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Singlet pairing gaps of neutrons and protons in hyperonic neutron stars *

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Abstract The $^1S_0$ nucleonic superfluids are investigated within the relativistic mean-field model and Bardeen-Cooper-Schrieffer theory in hyperonic neutron stars. The $^1S_0$ pairing gaps of neutrons and protons are calculated based on the Reid soft-core interaction as the nucleon-nucleon interaction. In particular, we have studied the influence of degrees of freedom for hyperons on the $^1S_0$ nucleonic pairing gap in neutron star matter. It is found that the appearance of hyperons has little impact on the baryonic density range and the size of the $^1S_0$ neutronic pairing gap; the $^1S_0$ protonic pairing gap also decreases slightly in this region where $\rho_B = 0.0$–0.393 fm^{-3}. However, if baryonic density becomes greater than 0.393 fm^{-3}, the $^1S_0$ protonic pairing gap obviously increases. In addition, the possible range for a protonic superfluid is obviously enlarged due to the presence of hyperons. In our results, the hyperons change the $^1S_0$ protonic pairing gap, which must change the cooling properties of neutron stars.

Key words: dense matter — (stars:) pulsars: general — equation of state

1 INTRODUCTION

In recent years, neutron stars (NSs) have become one of the hottest scientific problems in the domain of astrophysics. The reasoning is that the nucleonic energy gap and the corresponding critical temperature for superfluidity (SF) can greatly affect the emission of neutrinos, which dominate about 10^5–10^6 years of the cooling phase for NSs (Zuo et al. 2004; Zuo & Lombardo 2010; Gao et al. 2011; Tanigawa et al. 2004; Kaminker et al. 2002; Xu et al. 2013, 2012a; Tang et al. 2013; Liu & Wang 2013). Neutrons and protons in the interior of NSs can transition into superfluid states due to the attraction between two neutrons or protons. Neutrons in the NS crust probably form $^1S_0$ pairings and in the NS core, mainly form $^3P_2$ pairings. Protons in the NS core can suffer $^1S_0$ pairings, which appear in NS matter with supranuclear density. Such a dense region is closely related to the direct Urca processes that affect nucleons (Yakovlev et al. 1999; Shternin et al. 2011; Chen et al. 2006). It is well known that direct Urca processes for nucleons produce very powerful neutrino energy losses

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(Haensel & Gnedin 1994; Gnedin et al. 1994; Yakovlev et al. 2008; Xu et al. 2014). Thus nucleonic superfluids must affect NS cooling.

The $^1S_0$ nucleonic pairing gap has been considered using different model potentials of the nucleon-nucleon (NN) interaction. These theoretical calculations based on qualitative models give similar ranges for the presence of the $^1S_0$ nucleonic pairing. Nevertheless, due to many uncertain factors about the NN interaction such as non-direct observational data in extreme conditions, which lead to approximations used in the calculations, this process cannot obtain accurate results for the pairing gap and estimate the quantitative influence of the superfluid on NSs. Nowdays, many relativistic models draw attention in studies on NSs because they are particularly well suited for describing NSs according to special relativity. The most common among them is the relativistic mean field (RMF) theory, which could give very successful descriptions of nuclear matter and research about finite nuclei (Glendenning 1985). In 1993, to adapt to the effective $\Lambda\Lambda$ interaction $\Delta B_{\Lambda\Lambda} \sim 5$ MeV inferred from an earlier measurement and avoid the high-density instability in numerical calculations, Schaffner et al. extended the standard RMF model by adding strange mesons σ^* and ϕ (Schaffner et al. 1993). That is, baryons interact by exchanging the σ, ω, ρ, σ^* and ϕ mesons. However, recent measurements suggest that $\Delta B_{\Lambda\Lambda}$ should be $1.01 \pm 0.20^{+0.18}_{-0.11}$MeV (Bednarek & Manka 2005; Yang & Shen 2008; Wang & Shen 2010; Xu et al. 2012b). In addition, when the baryonic density is lower than $2\rho_0$ (ρ_0 is the saturation density of nuclear matter), NSs are generally made up of only neutrons, protons and leptons. Yet if the baryonic density exceeds about $2\rho_0$, hyperons, as new degrees of freedom, will appear in the cores of NSs. This must result in changing the equation of state (EOS); the nucleonic Fermi momenta and single particle energies tend to change, so the appearance of hyperons would affect the nucleonic SF. Σ hyperons are ruled out due to the debatable Σ potential at ρ_0 in nuclear matter (Batty et al. 1994). This paper is primarily focused upon the influence of hyperons on the $^1S_0$ nucleonic pairing gap.

