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Model simulation for periodic double-peaked outbursts in blazar OJ 287: binary black hole plus lighthouse effect

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Abstract The mechanism of formation for double-peaked optical outbursts observed in blazar OJ 287 is studied. It is shown that they could be explained in terms of a lighthouse effect for superluminal optical knots ejected from the center of the galaxy that move along helical magnetic fields. It is assumed that the orbital motion of the secondary black hole in the supermassive binary black hole system induces the 12-year quasi-periodicity in major optical outbursts by the interaction with the disk around the primary black hole. This interaction between the secondary black hole and the disk of the primary black hole (e.g. tidal effects or magnetic coupling) excites or injects plasmons (or relativistic plasmas plus magnetic field) into the jet which form superluminal knots. These knots are assumed to move along helical magnetic field lines to produce the optical double-peaked outbursts by the lighthouse effect. The four double-peaked outbursts observed in 1972, 1983, 1995 and 2005 are simulated using this model. It is shown that such lighthouse models are quite plausible and feasible for fitting the double-flaring behavior of the outbursts. The main requirement may be that in OJ 287 there exists a rather long ($\sim$40–60 pc) highly collimated zone, where the lighthouse effect occurs.

Key words: optical continuum — galaxies: jets — galaxies: kinematics — galaxies: variability — galaxies: individual (blazar OJ 287)

1 INTRODUCTION

Research on blazars is an important field in extragalactic astrophysics, in which extensive observations of their radiation from radio through γ-ray are carried out, and the mechanisms of radiation (including polarization) are studied (for recent progress, refer to, e.g., Marscher 2011; Marscher et al. 2011; Abdo et al. 2010; Raiteri et al. 2007, 2011; Schinzel et al. 2010, 2012; Vercellone et al. 2010; Marscher et al. 2012; Marscher 2014; Qian 2011, 2012, 2013; Qian et al. 2014). Through studies of the spectral energy distribution (SED) and variation of the SED of radiation from the blazar (e.g, Ghisellini et al. 2009a,b,c, 2010; Ghisellini & Tavecchio 2010; Tavecchio et al. 2007, 2010; Joshi et al. 2012a,b; Jorstad et al. 2010, 2012; Aleksić et al. 2011), the radiation mechanisms have been determined to be synchrotron and inverse-Compton processes. The strong radiation and its rapid variation in blazars are closely related to their relativistic jets with bulk Lorentz factors of $\sim$10–30, which are directed towards us at small viewing angles, thus relativistic beaming and Doppler boosting dominate blazar phenomena. Relativistic jets associated with blazars are believed to be produced in the supermassive black hole-accretion disk system existing in the center of their host galaxies. Since the *Fermi* satellite was launched in 2008, studies of blazars have achieved significant progress. In particular, based on the coordinated study of γ-ray and mm outbursts along with VLBI monitoring, it has been found that γ-rays can be emitted from the regions of a jet much beyond 1 pc, reaching 10–40 pc from the black hole (e.g. Schinzel et al. 2010; Agudo et al. 2011a, 2012b).

OJ 287 ($z = 0.306$) is one of the most well studied prominent blazars. It is an optically violent variable BL Lac object (BLO) with large and rapid polarized outbursts that radiate across the entire electromagnetic spectrum from radio through optical and X-ray to γ-ray. Very strong variability is observed in all these wavebands with various timescales (hours/days to years). OJ 287 was one of the bright *Fermi* γ-ray sources (Nolan et al. 2012; Ackermann et al. 2011). Since the *Fermi* satellite was launched in 2008, further investigations of OJ 287 have been carried out. Multifrequency observations, the study of its SED and correlations between different wavebands have revealed important clues about the radiation mechanisms, especially for X-ray and γ-ray emission and their emission positions in the jet. Agudo et al. (2011b) showed that its γ-ray emission was produced at a distance $>$14 pc from the core. Marscher & Jorstad (2011) discovered its large-scale (Mpc) X-ray jet.

OJ 287 is one of the most well-studied blazars and has been monitored for a long time through radio, IR, optical, UV, X-ray and γ-ray observations (e.g. Agudo et al. 2011a,c; Villforth et al. 2010a; Valtaoja et al. 2000; Sillanpää et al. 1988; Marscher & Jorstad 2011; Valtonen & Sillanpää 2011; Valtonen et al. 2009; Ciprini et al. 2007; Agudo et al. 2011b, 2012b). It is highly variable in all these wavebands.

OJ 287 is also a well-studied superluminal source that exhibits behavior on parsec scales, with a core-jet structure and superluminal components that are steadily ejected from the core. Angle swings in the jet position (both long-term and sharp changes) have been observed (e.g. Agudo et al. 2012a; Tateyama & Kingham 2004; Moór et al. 2011; Valtonen & Wiik 2012; Valtonen & Pihajoki 2013). Valtonen & Pihajoki explain the jump in optical polarization position angle in terms of the precession of the helical structure of the optical emission region.

In particular, its optical behavior shows exceptional properties. The most interesting features are the (quasi-) periodic outbursts observed in optical wavebands during a long period ($\sim$120 yr). (Sillanpää et al. 1988, 1996a,b; Lehto & Valtonen 1996; Sundelius et al. 1997; Valtonen 2007; Valtonen et al. 2009, 2011; Valtonen & Ciprini 2012; for a recent review see Villforth et al. 2010a).

The record of optical observations since 1891 shows that optical outbursts occur in OJ 287 with a (quasi-) periodicity of $\sim$12 yr. It also shows long-term optical variability with a (quasi-) periodicity of $\sim$60 yr. The range of variability reaches $\sim$4.5 magnitude (optical flux density ranges from $\sim$1 mJy to $\sim$60 mJy). Rapid variations in the optical emission often occur on timescales of less than a few weeks with flux density fluctuations up to 1–3 magnitude. In particular, some (maybe each) of the observed optical outbursts were actually constituted of two flares separated by $\sim$1–2 yr. Up to now four outbursts with double-peaked flares occurring in 1972, 1983, 1995 and 2005 have been clearly recorded.

The plan of the paper is: Section 2 gives the interpretations; Section 3 describes a new scenario; Section 4 gives the formalism of the model simulation; Section 5 describes the model simulation; Section 6 gives the summary and discussion.

2 INTERPRETATIONS

The observational properties listed above put OJ 287 in an important position for studies of blazars, because such remarkable, clearly determined, long lasting (quasi-) periodicities have never been observed in other blazars. These quasi-periodicities have been assumed to be related to the regular behavior of the binary black hole-accretion disk system in the center of its host galaxy.

Sillanpää et al. (1988) firstly suggested that the regularly appearing optical outbursts were produced by a close binary black hole system in which the pericenter passage of the secondary black hole induces tidal disturbances in the accretion disk of the primary. The orbital period of the binary black hole was assumed to be 12 yr to explain the observed 12-year cyclic optical outbursts.

Lehto & Valtonen (1996) further developed this model and suggested that the orbit of the secondary black hole is highly eccentric and during each orbit the secondary black hole impacts the disk of the primary twice, causing outbursts that constitute double-peaked flares separated by a time interval of 1–2 yr; that is, the two flares are produced during the two crossings of the secondary black hole through the accretion disk of the primary. In particular, both the flares are assumed to be produced by the bremsstrahlung process (i.e. thermal flares). The profiles of the double flares are interpreted in terms of the rate of inflow of particles into the accretion disk of the primary black hole, and are not related to the relativistic jet (Sundelius et al. 1997; Valtonen et al. 2009). As Sillanpää et al. (1996b) commented, this model could explain the periodicity and the double peak structure, but it has problems in explaining the fact that the two flares had the same color (extremely stable color during the outbursts, e.g. observed in the 1995 outburst), because the energy production mechanism changed for the flares produced at different impact locations. In order to predict the exact times of the future optical outbursts and double-peaked flares, this binary black hole model (Lehto/Valtonen model) has been continually improved by taking into account the effect of general relativity (orbital precession) and the interaction mechanisms between the secondary black hole and the disk of the primary (Valtonen 2007; Valtonen et al. 2009, 2011; Valtonen & Sillanpää 2011; Valtonen & Wiik 2012; Valtonen & Pihajoki 2013). When both the gravitational precession and gravitational radiation were included, Valtonen (2007) accurately predicted the second flare that occurred during the 2007 outburst. Since this model is based on the theory of dynamical orbits of a binary black hole, it may have a good ability to predict the exact times of occurrences for the outbursts. Moreover, it can be used to derive the masses of both the primary and secondary black holes, and even to measure the spin of the primary black hole (Valtonen et al. 2010a,b), helping to test general relativity.

Other authors have suggested alternative models for the periodic optical outbursts in OJ 287, also based on the assumption that OJ 287 hosts a binary black hole, but they pay more attention to the relativistic beaming and Doppler boosting effects in the relativistic jet of this object. (a) Katz (1997) suggested that the 12-year cyclic optical outbursts were produced by the precession of the jet associated with the primary black hole. Since the jet is anchored in the accretion disk of the primary hole and the orbital motion of the companion (secondary) drives the precession of the accretion disk, the jet follows this precession and regularly sweeps through the line of sight with a period for the precession ($\sim$12 yr), causing periodic optical outbursts due to enhanced radiation through relativistic beaming and Doppler boosting. According to Katz's suggestion, the double-peaked structure of the optical outbursts was caused by the nodding of the jet (Katz et al. 1982). In this driven-precessing disk model, the orbital period was only $\sim$2.7–3 yr. Interestingly, Valtonen & Wiik (2012) recently followed Katz's model, suggesting that the precession of the jet of the primary black hole caused the 120 or 60-year variability (a period of the so called Kozai cycle) and the 12-year cyclic outbursts were due to nodding motion. The double flares separated by 1–2 yr are still caused by the crossing of the secondary black hole through the accretion disk of the primary.

Alternatively, Villata et al. (1998) suggested a double-jet scenario, in which both primary and secondary black holes produced a jet and the two jets swept through the line of sight at intervals of $\sim$1–2 yr causing optical outbursts constituting double-peaked flares. Villata et al. (1998) ascribed the 12-year periodic occurrence of the outbursts to the orbital motion of the binary (not due to precession). This model needed a very special geometry for the relative positions of the jets. The radiation mechanism for the optical outbursts invoked in this model, like in Katz's disk-driven precession model, is synchrotron, which is in agreement with general theoretical results for the radiation mechanisms (synchrotron and inverse-Compton) of generic blazars (including astrophysical phenomena in radio galaxies and quasars that are observable with VLBI).

Valtaoja et al. (2000) suggested a different radiation mechanism for the two flares from the outbursts: the first flare was caused by the crossing of the second black hole through the accretion disk of the primary and thus was a thermal flare without a correlated mm/radio counterpart. The second flare was produced in the jet by a relativistic shock (and thus was a synchrotron flare) with a polarized mm/radio outburst that followed. This scenario seems unable to explain why the first flare and the second flare have such similar properties that are observed (in terms of variations of flux density, polarization, profile, timescale of spikes and spectrum), because the first flares and the second flares are produced by completely different mechanisms: bremsstrahlung and synchrotron respectively. Valtaoya et al. also argued against the lighthouse effect as the mechanism producing the optical outbursts, based on the observational fact that the optical variability during the outbursts and during the quiet periods had similar timescales. However, this argument seems not so compelling because the quiet-jet and the shocked-jet could have similar variability timescales: e.g., during the quiet state, the variability timescales represent variations in the optical core or turbulent plasma flows crossing 'standing shocks,' but during the burst state, the variability timescales represent variations of the optical knots propagating through the turbulent jet[1] (Qian et al. 1991; Quirrenbach et al. 1989; Standke et al. 1996; Marscher et al. 1992; Marscher & Jorstad 2010). Both could have similar Doppler effects.

