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# Interacting viscous dark energy in a Bianchi type-III universe

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**Abstract** We study the evolution of the equation of state of viscous dark energy in the scope of Bianchi type III space-time. We consider a case where the dark energy is minimally coupled to the perfect fluid, as well as in direct interaction with it. The viscosity and the interaction between the two fluids are parameterized by constants $\zeta_0$ and $\sigma$, respectively. We have made a detailed investigation of the cosmological implications of this parametrization. To differentiate between different dark energy models, we have performed a geometrical diagnostic by using the statefinder pair $\{s, r\}$.

**Key words:** Bianchi type-III models — dark energy — statefinder

## 1 INTRODUCTION

Recent astronomical and astrophysical observations indicate that we live in an accelerating, expanding Universe (Perlmutter et al. 1997, 1999; Riess et al. 1998, 2001; Tonry et al. 2003; Tegmark et al. 2004). This fact opens a very fundamental question regarding the source that can produce such an accelerating expansion. Since ordinary matter (energy) generates an attractive gravitational force, there should be a kind of unknown, non-baryonic source of energy with negative pressure in order to accelerate the expansion of the Universe. Of course, the amount of this energy should be larger than the amount of ordinary matter (energy) since a fraction of this force has to first counterbalance the attractive force of ordinary matter and then the rest gives rise to acceleration.

According to recent observations, we live in a nearly spatially flat Universe composed of approximately $4\%$ baryonic matter, $22\%$ dark matter (DM) and $74\%$ dark energy (DE). We know that the ultimate fate of our Universe will be determined by DE but unfortunately our knowledge about its nature and properties is still very limited. We do not even know the current value of the DE effective equation of state (EoS) parameter $\omega^X = p^X/\rho^X$. We only know that a kind of exotic energy with negative pressure drives the current accelerating expansion of the Universe and, although it dominates the present Universe, it was small at early times. This is why so many candidates have been proposed for DE including: cosmological constant ($\omega^X = -1$) (Weinberg 1989; Carroll 2001; Padmanabhan 2003; Peebles & Ratra 2003), quintessence ($-1 < \omega^X < -\frac{1}{3}$) (Wetterich 1988; Ratra & Peebles 1988), phantom ($\omega^X < -1$) (Caldwell 2002), quintom ($\omega^X < -\frac{1}{3}$) (Feng et al. 2005), interacting DE models, Chaplygin gas as well as generalized Chaplygin gas models (Srivastava 2005; Bertolami et al. 2004; Bento et al. 2002; Alam et al. 2003), etc. A cosmological constant (or vacuum energy) seems to be a proper candidate for DE because it can explain the current acceleration in a natural way, although it would suffer from some theoretical problems, such as the fine-tuning and coincidence problems. Quintessence and phantom DE models are provided by scalar fields. However, these models also encounter some problems. For example, since recent observations (Hinshaw et al. 2009; Komatsu et al. 2009; Copeland et al. 2006; Perivolaropoulos 2006) indicate that $\omega^X < -1$ is allowed at the $68\%$ confidence level, quintessence with $\omega^X > -1$ may not be a proper candidate for DE. Phantom DE models also suffer from some fundamental problems, such as a future singularity problem called the Big Rip (Caldwell et al. 2003; Nesseris & Perivolaropoulos 2004) and the problem of ultraviolet quantum instabilities (Carroll et al. 2003). Since recent cosmological observations mildly favor models with a transition from $\omega^X > -1$ to $\omega^X < -1$ in the near past (Riess et al. 2004; Choudhury & Padmanabhan 2005), a combination of quintessence and phantom in a unified model called quintom has been proposed (Feng et al. 2005).