The content of the paper is arranged in this way. The properties of an NS and $^1S_0$ nucleonic pairing gap are described using RMF and Bardeen-Cooper-Schrieffer (BCS) theories in Section 2. The numerical results are described in Section 3. The summary is presented in Section 4.

2 THE MODELS

Baryonic interactions occur by exchanging σ, ω, ρ, σ^* and ϕ mesons in the RMF approach. In this paper, neutron (n), proton (p), Λ and Ξ baryons are considered in NSs. The contribution of the first three mesons to the Lagrangian is (Glendenning 1985),

$$\begin{aligned} L \;=\; & \sum_B \overline{\psi}_B\Big[i\gamma_\mu\partial^\mu - (M_B - g_{\sigma B}\sigma) - g_{\rho B}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu - g_{\omega B}\gamma_\mu\omega^\mu\Big]\psi_B \\ & +\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - U(\sigma) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu \\ & -\frac{1}{4}F^{\mu v}F_{\mu v} - \frac{1}{4}G^{\mu v}G_{\mu v} + \sum_l \overline{\psi}_l\Big[i\gamma_\mu\partial^\mu - m_l\Big]\psi_l. \end{aligned} \tag{1}$$

Here, the field tensors of the vector mesons ω and ρ are denoted as $F_{\mu v}$ and $G_{\mu v}$, respectively. $U(\sigma) = \frac{1}{3}a\sigma^3 + \frac{1}{4}b\sigma^4$. The baryon species are represented by B. The contribution of strange mesons σ^* and ϕ to the Lagrangian is,

$$\begin{aligned} L^{YY} \;=\; & \frac{1}{2}(\partial_v\sigma^*\partial^v\sigma^* - m_{\sigma^*}^2\sigma^{*2}) - \sum_B g_{\sigma^* B}\overline{\psi}_B\psi_B\sigma^* - \sum_B g_{\phi B}\overline{\psi}_B\gamma_\mu\psi_B\phi^\mu \\ & -\frac{1}{4}S^{\mu v}S_{\mu v} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu. \end{aligned} \tag{2}$$

Here, because σ^* and ϕ are not coupled with nucleons, they only affect the hyperonic properties.

The five meson fields are considered to be classical fields and the field operators are replaced by their expected values in the RMF model (Glendenning 1985; Bednarek & Manka 2005; Yang & Shen 2008; Wang & Shen 2010; Xu et al. 2012b). The meson field equations in NSs are as follows:

$$\sum_B g_{\sigma B}\rho_{SB} = m_\sigma^2\sigma + a\sigma^2 + b\sigma^3, \tag{3}$$

$$\sum_B g_{\omega B}\rho_B = m_\omega^2\omega_0, \tag{4}$$

$$\sum_B g_{\rho B}\rho_B I_{3B} = m_\rho^2\rho_0, \tag{5}$$

$$\sum_B g_{\sigma^* B}\rho_{SB} = m_{\sigma^*}^2\sigma^*, \tag{6}$$

$$\sum_B g_{\phi B}\rho_B = m_\phi^2\phi_0. \tag{7}$$

Here I_{3B} is the isospin projection of baryon species B. ρ_{SB} and ρ_B denote baryonic scalar and vector densities, respectively. They are

$$\begin{aligned}\rho_{SB} &= \frac{1}{\pi^2}\int_0^{k_{\rm F}} \frac{m_B^*}{\sqrt{k^2+m_B^{*2}}}k^2dk,\\ \rho_B &= \frac{k_{\rm F}^3}{3\pi^2}.\end{aligned} \tag{8}$$