Although the binary black hole system is clearly the most obvious approach for explaining the 12 yr periodicity, Villforth et al. (2010a) indicated that for the case of OJ 287, the properties of certain outbursts suggest the jet is a source of variations. Thus Villforth et al. (2010a) suggested a magnetic breathing model. This represents a resonance mechanism of magnetic field lines in the accretion disk. In this case, the periodicity and double peaked structure could be caused by resonance that occurred in the accretion disk and/or jet. Specifically, the outbursts could be related to accretion of magnetic field lines. The regularly appearing flares are signs that the accretion of the magnetic field happened in avalanches. Massive accretion of the magnetic field causes strong disturbances in the magnetic field of the accretion disk. These disturbances cause a resonance in the accretion disk (e.g. Ouyed et al. 1997; Ouyed & Pudritz 1997a,b)[2], which appears as a 'magnetic breathing' phenomenon in the disk. The resonance causes a regularly appearing avalanche of the accretion magnetic field. Each double-peaked outburst represents a phase of massive magnetic field accretion. Villforth et al. (2010a) argued that observations of polarization support this resonance model. The biggest caveat of this approach is the fact that it could not naturally explain the double peaked structure. Villforth et al. speculate that the first flare represents the accretion of the magnetic field and the second flare represents accretion of matter; the 1–2 yr time interval represents the delay of the matter accretion with respect to the field accretion. It does not explain why the radio counterparts in some flares are missing. It is not yet clear what decides if a disturbance in the jet will be observed in radio, therefore it is unclear if the magnetic breathing model can explain the radio behavior. Gupta et al. (2012) suggested that the periodic variations in the accretion disk could translate to variations in the jet with the observed timescale shortened by a factor of Γ (jet Lorentz factor, also Valtonen & Pihajoki 2013). This ingredient should be taken into account when the relation between the thermal emission and synchrotron emission of the binary hole-accretion disk system is studied.

At present, arguments for the precessing binary hole model suggested by Lehto & Valtonen (1996) (and its variants) seem to be prevailing and it may be the most promising one to understand the phenomenon of periodic double-peaked outbursts observed in OJ 287. If this is really so, based on this model, OJ 287 could become a testbed for general relativity (e.g. orbital precession and gravitational waves). Although binary black hole models (Sillanpää et al. 1988; Lehto & Valtonen 1996; Sundelius et al. 1997) have achieved some success (which perform remarkably well in explaining the timing of the outbursts), there are still some aspects that need to be clarified and tested: For example, based on observational aspects, among the issues discussed by Villforth et al. (2010a) about

[1] Rapid radio variability could be explained in terms of shocks propagating through the turbulent jet, which was firstly suggested by Qian et al. (1991) and this concept could also be used to interpret variability in other wavebands (Melrose 1994).

[2] Strong magnetic disturbances could also be produced in the magnetosphere of the black hole due to instabilities, e.g. Tomimatsu et al. (2001).

the precessing binary model (e.g. mass of the black holes, crossing timescale, propagation time from the impacts to the jet and the transformation between the turbulence by tidal effect and the injection into the jet, etc.), the main issue might be how to separate the constributions of jet-synchrotron and bremsstrahlung (by crossing and tidal effect) in the observed optical lightcurves. Taking the 2005–2007 outburst as an example we find that

(1) During the period of September 2007–February 2008, the optical flare consisted of several spikes with timescales of $\sim$1–2 weeks and the most prominent one occurred in late November. Interestingly, the polarization measurements show that $\sim$60% data points have polarization degree larger than 15% and only $\sim$20% of data points have polarization degree less than 10% (Villforth et al. 2010a). If assuming that the emission with low polarization degrees was due to bremsstrahlung and the emission with high polarization degrees was due to jet-synchrotron, then the 'alternative occurrence' of the low- and high- polarization degrees during the period of September 2007–February 2008 would imply the 'alternative functioning' of the two mechnisms. This seems impossible, because the crossings of the secondary hole could only produce the thermal emission of the first spike (with timescale of $\sim$1–2 weeks).

(2) Moreover, the thermal spikes and the jet-synchrotron spikes have very different properties: the observed flux density (or radiative energy) of the jet-synchrotron spikes are augmented by Doppler boosting (by a factor of $\sim 10^2 - 10^3$) with timescales shortened by relativistic effects (by a factor of $\sim$5–10), but those of the thermal spikes are not. Therefore, the optical lightcurves observed in OJ 287 could not be directly compared with those predicted by the accretion flow of particles due to crossing and tidal effect of the secondary black hole, and the effects caused by time delay, Doppler boosting and time-shortening should first be taken into account. In addition, the Doppler boosting of the jet-synchrotron originated from the bulk acceleration of the jet which is produced by electromagnetically extracting the rotation energy of the accretion disk and/or the black hole. Thus if some significant portion of the observed synchrotron emission were somehow "mis-identified" as bremsstrahlung, then the masses in the binary hole of OJ 287 could be significantly overestimated, because the rotation energy dissipated into the jet formation could have been counted into the impacting and tidal energy. Actually, the mass of the primary black hole estimated by the precessing binary model is $\sim 1.8 \times 10^{10} M_{\odot}$, which is much higher (about an order of magnitude) than those measured for generic BL Lac objects and quasars. Accurate measurements of the mass of the black hole in OJ 287 is crucial.

(3) The optical emission observed in 3–4 November (2005) was argued as bremsstrahlung in origin (Valtonen et al. 2012b), but the polarization degree measured in R-band in 2005 November 2 (JD 2453676.669) by Villforth et al. (2010b) is 30.4$\pm$0.3%.[3] Such a high polarization degree could only be produced by a synchrotron mechanism. Moreover, if subtracting the 'bremsstrahlung flux' ($\sim$10 mJy in R-band) associated with the X-ray component (observed in 2005 Nov. 3–4), the 'residual spike' would have a polarization degree reaching $\sim$150% (polarized flux exceeds total flux), which is obviously a wrong value (the maximum polarization degree is 75% for an optically thin synchrotron source with a spectral index $\alpha_{\rm opt}$=1.0; Pacholczyk 1970). For the datapoints with a high polarization degree during the period Sept. 2007–Feb. 2008, a similar problem could happen (for example, for the datapoint with polarization degree 34.1% observed at JD 2454384.71).

(4) During the period 16 October–11 November 2005 (JD 2453660–2453685) the optical flare contains three major spikes which have associated mm-outbursts with some time delays (Ciprini et al. 2007, private communication). Normally, such optical-mm correlation implies that both optical and mm-flares are regarded as produced by superluminal knots in the jet, and thus have a synchrotron origin. Note that during the 1983–1984 and 1994–1996 outbursts observed in OJ 287 the occurrence of the associated radio countrparts have been regarded as evidence for

[3] The observed polarization degree at JD 2453684.755 (10 November 2005) is 20.7$\pm$0.4% (Villforth et al. 2010b).

the synchrotron mechanism: the radio outbursts are regarded as the evolutional product of the optical outbursts (Valtaoja et al. 2000).

(5) Strong rapid optical spikes with timescales of days or 1–2 weeks could be caused by impact-crossing of the secondary black hole through the disk of the primary, but also could be due to relativistic shocks (knots) propagating through plasma turbulence (Qian et al. 1991b; Merlose 1994; Standke et al. 1996; Marscher et al. 1992; Marscher 2014). The difference is that the former is unpolarized, but the latter has polarization degree varying in a wide range (e.g. from $<$5% to $>$30%, especially if taking into account the existence of multi polarized components). We would also note that the properties of the optical activity observed in OJ 287 are very similar to those in generic blazars (variability in flux density, polarization and timescales), with no seeming difference, except for the 12-year cycle and double structure. In the γ-ray and radio bands there is also similarity.

Based on the above arguments we would consider the possibility of whether the double-peaked optical outbursts observed in OJ 287 could be produced by a binary black hole system plus lighthouse effect.

3 A NEW SCENARIO

We propose a new scenario (binary hole system plus lighthouse effect) for interpreting the optical phenomena observed in OJ 287. We assume that the optical light curve observed in OJ 287 is formed from five processes.

(1) OJ 287 hosts a binary black hole system and the orbit of the secondary black hole has a modest eccentricity and an orbital period of 12 yr. The orbital motion of the secondary black hole around the primary induces large amplitude disturbances near the pericenter passage in the accretion disk (accretion flow) of the primary and in the injection of the plasma/magnetic field into the jet of the primary black hole, causing the 12 yr quasi-periodicity of optical outbursts. This assumption is similar to what was originally suggested by Sillanpää et al. (1988). We do not assume an extreme eccentricity for the disk-crossing process to explain the double-peaked flares with a time interval of 1–2 yr. The 60-year cyclic variability is assumed to be caused by the precession of the jet driven by the orbital motion of the secondary black hole, as suggested by Valtonen & Wiik (2012).

(2) We assume that the double peaks of the outbursts with time intervals of $\sim$1–2 yr are caused by the lighthouse effect. The lighthouse effect has been suggested to interpret the (quasi-) periodic optical flares in some prominent blazars (e.g. Camenzind & Krockenberger 1992, 3C 273; Schramm et al. 1993, 3C 345; Wagner et al. 1995, PKS0420–140). This phenomenon occurs when superluminal optical knots move along helical magnetic field lines in relativistic jets and periodically sweep through the line of sight, producing cyclic optical flares. In the case of OJ 287, the helical structure in its jet could be very stable for a long term period of (e.g.) over $\sim$100 yr. Thus the lighthouse effect could help to explain the double-peaked structure of the outbursts observed in OJ 287 and its reoccurrence during the past decades. Helical motion has also been suggested to interpret VLBI observations, e.g. Qian et al. (1992); Steffen et al. (1995); Ostorero et al. (2004); Perucho et al. (2012a,b, 2013); Valtonen & Pihajoki (2013).

(3) The formation and evolution of the optical knots causing the outbursts should also be included in the interpretation of the optical light curve, since the radiation lifetimes of the optical knots could have similar timescales as observed in the light curve.

(4) There is also a thermal component produced by the accretion disk of the primary black hole through bremsstrahlung. The synchrotron emission of the ‘quiet optical core’ of the jet should also be taken into account. For simplification, we assume that both components are constant during the outbursts, but have different levels for different outbursts.

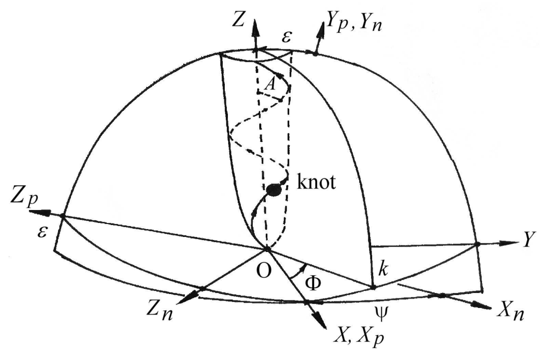

Fig. 1 Geometry for the model simulation of the four double-peaked optical outbursts.

(5) In blazars, especially in BLO-blazars, very rapid variations in brightness have often been observed. In the case of OJ 287, the optical flux density can vary by (e.g.) a factor of ten in about 1–3 weeks (see light curves given below). This short term variability could be related to the variability in the optical core of the jet and the optical knots propagating along the turbulent jet with large amplitude fluctuations of plasma density and field strength. However, in this paper, we do not intend to include this short term variability, but rather concentrate on the study of some 'mean' profiles of the flares. Of course, appropriately dealing with the rapid flux (and polarization) variations would improve our understanding of the optical phenomena observed in OJ 287. However, in this paper, we only deal with the three processes labeled (2) to (4).