Recently, dissipative DE models in which the negative pressure, which is responsible for the current acceleration, is an effective bulk viscous pressure have been proposed in order to avoid the occurrence of the Big Rip (McInnes 2002; Barrow 2004). The general theory of dissipation in a relativistic imperfect fluid was first suggested by Eckart (1940), Landau & Lifshitz (1987). Although this is only the first-order deviation from equilibrium and may suffer from a causality problem, one can still apply it to phenomena which are quasi-stationary; that is, slowly varying in space and time characterized by the mean free path and the mean collision time. It is worth mentioning that the second-order causal theory was proposed by Israel (1976) and developed by Israel & Stewart (1976). The effect of bulk viscosity on the background expansion of the Universe has been investigated from different points of view (Cataldo et al. 2005; Brevik & Gorbunova 2005; Szydlowski & Hrycyna 2007; Singh 2008; Feng & Li 2009; Piattella et al. 2011; Amirhashchi 2013a,b). There is also some astrophysical observational evidence which indicates that the cosmic media is not a perfect fluid (Jaffe et al. 2005). Therefore, the viscosity effect could be involved in the evolution of the Universe. The role of viscous pressure as an agent that drives the present acceleration of the Universe has also been studied (Zimdahl et al. 2001; Balakin et al. 2003). The possibility of a viscosity dominated late epoch of the Universe with accelerated expansion was already mentioned by Padmanabhan & Chitre (1987).

Interaction between DE and DM has been proposed as a possible solution to the coincidence problem (Setare 2007d; Jamil & Rashid 2008, 2009; Chimento et al. 2003). Moreover, DE-DM interaction provides the possibility of detecting DE in a natural way. It is worth mentioning that the possibility of such an interaction has been supported by recent observations (Bertolami et al. 2007; Le Delliou et al. 2007; Berger & Shojaei 2006). Interacting DE models have been widely investigated in literature (see, for example, Amirhashchi et al. 2011a,b; Amirhashchi et al. 2013a; Amirhashchi et al. 2013b; Amirhashchi 2013a,b, 2014; Saha et al. 2012; Yadav & Sharma 2013; Yadav 2012; Pradhan et al. 2011; Setare 2007a,b,c; Setare et al. 2009; Sheykhi & Setare 2010; Jamil & Farooq 2010; Zhang 2005; Sadjadi & Vadood 2008). A full dynamic analysis of anisotropic scalar-field cosmology with arbitrary potentials has been studied by Fadragas et al. (2013). Recently, Zu et al. (2014) investigated a class of transient acceleration models that are consistent with Big Bang cosmology.

In this paper, we study the behavior of the viscous DE EoS parameter in an anisotropic space-time, namely a Bianchi type III Universe in the following two cases: (i) when DE and DM are minimally coupled, that is, when there is no interaction between these two dark components; and, (ii) when there is an interaction between viscous DE and DM. We parameterize the interaction by a constant $\sigma$ and viscosity by $\zeta_0$. A detailed investigation of the cosmological implications of this parametrization will then be provided by assuming an energy flow from DE to DM. Finally, to discriminate the different interaction parameters, as usual, a statefinder diagnostic is also performed.

The outline of the paper is as follows: In Section 2, the metric and the field equations are described. Section 3 deals with the solutions of the field equations. In Sections 4 and 5 we study non-interacting and interacting viscus DE respectively. A geometrical diagnostic also performed in Section 6. Finally, concluding remarks summarized in Section 7.

## 2 THE METRIC AND FIELD EQUATIONS

We consider the Bianchi type-III metric as

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)\mathrm{e}^{-2\alpha x}dy^2 + C^2(t)dz^2\,, \tag{1}$$

where $A(t), B(t)$ and $C(t)$ are only functions of time.

We define the following physical and geometric parameters to be used in formulating the law and further in solving Einstein's field equations for the metric (1).

The average scale factor $a$ of the Bianchi type-III model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}\,. \tag{2}$$

A volume scale factor $V$ is given by

$$V = a^3 = ABC\,. \tag{3}$$

We define the generalized mean Hubble's parameter $H$ as

$$H = \frac{1}{3}(H_x + H_y + H_z)\,, \tag{4}$$

where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of $x$, $y$ and $z$, respectively. A dot stands for differentiation with respect to cosmic time $t$.