Here $k_{\rm F}$ is the baryonic Fermi momentum and $m_B^* = m_B - g_{\sigma B}\sigma_0 - g_{\sigma^* B}\sigma_0^*$ is the baryonic effective mass.

A description of NS matter with a uniform distribution is obtained through the conditions of electrical neutrality and β equilibrium. The electrical neutrality condition is

$$\rho_p = \rho_{\Xi^-} + \rho_e + \rho_\mu. \tag{9}$$

The baryonic chemical potential is expressed by

$$\mu_B = \mu_n - q_B\mu_e, \tag{10}$$

where q_B is the baryonic electric charge number. Then the β equilibrium conditions are given by

$$\mu_n = \mu_p + \mu_e, \quad \mu_{\Xi^-} = \mu_n + \mu_e, \quad \mu_n = \mu_\Lambda = \mu_{\Xi^0}, \quad \mu_e = \mu_\mu. \tag{11}$$

The nucleonic single-particle energy in the model is

$$E_N(k) = \sqrt{k^2 + {m_N^*}^2} + g_{\omega N}\omega_0 + g_{\rho N}\rho_{03}I_{3N}. \tag{12}$$

The BCS gap equation is (Zuo et al. 2004; Chen et al. 2006; Xu et al. 2013),

$$\Delta_N(k) = -\int \frac{V_{\rm NN}(k,k')\Delta_N(k')k'^2dk'}{4\pi^2\sqrt{\left[(E_N(k') - E_N(k_{\rm F})\right]^2 + \Delta_N^2(k')}}\,. \tag{13}$$

The $^1S_0$ pairing gaps of neutrons and protons are calculated based on the Reid soft-core (RSC) interaction (Nishizaki et al. 1991; Sprung & Banerjee 1971; Amundsen & Østgaard 1985; Wambach et al. 1993). The $^1S_0$ channel interaction between two neutrons or protons is

$$V_{\rm NN}(k,k') = 4\pi\int r^2 dr j_0(kr)V_{\rm NN}(r)j_0(k'r)\,, \tag{14}$$

where $V_{\rm NN}(r)$ is the ${}^1S_0$ NN interaction potential in coordinate space and $j_0(kr)$ is the zero order spherical Bessel function.

The nucleonic critical temperature $T_{\rm CN}$ of the ${}^1S_0$ pairing SF is (Takatsuka & Tamagaki 2003),

$$T_{\rm CN} \approx 0.66\Delta_N(k_{\rm F}) \times 10^{10}. \tag{15}$$

According to the discussion of the RMF approach above, we can obtain the EOS and NS composition as well as the nucleonic Fermi momenta and single particle energies, which are vitally important in research about the NN pairing gap.

3 DISCUSSION

Due to uncertainty in the interior constitution of NSs, we research NSs in two cases: (i) NS composition is n, p, e, μ(npeμ), (ii) n, p, Λ, Ξ^0, Ξ^-, e, μ(npHeμ). This work focuses on the influence of hyperons on the ${}^1S_0$ nucleonic pairing gap in NSs. The appearance of hyperons changes the EOS and NS composition as well as the ${}^1S_0$ nucleonic pairing SF. It is widely accepted that ${}^1S_0$ nucleonic superfluids should be controlled by the pairing gap $\Delta_N(k)$. Next, we will show the numerical results for the ${}^1S_0$ nucleonic pairing gaps in npeμ and npHeμ matter. The NSs' properties are found using the set of parameters displayed in Tables 1 and 2. We use $U_\Lambda^N = -30$ MeV, $U_\Sigma^N = +30$ MeV, $U_\Xi^N = -18$ MeV and $U_\Lambda^\Lambda = -5$ MeV, which are obtained based upon the recent measurement $\Delta B_{\Lambda\Lambda} \sim 1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV to decide the hyperonic scalar coupling constants. We use

