| - \sigma_{\rm R23}/A$ correlations have been interpreted as evidence that the observed anomalous values of $\ddot{\nu}$ for most canonical pulsars are severely contaminated by TN fluctuations (Chukwude 2003, 2007; Chukwude et al. 2010).

However, the validity of the assumption is somewhat doubtful when considering large heterogeneous samples, consisting of objects ranging from normal to old recycled pulsars. For such a population of pulsars, one would envisage a considerable spread in both strengths and characters of the underlying timing activity. Hence, a more objective approach in the analysis of a large population of pulsars must take into account the competing effects of fluctuations from TN and WN processes. In principle, the observed timing activity of a typical non-glitching pulsar could possibly be dominated by either TN or WN fluctuations. Following Equations (11) and (12), three plausible scenarios are envisaged in analysis of $\ddot{\nu}$ of large samples of radio pulsars.

(i) If the coefficient of the cubic term sufficiently models the pulsar's deterministic spin-down via pure magnetic dipole braking, then the irregular component of $\ddot{\nu}$ is negligible. In other words, we have $\ddot{\nu}_{\rm dip} \gg \delta\ddot{\nu}$ and $\ddot{\nu}_{\rm obs} \approx \ddot{\nu}_{\rm dip}$. Otherwise, the irregular component is dominant and we have $\ddot{\nu}_{\rm obs} \approx \delta\ddot{\nu}$.

(ii) For radio pulsars whose observed timing activity over long timescales is characterized by intrinsic (or genuine) rotational fluctuations (TN), it is expected, among other things, that $\ddot{\nu}_{\rm dip} \ll \delta\ddot{\nu}$, $\ddot{\nu}_{\rm wno} \ll \ddot{\nu}_{\rm tno}$ and the observed parameter largely measures the TN fluctuations (i.e. $\ddot{\nu}_{\rm obs} \approx \ddot{\nu}_{\rm tno}$).

(iii) On the other hand, if the observed timing activity of a radio pulsar is significantly dominated by fluctuations in WN character, it is expected that $\ddot{\nu}_{\rm tno} \ll \ddot{\nu}_{\rm wno}$ and the measured $\ddot{\nu}$ only quantifies the WN fluctuations in the data (i.e. $\ddot{\nu}_{\rm obs} \approx \ddot{\nu}_{\rm wno}$).

The first scenario requires the observed frequency second derivative ($\ddot{\nu}_{\rm obs}$) to be reasonably stationary, yielding a braking index $n_{\rm obs} \approx 3$. To date, this has been sufficiently confirmed to be the case in only about eight very young (with $\tau_{\rm c} < 20$ kyr, where $\tau_{\rm c} = -\frac{\nu}{2\dot{\nu}}$ is the pulsar characteristic age) pulsars (e.g. Pons et al. 2012, and references therein). The second and third scenarios will generally be characterized by anomalous and highly variable values of $\ddot{\nu}_{\rm obs}$. For the second scenario, we have: $\sigma_{\rm TN} \gg \sigma_{\rm WN}$, $A_{\rm TN} \gg A_{\rm WN}$ and $\sigma_{\rm z[TN]} \gg \sigma_{\rm z[WN]}$. The implication of this is that $\sigma_{\rm TN} \approx \sigma_{\rm R23}$, $A \approx A_{\rm TN}$, $\sigma_{\rm z} \approx \sigma_{\rm z[TN]}$ and $\ddot{\nu}_{\rm obs} \approx \ddot{\nu}_{\rm tno}$ and the resulting activity parameters are mere measures of TN fluctuations (hereafter TN activity parameters (TNAPs)). In this case, meaningful relationships would be expected between the anomalous frequency second derivatives ($\ddot{\nu}_{\rm obs}$) and the TNAPs ($\sigma_{\rm R23}$, $\sigma_{\rm z}$ and A) and, perhaps, the pulsar spin-down rate and other parameters derived from it (see e.g. Chukwude et al. 2010; Hobbs et al. 2010). The third scenario requires, in addition, that $\sigma_{\rm TN}(T) \ll \sigma_{\rm W}$, $A_{\rm TN}(T) \ll A_{\rm WN}$ and $\sigma_{\rm z[TN]} \ll \sigma_{\rm z[WN]}$. By implication, we have that: $\sigma_{\rm R23}(T) \approx \sigma_{\rm WN}$, $A \approx A_{\rm WN}$, $\sigma_{\rm z} \approx \sigma_{\rm z[WN]}$ and $\ddot{\nu}_{\rm obs} \approx \ddot{\nu}_{\rm wno}$ (hereafter referred to as WN-dominated activity parameters (WNAPs)). This scenario might be distinguished by lack of meaningful relationships between the anomalous $\ddot{\nu}$, the calculated WNAPs and the pulsars' spin-down parameters.

