RAA 2016 Vol. **16** No. **1**, 7 (8pp) doi: 10.1088/1674–4527/16/1/007
http://www.raa-journal.org *http://iopscience.iop.org/raa*

Research in Astronomy and Astrophysics

Automatic generation of optical initial configuration based on Delano diagram

Kai-Yuan Zhang[1,2,3], Xiang-Yan Yuan[1,2] and Xiang-Qun Cui[1,2]

[1] National Astronomical Observatories/Nanjing Institute of Astronomical Optics & Technology, Nanjing 210042, China; *xyyuan@niaot.ac.cn*
[2] Key Laboratory of Astronomical Optics & Technology, Nanjing Institute of Astronomical Optics & Technology, Chinese Academy of Sciences, Nanjing 210042, China
[3] University of Chinese Academy of Sciences, Beijing 100049, China

Received 2015 March 9; accepted 2015 May 21

Abstract This paper presents a method to automatically initialize an optical system based on the Delano diagram. The process to generate the optical initial configuration is constrained by the control points, which are deduced from parameters related to basic design requirements. We present the theory and method to generate the optical initial configuration automatically when the basic design requirements are known. Two optical systems are taken as examples to demonstrate the proposed method.

Key words: telescopes — methods: analytical

1 INTRODUCTION

The design of an optical system is convenient with the aid of optical optimization software, provided that a reasonable initial configuration has been obtained. There are many references (Fischer et al. 2008; Kingslake & Johnson 2010; Geary 2002) regarding how to optimize and improve the performance of an initial configuration.

There has been, however, no general method about how to calculate initial values of an optical system. In this paper, we present a method to automatically generate an optical initial configuration when the basic design requirements are known. This method is based on the Delano diagram, which was proposed by Erwin Delano in 1963 (Delano 1963). The Delano diagram is an excellent tool to illustrate the first-order parameters for a given system, and has been utilized in optical designs since then (Shack 1974; Bauman 2003; Pegis et al. 1967; Kessler & Shack 1992; Zhuang et al. 1982). However, there is no general method or process described in previous research about the Delano diagram. The general method in this paper is constrained by control points in the Delano diagram, which are deduced directly from the parameters of the design requirements and can thus ensure that the configuration obtained can fulfill the design requirements. The theoretical relationship between the control points and the parameters of the design requirements is deduced and demonstrated in this paper. Based on the relationship between the control points and the design goal of a given optical system, we have written the program OMAX (one key to generate the optical initial configuration in Zemax) that acts as an interface between Matlab and Zemax. In Section 2, the basic properties of the Delano diagram are introduced, which are used to deduce the control points in Section 3; in Section 3, the theoretical relationship between the control points and the parameters of the design requirements is demonstrated for systems with a finite object distance and an infinite object distance; in Section 4, the automatic process to generate the initial configuration under the constraints of the control points is introduced, and in Section 5, two optical systems designed by the proposed method are presented.

2 BASIC PROPERTIES OF THE DELANO DIAGRAM

The Delano diagram, $(\bar{y}_i, y_i)$ is used to illustrate the heights of the principal ray and the marginal ray propagating through an optical system (Fig. 1). When the Lagrange invariant of the system is known, all of the other first-order parameters can be obtained from the Delano diagram. For the optical system, Lagrange invariant Q is defined in Equation (1) (Delano 1963)

$$Q = ny\bar{u} - n\bar{y}u. \tag{1}$$

If Equation (1) is changed to the form given in Equation (2)

$$y = \frac{u}{\bar{u}}\bar{y} + \frac{Q}{n\bar{u}}, \tag{2}$$

then it is clear that the Delano diagram is the diagram illustrating the Lagrange invariant in different positions of the optical system, and the slope of the line in the diagram represents the ratio of the angle of the marginal ray to the angle of principal ray.

Figure 1 is a Delano diagram, which represents an optical system containing three optical components. The components labeled J and M are the object and image plane in the Delano diagram, and A, B and C are three optical components. JA is the line that connects the object plane and the first component, and CM is the line that connects the last component and the image plane. The component P is the intersection point of the line JA and CM, and it represents the principal plane of the system. The line OF' can be constructed to be parallel to line JA and to intersect line CM at the point F'. Then F' represents the image focal plane of the system. The focal length of the system can be obtained by the cross product of vectors as follows (Delano 1963)

$$f' = \frac{OF' \times OP}{Q}. \tag{3}$$

JA intersects the y axis at point S', which represents the position of the entrance pupil of the system. CM intersects the y axis at point S'', which represents the position of the exit pupil. The distance between two successive components is (Delano 1963)

$$d_{i,i+1} = \frac{(\bar{y}_{i+1}, y_{i+1}) \times (\bar{y}_i, y_i)}{Q}. \tag{4}$$