$$\frac{2}{3}g_{\omega N} = g_{\omega\Lambda} = 2g_{\omega\Xi}, \qquad g_{\rho N} = g_{\rho\Xi}, \qquad g_{\rho\Lambda} = 0, \qquad 2g_{\phi\Lambda} = g_{\phi\Xi} = -\frac{2\sqrt{2}}{3}g_{\omega N}$$

to calculate the hyperonic vector coupling constants (Bednarek & Manka 2005; Yang & Shen 2008; Wang & Shen 2010; Xu et al. 2012b).

As mentioned above, baryons interact by exchanging mesons. More specifically, the attraction, repulsion and isospin interaction between two baryons are supplied by σ, ω and ρ, respectively. The additional attraction and repulsion between two hyperons are supplied by strange mesons σ^* and ϕ, respectively.

Figure 1 gives the EOS, namely pressure P and energy density ε as a function of baryonic density ρ_B in npeμ and npHeμ matter. As shown in Figure 1, one can see that the pressure P and energy density ε remain unchanged at lower densities in both cases. However, with increasing baryonic density, the appearance of hyperons makes the pressure P and energy density ε decrease. That is, the EOS is softened, which will inevitably cause the NS's bulk property to change. The crucial physical quantities for the ${}^1S_0$ nucleonic pairing gap are the nucleonic Fermi momenta and single-particle energies.

Figure 2 shows the numerical results of NS composition as a function of baryonic density ρ_B in npeμ and npHeμ matter. As shown in Figure 2, the threshold densities of Λ, Ξ^- and Ξ^0 hyperons are 0.320 fm^{-3}, 0.389 fm^{-3} and 0.734 fm^{-3}, respectively. One can also see that nucleonic fractions are

Table 1 The TM1 Set with Masses in the Unit of MeV (Yang & Shen 2008)

m_σ	$g_{\sigma N}$	m_ω	$g_{\omega N}$	m_ρ	$g_{\rho N}$	c_3	g_2(fm^{-1})	g_3	m_N	m_ϕ
511.198	10.029	783.0	12.614	770.0	4.632	71.308	7.233	0.618	983.0	1020.0

Table 2 The Scalar Coupling Constants of Hyperons with Masses in the Unit of MeV

	m_{σ^*}	m_{ϕ^*}	$g_{\sigma\Lambda}$	$g_{\sigma\Xi}$	$g_{\sigma^*\Lambda}$	$g_{\sigma^*\Xi}$
with $\sigma^*\phi$	975.0	1020.0	6.170	3.202	5.412	11.516

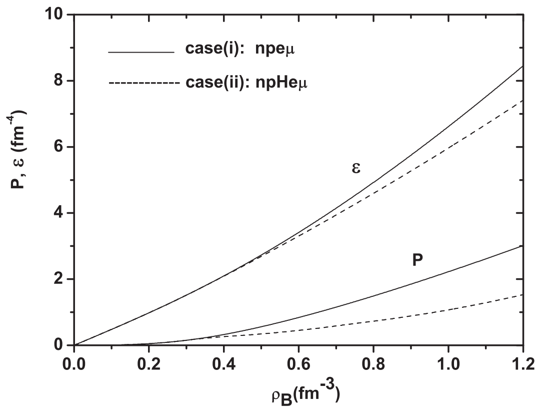

Fig. 1 Pressure P and energy density ε as a function of baryonic density ρ_B in npeμ and npHeμ matter.

