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Thermal evolution of neutron stars with decaying magnetic fields

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Abstract Rotochemical heating originates in the deviation from beta equilibrium due to spin-down compression, which is closely related to the dipole magnetic field. We numerically calculate the deviation from chemical equilibrium and thermal evolution of neutron stars with decaying magnetic fields. We find that the power-law long term decay of the magnetic field slightly affects the deviation from chemical equilibrium and surface temperature. However, the magnetic decay leads to older neutron stars that could have a different surface temperature with the same magnetic field strength. That is, older neutron stars with a low magnetic field (10^8 G) could have a lower temperature even with rotochemical heating in operation, which probably explains the lack of other observations on older millisecond pulsars with higher surface temperature, except millisecond pulsar J0437–4715.

Key words: stars: neutron — stars: magnetic field — radiation mechanisms: thermal

1 INTRODUCTION

Neutron star cooling is an important tool to probe the inner structure of neutron stars. A neutron star initially loses thermal energy through neutrino emission, but this process is taken over by surface photon radiation about 10^5 yr after birth. The observations of surface thermal emission from a neutron star have the potential to provide constraints on its inner structure, dense matter inside it and magnetic field properties (Yakovlev & Pethick 2004; Page et al. 2006).

Recently, the observation of millisecond pulsar J0437–4715 shows that an old neutron star (after 10^7 yr) could have a high temperature (above 10^5K) (Kargaltsev et al. 2004), which suggests the importance of heating mechanisms in the thermal evolution of neutron stars. Several mechanisms have been studied, such as crust cracking (Cheng et al. 1992; Baym & Pines 1971) and superfluidity vortex creep (Alpar et al. 1984; Larson & Link 1999; Shibazaki & Lamb 1989), among which rotochemical heating is important in old neutron stars (Reisenegger 1995).

Rotochemical heating originates in the deviation from beta equilibrium due to spin-down compression. As a neutron star spins down, the decrease of centrifugal force leads to compression and increases the star's internal density, which results in a deviation from chemical equilibrium. The beta decay and its inverse process drive the system to a new equilibrium state, but as the reaction rate is slower than the deviation process due to compression, the star will never exactly be in chemical equilibrium. Thus energy is stored and dissipated by enhanced neutrino emission and heat generation. In an isolated neutron star, the heating that is generated comes from the kinetic energy of rotation.

It is generally recognized that neutron stars spin down due to magnetic dipole radiation, and the strength of the magnetic field affects the deviation from chemical equilibrium and heating efficiency (Reisenegger 1995). In previous studies, the magnetic fields are assumed to be constant in the evolution (Cheng & Dai 1996; Fernández & Reisenegger 2005; Petrovich & Reisenegger 2010; Wei & Zheng 2011). However, some studies indicated that the magnetic fields of neutron stars decay (Pons & Geppert 2007; Dall'Osso et al. 2012; Kojima & Kisaka 2012; Igoshev & Popov 2013). Recently, Zhang & Xie (Xie & Zhang 2011; Zhang & Xie 2012) tested models of the magnetic field evolution of neutron stars with the statistical properties of their spin evolution. They constructed a phenomenological model of the evolution of B, which contains a long term decay (LTD) modulated by shorter term oscillations (STO). A simple power-law LTD can explain all the observed statistical properties of the second derivatives of the spin frequencies. Motivated by this investigation, the aim of this work is to probe the deviation from chemical equilibrium and heating process in neutron stars with decaying magnetic fields. The paper is arranged as follows. In Section 2 we derive the equations of chemical and thermal evolution, and provide an application in a uniform neutron star model. The results and corresponding explanations are presented in Section 3. In Section 4 we give our conclusions and discussions.

2 CHEMICAL AND THERMAL EVOLUTION

For a neutron star, the relative concentrations of n, p and e matter are adjusted by the following weak reactions,

$$p + e^- \to n + \nu_e \,, n \to p + e^- + \overline{\nu}_e \,, \tag{1}$$

$$p + p + e^- \to n + p + \nu_e, n + p \to p + p + e^- + \overline{\nu}_e \,, \tag{2}$$

$$p + n + e^- \to n + n + \nu_e, n + n \to p + n + e^- + \overline{\nu}_e \,. \tag{3}$$

Equation (1) is a Durca reaction. Equations (2) and (3) are p branch and n branch Murca reactions, respectively.