Line OF'_c can be constructed to be parallel to line BC, and to intersect the line CM at point F'_c. F'_c represents the image focal plane of the component C, and the focal length of the component C is given below (Delano 1963)

$$f'_c = \frac{OF'_c \times OC}{Q}. \tag{5}$$

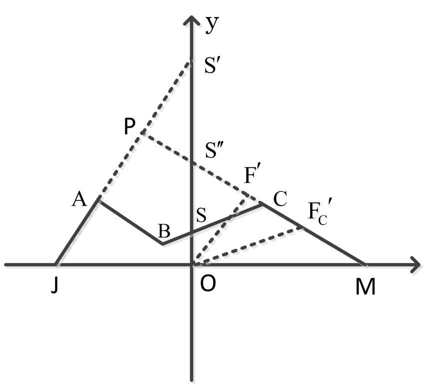

Fig. 1 The Delano diagram of an optical system with three components.

3 THE RELATIONSHIP BETWEEN THE CONTROL POINTS AND DESIGN GOAL

In this section, the relationship between the control points and the basic design goal is deduced. For a system with a finite object distance and an infinite object distance, the control points are different. In Section 3.1, the control points of the system with a finite object distance will be introduced, and in Section 3.2, the control points of the system with an infinite object distance will be introduced. The control

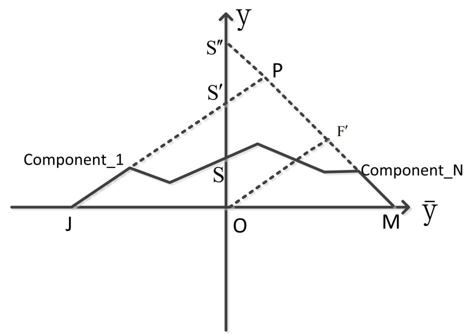

Fig. 2 Control points of a system with a finite object distance are shown in the Delano diagram.

points and design goal parameters are necessary and sufficient conditions to undertake this process. From the design goal, we can obtain accurate values of control points, and we can achieve the design goal by making the system fulfill the constraint conditions by using the control points.

3.1 Control Points of a System with a Finite Object Distance

For an optical system with a finite object distance, we establish the basic design requirements as follows:

– The object height: h_J.
– The image height: h_M.
– The object numerical aperture: NA.
– The focal length: f'.
– The entrance pupil diameter: D.

Figure 2 shows the control points J, P and M of the optical system with a finite object distance. J in Figure 2 represents the object plane, P represents the principal plane and M represents the image plane. From the basic design goal of the object height and image height, the coordinates of J and M can be obtained as:

$$\begin{cases} \bar{y}_J = h_J \,, \\ y_J = 0 \,, \end{cases} \tag{6}$$

and

$$\begin{cases} \bar{y}_M = h_M \,, \\ y_M = 0 \,. \end{cases} \tag{7}$$

From the relationship between the numerical aperture NA and the object aperture angle u_J given in Equation (8)

$$NA = n_J \sin u_J \,, \tag{8}$$

the object aperture angle can be calculated as

$$u_J = \arcsin \frac{NA}{n_J} \,. \tag{9}$$

Line JS' is the line that connects the object plane and the first optical component, thus the slope of JS' can be obtained by Equation (2) as follows

$$K_{JS'} = \frac{u_J}{\bar{u}_J} \,. \tag{10}$$

In the Delano diagram, the coordinates of the entrance pupil can be obtained from the basic design goal

$$\begin{cases} \bar{y}_{s'} = 0\,, \\ y_{s'} = \dfrac{D}{2}\,. \end{cases} \tag{11}$$

The slope of JS' can thus be denoted as

$$K_{JS'} = -\frac{D}{2h_J}\,. \tag{12}$$

Combining Equations (9), (10) and (12), the angles of the marginal ray and principal ray at the object plane can be obtained as

$$\begin{cases} u_J = \arcsin\dfrac{NA}{n_J}\,, \\ \bar{u}_J = -\dfrac{2h_J}{D}\arcsin\dfrac{NA}{n_J}\,. \end{cases} \tag{13}$$

The Lagrange invariant of a system with a finite object distance can be calculated at the object plane by Equations (1), (6) and (13)

$$Q = -n_J h_J \arcsin\frac{NA}{n_J}. \tag{14}$$

Due to the fact that JS' and $S'P$ are on the same line, the slopes of JS' and the $S'P$ are therefore equal, and satisfy the relation

$$\frac{y_p - \frac{D}{2}}{\bar{y}_p} = \frac{\frac{D}{2} - 0}{0 - h_J}. \tag{15}$$