From Equations (2)–(4), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\,. \tag{5}$$

The physical quantities of observational interest in cosmology, which are the expansion scalar $\theta$, the average anisotropy parameter $A_m$ and the shear scalar $\sigma^2$, are defined as

$$\theta = u^i_{;i} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\,, \tag{6}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right)\,, \tag{7}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2\,, \tag{8}$$

where $\Delta H_i = H_i - H (i = x, y, z)$ represent the directional Hubble's parameters in the directions of $x$, $y$ and $z$ respectively. $A_m = 0$ corresponds to isotropic expansion.

Einstein's field equations (in gravitational units $8\pi G = c = 1$) read as

$$R^i_j - \frac{1}{2}Rg^i_j = -T^{(m)i}_j - T^{(X)i}_j\,, \tag{9}$$

where $T^{(m)i}_j$ and $T^{(X)i}_j$ are the energy momentum tensors of perfect fluid and viscous DE, respectively. These are given by

$$\begin{aligned} T^{(m)i}_j &= \mathrm{diag}[-\rho^m, p^m, p^m, p^m]\,,\\ &= \mathrm{diag}[-1, \omega^m, \omega^m, \omega^m]\rho^m\,, \end{aligned} \tag{10}$$

and

$$T_j^{(X)i} = \mathrm{diag}[-\rho^X,\ p^X,\ p^X,\ p^X]\,,$$
$$= \mathrm{diag}[-1, \omega^X,\ \omega^X,\ \omega^X]\rho^X\,, \tag{11}$$

where $\rho^m$ and $p^m$ are, respectively, the energy density and pressure of the perfect fluid component or ordinary baryonic matter while $\omega^m = p^m/\rho m$ is its EoS parameter. Similarly, $\rho^X$ and $p^X$ are, respectively, the energy density and effective pressure of the DE component while $\omega^X = p^X/\rho^X$ is the corresponding EoS parameter. In Eckart's theory (Eckart 1940) a viscous DE EoS is specified by

$$p^X_{\mathrm{eff}} = p^X + \Pi\,. \tag{12}$$

Here $\Pi = -\xi(\rho^X)u^i_{;i}$ is the viscous pressure and $H = \frac{u^i_{;i}}{3}$ is the Hubble's parameter. On thermodynamical grounds, in conventional physics $\xi$ has to be positive. This is a consequence of the positive sign of the entropy change in an irreversible process (Nojiri & Odintsov 2003). In general, $\xi(\rho^X) = \xi_0(\rho^X)^\tau$, where $\xi_0 > 0$ and $\tau$ are constant parameters.

In a co-moving coordinate system ($u^i = \delta^i_0$), Einstein's field equations (9) with (10) and (11) for the Bianchi type-III metric (1) subsequently lead to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\omega^m\rho^m - \omega^X_{\mathrm{eff}}\rho^X + \Pi\,, \tag{13}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\omega^m\rho^m - \omega^X_{\mathrm{eff}}\rho^X + \Pi\,, \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -\omega^m\rho^m - \omega^X_{\mathrm{eff}}\rho^X + \Pi\,, \tag{15}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = \rho^m + \rho^X\,, \tag{16}$$

$$\alpha\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0\,. \tag{17}$$

The conservation of energy equation ($T^{ij}_{;j} = 0$) yields

$$\dot{\rho}^m + 3(1+\omega^m)\rho^m H + \dot{\rho}^X + 3(1+\omega^X_{\mathrm{eff}})\rho^X H = 0\,. \tag{18}$$

The Raychaudhuri equation for a given distribution is found to be

$$\frac{\ddot{a}}{a} = \frac{1}{2}\xi\theta - \frac{1}{6}(\rho^X + 3p^X) - \frac{1}{6}(\rho^m + 3p^m) - \frac{2}{3}\sigma^2\,. \tag{19}$$

## 3 SOLUTION OF THE FIELD EQUATIONS

The field Equations (13)–(17) are a system of five linearly independent equations with seven unknown parameters: $A$, $B$, $C$, $\rho^m$, $p^X$, $\rho^X$ and $\omega^X$. Two additional constraints relating these parameters are required to obtain explicit solutions of the system. Equation (17) obviously leads to

$$B = \ell_0 A\,, \tag{20}$$

where $\ell_0$ is a constant of integration.