3 DATA ANALYSIS

The analysis presented in this work employed a large heterogeneous sample of 366 Jodrell Bank Observatory (JBO) radio pulsars with significantly measured spin-down and TAPs (Hobbs et al. 2010). The relevant parameters (both timing activity and spin-down) of the pulsars were estimated from radio timing data with timespan length of $\sim 10 - 40$ yr (see Hobbs et al. 2004 for details of the timing solution).

A careful inspection of the observed timing residuals of the 366 JBO radio pulsars (Hobbs et al. 2010, Fig. 3) suggests the presence of two dominant types of fluctuations in the long-term timing behavior of the objects. For instance, while the timing residuals of some pulsars over timescales of $\sim 10 - 40$ yr are characterized by large amplitude scatters, those of others are associated with a wide spectrum of unmodeled structures. These observations combine to suggest that the long-term spin-down history of the current set of pulsars is dominated by either TN (for objects with discernible structures in their timing residuals) or WN (for those characterized by noisy timing residuals).

Consequently, the first step in the current analysis is to classify the 366 radio pulsars into two subsets based on whether the observed timing activity is dominated by intrinsic rotational fluctuations (subset I) or by WN (subset II). To accomplish this, we introduced a parameter $\beta = (\sigma_{\rm R}(2,T) - \sigma_{\rm R}(3,T))/T$, where T is the entire observation timespan in years and $\sigma_{\rm R}(2,T)$ and $\sigma_{\rm R}(2,T)$ are, respectively the rms timing residuals from 2nd- and 3rd-order polynomial fits to the arrival time data, as a quantitative measure of fluctuations in excess of measurement errors introduced by either TN or WN activities. The calculated values of β were found to vary between $\simeq 0$ (for objects whose observed timing activity is sufficiently dominated by WN) to ~ 400 (for pulsars exhibiting the strongest TN in their timing residuals). In particular, we find that $\beta = 0$ for 60 pulsars in which TN fluctuations are generally at the level of measurement errors (i.e. $\sigma_{\rm R}(2,T) \simeq \sigma_{\rm R}(3,T)$). The 366 pulsars were categorized into two subsets using a value of $\beta = 0.01$ ms yr^{-1}. Subset I contains 223 objects with $\beta \geq 0.01$ ms yr^{-1}, while the remaining 143 pulsars with $\beta < 0.01$ make up subset II.

Although the choice of $\beta = 0.01$ is somewhat arbitrary, it nonetheless proved successful in isolating $\sim 40\%$ of the pulsars, whose observed timing activity was previously attributed to WN processes (see Hobbs et al. 2010).

The activity parameter (A) was calculated from Equations (3) and (7). The assumption here is that the novel timing technique implemented by Hobbs et al. (2010) effectively whitened the timing residuals of the pulsars in the sample. In other words, the rms post-fit timing residuals after whitening (σ_3) are taken as fair representations of the rms WN ($\sigma_{\rm WN}$) in the data. Hence, $\sigma_3 \approx \sigma_{\rm WN}$ generally represents upper limits on the expected rms WN in the data (see also D'Amico et al. 1998). It is noteworthy that subset II contains about 30 recycled millisecond radio pulsars, whose intrinsic spin-down evolution is widely considered to be significantly different from those of normal radio pulsars. However, current analysis is premised on the assumption that the intrinsic spin properties of all the radio pulsars are strongly dominated by timing fluctuations. By implication, the observed rotational parameters and TAPs of the objects in subset II merely probe the same phenomenon (i.e. measurement uncertainty) and would be largely indistinguishable for both recycled millisecond and normal radio pulsars.

Furthermore, the spread in values of the spin-down parameters of pulsars in the subsets appears significantly reduced when compared with the entire sample. This suggests that the subsets of pulsars are generally more homogeneous than the entire sample. Subsequently, the subsets of data were analyzed separately. In this work, we consider $\sigma_{\rm R23}$ to be in units of time rather than in cycles, given that one of the main objectives of the current analysis is to ascertain if this statistic correlates generically with pulsar spin frequency ($\nu = \frac{1}{P}$). It is obvious that expressing $\sigma_{\rm R23}$ in cycles would automatically impose a period dependence on the parameter (e.g. Cordes & Helfand 1980).