The coordinates of the principal plane calculated from Equation (15) can be denoted as

$$\begin{cases} \bar{y}_p = h_J - \dfrac{2h_J y_p}{D}\,, \\ y_p = y_p\,. \end{cases} \tag{16}$$

Constructing the line OF' parallel to the line JP, and setting OF' to intersect PM at F', the relation for F' can be obtained as Equation (17), which is the focal plane of the system

$$\begin{cases} y_{F'} = -\dfrac{D}{2h_J}\bar{y}_{F'}\,, \\ y_{F'} = \dfrac{y_p}{h_J - \frac{2h_J y_p}{D} - h_M}(\bar{y}_{F'} - h_M)\,. \end{cases} \tag{17}$$

After solving Equation (17), the coordinates of F' can be defined as

$$\begin{cases} \bar{y}_{F'} = \dfrac{2y_p h_M}{D(1 - \frac{h_M}{h_J})}\,, \\ y_{F'} = \dfrac{y_p h_M}{h_M - h_J}\,. \end{cases} \tag{18}$$

The focal length can be obtained in the Delano diagram by solving Equation (3). Combining Equations (3), (16) and (18), the coordinates of the principal plane can be obtained as shown below

$$\begin{cases} \bar{y}_p = h_J - \dfrac{2Qf'(h_J - h_M)}{Dh_M}\,, \\ y_p = \dfrac{Qf'(h_j - h_M)}{h_M h_J}\,. \end{cases} \tag{19}$$

For a given system, the coordinate in the Delano diagram of the control points J, P and M have been obtained as described by Equations (6), (7) and (19), which are entirely determined by the parameters for the design requirements (h_J, h_M, NA, f', D), regardless of the values of the design parameters. However, if the designers set the control points J, P and M as constraints during the design process, the final outcome must fulfill the design requirements as shown in this section.

3.2 Control Points of a System with an Infinite Object Distance

For an optical system with an infinite object distance, we have established the basic design requirements that are listed below:

- The half field of view: θ.
- The focal length: f'.
- The entrance pupil diameter: D.

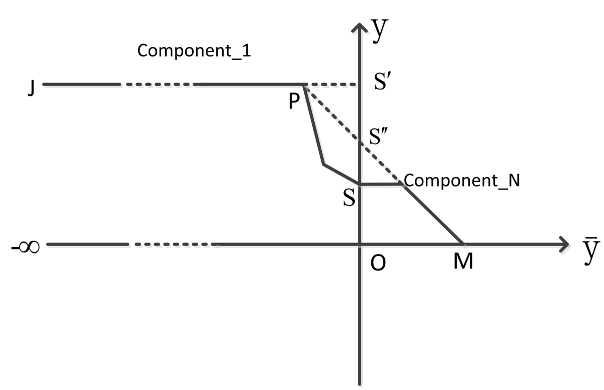

Fig. 3 Control points of a system with an infinite object distance.

Figure 3 is the Delano diagram corresponding to a system with an infinite object distance. J represents the object plane, and JP is the line connecting the object plane to the principal plane in the diagram. For this type of system, JP is parallel to the $\bar{y}$ axis. The entrance pupil plane is represented by S', and its coordinates can be obtained from the design requirements, as shown in Equation (11), where D is also the diameter of the entrance pupil. Because JP and PS' are on the same line, and J represents the object plane with an infinite distance, the coordinates defining object plane J are as follows

$$\begin{cases} \bar{y}_J = -\infty\,, \\ y_J = \dfrac{D}{2}\,. \end{cases} \tag{20}$$

The angles of the principal ray and the marginal ray at the object plane J are

$$\begin{cases} \bar{u}_J = \theta\,, \\ u_J = 0\,. \end{cases} \tag{21}$$

At the image plane M, the principal ray height can be obtained by the relationship between the image height and the half field of view (Bentley & Olson 2012), and the marginal ray height is 0 on the image plane. The coordinates for the image plane can be defined as follows

$$\begin{cases} \bar{y}_M = f' \tan\theta\,, \\ y_M = 0\,. \end{cases} \tag{22}$$

By Equations (1), (20) and (21), the Lagrange invariant can be obtained at the object plane as

$$Q = n\frac{D}{2}\theta. \tag{23}$$

By Equations (20) and (22), information about the design requirements can be fully included. We thus set J and M as the control points for the system with an infinite object distance. If the designers want the design outcome to fulfill the basic design requirements that are described in this section, the relationship shown by Equations (20) and (22) must be fulfilled during the design process, and vice versa.