Firstly, we assume that the scalar expansion $\theta$ in the model is proportional to the shear scalar. This assumption is in accord with the Thorne study (Thorne 1967), which says that the observations of the redshift for extragalactic sources suggest that Hubble expansion of the Universe is isotropic today to within approximately 30% (Kantowski & Sachs 1966; Kristian & Sachs 1966; Mohanty et al. 2007). More precisely, redshift studies place the limit $\frac{\sigma}{H} \leq 0.3$. Therefore, from Equations (5)–(7) and (20) we obtain

$$A = C^n \,, \tag{21}$$

where $n$ is a constant.

Secondly, following Amirhashchi et al. (2011b) we consider the following ansatz for the scale factor

$$a(t) = \sinh(t)\,. \tag{22}$$

By assuming a time varying deceleration parameter one can generate such a scale factor. It has also been shown that this scale factor is stable under metric perturbation (Chen & Kao 2001). In terms of redshift the above scale factor turns into

$$a = \frac{1}{1+z}, \quad z = \frac{1}{\sinh(t)} - 1\,. \tag{23}$$

Now, by using (13), (14) and (20)-(23) we can find the metric components as

$$A = \ell_1 \sinh^{\frac{3n}{2n+1}}(t) = \ell_1(1+z)^{-\frac{3n}{2n+1}}\,, \tag{24}$$

$$B = \ell_2 \sinh^{\frac{3n}{2n+1}}(t) = \ell_2(1+z)^{-\frac{3n}{2n+1}}\,, \tag{25}$$

$$C = \ell_3 \sinh^{\frac{n}{2n+1}}(t) = \ell_3(1+z)^{-\frac{3}{2n+1}}\,, \tag{26}$$

where $\ell_1 = K^{-\frac{3n}{(2n+1)}}$, $\ell_2 = \ell_0\ell_1$, $\ell_3 = \ell_1^{\frac{1}{n}}$ and $K$ is a constant of integration.

Therefore, the metric (1) reduces to

$$\begin{aligned} ds^2 \;=\; & -dt^2 + \ell_1^2 \sinh^{\frac{6n}{2n+1}}(t)dx^2 + \ell_2^2 \sinh^{\frac{6n}{2n+1}}(t)\mathrm{e}^{-2\alpha x}dy^2 \\ & + \ell_3^2 \sinh^{\frac{6}{2n+1}}(t)dz^2\,. \end{aligned} \tag{27}$$

One can write the above metric in terms of redshift as

$$\begin{aligned} ds^2 \;=\; & -dt^2 + \ell_1^2(1+z)^{-\frac{6n}{2n+1}}dx^2 + \ell_2^2(1+z)^{-\frac{6n}{2n+1}}\mathrm{e}^{-2\alpha x}dy^2 \\ & + \ell_3^2(1+z)^{-\frac{6}{2n+1}}(t)dz^2\,. \end{aligned} \tag{28}$$

In the following sections we deal with two cases: (i) viscous non-interacting two-fluid model; and, (ii) viscous interacting two-fluid model.