4 RESULTS

Figure 1 shows the distributions of the absolute values of the observed frequency second derivative ($|\ddot{\nu}_{\rm obs}|$: Fig. 1(a)) and the ratio of $|\ddot{\nu}_{\rm obs}|$ with the systematic frequency second derivative expected from the standard spin-down model ($\ddot{\nu}_{\rm dip}$: Fig. 1(b)) for the 366 JBO radio pulsars. Figure 1(a) shows that the distribution of $|\ddot{\nu}_{\rm obs}|$ is sufficiently lognormal for both subsets of data. The data in subsets I and II appear to be skewed towards, respectively, the right and left corners of the graph. This result suggests that radio pulsars in subset I, on average, have larger values of $|\ddot{\nu}_{\rm obs}|$ than those in subset II. Figure 1(b) clearly demonstrates the anomalous nature of $\ddot{\nu}_{\rm obs}$ for the current pulsars. In absolute terms, it is shown that $\ddot{\nu}_{\rm obs}$ can differ from $\ddot{\nu}_{\rm dip}$ by as much as 5 and 9 orders of magnitude for pulsars in subsets I and II, respectively.

The absolute values of the observed frequency second derivatives ($|\ddot{\nu}_{\rm obs}|$) are plotted against the basic timing activity diagnostics (A, $\sigma_{\rm R23}$ and $\sigma_{\rm z}(\tau)$) and the pulsar spin-down parameters ($\dot{\nu}$, $\tau_{\rm c}$ and $\ddot{\nu}_{\rm dip} = 3\dot{\nu}^2/\nu$) on logarithmic scales in Figures 2 and 3. The highlights of the results of simple regression analyses of the two subsets of data are summarized as follows:

(i) Generally, pulsars in subset II appear concentrated in a relatively smaller region in the lower left corner of the plots, corresponding to relatively smaller parameter values, while objects in subset I occupy an extended region up to the top right corner (corresponding to larger parameter values). Evidently, the spread in absolute values of the TAPs is significantly (> 2 orders of magnitude) smaller for objects in subset II than those in subset I.

(ii) For the 223 radio pulsars in subset I, whose long-term timing activities are attributable to TN fluctuations, the scatter plots reveal very strong relationships between $|\ddot{\nu}_{\rm obs}|$ and the TNAPs ($A_{\rm TN}$, $\sigma_{\rm TN}$ and $\sigma_{\rm z[TN]}$). Simple regression analyses of the $|\ddot{\nu}_{\rm obs}| - A$, $|\ddot{\nu}_{\rm obs}| - \sigma_z(\tau)$ and $|\ddot{\nu}_{\rm obs}| - \sigma_{\rm R23}$ data yield $r \simeq +0.91$, +0.92 and +0.88, respectively, as the correlation coefficients.

(iii) Furthermore, the analysis reveals significant dependence of $|\ddot{\nu}_{\rm obs}|$ on pulsar spin-down parameters ($\dot{\nu}$, $\ddot{\nu}_{\rm dip}$ and $\tau_{\rm c}$) for objects in the subset. In particular, it is found that while the $|\ddot{\nu}_{\rm obs}| - \dot{\nu}$ and $|\ddot{\nu}_{\rm obs}| - \ddot{\nu}_{\rm dip}$ are characterized by strong correlations (with $r = +0.85$ and +0.82, respectively), the $|\ddot{\nu}_{\rm obs}| - \tau_{\rm c}$ data reveal a strong anti-correlation, with $r = -0.78$.

(iv) Similar analyses of the 143 pulsars in subset II, whose observed timing activities over the period of $\sim 10 - 30$ yr appear to be dominantly WN, show that the relationships between $|\ddot{\nu}_{\rm obs}|$ and both the timing activity diagnostics ($\sigma_{\rm WN}$, $A_{\rm WN}$ and $\sigma_{\rm z[WN]}$) and the spin-down parameters ($\dot{\nu}$, $\ddot{\nu}_{\rm dip}$ and $\tau_{\rm c}$) are either nonexistent or, at most, only marginal (with $|r| \sim 0.02 - 0.30$).