4 PROCEDURE FOR AUTOMATIC GENERATION OF THE OPTICAL CONFIGURATION

For an optical system with a finite object distance, we set the number of optical components as N. If the basic design requirements in Section 3 are satisfied, the control points must satisfy Equations (6), (7) and (19). Because JP is the line that connects the object plane to the first optical plane, and PM is the line that connects the last optical component and to the image plane, to make the control points fulfill Equations (6), (7) and (19), the coordinates of the first optical component must be located on the line JP and coordinates of the last optical component must be located on the line PM in Figure 2. Then the necessary constraint conditions which would ensure that the configuration would fulfill the design requirements can be obtained as shown in Equation (24) below

$$\begin{cases} \bar{y}_J = h_J\,, \\ y_J = 0\,, \\ \bar{y}_M = h_M\,, \\ y_M = 0\,, \\ \dfrac{y_1 - y_J}{\bar{y}_1 - \bar{y}_J} = \dfrac{y_p - y_J}{\bar{y}_p - \bar{y}_J}\,, \\ \dfrac{y_N - y_p}{\bar{y}_N - \bar{y}_P} = \dfrac{y_p - y_M}{\bar{y}_p - \bar{y}_M}\,. \end{cases} \tag{24}$$

Equation (24) just defines the constraint condition of the object plane, image plane and the first and last optical components in the Delano diagram. The coordinate of the other optical component needs to be designed or arranged based on the following direction rule, which is clockwise for $Q > 0$ and counterclockwise for $Q < 0$.

Thus for a system with N optical components, there are a total of $(2N - 2)$ degrees of freedom for y and $\bar{y}$ that can be arranged freely.

For an optical system with an infinite object distance, it is required that the first component is on the line JS' as shown in Figure 3, and then the constraint condition can be obtained as

$$\begin{cases} \bar{y}_J = -\infty\,, \\ y_J = \dfrac{D}{2}\,, \\ \bar{y}_M = f' \tan\theta\,, \\ y_M = 0\,, \\ y_1 = \dfrac{D}{2}\,. \end{cases} \tag{25}$$

The basic design requirements will be satisfied under the constraints defined by Equation (25) for a system with an infinite object distance. The direction for the system with an infinite object distance is the same as that for a system with a finite object distance. For an optical system with N optical components, there will be a total of $(2N - 1)$ degrees of freedom for y and $\bar{y}$.

If the number of optical components is larger than two, there would be an infinite number of possible configurations that can fulfill the basic design requirements under the constraints defined by Equation (24) for a system with a finite object distance, or for constraints defined by Equation (25) for a system with an infinite object distance. It is known that aberrations are induced by reflection or refraction of a ray on each optical component. One excellent initial configuration can be obtained if an objective function that measures the total deflection angle from all of the optical components is minimized. The objective function that optimizes objective as

$$\begin{aligned} &\mathrm{objective}\ (y_i, \bar{y}_i) = \\ &\textstyle\sum_{i=1}^{N} \left(|u_{i1} - u_{i0}| + |\bar{u}_{i1} - \bar{u}_{i0}|\right). \end{aligned} \tag{26}$$

All of the marginal and principal ray angles can be obtained at each optical component by the paraxial ray trace Equation (3). We put the ray trace equation in an iterative form as shown in Equation (27)

$$\begin{cases} u_{10} = u_J\,, \\ \bar{u}_{10} = \bar{u}_J\,, \\ u_{i1} = \dfrac{n_i u_{i0} - y_i \varphi_i}{n'_i}\,, \\ \bar{u}_{i1} = \dfrac{n_i \bar{u}_{i0} - \bar{y}_i \varphi_i}{n'_i}\,, \\ u_{(i+1)0} = u_{i1}\,, \\ \bar{u}_{(i+1)0} = \bar{u}_{i1}, \end{cases} \tag{27}$$

where u_{i1} represents the marginal ray angle after propagating through the i^{th} component, and u_{i0} means the marginal