As the neutron star spins down, the decrease in centrifugal force leads to compression and increases the star's internal density, which changes the chemical potential of each particle species and results in a deviation from chemical equilibrium. To express the degree of deviation from chemical equilibrium, we define the chemical potential difference in the npe matter

$$\delta\mu = \delta\mu_n - \delta\mu_p - \delta\mu_e \,, \tag{4}$$

where $\delta\mu_i = \mu_i - \mu_i^{\rm eq}$ is the deviation from the equilibrium chemical potential of species i.

Following Reisenegger (Reisenegger 1995), the time evolution equation of the chemical potential difference is

$$\frac{d\delta\mu}{dt} = -E_{xx}\left(n\frac{E_{nx}}{E_{xx}}\frac{\Omega\dot{\Omega}}{G\rho_c}\right) - \Gamma \,, \tag{5}$$

where G is the gravitational constant, ρ_c is the central density of the star, and Ω and $\dot{\Omega}$ are angular velocity and the derivative of angular velocity, respectively. n is the baryon number density, E_{nx} and E_{xx} are the partial derivatives of chemical energy per baryon with respect to baryon number density and composition parameter $x = n_p/n$ (e.g. $E_{nx} \equiv \partial^2 E/\partial n \partial x$) respectively, and Γ is the reaction rate per baryon for Murca reactions. In the last expression, the first term in parentheses accounts for the change in the chemical equilibrium state due to spin-down, and the second term for the change in the actual chemical state due to the reactions.

To solve this equation, a thermal equation is necessary, which reads

$$c_v \frac{dT}{dt} = \Gamma \delta\mu - \epsilon - \dot{E}_\gamma \, , \tag{6}$$

where the three terms on the right hand side, from left to right respectively, represent the chemical heating released, the energy radiated by neutrinos and antineutrinos, and the energy in photons from the surface of the star. T is the temperature in the neutron star, c_v is the specific heat of dense matter, and Γ and ϵ are the reaction rate and the neutrino emissivity of the Murca reactions in the non-equilibrium state respectively.

Assuming pure magnetic dipole radiation as the braking mechanism for the neutron star's spin-down, both the angular velocity and the moment of inertia change in the spin-down process, and we need to integrate the coupled equations of the angular momentum balance and the energy balance simultaneously. As the change in the moment of inertia is very small, we use the approximate treatment given by Shapiro & Teukolsky (1983) to calculate the angular velocity, which reads

$$I\Omega\dot{\Omega} = -\frac{(BR^3)^2}{6c^3}\Omega^4 \, , \tag{7}$$

where B is its dipole magnetic field, R is its radius, and I is its moment of inertia. The heating energy comes from the kinetic energy of rotation, whereas the heat dissipation is smaller than the term $\frac{(BR^3)^2}{6c^3}\Omega^4$ by at least three orders of magnitude and is ignored in the equation.

In previous studies, the magnetic fields were assumed to be constant so that $\dot{B} = 0$, namely, the loss rate of rotational kinetic energy is constant. However, recent studies on magnetic field evolution of neutron stars give different results. Zhang & Xie (2012) tested models of magnetic field evolution with statistical properties of their spin evolution, and found that a simple power-law LTD modulated by STO could explain all observed statistical properties of $\ddot{\nu}$. The phenomenological model of the B-evolution is

$$B(t) = B_d(t)\left[1 + k\sin\left(\phi + 2\pi\frac{t}{T_{\rm o}}\right)\right] \, , \tag{8}$$

where t is the neutron star's age, and k, ϕ and $T_{\rm o}$ are the amplitude, phase and period of the oscillating magnetic field, respectively. Since the cooling of neutron stars is a process that follows a long time scale, but the time scale for oscillation is on the order of years, the effect of the oscillating magnetic field on thermal evolution is not obvious and the oscillation component in Equation (8) can be neglected. Therefore, we only consider the LTD of the magnetic field $B(t) = B_d(t)$ in our calculation of the chemical deviation and thermal evolution.

$B_d(t)$ follows a simple power-law decay, which reads

$$B_d(t) = B_0\left(\frac{t}{t_0}\right)^{-\alpha} \, , \tag{9}$$

where B_0 is the initial surface magnetic field strength and t_0 is the time when the decay begins. According to Zhang & Xie's results, $\alpha \geq 0.5$ is required, but its upper limit is unconstrained. Figure 4 in Zhang & Xie (2012) shows the correlations of α with t_0 and B_0.