(v) The deviation of the measured frequency second derivative from the deterministic value is highest (up to 9 orders of magnitude) for objects with the smallest values of $\ddot{\nu}_{\rm dip}$.

The correlations reported in this paper are very consistent with the behaviors displayed by the timing residuals of these pulsars (Hobbs et al. 2010). The sub-sample of 223 pulsars that is characterized by strong correlations between the TN properties corresponds to those pulsars whose timing residuals showed evidence of TN activity at levels well above measurement errors. The timing residuals of this category of pulsars are dominated by structures whose patterns vary from simple cubic polynomials, higher-order polynomials to quasi-periodicities (Hobbs et al. 2010). The presence of meaningful TN activity in these objects equally explains the strong correlations found between TNAPs and the pulsars' spin-down parameters. On the other hand, subsample II, in which the timing activity and spin-down properties were largely uncorrelated ($r < 0.3$), corresponds to pulsars whose observed timing residuals appeared to be strongly dominated by large amplitude scatters. This behavior of timing residuals has been widely attributed to the presence of TN activity at levels sufficiently below the measurement errors (Hobbs et al. 2010; Chukwude 2002; D'Alessandro et al. 1995; Cordes & Downs 1985).

5 DISCUSSION

We have characterized the phenomenon of timing activity in radio pulsars over timescales of $\sim 10 - 40$ yr using published values of anomalous frequency second derivatives in a sample 366 radio pulsars (Hobbs et al. 2010). The analysis is predicated on the assumption that while TN processes (genuine fluctuations in the rotation of the underlying neutron stars) dominate the observed long-term timing activity of a subsample of 223 pulsars (subset I), WN (an aggregation of fluctuations produced outside the pulsar system) is predominant in the remaining 143 objects (subset II). Irrespective of sign, the distributions of $\ddot{\nu}_{\rm obs}$ were sufficiently lognormal, spanning about 5 orders of magnitude for both subsets of pulsars. In particular, it is found that the magnitude of $\ddot{\nu}_{\rm obs}$ has values in the range of $\sim$ (0.003 $-$ 2000) $\times 10^{-25}{\rm s}^{-3}$ and (0.000009 $-$ 5)$\times 10^{-25}{\rm s}^{-3}$ for pulsars in subsets I and II, respectively, with $|\ddot{\nu}_{\rm obs}|_{\rm mean(I)}/|\ddot{\nu}_{\rm obs}|_{\rm mean(II)}$ of 200. A two-sample Kolmogorov-Smirnoff (K-S) statistical test on the anomalous frequency second derivatives shows that pulsars in subset I have significantly larger values of $|\ddot{\nu}_{\rm obs}|$ than those in subset II with a chance probability $\rho \leq 0.023$. This suggests that there is up to 99% confidence that the underlying distributions of the parameter are significantly different for the pulsars in the current subsets. An obvious implication of this is that the underlying processes responsible for the observed anomalous values of $\ddot{\nu}$ are appreciably different for the two subsets of radio pulsars.

The scatter plots of $|\ddot{\nu}_{\rm obs}|$ against the TAPs (A, $\sigma_{\rm R23}$ and $\sigma_{\rm z}(\tau)$) reveal that the parameters are concentrated in the lower left corner of the plots with a spread in parameters values less than 3 orders of magnitude for pulsars in subset II. Apparently, this suggests that the parameter values are relatively smaller and do not vary widely. For objects in subset I, the parameters (both TNAPs and spin-down parameters) occupy regions extending from the lower left corner to the top right corner of the plots (covering $\sim 4 - 6$ orders of magnitude in parameter space). This corresponds to relatively larger and widely different parameter values. This result can be better understood if the anomalous values of $\ddot{\nu}$ and the TAPs are regarded as quantitative measures of pulsar timing fluctuations (e.g. Cordes & Downs 1985; Chukwude 2003, 2007; Hobbs et al. 2010). In this context, timing fluctuations basically contaminate the deterministic frequency second derivative, causing the measured $\ddot{\nu}$ to have spurious values. Similarly, the TAPs merely quantify the strengths of fluctuations produced by TN or WN. In principle, the calculated values of $|\ddot{\nu}_{\rm obs}|$ and the TAPs will depend, and to a large extent directly, on the amplitudes (or strengths) of timing fluctuations. In general, the amplitudes of fluctuations arising from WN processes are expected to be significantly smaller both in real size and the range of values than those caused by TN.