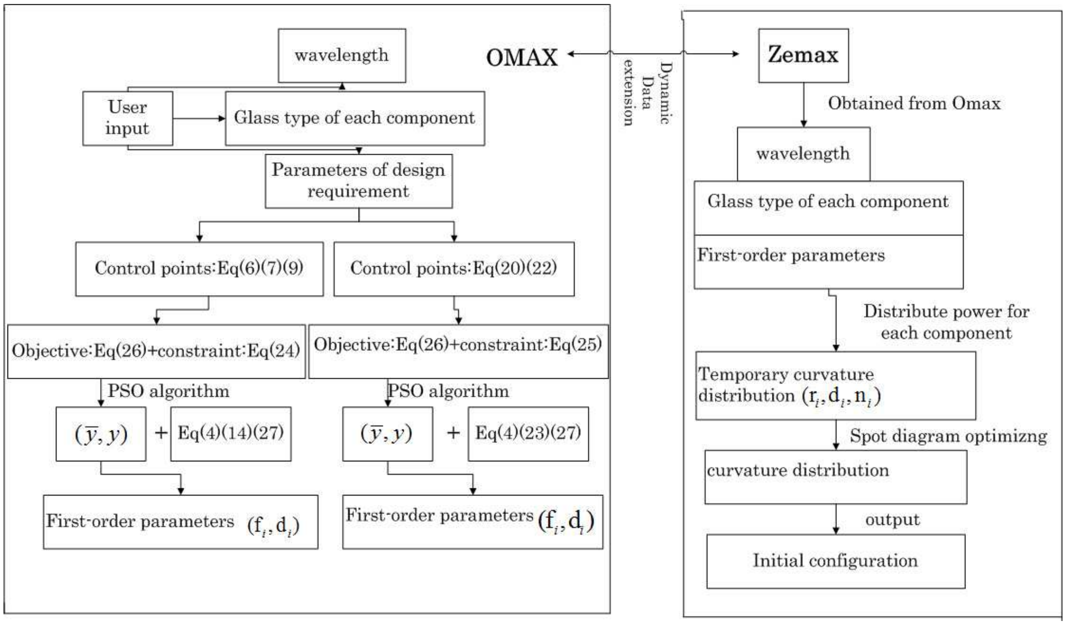

Fig. 4 The process to automatically obtain the initial configuration.

ray angle before reaching the i^{th} component. It has the same meaning for the principal ray angle $\bar{u}_i$. $(\bar{u}_J, u_J)$ is defined by Equation (13) for a system with a finite object distance, and defined by Equation (21) for a system with an infinite object distance. The focal power of the i^{th} optical component is φ_i, and can be calculated as

$$\varphi_i = \frac{Q(k_{i-1,i} - k_{i,i+1})}{(y_i - k_{i-1,i}\bar{y}_i)(y_i - k_{i,i+1}\bar{y}_i)}, \tag{28}$$

where

$$k_{i,i+1} = \frac{y_{i+1} - y_i}{\bar{y}_{i+1} - \bar{y}_i}. \tag{29}$$

Q is given by Equation (14) for a system with a finite object distance, and given by Equation (23) for a system with an infinite object distance. From Equations (27), (28) and (29), it can be shown that the variables defined by the objective function in Equation (26) are $(\bar{y}_i, y_i)$. Utilizing the optimization algorithm, it is easy to search for values related to every optical component that makes Equation (26) achieve a local minimum, and $(\bar{y}_i, y_i)$ can make the system fulfill the basic design goal. After automatically finding $(\bar{y}_i, y_i)$ by the optimization algorithm, all of the first-order parameters of the system can be obtained by Equations (4) and (28). The optimization algorithm we used is Particle Swarm Optimization (PSO). Knowing all of the first-order parameters, we just need to implement the curvatures for every optical component to get the initial configuration. We have implemented this as a dynamic data extension between Matlab and Zemax. By solving for the focal power of each component, the temporary curvatures of each component are implemented. Then we run several loops of the optimization process in Zemax to get better values for curvature. The initial configuration can be obtained in Zemax by the interface program OMAX. The flow chart for finding the first-order parameters that can generate the initial configuration in Zemax is shown in Figure 4. This process is controlled by the interface program OMAX.

5 EXAMPLES OF THE AUTOMATIC GENERATION OF THE INITIAL CONFIGURATION

In this section, two examples are presented to show the process of generating an optical initial configuration when the parameters related to the design requirement are known.

5.1 Telescope System with Two Mirrors and Three Corrector Lenses

The first system is a near infrared (NIR) survey telescope and the parameters related to the design requirement are:

- The half field of view: $\theta = \frac{1}{180}$ rad.
- The focal length: $f' = 3400$ mm.
- The entrance pupil diameter: $D = 680$ mm.

The wave band was set from 1 μm to 2.3 μm, and the glass type of the corrector lens was fused silicon. After we input the parameters related to the design requirement, as well as those related to glass and wavelength in the OMAX interface, the first-order parameters were obtained by the PSO algorithm, and the initial configuration was generated in Zemax. The first-order parameters are presented in Table 1.

In Figure 5, the Delano diagram of the first-order configuration, initial configuration and final configuration are . The final image quality is diffraction limited, as illustrated in Figure 6.