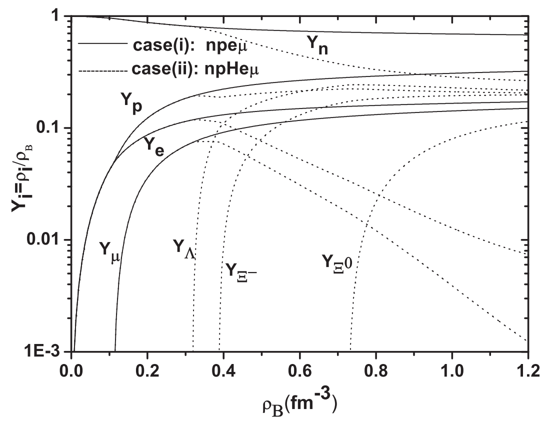

Fig. 2 Composition of NSs as a function of baryonic density ρ_B.

suppressed due to hyperons appearing in NSs through the conditions of electrical neutrality and β equilibrium (see Eqs. (9)–(11)). Therefore, according to Equation (8), we can see that when hyperons appear in NSs, the individual Fermi momenta of neutrons and protons are all much less than their values in npeμ matter.

Figure 3 displays the nucleonic single particle energy $E_N(k_\mathrm{F})$ at the Fermi surface as a function of baryonic density ρ_B in npeμ and npHeμ matter. As Figure 3 shows, the nucleonic single particle energies in npHeμ matter are obviously less than their values in npeμ matter, which is because the reduction in nucleonic Fermi momenta results in $E_n(k_\mathrm{F})$ and $E_p(k_\mathrm{F})$ all decreasing in npHeμ matter (see Eq. (12)).

So far, due to the uncertainty of NN interaction, the $^1S_0$ nucleonic pairing gap is also uncertain. In this work, we calculate $\Delta_N(k)$ based on the RSC potential. Our main concentration is the hyperonic influence on the $^1S_0$ nucleonic pairing gap.

Figure 4 presents the $^1S_0$ nucleonic pairing gap at the Fermi surface as a function of baryonic density ρ_B in npeμ and npHeμ matter. In Figure 4, one can see that the $^1S_0$ neutronic pairing gap always exists in the region with lower densities in both cases. The region with neutronic superfluids only affects the NS's surface cooling. The $^1S_0$ protonic superfluid can reach relatively high densities. The region with protonic superfluids is closely related to the direct Urca processes on nucleons, which govern almost all the cooling processes of NSs. In addition, one can also see that the appear-

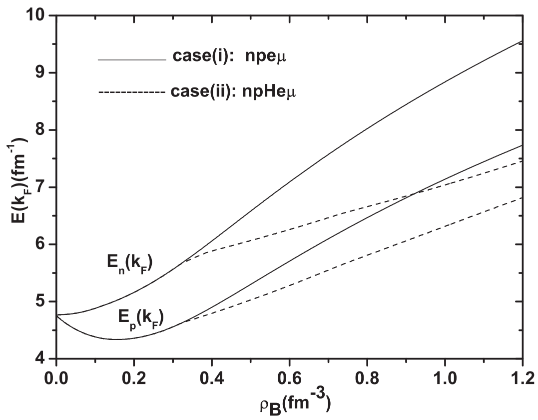

Fig. 3 The nucleonic single particle energy $E_N(k_\mathrm{F})$ at the Fermi surface vs. baryonic density ρ_B in npeμ and npHeμ matter.

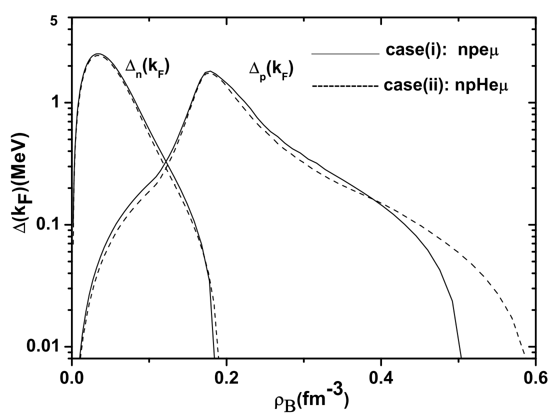

Fig. 4 The $^1S_0$ nucleonic pairing gap $\Delta_N(k_\mathrm{F})$ at the Fermi surface as a function of baryonic density ρ_B in npeμ and npHeμ matter.