Our main purpose is to perform a simulation with this model to look for an explanation of the double-peaked structure of the outbursts observed in OJ 287 through the lighthouse effect. Specifically, we look for appropriate helical motion in the optical knots and appropriate parameters to describe their evolution. We will use the model to simulate the profiles of the double flares for the outbursts observed in 1972, 1983, 1995 and 2005 for OJ 287.

In this paper, we will adopt the concordant cosmological model (ΛCDM model) with $\Omega_m = 0.27$, $\Omega_\lambda = 0.73$ and Hubble constant $H_0 = 71\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$ (Spergel et al. 2003). Thus for OJ 287, ($z = 0.306$), its luminosity distance is $D_\mathrm{l} = 1.58$ Gpc (Hogg 1999; Pen 1999) and angular diameter distance $D_\mathrm{a} = 0.9257$ Gpc. The angular scale 1 mas = 4.487 pc, and the proper motion of 1 mas yr^{-1} is equivalent to an apparent velocity of $19.1c$ (c is the speed of light).

4 FORMALISM OF MODEL SIMULATION

In order to study the formation of the double-peaked structure of the optical outbursts observed in blazar OJ 287, we will consider the lighthouse effect caused by optical knots moving along magnetic field lines of the jet. We will apply the formalism given by Qian et al. (1992), which has been used to study the kinematics of radio superluminal knots on parsec scales in blazars, e.g. 3C 345, 3C 279 and 3C 454.3 (Qian et al. 2009; Qian 2011, 2012, 2013; Qian et al. 2014).

The geometry of the model for helical motion is shown in Figure 1. Three coordinate frames are shown: (X, Y, Z), (X_p, Y_p, Z_p) and (X_n, Y_n, Z_n). The Y_n axis is directed towards the observer and (X_n, Z_n) defines the plane of the sky with the X_n-axis pointing towards the negative right ascension and the Z_n-axis towards the north pole. Z-axis represents the jet-axis defined by parameters (ϵ,ψ). Φ represents the phase of the optical knot. The trajectory of a superluminal knot is described in cylindrical coordinates (Z, $A(Z)$, $\Phi(Z)$): Z– distance from the origin along the jet axis. $A(Z)$ represents the amplitude of the knot's path; $\Phi(Z)$ is the azimuthal angle or the phase of the knot. Z and $A(Z)$ are measured in the unit of milliarcsecond (mas) and Φ is measured in the unit of radian.

For the helical motion of a knot along magnetic field lines, the trajectory (or orbital) phase of the knot can be defined as

$$\Phi(Z) = \Phi_0 + R_\phi(Z)\times Z \,, \tag{1}$$

where R_ϕ is the rotation rate (rad mas^{-1}) and Φ_0 is the initial phase of the knot at $Z = 0$. When functions $A(Z)$ and $\Phi(Z)$ (or $R_\phi(Z)$) are given and parameters ϵ, ψ, Φ_0 and Γ (bulk Lorentz factor of the knot) are set, the kinematics of the knot (projected trajectory, apparent velocity, Doppler factor and viewing angle as functions of time) can then be calculated. The formulas are listed as follows.

$$X(Z,\Phi) \;=\; A(Z)\mathrm{cos}\Phi(Z)\,, \tag{2}$$

$$Y(Z,\Phi) \;=\; A(Z)\mathrm{sin}\Phi(Z)\,. \tag{3}$$

The projected trajectory on the plane of the sky is represented by

$$X_n(Z,\Phi) \;=\; X(Z,\Phi)\mathrm{cos}\psi - \Big[Z\mathrm{sin}\epsilon - Y(Z,\Phi)\mathrm{cos}\epsilon\Big]\mathrm{sin}\psi\,, \tag{4}$$

$$Z_n(Z,\Phi) \;=\; X(Z,\Phi)\mathrm{sin}\psi + \Big[Z\mathrm{sin}\epsilon - Y(Z,\Phi)\mathrm{cos}\epsilon\Big]\mathrm{cos}\psi\,. \tag{5}$$

Introducing the following functions:

$$\Delta \;=\; \arctan\Big[\frac{dX}{dZ}^2 + \frac{dY}{dZ}^2\Big]^{\frac{1}{2}}\,, \tag{6}$$

$$\Delta_p \;=\; \arctan\Big[\frac{dY}{dZ}\Big]\,, \tag{7}$$

$$\Delta_s \;=\; \arccos\Big[1 + \Big(\frac{dX}{dZ}\Big)^2 + \Big(\frac{dY}{dZ}\Big)^2\Big]^{-\frac{1}{2}}\,, \tag{8}$$

we can then calculate the elapsed time T_0 (at which the knot reaches axial distance Z), apparent velocity β_{a}, Doppler factor δ and viewing angle θ of the knot

$$T_0 = \int_0^Z \frac{1+z}{\Gamma\delta v\mathrm{cos}\Delta_s}dZ\,, \tag{9}$$

$$\theta = \arccos[\cos\epsilon(\cos\Delta + \sin\epsilon\tan\Delta_p)]\,, \tag{10}$$

$$\delta = \frac{1}{\Gamma(1-\beta\mathrm{cos}\theta)}\,, \tag{11}$$

$$\beta_{\mathrm{a}} = \frac{\beta\mathrm{sin}\theta}{1-\beta\mathrm{sin}\theta}\,, \tag{12}$$

where $\beta = v/c$ (here v is the speed of the knot) and $\Gamma = (1-\beta^2)^{-\frac{1}{2}}$.

5 MODEL SIMULATION

In the following, we will make models to simulate the light curves of the four optical outbursts with double peaks that occurred in blazar OJ 287: 1972, 1983, 1995 and 2005 outbursts described in Valtaoja et al. (2000) and Villforth et al. (2010a).

As indicated in the introduction, it is assumed in the proposed model that the optical light curves contain different physical processes (see Fig. 2): (1) the radiation from the accretion disk of the primary black hole, produced by bremsstrahlung; (2) the injection of magnetized plasma into the jet, which is modulated by the orbital motion of the secondary black hole (through a tidal effect as suggested by Sillanpää et al. 1988). The interaction between the accretion disk of the primary

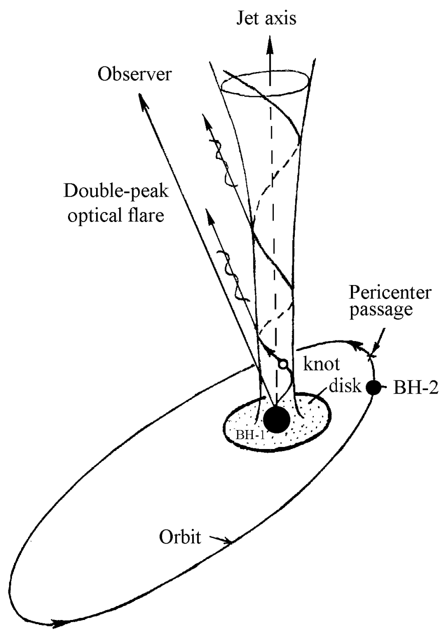

Fig. 2 A sketch of the lighthouse model for double-peaked optical outbursts occurring in blazar OJ 287 (not to scale). Precession of the pericenter of the secondary black hole (see Sillanpää 1988) is not included for simplicity.

black hole and the companion black hole is very complex. We assume that the optical outbursts are excited by the pericenter passage of the companion black hole. Thus an orbital period of $\sim$12 yr is assumed to cause the periodic optical outbursts. The precession of the jet-axis derived by the orbital motion ($\sim$60 or 120 yr) could be the cause of the long-term quasi-periodic optical variability; (3) the formation and evolution (including emission and kinematic properties) of the superluminal optical knots are responsible for the optical radiation through a synchrotron mechanism; (4) the superluminal motion of the optical knots along the helical magnetic field lines of the jet causes the lighthouse effect through relativistic beaming of radiation. Thus there are many physical parameters involved which can be adjusted to explain the light curves of the outbursts. On the other hand, for superluminal optical knots, no data on their kinematics can be obtained. (This is different from radio superluminal knots for which VLBI data can provide important information.) Thus in the following, we would not attempt to make detailed "purely physical" models (e.g. Camenzind & Krockenberger 1992; Schramm et al. 1993) to interpret the double-peaked outbursts of OJ 287. Instead, we will only propose a "formal" model (tentative and qualitative) and use very simple numerical simulations to show the possibility of how our model can explain the formation of the double-peaked optical outbursts (flare profiles and the time interval of the double flares). The four double-peaked outbursts will be treated individually.

Since the model contains many parameters and functions, we will make a few assumptions to simplify the description of the model as follows:

(1) We assume that $\epsilon = 3^\circ$ and $\psi = 0$, which define the direction of the jet axis;
(2) We assume $\Gamma = 10$ which defines the bulk Lorentz factor of all (four) optical knots;
(3) We will not consider detailed physical models for the formation and evolution of the emission of the knots which involve the acceleration of relativistic electrons and field magnification by

(e.g.) magnetic turbulence, but only assume that the flux density of the knot is described by

$$F(\nu_{\rm obs}, Z) = F_*(\nu_{\rm obs}, Z)\times\delta^{3+\alpha} + F_0(\nu_{\rm obs})\,, \tag{13}$$

where F_* ($\nu_{\rm obs}$, Z) is the comoving flux density of the knot. The factor $\delta^{3+\alpha}$ describes the Doppler boosting (Blandford & Königl 1979). The variation in Doppler factor δ caused by the helical motion of the superluminal optical knot produces the lighthouse effect. α is the spectral index ($F_\nu \propto \nu^{-\alpha}$). $F_0(\nu_{\rm obs}) = F_{\rm d}(\nu_{\rm obs}) + F_{\rm c}(\nu_{\rm obs})$ describes the emission from the accretion disk of the primary black hole and the "quiet" optical core of the jet. We assume F_0 = constant during the period of the outbursts; actually, they are variable and inclusion of these variations would improve the simulations of the light curves of the flares, especially for the periods between the two flares.

Optical spectral index α has been measured for the outbursts in OJ 287 (Villforth et al. 2010a; Hagen-Thorn et al. 1998). It depends on the brightness of OJ 287, varying between ∼1.6 and 1.1. We take $\alpha = 1.0$ here.

(4) Since we do not apply a physical model to describe the relation between the functions $A(Z)$ and R_Φ that define the helical trajectory, and the evolution the optical knot radiation ($F_*(Z)$), we will use separate step functions with irregular lengths to describe the functions $A(Z)$, $\Phi(Z)$, $R_\phi(Z)$ and $F_*(Z)$. However, a few conditions are set as follows.

(5) The amplitude function $A(Z)$ should contain three regions with an initial opening and then collimation and expansion. The collimation region is the region in which double flares with similar intensity could be produced through the lighthouse effect. As Schramm et al. (1993) indicate, the jet must be perfectly collimated at the base in agreement with expectations for self-collimated current-carrying jets (Appl & Camenzind 1993). Thus we assume that the jet becomes collimated at axial distance $Z <\sim 0.1$ mas (0.45 pc). The amplitude of the trajectory $A(Z) \sim 0.02 - 0.03$ mas in the collimation region. In the expansion regions, the optical knots evolve into radio knots and radio counterparts appear following the second optical flare.