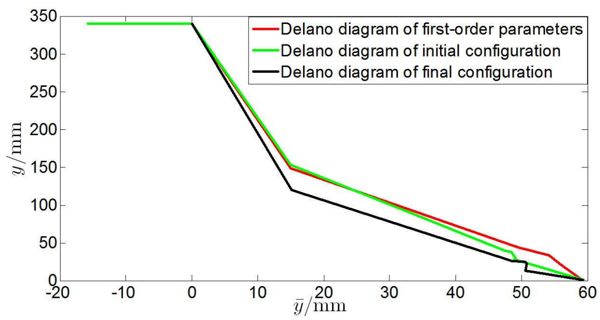

Fig. 5 Plotted values for the first-order parameters, initial configuration, and final configuration derived from the Delano diagram.

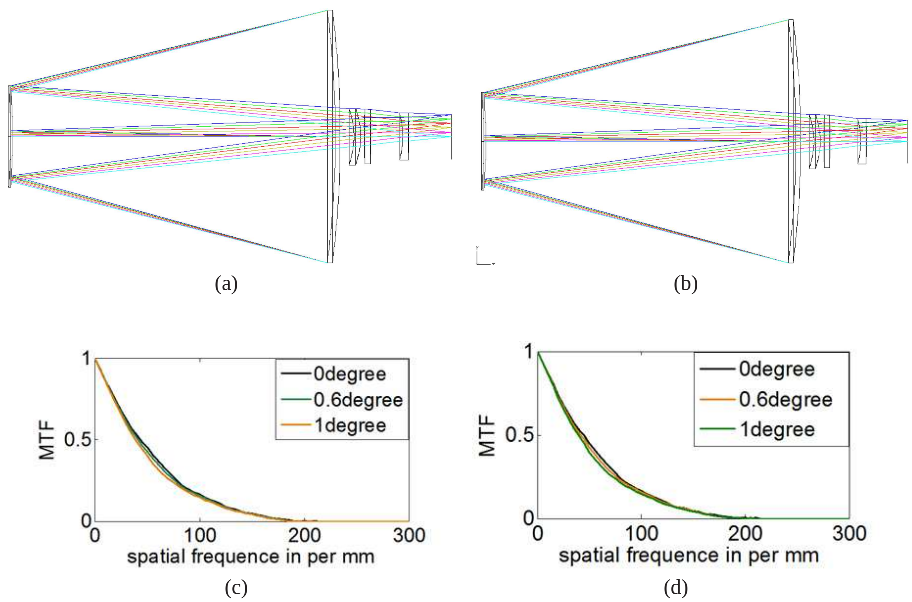

Fig. 6 (a) Initial configuration generated by OMAX; (b) final configuration optimized from Fig. 5; (c) Modulation Transfer Function (MTF) of the initial configuration; (d) MTF of the final configuration.

Table 1 The Outcome of Automatic Initialization for the NIR Survey Telescope

Item	$\bar{y}$ (mm)	y (mm)	f_i' (mm)	$d_{i,i+1}$ (mm)
Object	$-\infty$	340.0000	0	Inf
Mirror 1	0	340	1.5206×10^3	–856.8589
Mirror 2	14.9565	148.4070	-1.1335×10^3	1.1281×10^3
Lens 1	49.5320	43.8601	-8.5910×10^3	126.1984
Lens 2	54.1275	32.8089	2.6800×10^3	100.0014
Lens 3	55.7494	22.8275	1.2183×10^5	228.2747
image	59.3472	0		

5.2 Lithographic Lens System with 28 Components

In this section, generation of an optical system with finite object distance is presented. To demonstrate the effectiveness, we designed a lithographic system with the same design requirements as an example with an existing patent, and then compared the image quality. In the optical system patent library (Lensview), we found a lithographic system (the patent number is 6084723), and the parameters of its design requirement were:

– The object height: $h_J = -56$ mm.
– The image height: $h_M = 22.4$ mm.
– The object numerical aperture: $NA = 0.12$.
– The focal length: $f' = 880$ mm.
– The entrance pupil diameter: $D = 740$ mm.

Although it was designed for a wavelength of 365 nm, fused silica was used in the lens system. Because of the strict requirements related to field curvature and distortion, the system was designed with 28 components and became very complicated. We also designed it with 28 components. Using the OMAX interface, we directly obtained the first-order parameters as listed in Table 2, and then by distributing the curvature in the Zemax, which is also controlled by OMAX, we obtained the initial configuration. After obtaining the initial configuration, we can optimize its image quality to get the final configuration.