ance of hyperons has little impact on the range of baryonic density and the size of the $^1S_0$ neutronic pairing gap $\Delta_n(k_\mathrm{F})$ in Figure 4. This is because the $^1S_0$ neutronic superfluids only appear in the region with lower densities where hyperons do not appear in NS matter. To clearly see the influence of hyperon degrees of freedom on the $^1S_0$ nucleonic pairing gap more intuitively, baryonic density ranges for the $^1S_0$ nucleonic pairing gap $\Delta_N(k_\mathrm{F})$ at the Fermi surface in npeμ and npHeμ matter are listed in Table 3. As seen in Figure 4 and Table 3, when hyperons appear in NS matter, the $^1S_0$ protonic pairing gap $\Delta_p(k_\mathrm{F})$ decreases slightly in the region with $\rho_B = 0.0 - 0.393\ \mathrm{fm}^{-3}$, and obviously increases in the region $\rho_B = 0.393 - 0.588\ \mathrm{fm}^{-3}$, which is because of the appearance of Λ and Ξ^- hyperons in the NS core (see Fig. 2 for details). The increase of the $^1S_0$ protonic pairing gap must lead to the increase in protonic critical temperature T_CP (see Eq. (15)), so the neutrino energy losses from direct Urca process on nucleons would be further suppressed in the region with $\rho_B = 0.393 - 0.588\ \mathrm{fm}^{-3}$. The range of the $^1S_0$ protonic SF is obviously enlarged due to the presence of hyperons, which can achieve coverage or partial coverage in the cores of NSs. That is, if we

Table 3 Baryonic Density Ranges for the $^1S_0$ Nucleonic Pairing Gap $\Delta_N(k_{\rm F})$ at the Fermi Surface in npeμ and npHeμ Matter

	npeμ	npHeμ
$\Delta_n(k_{\rm F})$	$0.0 \le \rho_B \le 0.188$	$0.0 \le \rho_B \le 0.191$
$\Delta_p(k_{\rm F})$	$0.0 \le \rho_B \le 0.509$	$0.0 \le \rho_B \le 0.588$

do not consider the contributions of the direct Urca processes on hyperons involved in NS cooling, the presence of hyperons must decrease the cooling rate of NSs.

4 CONCLUSIONS

We study the effects of hyperons on the $^1S_0$ nucleonic SF by adopting the RMF and BCS theories in NSs. The results indicate that the appearance of hyperons has little influence on the baryonic density range and size for the $^1S_0$ neutronic SF. However, the $^1S_0$ protonic pairing gap (and the $^1S_0$ protonic critical temperature) in npHeμ matter is much larger than their values in npeμ matter in the region where $\rho_B = 0.393 - 0.588$ fm^{-3}. The baryonic density range of the $^1S_0$ protonic SF is also enlarged from $\rho_B = 0.0 - 0.509$ fm^{-3} to $\rho_B = 0.0 - 0.588$ fm^{-3} on account of the presence of hyperons. The changes in the $^1S_0$ protonic SF can further suppress the NS's cooling rate. Hyperons in NSs change the $^1S_0$ protonic pairing gap, which must affect the cooling properties of NSs.

Our model may be a simplification because it adopts the lowest level of approximation in the BCS equation as well as neglecting the possible influence of inhomogeneity in the NS crust and $^1S_0$ hyperonic pairing in the NS core on the $^1S_0$ nucleonic energy gap. However, it can still clearly describe influence of hyperon degrees of freedom on the $^1S_0$ nucleonic pairing. We will analyze more complicated models in future studies.

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