(6) Rotation rate $R_\phi(Z)$ in the collimation region should be large enough to produce the helical rotation of the optical knots, causing double flares. Generally, $R_\phi(Z)$ should conform to the amplitude $A(Z)$: when $A(Z)$ increases, $R_\phi(Z)$ decreases. Only two values of R_Φ are taken for each of the four outbursts.

(7) The evolution of the synchrotron radiation of the optical knot should contain three stages: an initial rapid increase, a plateau and a decreasing stage (or correspondingly, Compton-, synchrotron- and adiabatic stage). This requirement is consistent with the normal evolution of a superluminal knot (Marscher & Gear 1985). Due to pressure effects and dissipation, the jet would expand sideways when it emerges beyond the ∼10 mas-scale. Driven by this expansion, the knot spectrum moves to lower frequencies so that the optical synchrotron flux decays. We would not specifically set the properties of the optical knots (e.g. density and energy spectrum of the relativistic electrons, magnetic field strength, knot size, etc.) and only choose some type of flux variations for making simulations; e.g. within the collimated regions of the jet, the optical flux of the knots remains stable to assure the production of quasi-equal intensity double flares through the lighthouse effect (Schramm et al. 1993).[4] The expansion of the jet leads to the optical knots evolving into radio knots and producing mm/radio outbursts.

(8) We will not consider the rapid, short-term variations (on timescales of weeks) of the optical knots. In the case of OJ 287, these variations make the profiles of the double-peaked outbursts difficult to determine; we only simulate the 'average' (smoothed) profiles of the optical outbursts. The rapid variations in flux density on timescales of weeks could be due to the propagation of the relativistic shocks (optical knots superluminally moving through the very turbulent jet, e.g. Qian et al. 1991; Standke et al. 1996; Marscher et al. 1992; Marscher & Jorstad 2010).

[4] Within the collimated regions, radiation losses through synchrotron and inverse-Compton processes are compensated by efficient acceleration of electrons and the expansion loss is negligible, see below.

(9) We include the optical emission from the disk of the primary black hole ($F_{\rm d}$, bremsstrahlung) and that from the optical core of the jet ($F_{\rm c}$). In the numerical simulation, we assume that both components are stable (non-variable: $F_0 = F_{\rm d} + F_{\rm c} =$ constant) during the outbursts in the following numerical simulations. Actually, F_0 is rapidly variable on short timescales of weeks (in particular for the jet core, the flux from which is Doppler-boosted) and inclusion of this component would help to explain the optical variations during the periods between the double flares.

These conditions are required for assuring obtaining an explanation of the double-peaked flares.

Although our model simulation is qualitative, the projected trajectory and timescale obtained in the following are similar to those obtained by Schramm et al. (1993) for the optical knot of blazar 3C 345 in their lighthouse model (see below).

5.1 Model Simulation of the 1972 Flare

The parameters ($A(Z)$, $\Phi(Z)$, $R_\phi(Z)$) for the model simulation are given as follows:

(1) $Z(\rm mas) \leq 0.1$: $A(Z)(\rm mas) = 0.0278(Z/0.1)$; $\Phi(Z)(\rm rad) = 1.552 + R_\phi(Z) \times Z$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.7$.
(2) $Z(\rm mas) = 0.1 - 14$: $A(Z)(\rm mas) = 0.0278$; $\Phi(Z)(\rm rad) = 1.622 + R_\phi \times (Z - 0.1)$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.7$.
(3) $Z(\rm mas) = 14 - 24$: $A(Z)(\rm mas) = 0.0278$; $\Phi(Z)(\rm rad) = 11.352 + R_\phi(Z) \times (Z - 14)$; R_ϕ (rad mas^{-1}) $= 0.7$.
(4) $Z(\rm mas) > 24$: $A(Z)(\rm mas) = 0.0278 \times [1 + 0.5(Z - 18)]$; $\Phi(Z)(\rm rad) = 18.352 + R_\phi \times (Z - 24)$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.3$.

The flux density from the quiet background component (accretion disk of the primary black hole plus the quiet core of the jet) is assumed to be $F_0 = 5.5$ mJy.

It should be noted that the position $(Z, A) = (0, 0)$ is only a "mathematical origin;" it does not represent the location of the central supermassive black hole or the location of the optical core. A reasonable choice may be that the location of the first peak produced by the superluminal knot is regarded as the site of the optical core of the jet and the black hole is located a bit inwards.

The comoving flux densities are given as follows:

(1) $Z(\rm mas) \leq 0.1$: $F_*(Z)(\rm mJy) = 5.10 \times 10^{-4} Z/0.1$.
(2) $Z(\rm mas) = 0.1 - 12$: $F_*(Z)(\rm mJy) = 5.10 \times 10^{-4}[1 - 0.0196(Z - 0.1)]$.
(3) $Z(\rm mas) = 12 - 20$: $F_*(Z)(\rm mJy) = 3.91 \times 10^{-4}[1 - 0.125(Z - 12)]$.
(4) $Z(\rm mas) > 20$: $F_*(Z)(\rm mJy) = 0$.

The results of the model simulation are shown in Figures 3–5. It can be seen that the double-peaked optical outburst of OJ 287 observed in 1971–1972 is well simulated by our simple numerical model, including the peak flux densities, the widths of the flare profiles, the time interval between the two flares and the radiation level of the quiet background component. The model simulation for this outburst is a very typical example: both the double flares of the outburst are simulated to occur within the collimated region of the jet and have a uniform rotation rate of 0.7 rad mas^{-1}. The radial distances of the two peaks occur at 6.7 mas and 15.2 mas from the origin ($Z = 0$). If the first flare peak is approximately regarded as occurring near the core of the optical jet, then the second flare is emitted at ∼8.5 mas (38 pc) from the core. The width of the jet at both positions of the peaks is ~ 0.0278 mas ($= 0.13$ pc).

5.2 Model Simulation of the 1983 Flare

The parameters $A(Z)$, $\Phi(Z)$ and $R_\phi(Z)$ for the model simulation are given as follows:

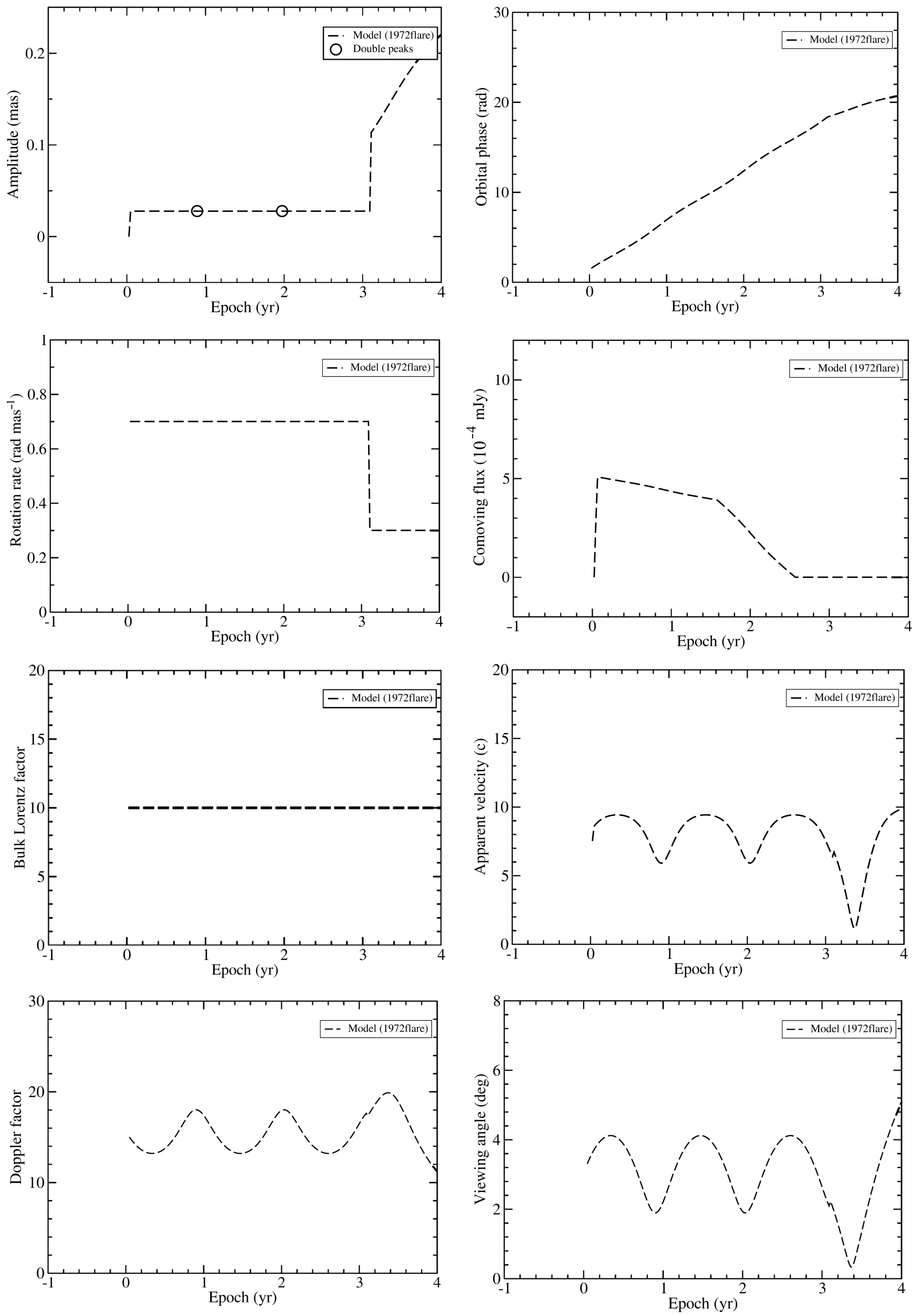

Fig. 3 Model simulation for the 1972 outburst. The upper four panels are for amplitude $A(t)$, orbital phase $\Phi(t)$, rotation rate $R_\phi(t)$ and comoving flux density $F_*(t)$. The lower four panels (for the kinematic properties of optical knot): bulk Lorentz factor $\Gamma(t)$, apparent velocity $\beta_{\rm a}(t)$, Doppler factor $\delta(t)$ and viewing angle $\theta(t)$. Epoch zero corresponds to 1970.58.

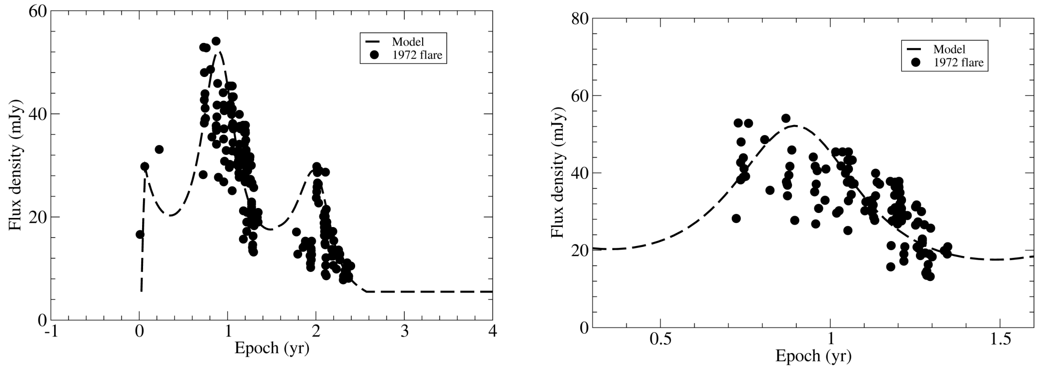

Fig. 4 The left panel shows the 1972 outburst: simulation of the double-peaked light curve. The right panel shows the simulation of its first flare on an expanded timescale. The origin of the epoch is 1970.58.