## 4 VISCOUS DE (NON-INTERACTING CASE)

In this section we assume that two fluids do not interact with each other. Therefore, the general form of conservation, given by Equation (18), leads us to write the conservation equation for the barotropic and dark fluid separately as,

$$\dot{\rho}^m + 3\frac{\dot{a}}{a}\left(\rho^m + p^{(m)}\right) = \dot{\rho}^m + (1+\omega^m)\rho^m(2n+1)\frac{\dot{C}}{C} = 0\,, \tag{29}$$

and

$$\dot{\rho}^X + 3\frac{\dot{a}}{a}\left(\rho^X + p^X_{\rm eff}\right) = \dot{\rho}^X + (1+\omega^X_{\rm eff})\rho^X(2n+1)\frac{\dot{C}}{C} = 0\,. \tag{30}$$

Integration of (29) leads to

$$\rho^m = \rho_0 C^{-(2n+1)(1+\omega^m)} = \rho_0 l_0 \sinh^{-3(1+\omega^m)}(t) = \rho_0 l_0 (1+z)^{3(1+\omega^m)}\,, \tag{31}$$

where $\rho_0$ is a constant of integration and $l_0 = \ell_3^{-(2n+1)(1+\omega^m)}$.

By using Equations (20), (21) and (31) in Equations (16) and (13), we obtain

$$\rho^X = n(n+2)\frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{C^{2n}} - \rho_0 l_0 \sinh^{-3(1+\omega^m)}(t)\,, \tag{32}$$

and

$$p^X = -\left[2n\frac{\ddot{C}}{C} + n(3n-2)\frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{C^{2n}}\right] - \omega^m \rho_0 l_0 \sinh^{-3(1+\omega^m)}(t)\,. \tag{33}$$

Using Equation (23) in Equations (32) and (33), we obtain the energy density and pressure of DE (i.e. $\rho^X$ and $p^X_{\text{eff}}$) as

$$\begin{aligned}\rho^X &= \frac{9n(n+2)}{(2n+1)^2}\coth^2(t) - \alpha^2\ell_3^{-2n}\sinh^{-\frac{6n}{(2n+1)}}(t) - \rho_0 l_0 \sinh^{-3(1+\omega^m)}(t)\\ &= \frac{9n(n+2)}{(2n+1)^2}\left[1+(1+z)^2\right] - \alpha^2\ell_3^{-2n}(1+z)^{\frac{6n}{(2n+1)}} - \rho_0 l_0 (1+z)^{3(1+\omega^m)}\end{aligned} \tag{34}$$

$$\begin{aligned}p^X_{\text{eff}} &= -\left[\frac{9(n^2+n+1)}{(2n+1)^2}\coth^2(t) - \frac{3(n+1)}{(2n+1)}\cosh^2(t)\right]\\ &\quad -\omega^m\rho_0 l_0\sinh^{-3(1+\omega^m)}(t) - 3\xi_0 H(\rho^X)^\tau\\ &= -\left\{\frac{9(n^2+n+1)}{(2n+1)^2}\left[1+(1+z)^2\right] - \frac{3(n+1)}{(2n+1)}\left[1+(1+z)^{-2}\right]\right\}\\ &\quad -\omega^m\rho_0 l_0(1+z)^{3(1+\omega^m)} - 3\xi_0 H(\rho^X)^\tau\,.\end{aligned} \tag{35}$$

Using the above two equations we finally find the effective EoS parameter of DE as