In Figure 7, the Delano diagram of the control points is shown with the Delano diagram of first-order parameters, the final configuration and corresponding values from the case of patent 6084723. As Figure 8 illustrates, the im-

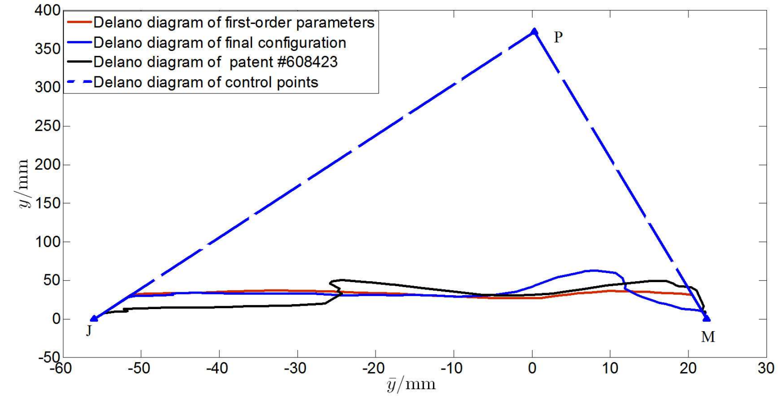

Fig. 7 Delano diagram of the first-order parameters, final configuration, patent configuration, and control points of the lithographic lens system.

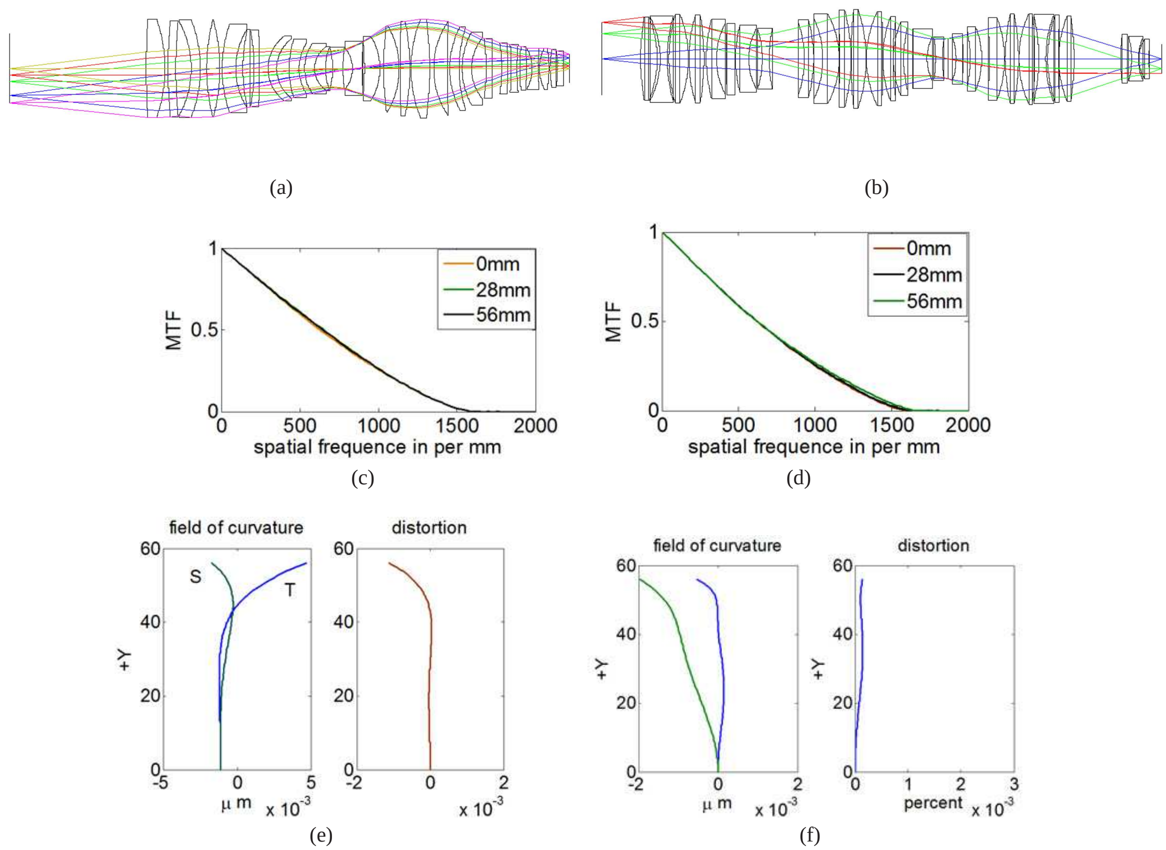

Fig. 8 (a) Design of a lithographic lens system based on the Delano diagram; (b) lithographic lens system of patent #6084723; (c), (e) the MTF, field curvature and distortion of the system in Fig. 8(a); (d), (f) the MTF, field curvature and distortion of the system in Fig. 8(b).

age quality of the system designed by the proposed method is as good as that done by the system with an existing patent. For a complicated optical system, there are an infinite number of solutions the designers can obtain if only a reasonable initial configuration can be obtained. Now by OMAX, the initial configuration can be generated automatically when the design requirement is known. The designers can even finish a complicated lithographic lens system in a very short time after obtaining the initial configuration by the proposed method.