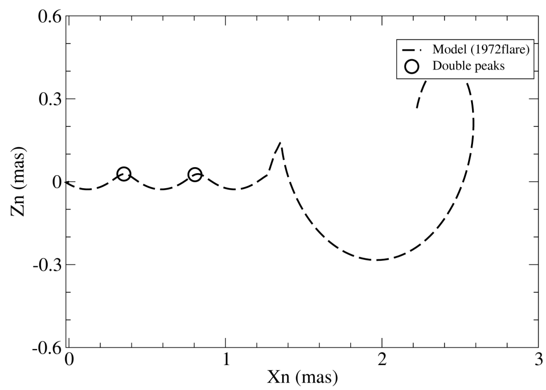

Fig. 5 Model simulation of the projected trajectory for the 1972 outburst. The circles show the position where the double flares are emitted.

(1) $Z(\text{mas}) \leq 0.1$: $A(Z)(\text{mas}) = 0.0278(Z/0.1)$; $\Phi(Z)(\text{rad}) = 1.552 + R_\phi(Z) \times Z$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.7$.
(2) $Z(\text{mas}) = 0.1 - 15$: $A(Z)(\text{mas}) = 0.0278$; $\Phi(Z)(\text{rad}) = 1.622 + R_\phi(Z) \times (Z - 0.1)$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.7$.
(3) $Z(\text{mas}) = 15 - 24$: $A(Z)(\text{mas}) = 0.0278$; $\Phi(Z)(\text{rad}) = 12.052 + R_\phi(Z) \times (Z - 15)$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.7$.
(4) $Z(\text{mas}) > 24$: $A(Z)(\text{mas}) = 0.0278[1 + 0.5(Z - 18)]$; $\Phi(\text{rad}) = 18.352 + R_\phi(Z) \times (Z - 24)$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.4$.

The flux density of the quiet background component is constant, $F_0 = F_\text{d} + F_\text{c} = 4.0$ mJy.
The comoving flux density of the optical knot is given as follows:

(1) $Z(\text{mas}) \leq 3$: $F_*(Z)(\text{mJy}) = 7.15 \times 10^{-6} \times Z/3$.
(2) $Z(\text{mas}) = 3 - 6$: $F_*(Z)(\text{mJy}) = 7.15 \times 10^{-6}[1 + 9.67(Z - 3)]$.
(3) $Z(\text{mas}) = 6 - 13$: $F_*(Z)(\text{mJy}) = 2.15 \times 10^{-4}$.
(4) $Z(\text{mas}) = 13 - 16$: $F_*(Z)(\text{mJy}) = 2.15 \times 10^{-4}[1 - 0.2(Z - 13)]$.

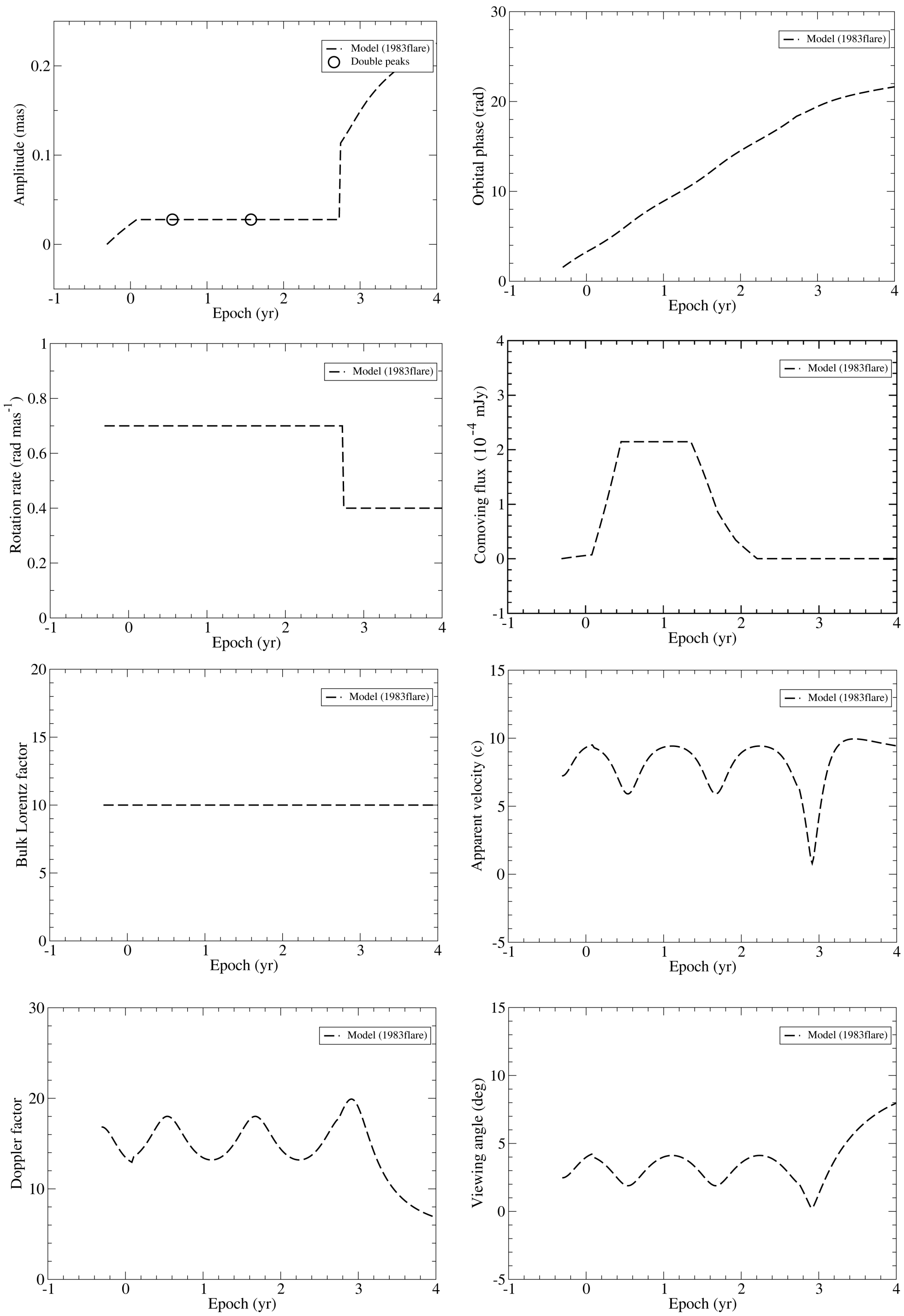

Fig. 6 Model simulation for the 1983 flare. The upper four panels show the amplitude A(t), orbital phase $\Phi(t)$, rotation rate $R_\phi(t)$ and comoving flux density $F_*(t)$. The lower four panels show the kinematic properties of the optical knot: bulk Lorentz factor $\Gamma(t)$, apparent velocity $\beta_{\rm a}(t)$, Doppler factor $\delta(t)$ and viewing angle $\theta(t)$. Epoch zero corresponds to 1982.58.

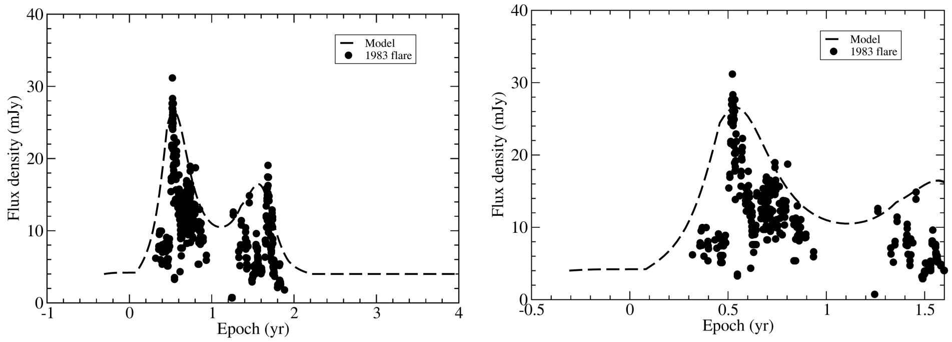

Fig. 7 The left panel shows the 1983 outburst with a model simulation of the light curve for the optical double-peaked outburst. The right panel shows the simulation of its first flare on an expanded timescale. Epoch zero $= 1982.58$.

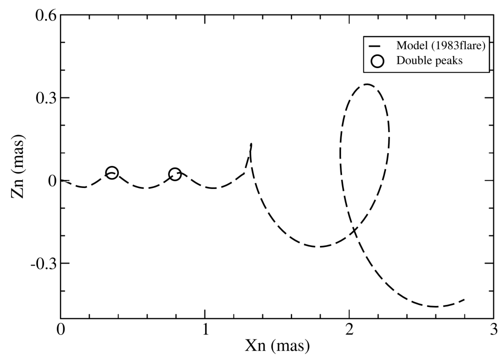

Fig. 8 Simulation of the projected trajectory of the optical knot of the 1983 outburst. The circles indicate the positions where the double flares are emitted.

(5) $Z(\mathrm{mas}) = 16 - 18$: $F_*(Z)(\mathrm{mJy}) = 8.58 \times 10^{-5}[1 - 0.3(Z - 16)]$.
(6) $Z(\mathrm{mas}) = 18 - 20$: $F_*(Z)(\mathrm{mJy})(= 3.43 \times 10^{-5}[1 - 0.5(Z - 18)]$.
(7) $Z(\mathrm{mas}) > 20$: $F_*(Z)(\ \mathrm{mJy}) = 0$.

The model simulation results are shown in Figures 6–8. Like the 1972 outburst, the 1983 outburst is also a typical example that exhibits the lighthouse effect. Both the double flares from the outburst were simulated to occur within the collimated region and have a uniform rotation rate $R_\Phi = 0.7$ rad mas^{-1}. The radial distances of the peaks occur at 6.8 mas and at 14.8 mas. If the first flare is assumed to occur near the core of the optical jet (or the core that is polarized in optical, described by Villforth et al. 2010a), then the second flare occurs at radial distance of $\sim$8 mas ($\sim$40 pc) from the core. (Actually, we do not know how far the optical core is located from the black hole.) The width of the jet at these sites is 0.0278 mas ($\sim$0.125 pc).

5.3 Model Simulation for the 1995 Outburst

For the model simulation, the parameters $A(Z)$, $\Phi(Z)$ and $R_\phi(Z)$ are given as follows:

(1) $Z(\mathrm{mas}) \leq 0.1$:
$A(Z)(\mathrm{mas}) = 0.0325(Z/0.1)$; $\Phi(Z)(\mathrm{rad}) = 1.552 + R_\phi \times Z$; R_ϕ (rad mas^{-1}) $= 1.30$.