Recently, Shannon & Cordes (2010) proposed the use of the rms timing residuals after a second order fit ($\sigma_{\rm TN}$) as an appropriate measure of the strength (or the amplitude) of TN in pulsars. It is found that the parameter values span $\sim$ 4 orders of magnitude (0.5$-$3050 ms) for pulsars with dominant TN activity, with only about 10 pulsars having $\sigma_{\rm TN} < 2$ ms. For pulsars in subset II, $\sigma_{\rm TN}$ has values in the range of 0$-$8 ms. Furthermore, over 130 pulsars in this subset are characterized by $\sigma_{\rm TN} < 1$ ms, with 60 of them having $\sigma_{\rm TN} = 0$. These results are generally consistent with our proposition that the observed timing history of pulsars in subsets I and II is strongly dominated by TN and WN fluctuations, respectively. The relatively large dispersion seen in the values of $\sigma_{\rm TN}$ for TN-dominated pulsars is a direct consequence of the fact that genuine spin fluctuations arise from a mixture of several timing activities, resulting in a wide range of fluctuation amplitudes (e.g. Cordes & Downs 1985; D'Alessandro 1996; Chukwude 2002). The expected wide range of TN fluctuation amplitudes could plausibly account for the observed large dispersion in the TAPs of these pulsars. Hence, radio pulsars in subset I are expected to be associated with relatively larger and widely different values of $|\ddot{\nu}_{\rm obs}|$ and TNAPs. On the other hand, the small spread in the timing fluctuation amplitudes ($\sigma_{\rm TN} \sim 0.1 - 1$ ms) of pulsars in subset II could be attributed to very limited sources of WN fluctuations, which in our case are largely measurement errors (e.g. Hobbs et al. 2010). This result naturally explains the concentration of the estimated TAPs of pulsars in subset II in the lower left corner of the graphs. The small dispersions of $\sim$ 2 orders of magnitude observed in the values of $\sigma_{\rm TN}$, which translate to 2$-$3 orders of magnitude spread in WNAPs, could be attributed to differences in the signal-to-noise ratio (SNR) of the pulsars.

Furthermore, the very strong ($r \sim +0.9$) and very weak ($|r| < 0.1$) correlations, which characterize the relationships between $|\ddot{\nu}_{\rm obs}|$ and the TAPs (A, $\sigma_{\rm R23}$ and $\sigma_{\rm z}$) of pulsars in subsets I and II, respectively, are as expected if they merely quantify timing fluctuations in these objects. In principle, if the anomalous $\ddot{\nu}$ and the TNAPs are regarded as simply probes of a similar phenomenon, the low-frequency structures in timing residuals of pulsars in subset I, the observed magnitudes of the parameters would depend directly on the strength of TN fluctuations. On this premise, the estimated values of the TNAPs and $\ddot{\nu}_{\rm obs}$, irrespective of signs, would be expected to correlate strongly, as strong TN would lead to large values of the $|\ddot{\nu}_{\rm obs}|$ and the TNAPs (e.g. Chukwude 2003; Hobbs et al. 2010). On the other hand, the timing activity of pulsars in subset II is believed to be dominated by WN fluctuations (largely measurement errors), with statistics that are characterized by equal power at all fluctuation frequencies (Cordes & Downs 1985; D'Alessandro et al. 1995). For this set of objects, we argue that the parameters $\ddot{\nu}_{\rm obs}$ and WNAPs represent quantitative assessments of measurement errors (i.e. fluctuations of nearly equal amplitudes). In general for these 143 pulsars in subset II, we have that $\sigma_{\rm R23} \approx \sigma_{\rm WN}$,