Table 2 Values Generated by the Automatic Initialization of the Lithographic Lens System

Item	$\bar{y}$ (mm)	y (mm)	f'_i (mm)	$d_{i,i+1}$ (mm)
Object	–56	0	0	256.6667
Lens 1	–51.3044	31.0242	646.9688	20.4365
Lens 2	–49.3100	32.5144	561.9252	22.9195
Lens 3	–45.0619	32.8595	-1.2355×10^{3}	18.9603
Lens 4	–42.2392	33.6493	-2.1162×10^{3}	19.6659
Lens 5	–39.7040	34.7812	-2.4069×10^{3}	14.1522
Lens6	–38.1131	35.8002	1.1377×10^{3}	35.7461
Lens7	–32.8971	37.2492	404.6384	30.8830
Lens8	–25.8799	35.6582	634.6504	13.8825
Lens9	–22.1594	34.1630	2.4536×10^{8}	18.1224
Lens10	–17.3027	32.2112	703.7760	13.0042
Lens11	–13.4979	30.2154	–419.7836	17.1165
Lens12	–9.0403	28.8205	448.6767	14.8592
Lens13	–4.8712	26.6551	–183.1181	23.9051
Lens14	1.2001	26.6511	–57.3005	4.0447
Lens15	2.3121	28.5316	320.3266	4.8761
Lens16	3.6174	30.3645	576.3471	4.4752
Lens17	4.7874	31.8108	1.5583×10^{3}	4.8977
Lens18	6.0527	33.2937	255.1710	5.7713
Lens19	7.4068	34.2881	-3.4389×10^{3}	10.8001
Lens20	9.9641	36.2567	140.3043	9.1981
Lens21	11.4889	35.5563	1.2887×10^{5}	3.2493
Lens22	12.0272	35.3080	-1.3473×10^{3}	10.4253
Lens23	13.8476	34.7845	–352.1959	5.0033
Lens24	14.9179	35.0274	194.9064	7.4384
Lens25	15.9398	34.0518	–886.8182	8.9030
Lens26	17.3230	33.2259	-3.9396×10^{3}	10.4994
Lens27	19.0003	32.3405	961.0504	11.1573
Lens28	20.5622	31.0242	168.4239	102.6667
image	22.4000	0		

6 CONCLUSIONS

This paper has presented a method and process for automatically generating the optical initial configuration. This method is based on the theory between control points in a Delano diagram and design requirements. We have written the interface program OMAX, which can be used to generate the initial configuration in Zemax, making the optical design process much more convenient. This method can be used to generate a system that has rotational symmetry with finite focal length and an entrance pupil, and two design examples as shown. For an afocal system or a telecentric system, we have also deduced the relationship between the control points and design requirements, and are writing the associated interface program. After finishing it, we will publish an updated description of the method.

Acknowledgements This research was supported by the National Natural Science Foundation of China (Grant No. 11190011) and the National Basic Research Program of China (973 program, 2013CB834901).

References

Bauman, B. J. 2003, Optical Design for Extremely Large Telescope Adaptive Optics Systems, PhD Thesis, The University of Arizona

Bentley, J., & Olson, C. 2012, Field guide to lens design (SPIE)

Delano, E. 1963, Applied Optics, 2, 1251

Fischer, R. E., Tadic-Galeb, B., & Yoder, P. R. 2008, Optical System Design: Second Edition (The McGraw-Hill Companies)

Geary, J. M. 2002, Introduction to Lens Design: with Practical ZEMAX Examples (Willmann-Bell Richmond)

Kessler, D., & Shack, R. V. 1992, Applied optics, 31, 2692

Kingslake, R., & Johnson, R. B. 2010, Lens Design Fundamentals, Second Edition (SPIE and Academic Press)

Pegis, R., Vogl, T., Rigler, A., & Walters, R. 1967, Applied optics, 6, 969

Shack, R. V. 1974, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 39, 127

Zhuang, S., Zheng, Q., & Yu, F. T. 1982, Optics letters, 7, 581