3 RESULTS

We consider a uniform star model in the simulation and take a baryon number density $n = 1.0\,\mathrm{fm}^{-3}$. The initial surface magnetic field strength B_0, the beginning time of decay t_0 and the value of $\alpha = 1$ are given according to the results of Zhang (2012). After giving an initial rotation period $Pi = 1\,\mathrm{ms}$, we integrate the coupled evolution Equations (5)–(7) and (9).

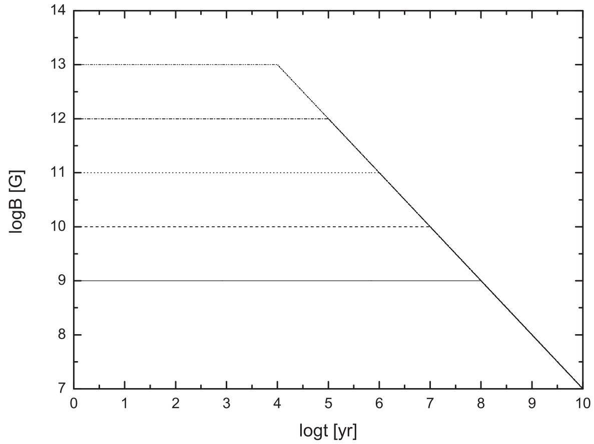

Fig. 1 Surface magnetic field as a function of time, with different initial magnetic field strengths $B_0 = 10^9$ G (*solid lines*), $B_0 = 10^{10}$ G (*dashed line*), $B_0 = 10^{11}$ G (*dotted line*), $B_0 = 10^{12}$ G (*dash-dotted line*) and $B_0 = 10^{13}$ G (*dash-dot-dotted line*).

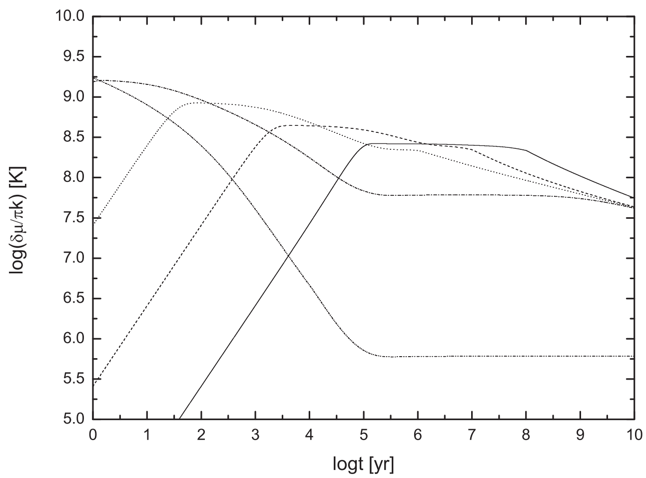

Fig. 2 Chemical evolution. The variable $\delta\mu/\pi k$ (a measure for the departure from chemical equilibrium) is plotted logarithmically as a function of time. The surface magnetic fields follow a simple power-law decay, with different initial magnetic field strengths $B_0 = 10^9$ G (*solid lines*), $B_0 = 10^{10}$ G (*dashed line*), $B_0 = 10^{11}$ G (*dotted line*), $B_0 = 10^{12}$ G (*dash-dotted line*) and $B_0 = 10^{13}$ G (*dash-dot-dotted line*).

Figure 1 shows the evolution of the surface magnetic field with different initial magnetic field strengths B_0 and the corresponding beginning times for decay t_0. Although the initial magnetic field strengths are different, the magnetic fields are the same after 10^8 yr. The magnetic fields of the stars are about 10^8 G at 10^9 yr, which agrees with the observations of old millisecond pulsars.

Figure 2 shows chemical evolution of the neutron stars with different initial magnetic fields, which follow the simple power-law decay as illustrated in Figure 1. Accordingly, Figure 3 depicts the thermal evolution of the neutron stars with the decaying magnetic fields. As is the case for neutron