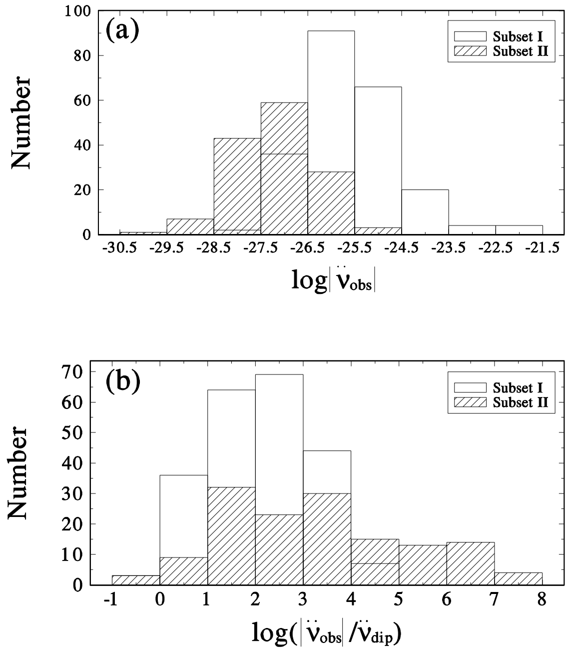

Fig. 1 Plots showing the distributions of (a) absolute values of the observed frequency second derivatives ($|\ddot{\nu}_{\rm obs}|$) and (b) ratio of $|\ddot{\nu}_{\rm obs}|$ and model-predicted systematic frequency second derivative ($\ddot{\nu}_{\rm dip}$) for the 366 JBO pulsars on logarithmic scales.

$\ddot{\nu}_{\rm obs} \approx \ddot{\nu}_{\rm wno}$ and $\sigma_{\rm z} \approx \sigma_{\rm z[WN]}$, implying that the diagnostics have random values that are independent of dataspan length. Hence, the parameters used to characterize these fluctuations are expected to have nearly equal values for a particular approach and the values from different approaches would be largely uncorrelated. This possibly explains both the small dispersion in parameter values and lack of any meaningful correlation between $|\ddot{\nu}_{\rm obs}|$ and the WNAPs.

The strong correlations ($r > +0.80$) found between $|\ddot{\nu}_{\rm obs}|$ and the spin-down parameters (the spin-down rate ($\dot{\nu}$), characteristic age ($\tau_{\rm c}$) and the systematic frequency second derivative ($\ddot{\nu}_{\rm dip} = \frac{3\dot{\nu}^2}{\nu}$)) in subset I can be explained if the TN of pulsars is regarded as an intrinsic process linked directly to the spin-down of the underlying neutron stars. Several previous analyses have demonstrated that pulsars that appear to spin-down more rapidly are characterized by large values of both TN diagnostics and $|\ddot{\nu}_{\rm obs}|$ (Cordes & Downs 1985; D'Alessandro et al. 1993; Arzoumanian et al. 1994; Urama et al. 2006 Chukwude 2007; Beskin et al. 2006; Hobbs et al. 2010). A plausible scenario is that TN is a direct consequence of some form of intrinsic stochastic fluctuations in pulsars' spin-down torque (Urama et al. 2006). Several authors have described in detail how TN fluctuations could result from either internal torque variations arising from complex vortex dynamics involving irregular transfer of angular momentum between the superfluid interior and stellar crust (e.g. Alpar et al. 1986; Jones 1990) or external torque fluctuations involving changes in magnetospheric current of pulsars (e.g. Cheng 1987; Kramer et al. 2006; Lyne et al. 2010; Chukwude & Buchner 2012). Another school of thought believes that strong $\ddot{\nu} - |\dot{\nu}|$ dependence could be sufficiently explained as an evolutionary process, in which the pulsar spin rates oscillate with widely different amplitudes on timescales of $\sim 100 - 1000$ years (Beskin et al. 2006; Biryukov et al. 2007, 2012). In both scenarios, the amplitudes of the variations would significantly depend on magnitude of the spin-down torque, which is directly proportional to $|\dot{\nu}|$, such that pulsars with larger $|\dot{\nu}|$ will generally exhibit enhanced TN activity. In principle, this translates directly to larger values of TNAPs ($\sigma_{\rm TN}, A_{\rm TN}, \sigma_{\rm z[TN]}$) and $|\ddot{\nu}_{\rm obs}| \equiv \ddot{\nu}_{\rm tno}$ for pulsars with relatively short characteristic ages. For the relatively older and less rapidly spinning down pulsars in subset II, both the WNAPs and $|\ddot{\nu}_{\rm obs}|$ quantify the measurement noise, which is unrelated to the spin-down mechanism. This plausibly explains the lack of any meaningful correlation between the WNAPs and the spin-down properties of the pulsars.