$$\begin{aligned}\omega^X_{\text{eff}} &= -\left[\frac{\frac{9(n^2+n+1)}{(2n+1)^2}\coth^2(t) - \frac{3(n+1)}{(2n+1)}\cosh^2(t) + 3\Omega_0^m l_0\omega^m\sinh^{-3(1-\omega^m)}(t)}{\frac{9n(n+2)}{(2n+1)^2}\coth^2(t) - \alpha^2\ell_3^{-2n}\sinh^{-\frac{6n}{(2n+1)}}(t) - 3\Omega_0^m l_0\sinh^{-3(1+\omega^m)}(t)}\right]\\ &\quad -\zeta_0 H^\tau(\Omega^X)^{\tau-1}\\ &= -\left\{\frac{\frac{9(n^2+n+1)}{(2n+1)^2}\left[1+(1+z)^2\right] - \frac{3(n+1)}{(2n+1)}\left[1+(1+z)^{-2}\right] + 3\Omega_0^m l_0\omega^m(1+z)^{3(1-\omega^m)}}{\frac{9n(n+2)}{(2n+1)^2}\left[1+(1+z)^2\right] - \alpha^2\ell_3^{-2n}(1+z)^{\frac{6n}{(2n+1)}} - 3\Omega_0^m l_0(1+z)^{3(1+\omega^m)}}\right\}\\ &\quad -\zeta_0 H^\tau(\Omega^X)^{\tau-1}\,.\end{aligned} \tag{36}$$

Here, $\zeta_0 = 3^\tau\xi_0$, and $\Omega^m$ and $\Omega^X$ are the energy density of matter and DE, respectively (note that the subscript 0 indicates the present value of any parameter).

The behavior of the EoS parameter for DE in terms of redshift $z$ is shown in Figure 1. Since we are interested in the late time and future evolution of DE, we plot the range of redshift $z$ from $-1$ to $z = 5$. The parameter $\Omega^m$ is taken to be 0. This figure shows that $\omega^X_{\text{eff}}$ of non-viscous DE ($\xi_0 = 0$) is only varying in the quintessence region whereas the variation of viscous DE starts from the quintessence region, crossing the phantom divided line (PDL), and varies in the phantom region. However, the EoS of both non-viscous and viscous DE ultimately approaches the cosmological constant region ($\omega^X_{\text{eff}} = -1$) independent of the value of $\xi_0$. This behavior clearly shows that the phantom phase (i.e. $\omega^X_{\text{eff}} < -1$) is an unstable phase and there is a transition from phantom to the cosmological constant phase at late time. The variations of energy density $\rho^X$, mean anisotropy parameter $A_m$ and bulk viscosity $\xi(\rho^X)$ are depicted in Figure 2. As expected, all these parameters are decreasing functions and approach zero at late time ($z = -1$).

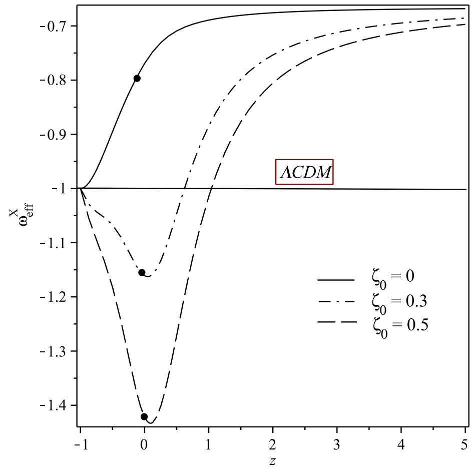


**Fig. 1** The EoS parameter $\omega^X_{\mathrm{eff}}$ versus $z$ for $n = \beta = \alpha = \ell_3 = l_0 = 1$ and $\Omega^m_0 = 0.3$. The dots signify the current values of $\omega^X_{\mathrm{eff}}$.

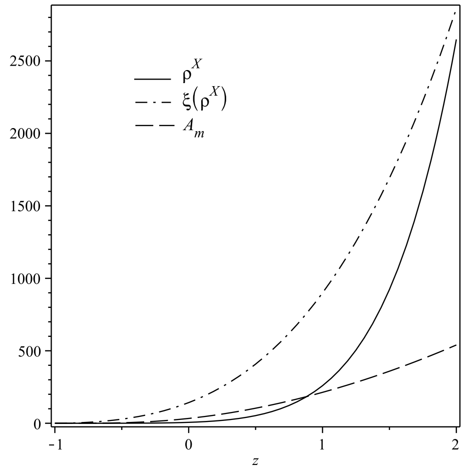


**Fig. 2** A plot of the DE energy density $\rho^X$, average anisotropy parameter $A_m$ and the bulk viscosity $\xi(\rho^X)$ vs. $z$ for $\alpha = \ell_3 = \ell_0 = 1$, $\Omega^m = 0.3$ and $\xi_0 = 0.1$.