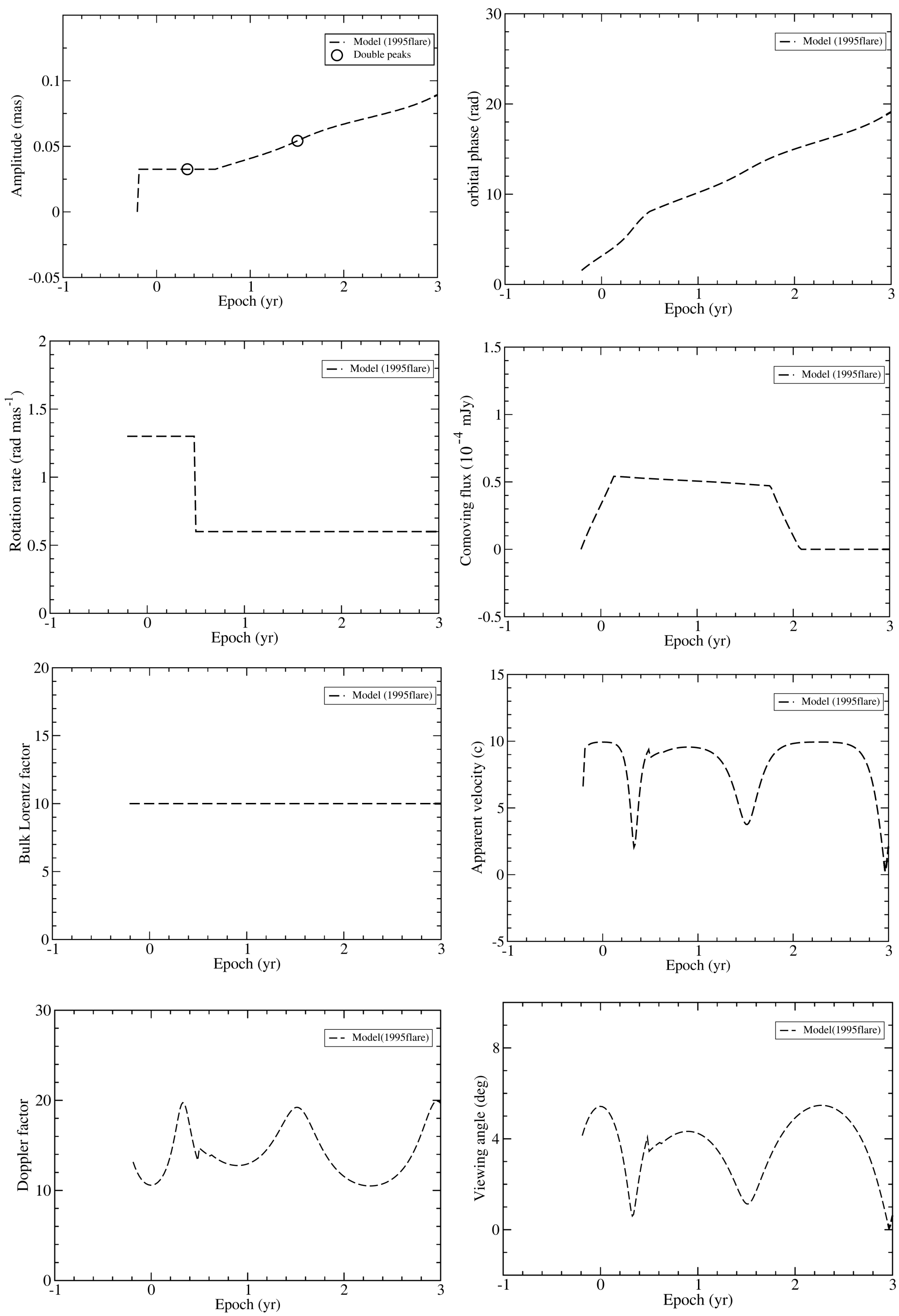

Fig. 9 Model simulation for the 1995 outburst. The upper four panels: amplitude A(t), orbital phase $\Phi(t)$, rotation rate $R_\phi(t)$ and comoving flux density $F_*(t)$. The lower four panels (for kinematic properties of the superluminal motion of the optical knot): bulk Lorentz factor $\Gamma(t)$, apparent velocity $\beta_{\rm a}(t)$, Doppler factor δ and viewing angle $\theta(t)$. Epoch zero = 1994.50

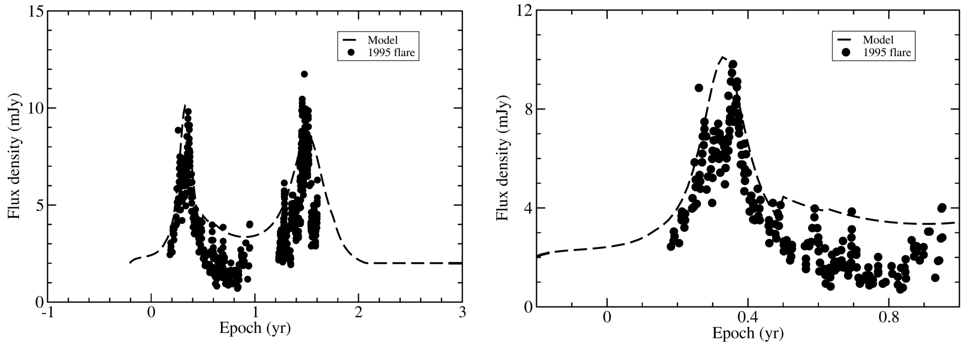

Fig. 10 The left panel shows the model simulation of the light curve from the 1995 outburst. The right panel shows the simulation of its first flare on an expanded timescale. Epoch zero = 1994.50.

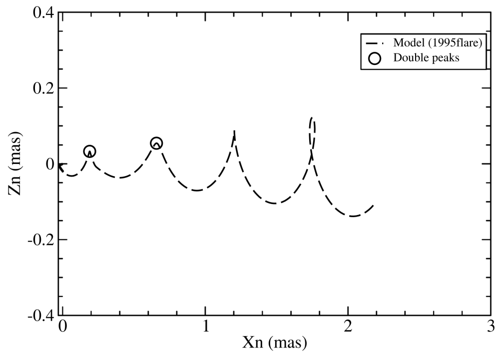

Fig. 11 The projected trajectory of the optical knot simulated for the 1995 outburst. The circles show the positions where the double flares are emitted.

(2) $Z(\mathrm{mas}) = 0.1 - 5$:
$A(Z)(\mathrm{mas}) = 0.0325$; $\Phi(Z)(\mathrm{rad}) = 1.682 + R_\phi(Z - 0.1)$; $R_\phi(Z)$ (rad mas^{-1}) $= 1.3$.
(3) $Z(\mathrm{mas}) = 5 - 6$:
$A(Z)(\mathrm{mas}) = 0.0325$; $\Phi(Z)(\mathrm{rad}) = 8.052 + R_\phi \times (Z - 5)$; R_ϕ (rad mas^{-1}) $= 0.6$.
(4) $Z(\mathrm{mas}) > 6$:
$A(Z)(\mathrm{mas}) = 0.0325[1+0.1(Z-6)]$; $\Phi(Z)(\mathrm{rad}) = 8.652+R_\phi(Z-6)$; R_ϕ (rad mas^{-1}) $= 0.6$.

The flux density emitted by the quiet background component (accretion disk of the primary black hole plus the quiet optical core) is taken as a constant, $F_0 = F_\mathrm{d} + F_\mathrm{c} = 2.5$ mJy.

The comoving flux density of the optical knot is set as follows:

(1) $Z(\mathrm{mas}) \leq 2$: $F_*(Z)(\mathrm{mJy}) = 5.42 \times 10^{-5} \times Z/2$.
(2) $Z(\mathrm{mas}) = 2 - 15$: $F_*(Z)(\mathrm{mJy}) = 5.42 \times 10^{-5}[1 - 0.01(Z - 2)]$.
(3) $Z(\mathrm{mas}) = 15 - 17$: $F_*(Z)(\mathrm{mJy}) = 4.72 \times 10^{-5}[1 - 0.5(Z - 15)]$.
(4) $Z(\mathrm{mas}) > 17$: $F_*(Z)(\mathrm{mJy}) = 0$.

The results of model simulation for the 1995 outburst are shown in Figures 9–11. It can be seen that the double peaked structure of the outburst is well fitted. The double flares from this outburst exhibit narrower profiles than those from the 1972 and 1983 outbursts. Thus a larger rotation rate is needed for the first flare ($R_\Phi = 1.3$ rad mas^{-1}, in comparison with $R_\Phi = 0.7$ rad mas^{-1} for the 1972 and 1983 outbursts). The second flare is simulated to occur in the expansion region of the jet with $R_\Phi = 0.6$ rad mas^{-1}. The radial distances of the two flares are 3.6 mas and 13 mas from the origin $Z = 0$. If we define the location of the first flare peak as the core of the optical jet, then the second flare occurs at a radial distance of 9.4 mas (∼42 pc). The widths of the jet at the sites are simulated to be 0.033 mas and 0.053 mas, larger than those for the 1972 and 1983 outbursts (0.028 mas).

5.4 Model Simulation of the 2005 Outburst

For the model simulation of the 2005 outburst, the parameters $A(Z)$, $\Phi(Z)$ and $R_\phi(Z)$ are set as follows.

(1) $Z \leq 0.1$ mas:
$A(Z)(\mathrm{mas}) = 0.0278 \times (Z/0.1)$; $\Phi(Z)(\mathrm{rad}) = 0.7520 + R_\phi \times Z$; $R_\phi(Z)$ (rad mas^{-1}) $= 0.7$.
(2) $Z(\mathrm{mas}) = 0.1 - 14$:
$A(Z)(\mathrm{mas}) = 0.0278$; $\Phi(Z)(\mathrm{rad}) = 0.822 + R_\phi(Z - 0.1)$; R_ϕ (rad mas^{-1}) $= 0.7$.
(3) $Z(\mathrm{mas}) > 14$:
$A(z)(\mathrm{mas}) = 0.0278[1 + 0.40(Z - 14)]$; $\Phi(Z)(\mathrm{rad\,mas^{-1}}) = 10.552 + R_\phi(Z - 14)$; $R_\phi(\mathrm{rad\,mas^{-1}}) = 0.23$; At $Z > 14$ mas, rotation rate R_ϕ has a small value and is for obtaining a larger time interval between the two flare peaks (∼2 yr).

The flux density emitted by the accretion disk of the primary black hole and the quiet optical core is taken to be a constant, $F_0 = F_\mathrm{d} + F_\mathrm{c} = 2.5$ mJy. The comoving flux density of the optical knot is set as follows:

(1) $Z(\mathrm{mas}) \leq 4.5$: $F_*(Z)(\mathrm{mJy}) = 1.13 \times 10^{-4} \times (Z/4.5)$.
(2) $Z(\mathrm{mas}) = 4.5 - 30$: $F_*(Z)(\mathrm{mJy}) = 1.13 \times 10^{-4}[1 - 0.02(Z - 9)]$.
(3) $Z(\mathrm{mas}) = 30 - 35$: $F_*(Z)(\mathrm{mJy}) = 0.655 \times 10^{-4}[1 - 0.2(Z - 30)]$.
(5) $Z(\mathrm{mas}) > 35$: $F_*(Z)$ (mJy) $= 0$.

The results of the model simulation are shown in Figures 12–14. It can be seen that the double peaked structure of the 2005 outburst is reasonably well simulated. In this case, the time interval between the two flare peaks is ∼2 yr, two times longer compared with those in the cases of the 1972, 1983 and 1995 outbursts. Thus the second flare is simulated to occur in the expansion region with rotation rate $R_\Phi = 0.23$ rad mas^{-1}, while the first flare is in the collimated region with $R_\Phi = 0.7$ rad mas^{-1}, similar to the case for the 1972 and 1983 outbursts. The intensity evolution of the optical knot shows some different behavior compared to those in the cases of the 1972 and 1983 outbursts, that is, its rest-frame flux density always slowly decreases from the collimated region to the expansion region without a plateau stage. In this case, the two intensity peaks are simulated to be at radial distances 7.8 mas and 24 mas. If we define the location of the first flare peak occurring at the core of the optical jet, then the second flare occurred at a separation of ∼16.2 mas (73 pc) from the core. This seems to imply that a very efficient acceleration mechanism for relativistic particles exists in the jet at ∼100 pc from the central supermassive black hole, which forms a Compton/synchrotron loss zone that produces γ-rays and optical emission.