Variable amplitude intrinsic scatter in the timing residuals of some radio pulsars has been observed in some detail from several pulsar timing programs (Cordes & Downs 1985; D'Alessandro et al. 1995; Chukwude 2002; Hobbs et al. 2010). Perhaps, what is striking about the current JBO sample is the widespread dominance of WN fluctuations. About 140 objects ($\sim 40\,\%$ of the entire sample of 366 radio pulsars) have their timing residuals strongly dominated by fluctuations that are characteristic of measurement errors on timescales of $\sim 10 - 30$ yr (see fig. 1 of Hobbs et al. 2010). What factors could be responsible for this unusual high incidence of WN-dominated timing residuals in the sample? It is noteworthy that most of the pulsars in subset II are characterized by very low flux density ($S_{(1.4{\rm GHz})} < 1$ mJy: Gould & Lyne 1998). For

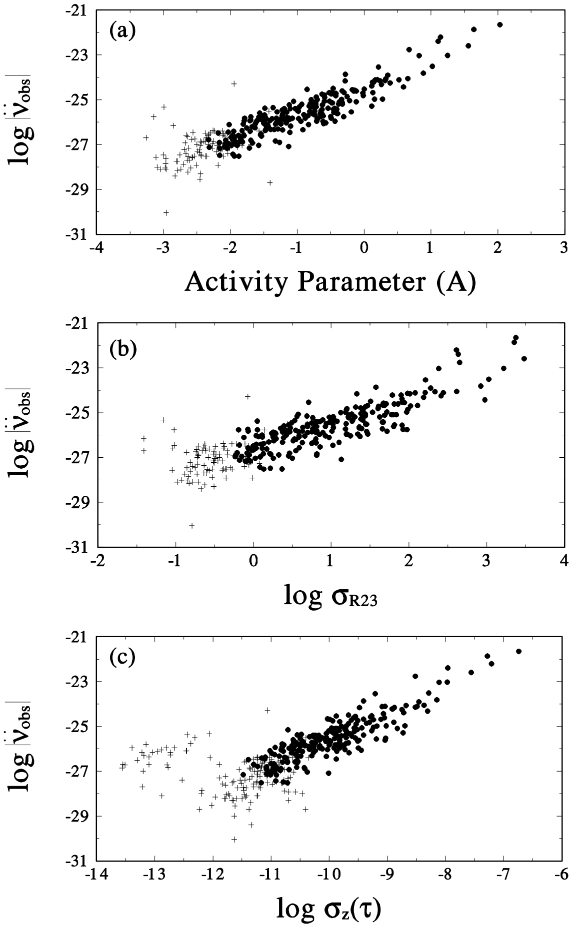

Fig. 2 Scatter plots showing the relationships between the anomalous frequency second derivatives, irrespective of sign, ($|\ddot{\nu}_{\rm obs}|$) and (a) the activity parameter (A), (b) the quantitative measure of the amount of timing fluctuation absorbed by the frequency second derivative ($\sigma_{\rm R23}$) and (c) the stability parameter ($\sigma_{\rm z}(T)$), on logarithmic scales, for the 366 JBO radio pulsars. Dots (•) represent pulsars with $\beta > 0.01$ ms yr^{-1} and pluses (+) represent pulsars with $\beta < 0.01$ ms yr^{-1}.

these pulsars, it is observed that the average flux density is generally less than 0.5 mJy. Remarkably, over 400 radio pulsars are routinely observed in the JBO pulsar timing program, necessitating very short (< 3 min) integration times (D'Amico et al. 1998; Hobbs et al. 2004). A combination of low flux density and short integration time could significantly reduce the quality of the data, resulting in very low SNR observations. Cordes & Downs (1985) have argued that while measurement uncertainty could be the dominant component of fluctuations seen in the timing residuals of pulsars with low SNR, it is inconsequential for objects with high SNR. However, few apparently bright pulsars ($S_{(1.4{\rm GHz})} > 1$ mJy (Lorimer et al. 1995)) still show timing residuals that have the imprint of WN. The observed timing fluctuations of this category of pulsars could be dominated by magnetospheric effects (e.g. random phase jitter of the pulse and pulse shape changes arising from mode changes (Cordes & Downs 1985)).