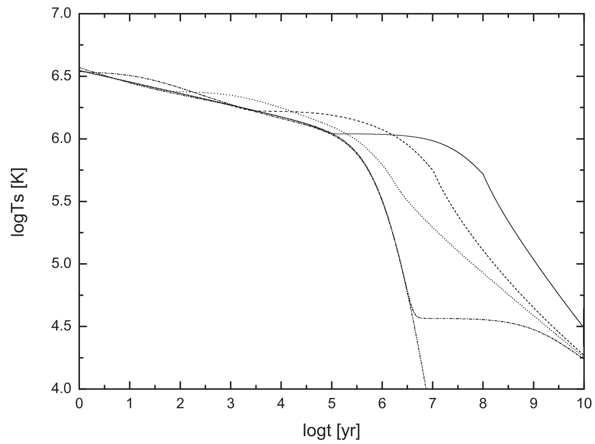

Fig. 3 Effective surface temperature as a function of time. The surface magnetic fields follow a simple power-law decay, with different initial magnetic field strengths $B_0 = 10^9$ G (*solid line*), $B_0 = 10^{10}$ G (*dashed line*), $B_0 = 10^{11}$ G (*dotted line*), $B_0 = 10^{12}$ G (*dash-dotted line*) and $B_0 = 10^{13}$ G (*dash-dot-dotted line*).

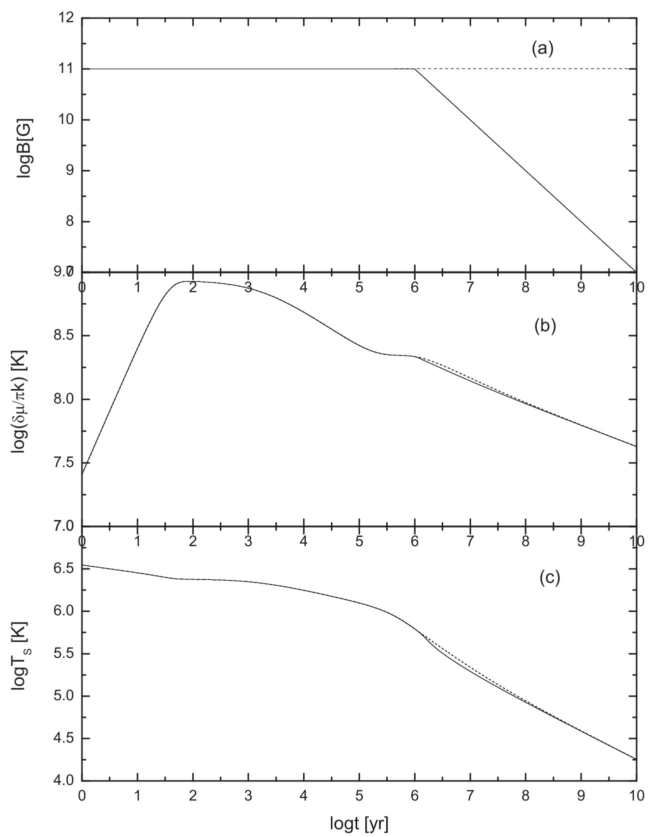

Fig. 4 Magnetic (a), chemical (b) and thermal evolution (c) with magnetic field $\dot{B} = 0$ (*dashed line*) and $\dot{B} \neq 0$ (*solid line*). The constant magnetic field is $B = 10^{11}$ G and the decaying magnetic field follows a simple power-law decay, with $\alpha = 1$ and initial magnetic field strength $B_0 = 10^{11}$ G.

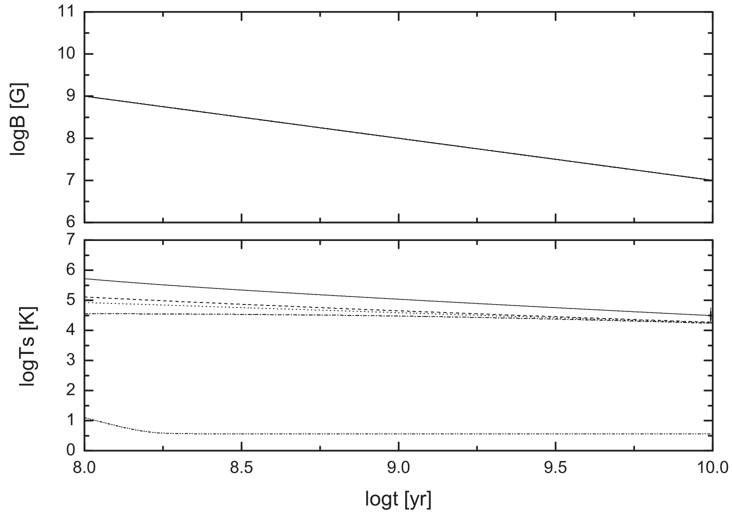

Fig. 5 The evolution of magnetic field and effective surface temperature during $10^8 - 10^{10}$ yr, with different initial magnetic fields. The linestyles in the lower panel denote the same quantities as in Fig. 3.

stars with constant magnetic fields, the higher the initial magnetic field strength is, the greater the kinetic energy of rotation that is converted into chemical energy is, the larger the chemical potential departure is and the earlier the appearance of the heating effect is.