The widths of the jet at these sites are 0.028 mas and 0.13 mas, respectively, and the projected separation of the second flare from the core is about 0.8 mas (3.6 pc).

The projected location $(Xn, Zn) = (1.2\,\mathrm{mas}, 0.13\,\mathrm{mas})$ obtained for the second flare here is similar to that obtained for the fourth peak of the light curve for blazar 3C 345 by Schramm et al. (1993) in their lighthouse model. (Note: in the case of 3C 345, the viewing angle of the jet is set as $0.95°$, thus the deprojected factor is larger than that in our model by a factor of ∼3.)

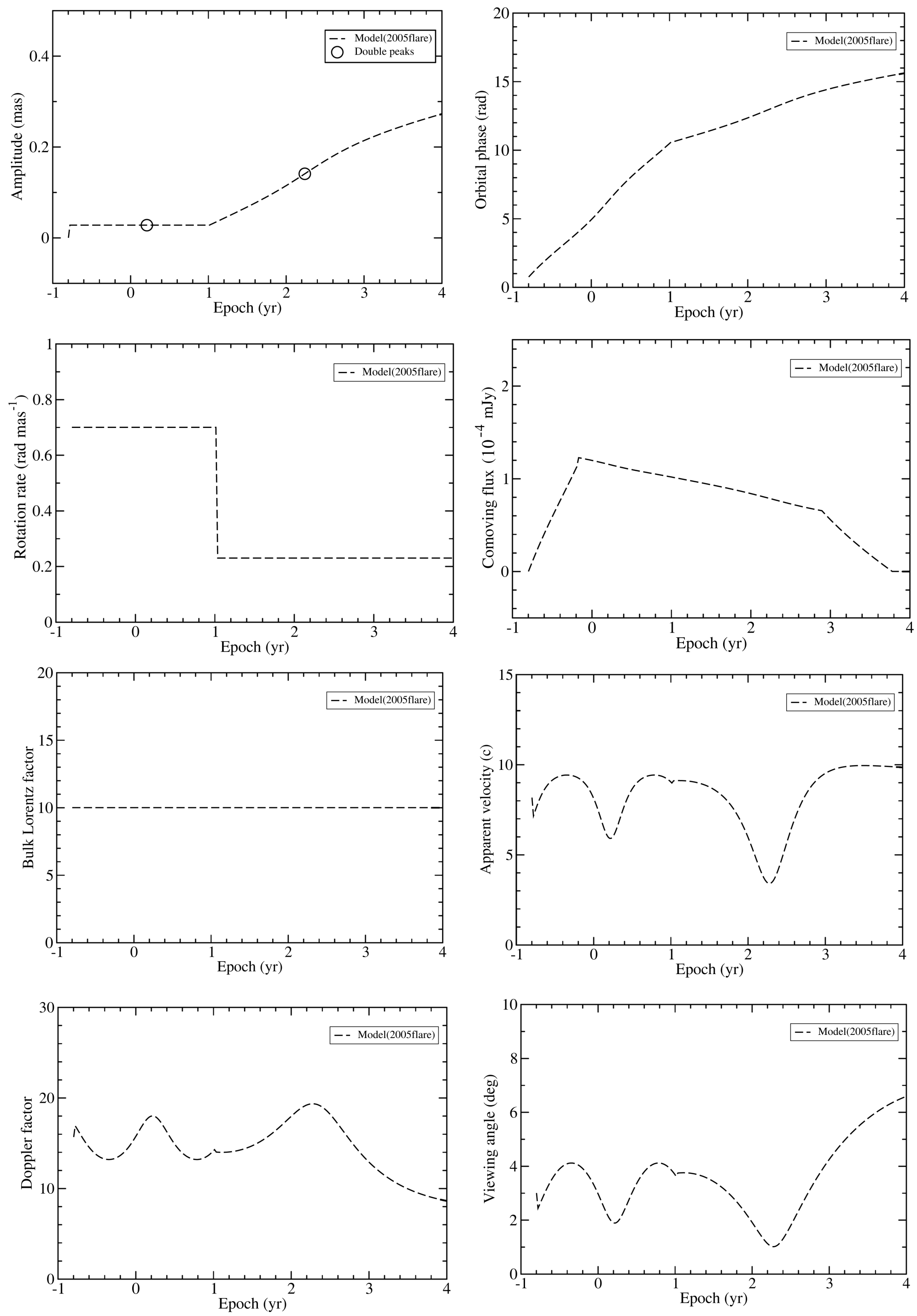

Fig. 12 Parameters used for the model simulation of the 2005 outburst. The upper four panels: amplitude A(t), orbital phase $\Phi(t)$, rotation rate $R_\phi(t)$ and rest-frame flux $F_*(t)$ of the optical knot; the lower four panels (for the kinematic properties of the superluminal motion of the optical knot): bulk Lorentz factor $\Gamma(t)$, apparent velocity $\beta_a(t)$, Doppler factor $\delta(t)$ and viewing angle $\theta(t)$. Epoch zero = 2005.60.

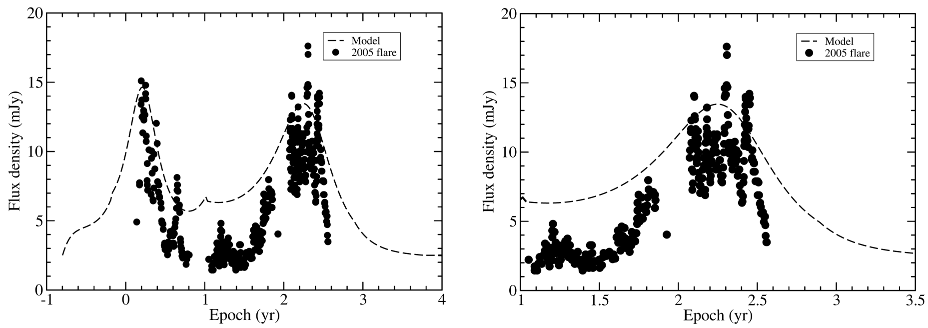

Fig. 13 The left panel shows the model simulation of the light curve of the 2005 outburst. The right panel shows the simulation of its second flare on an expanded timescale. Epoch zero = 2005.60.

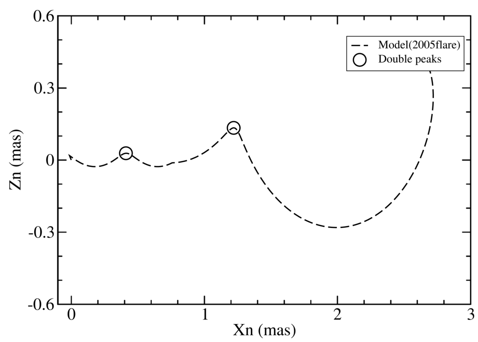

Fig. 14 The projected trajectory of the optical knot simulated for the 2005 outburst. The circles indicate the positions where the double flares are emitted.

6 SUMMARY AND DISCUSSION

We have done simulations using a consistent model for the light curves of the four double-peaked optical flares. Since we only chose a certain set of parameters and functions, these simulation results can only be regarded as particular solutions or examples, because different sets of parameters and functions would lead to different results (for example, for a different jet orientation, bulk Lorentz factor, helical pattern of trajectory, etc.). In the simulations, we have considered three emission components: emission from the superluminal optical knot and the 'quiet core' of the optical jet (both synchrotron) and emission from the accretion disk of the primary black hole (bremsstrahlung). Here we summarize the main ideas and assumptions involved in the simulations and the main results.

The orbital motion of the secondary black hole around the primary black hole strongly disturbs the accretion disk of the primary black hole and induces enhanced injection of plasmas and magnetic fields into the jet by tidal and electromagnetic effects near the pericenter passage. An adequate eccentricity and 12 yr orbital period are required. Thus superluminal knots are created (formed) periodically per 12 yr. The superluminal knots move along helical magnetic field lines and produce the double-peaked optical outbursts (optical light curves) through the lighthouse effect, due to the jet axis directed towards the observer with a small angle of $\sim 3^\circ$ and bulk Lorentz factor $\Gamma \sim 10$ (Doppler factor $\sim$12–18). These values are similar to those obtained by other researchers, e.g. (Tavecchio et al. 2010; Hovatta et al. 2009; Pihajoki et al. 2013; Ciprini et al. 2007). In the lighthouse model, the optical radiation is Doppler boosted by a factor of $\sim 10^4$, thus the comoving (rest-frame) radiation

Table 1 Comparison of the parameters used for the model simulations of the double flares of the four outbursts: amplitude A (mas), rotation rate R_ϕ (rad mas^{-1}) and axial distance between the location of the first and second flare D_{12} (mas). For all cases, the bulk Lorentz factor $\Gamma = 10$.

Outburst	Second flare		First flare		Distance
	A (mas)	R_ϕ (rad mas^{-1})	A (mas)	R_ϕ (rad mas^{-1})	D_{12} (mas)
1972	0.028	0.70	0.028	0.70	8.0
1983	0.028	0.70	0.028	0.70	8.0
1995	0.033	1.30	0.053	0.60	9.4
2005	0.028	0.70	0.13	0.23	16.2

energy is lower than the bremsstrahlung energy (in a binary black hole model with disk crossings) by the same factor. Significant energy is contained in the kinetic energy of the bulk relativistic motion of the optical knots. Thus the model simulations show that the binary black hole plus the lighthouse effect scenario may be useful for simplifying the excitation mechanism of the optical/radio outbursts, by only relying on gravitational tidal effects and electromagnetic interaction during the pericenter passage of the secondary black hole.

The four optical double-peaked outbursts that occurred in blazar OJ 287, in 1972, 1983, 1995 and 2005, are reasonably well simulated, including their peak intensity, profiles and the time interval between the two peaks. In the region where the double peaked outbursts occurred, the amplitude $A \simeq 0.03 - 0.05$ mas (i.e. width of the collimated jet region) and the rotation rate R_ϕ is $\sim 0.7 - 1.3$ rad mas^{-1}. The time intervals between the two peaks obtained in the model simulations are 1.08, 1.02, 1.17 and 2.03 yr for the four outbursts respectively. If the locations of the first flares are defined as the site of the core of the optical jet, then the second flares occur at axial separations of 38, 36, 32 and 73 pc for the four outbursts respectively. The simulations show that the parameters used for the four double-peaked flares are quite similar, as shown in Table 1 where a comparison of the parameters used for the modulations are given. This may indicate that the helical magnetic field structure of the jet in OJ 287 has been rather stable for a very long time period (e.g. $\sim$100 yr). This is possible because this helical field is anchored in the innermost region of the accretion disk of the primary black hole, whose gravitation keeps the helical field structure in the jet solid and stable.

In the simulations, the location of the first flares of the four double peaked outbursts is defined as the core of the optical jet of OJ 287 and the second flares occur at projected separations of $\sim$0.4 mas (for 1972, 1983 and 1995 outbursts) from the core (see Figs. 5, 8 and 11). These three outbursts have time intervals that correspond to double peaks of about 1–2 yr. The projected trajectories and timescales obtained by our model simulations are quite similar to those obtained by Schramm et al. (1993) for the model fitting to the periodic flares in blazar 3C 345 by applying a physical model of the lighthouse effect. We should point out that in our model simulations, the functions describing the helical trajectory ($A(Z)$, $R_\Phi(Z)$) and knot flux density evolution $F_*(Z)$ are given independently, thus our model could not be fully physically coherent. However, in searching for a possible explanation for the double-peaked structure of the outbursts by using the lighthouse effect, our simulations have obtained meaningful results, which can interpret the basic properties and require conditions for the lighthouse effect mechanism by applying the optical behavior observed in blazar OJ 287.