The result presented in this paper has an obvious implication on the current quest for realistic measurements of braking indices that sufficiently describe the intrinsic spin-down evolution of radio pulsars. So far, efforts in this direction have concentrated on pulsars with relatively small $\tau_{\rm c}$, which are generally associated with large $\dot{\nu}$ and $\ddot{\nu}$ (the so called young pulsars). Incidentally, this class of pulsars is notorious for supporting a wide range of intrinsic rotational instabilities (e.g. glitches, microglitches and TN) whose effects have seriously hampered realistic measurements of $\ddot{\nu}$ (e.g. Lorimer & Kramer 2004). Although pulsars with large $\tau_{\rm c}$ are generally associated with a relatively lower level of intrinsic rotational fluctuations, their expected systematic $\ddot{\nu}$ could be extremely small, making direct measurements very difficult (Cordes & Downs 1985; D'Alessandro et al. 1995; Hobbs et al. 2010). However, current analysis has sufficiently demonstrated that the measured values of $\ddot{\nu}$ for some of this category of pulsars could be severely contaminated by measurement noise. Interestingly, the amplitudes of measurement noise, unlike the intrinsic pulsar rotational instabilities, can be significantly reduced by adjusting some parameters of the observing hardware (e.g. increasing the observing bandwidth and/or the integration time (Lorimer & Kramer 2004)). Hence, relatively old radio pulsars, especially those whose long-term rotational history appears

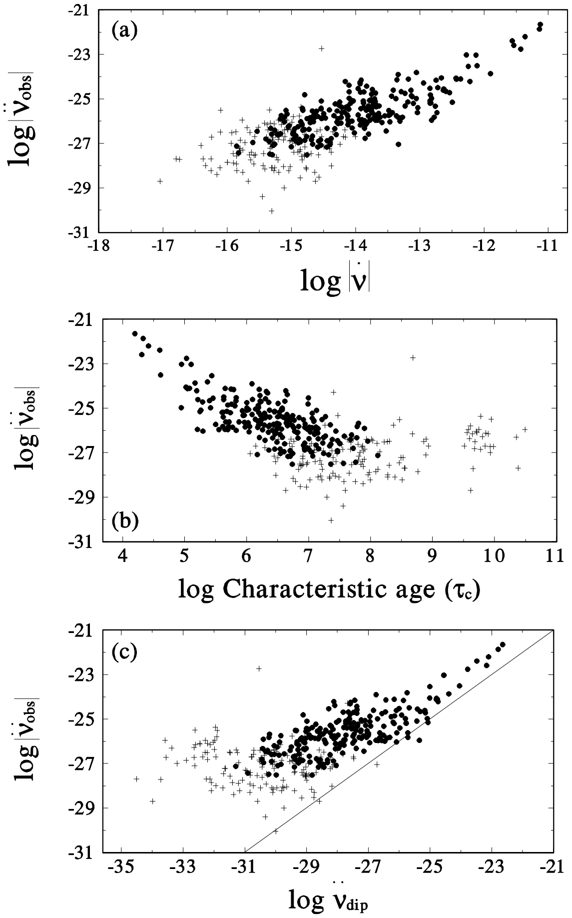

Fig. 3 Scatter plots showing the relationships between the absolute magnitude of the observed anomalous frequency second derivative ($|\ddot{\nu}_{\rm obs}|$) and (a) the absolute magnitude of the spin-down rate ($|\dot{\nu}|$), (b) the characteristic age (τ_c) and (c) the deterministic frequency second derivative predicted from the standard pure magnetic dipole breaking model ($\ddot{\nu}_{\rm dip}$), on logarithmic scales, for the 366 JBO radio pulsars. Dots (●) represent pulsars with $\beta > 0.01$ ms yr^{-1} and pluses ($+$) represent pulsars with $\beta < 0.01$ ms yr^{-1}.

to be dominated by measurement noise, could as well be explored for braking index measurements. For this category of pulsars, emphasis should be on appropriate instrumentation that minimizes measurement noise. In this case, the requirement is that the measurement noise would be at a level where $\ddot{\nu}_{\rm wno} \ll \ddot{\nu}_{\rm dip}$. This could be reasonably achieved by carefully selecting the most suitable parameters of the pulsar observing hardware to maximize the SNR of the radio pulsars. In practice, this would involve use of the world's largest telescopes and optimizing the observing bandwidth, integration time and effective receiver temperature (e.g. Lorimer & Kramer 2004).

6 CONCLUSIONS

We have demonstrated, using a large heterogeneous sample of 366 radio pulsars with accurately measured spin-down properties, that the timing behavior of pulsars on a wide range of timescales is sufficiently dominated by either TN or WN fluctuations. In particular, our results show that the anomalous values of frequency second derivative of most radio pulsars, obtained from the standard timing technique, are mere measures of the strength of timing fluctuations. The result also highlights the possibility of extending current efforts for realistic measurements of deterministic $\ddot{\nu}$ to relatively old high flux density radio pulsars that show dominant WN timing activity.

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