In order to illustrate the effect of magnetic evolution on the rotochemical heating, Figure 4 plots magnetic (*a*), chemical (*b*) and thermal evolution (*c*) of neutron stars with magnetic field $\dot{B} = 0$ (dashed line) and $\dot{B} \neq 0$ (solid line) for comparison. The chemical departure is a competitive process between spin-down compression and weak reactions. The high magnetic field causes large deviation from chemical equilibrium and an early appearance of the effect of heating. Namely, the rotational energy is converted to heating energy at the early stage, before the magnetic field begins to decay. Even though the magnetic field decays to 10^8 G, the effect of heating will not appear at a later stage. So, the decay of the magnetic field does not obviously influence the chemical and thermal evolution of the neutron star.

Although the influence of the magnetic evolution on the surface temperature is not dramatic in terms of the order of magnitude, the interpretation of the thermal evolution behavior is very different from the previous results. The previous works indicated that a low magnetic field (10^8 G) leads to a high surface temperature (10^5 K) after 10^8 yr in neutron stars with rotochemical heating (Reisenegger 1995). However, Figure 5 shows that the neutron stars could have different thermal evolution, even though the magnetic fields are low after 10^8 yr, and the surface temperature is related to the initial magnetic field strength. The different initial magnetic fields result in the different extraction rates of rotational energy and directly influence the thermal evolution of the stars, whereas the magnetic fields can be the same at the late stage as a result of the power-law decay. That is, older neutron stars with magnetic fields of about 10^8 G could have lower surface temperatures, even when rotochemical heating is occurring in them.

4 CONCLUSIONS AND DISCUSSION

We have calculated the chemical evolution and cooling process of neutron stars that have decaying magnetic fields and rotochemical heating. Based on these assumptions, we have found that the power-law LTD of the magnetic field slightly affects the deviation from chemical equilibrium and surface temperature. However, the decay of the magnetic field results in older neutron stars having different surface temperature with the same magnetic field strengths, which will change our understanding of cooling millisecond pulsars.

Depending on the existing observations, if the magnetic field does not decay, millisecond pulsars may have lower magnetic field strengths at their birth. The previous results indicated that rotochemical heating enhanced the surface temperature of neutron stars to their quasi-equilibrium state after 10^7 yr, with a magnetic field of 10^8 G (Reisenegger 1995), so the older millisecond pulsars should have a higher surface temperature. Currently, the only older millisecond pulsar that has been observed to have thermal emission of about 10^5 K is J0437–4715 (Kargaltsev et al. 2004). The lack of other detected sources may imply that the magnetic fields of millisecond pulsars experience a long term decay. If the magnetic fields follow the power-law decay, the millisecond pulsars with almost the same field strengths could have different initial magnetic fields and different effective surface temperatures. That is, the older millisecond pulsars could have a lower temperature even with rotochemical heating in operation. The surface temperatures of some millisecond pulsars are high enough to be detected, but some are too low to be detected.

In this work, we assume the magnetic field is decaying by a simple power-law, whereas the exponential decay is also discussed by some researchers (Igoshev & Popov 2013). The magnetic field follows an exponential decay that decreases slowly at the beginning, but damps very quickly at the end. A higher magnetic field at the beginning leads to the fast extraction of rotational energy, but the heating is not obvious, because the surface temperature of the neutron star is now too high. At the late stage, most of the rotational energy is converted into heat, and the magnetic field is so small that the heating is also not obvious. Thus the rotochemical heating is not apparent in neutron stars with magnetic fields that follow an exponential decay, which, to some extent, does not agree with the observation of an older millisecond pulsar with a higher surface temperature.

Even though our calculation is based on a uniform density model, the results demonstrate the thermal evolution behavior of neutron stars with decaying magnetic fields and may explain the lack of observations showing older millisecond pulsars that have higher surface temperature. Of course, accurate calculations are worth pursuing. By taking the structure of the star into account in the frame of general relativity, using a realistic equation of state for dense matter, presenting the superfluid state of the nucleons and considering the corresponding parameters of pulsars, a more realistic cooling model can be compared with the observations. This is our future goal.

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