Based on our scenario, the three basic properties of OJ 287 (as Sillanpää et al. 1996b suggested) can be explained consistently: (a) the 12-year cyclic optical outburst behavior, (b) the double-peaked structure of the cyclic outbursts (as described above) and (c) the extremely stable color (observed during the 1995 outburst with both flares having the same color, Sillanpää et al. 1996b). The lighthouse effect is an achromatic effect (a purely geometric effect). The optical outbursts are solely produced by a change in the Doppler boosting when the optical knots follow helical trajectories. Villforth et al. (2010a) found spectral changes during the 2005 outburst (May 2005–June 2009): OJ 287 was bluer when it was brighter. This flattening of the optical spectrum could be interpreted in terms of injection of higher-energy electrons into the optical knot. Thus both the stable color and spectral change could be explained in our scenario as consequences of outbursts occurring in generic blazars.

Since in the highly collimated zone, the optical knots do not experience expansion losses and the efficient acceleration of particles could keep the optical knots emitting in optical wavebands, the optical knots could not evolve into radio knots and the optical knots themselves are optically thick in the mm/radio wavebands. This explains why the first flares of the double-peaked outbursts observed in OJ 287 were not accompanied by strong radio counterparts. Only when the optical knots evolve into mm/radio knots by expansion and due to a lack of acceleration of particles, strong mm/radio outbursts could appear. This could occur when the optical knots have moved out of the collimated region into the expanding region of the jet. This explains why the second flares of the double-peaked outbursts observed in OJ 287 had mm/radio counterparts.

The optical outbursts observed in OJ 287 have short timescale spikes of (∼a few weeks) with fluctuations in intensity of $\sim 30\% - 40\%$ both during the quiet phase and the burst phase. Valtaoja et al. (2000) argued against the lighthouse model based on this behavior. Although we did not take this ingredient into account in this paper, this behavior (short timescale variations) could be explained in our scenario. For example, the short timescale variations during the burst phase could be due to the shocks (optical knots) propagating through the very turbulent jet and those during the quiet phase due to turbulent plasma flow passing through a standing shock or the optical core. Therefore, they could have similar timescales, because they could have similar Doppler boosting by relativistic effects (Qian et al. 1991; Standke et al. 1996; Quirrenbach et al. 1989; Marscher & Jorstad 2010).

The key point (or assumption) in our model simulations is that there may exist a highly collimated zone in the jet of OJ 287, where expansion losses are negligible and efficient acceleration of electrons could compensate the radiation losses through Compton/synchrotron processes to keep the optical knots emitting in γ-ray and optical wavebands. Thus the rotation of the optical knots along the helical trajectories could produce double-peaked outbursts with similar intensities.

Figures 4, 7, 10 and 13 illustrate good examples. The significant decay of radiation from the optical knots could only occur after the optical knots move out of the collimated zone. This explanation is consistent with the results obtained by Schramm et al. (1993) for the periodic optical flares observed in blazar 3C 345: the first three optical flares with almost equal-intensity and equal time width fitted by their lighthouse model occurred in the highly collimated region (initial opening angle of the jet was 0.05 degrees), and the fourth (predicted) flare occurred in the region where the jet expanded with decayed intensity. Therefore, in principle, if there exists a highly collimated zone in the jet, the lighthouse effect could play its role to explain the occurrence of double-peaked outbursts.

However, in this case, the highly collimated region should have a rather long length.[5] This has been regarded as a problem for the lighthouse model (e.g. Sillanpää et al. 1996b). However, recent γ-ray observations seem to provide some evidence for this requirement. Schinzel et al. (2010, 2012) report that gamma-rays from the blazar 3C 345 were produced in a region of the jet that is up to 23 pc (de-projected) in extent[6] and suggest the synchrotron self-Compton process as the most likely mechanism for the production of gamma-rays, and question the entire class of models that place the gamma-ray emission site within 1 pc from the central engine of the AGN. In a study that connected mm with gamma-rays, Agudo et al. (2011b, 2012b) argued that in blazars OJ 287 and AO0235+164, gamma-ray flares were produced at sites larger than 14 pc and 12 pc from the mm-core, respectively. These observational facts strongly support the possibility of the existence of a highly collimated zone in the jets of blazars having a length of a few tens of parsecs, which is a key requirement of the lighthouse model.

5 A large scale of about a few tens parsecs, e.g. for the 1972 outburst of OJ 287, the second flare occurs at a location ∼40 pc from the core in the model simulation of this paper.

6 This distance depends on the viewing angle chosen. If a smaller viewing angle of 2.7° (Jorstad et al. 2005, instead of 5.2°, Schinzel et al. 2010) is used, this distance would be 40 pc.

These observations show that optical knots, optical photons of which act as the seed-photon source of the self-Compton process, can propagate to tens of parsecs, implying this region of the jet could be highly collimated and the optical knots have not been diminished by expansion and radiative losses, constructing a stable Compton/synchrotron loss zone that produces γ-ray and optical emission up to $\sim$100 pc. This is just what the lighthouse model requires. Within the collimated regions (without adiabatic losses due to sideways expansion), efficient particle acceleration by shocks and magnetic turbulences (e.g. the turbulent cell model proposed by Marscher & Jorstad 2010; Marscher 2014) would keep the optical knots emitting in synchrotron self-Compton-gamma-rays and synchrotron optical wavebands. This would be the cause the long length of the Compton loss zone found by Schinzel et al. (2010). Qian et al. (2010) also suggested the existence of a highly collimated region in the jet of the blazar BL Lacertae[7] through the study of the evolution of its mm/cm outbursts in terms of a 3-stage evolutional model (Compton-synchrotron-adiabatic stages) in order to explain the lack of spectral flattening from the transition from the Compton stage to the adiabatic stage. Thus both the observational results and the theoretical results (Camenzind & Krockenberger 1992; Schramm et al. 1993; Wagner et al. 1995) are consistent in supporting the following idea: in blazars, a highly collimated zone (Compton-synchrotron loss zone) could exist up to radial distances in the range $\sim$40–80 pc from the core.

In our model simulations, we only calculated the profiles of the lightcurves caused by the lighthouse effects, without considering the fast variability. Rapid variations with timescales of 1–2 weeks (optical spikes) could be due to relativistic shocks (superluminal optical knots) propagating through extremely turbulent jet flows (Qian et al. 1991b; Standke et al.1996; Marscher et al.1992; Marscher 1994) and these rapid variations are enhanced under the lighthouse profiles lasting $\sim$0.3–0.4 years. However, these variations are produced through random processes (e.g. electron acceleration by magnetic turbulence) and thus their strength, structure and phase within the lighthouse profiles could not be predicted.

We point out that the proposed model does not require extraordinary physical conditions (e.g. crossing of the secondary black hole into the accretion disk of the primary black hole) and seems very feasible (flexible). The results obtained by this numerical simulation demonstrate the plausibility (or possibility) to understand the double-peaked outbursts in terms of a jet phenomenon. The parameters and functions chosen in our model simulation are only examples, and they are not unique. Different sets of parameters and functions could be chosen for the simulation. However, such a type of model simulation could not be used to make an accurate timing of future double-peaked events or other properties, because in this model, the occurrence of the radiation processes mentioned in Section 3 cannot be accurately predicted. (This is in contrast to the binary hole scenario of Lehto & Valtonen (1996) with two crossings of the secondary black hole into the accretion disk of the primary black hole). The biggest caveat is how to find a set of helical motions (A and R_Φ) and evolution of optical knots which are allowed by theoretical arguments based on jet formation and radiation theory. The numerical simulation does not take into account the physical connection between the helical trajectory and rest-frame intensity, which is not necessarily appropriate; they might be inconsistent with each other in some aspects: for example, the expanding helical trajectory and the evolution of the knots' emission. Of course, the model simulation of the profiles, interval lengths, peak intensity and peak-ratio should be based on consistent theoretical models.

Although losing the ability for predicting the timing of the second flares, the proposed scenario has the advantage of accommodating the explanations for the double peaked outbursts, including the time interval between the two peaks, flare profile width, peak intensity ratio, etc.[8] This scenario

[7] A similar phenomenon could occur in blazar 3C 273 and 3C 345 (Qian et al. 2010; Stevens et al. 1996, 1998).

[8] Our model simulation did not include the variation of the optical emission on timescales less than $\sim$1 month. These short-term variations could be due to variable conditions within the optical knots (e.g. acceleration of relativistic electrons by magnetic turbulences), while the shocks propagate through the turbulent jet, experiencing relativistic time shortening.

seems to be more consistent with the results of theoretical studies on optical flares in generic blazars (synchrotron plus relativistic beaming).

Although we did not give the physical details (models) for the four processes, it does not seem difficult to describe these processes individually in the framework of astrophysics for blazars. Based on the results of this model, a detailed theoretical model of a relativistic jet for the double-peaked outbursts could be established. Specifically, in the case of OJ 287, a detailed physical model should contain several theoretical aspects as follows. (1) The interaction between the secondary black hole and the accretion disk of the primary black hole should be detailed, and the enhanced injection of the magnetized plasma flow into the jet and the formation of the superluminally moving optical knot (including the calculation of the time delays between the pericenter passage of the secondary black hole and the first optical flare) (Sillanpää et al. 1988; Valtonen & Wiik 2012). (2) The helical field structure should be described by continuous functions. The amplitude function $A(Z)$ should conform to the rotation rate function $R_{\phi}(Z)$ through the rule of conservation of angular momentum. (Camenzind & Kronckenberg 1993). (3) The connection between the disk bremsstrahlung and the synchrotron radiation of the superluminal knot should be detailed. (4) Further investigations into the correlation between mm/radio and optical outbursts and that between the optical and γ-ray outbursts, including optical polarization and SED, should be carried out, which would help to clarify the nature of the optical emission and determine the physical parameters for the three processes. Future observations of double-peaked optical flares would test the model. Since we ascribe the double flares of the outbursts to the synchrotron of the optical knots in the jet, there are plenty of theoretical results for generic blazars that can be applied to OJ 287: for example, the theory of magnetohydrodynamics for relativistic jets and the accretion disk flow and the theory of how outbursts produce radiation in blazars.

The proposed model is oversimplified and is just preliminary, tentative and semi-qualitative. Our aim is to find some alternative clues to explain the formation of the double-peaked optical outbursts observed in blazar OJ 287. Future observations would test these ideas. Obviously, if the lighthouse effect is an appropriate model to interpret the double peaked outbursts of OJ 287, then theoretical and physical models (like Camenzind & Krockenberger 1992; Schramm et al. 1993; Wagner et al. 1995) should be constructed to establish the physical connections between the helical motion, evolution of the knot, interaction of the disk disturbances and plasma/field injection of the jet, bulk Lorentz factor, etc.

In summary, we have tentatively suggested that there might be a possibility to explain the 12-year cycle optical outbursts observed in blazar OJ 287 in terms of the framework proposed by Sillanpää et al. (1988): the pericenter passage (with a 12-year cycle) of the secondary black hole induces disturbances by tidal effects (and enhanced accretion) in the disk of the primary black hole, which are then transformed into superluminal knots in the jet after some time delay. The superliminal motion of these optical knots along the helical trajectory could cause the lighthouse effect, producing the double-peaked structure of the optical outbursts observed in OJ 287. As an alternative scenario, it might also possible that these knots propagate through two separate standing shocks (e.g. one optical core and one mm-core along the jet), producing the double-peaked structure of the optical outbursts.

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