The matter density $\Omega^m$ and DE density $\Omega^X$ are also given by

$$\Omega^m = \frac{\rho^m}{3H^2} = \frac{\rho_0 l_0 \sinh^{-3(1-\omega^m)}(t)}{3\coth^2(t)} = \Omega^m_0 l_0 (1+z)^{3(1-\omega^m)}, \tag{37}$$

and

$$\begin{aligned}\Omega^X &= \frac{\rho^X}{3H^2} = \frac{3n(n+2)}{(2n+1)^2} - \frac{\alpha^2 \ell_3^{-2n} \sinh^{-\frac{6n}{(2n+1)}}(t) + \rho_0 l_0 \sinh^{-3(1+\omega^m)}(t)}{3\coth^2(t)} \\ &= \frac{3n(n+2)}{(2n+1)^2} - \frac{\alpha^2 \ell_3^{-2n}(1+z)^{\frac{6n}{(2n+1)}}}{3\left[1+(1+z)^2\right]} - \Omega^m_0 l_0 (1+z)^{3(1-\omega^m)}.\end{aligned} \tag{38}$$

Adding Equations (37) and (38), we obtain total energy ($\Omega$)

$$\begin{aligned}\Omega = \Omega^m + \Omega^X &= \frac{9n(n+2)}{(2n+1)^2} - \frac{\alpha^2 \ell_3^{-2n} \sinh^{-\frac{6n}{2n+1}}(t)}{3\coth^2(t)} \\ &= \frac{9n(n+2)}{(2n+1)^2} - \frac{\alpha^2 \ell_3^{-2n}(1+z)^{\frac{6n}{2n+1}}}{3\left[1+(1+z)^2\right]}.\end{aligned} \tag{39}$$

Figure 3 shows the values of $\Omega^X_0$ and $\Omega^m_0$ which are permitted by our model. The line $1 = \Omega^X + \Omega^m$ represents a flat Universe separating open from closed Universes. From this figure we observe that for values $\alpha = 0$ and $n = 1$, which represent a spatially flat Universe ($\Omega = 1$), $\Omega^X_0 \approx 0.76$, and $\Omega^m_0 \approx 0.24$. Other models with different values of $\alpha \neq 0$ represent various open Universes ($\Omega < 1$).

The variation of density parameters $\Omega^m$ and $\Omega^X$ with redshift $z$ has been depicted in Figure 4. It is observed that $\Omega^X$ increases as redshift decreases and approaches 1 at late time whereas $\Omega^m$ decreases as $z$ decreases and approaches zero at late time.

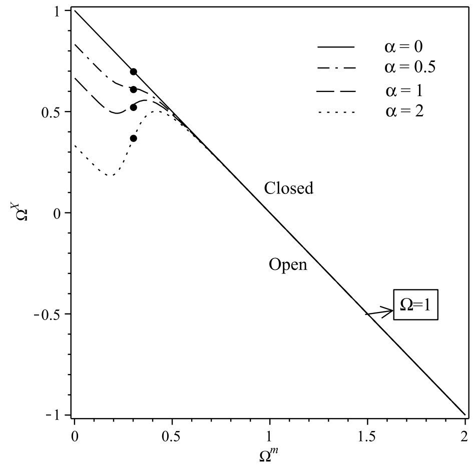


**Fig. 3** The plot of $\Omega^X$ versus $\Omega^m$ for $n = \ell_0 = \ell_3 = 1$ and $\Omega_0^m = 0.3$. The solid line indicates a flat Universe ($n = 1$, $\alpha = 0$). The dots signify the current values of $\Omega^X$ and $\Omega^m$.

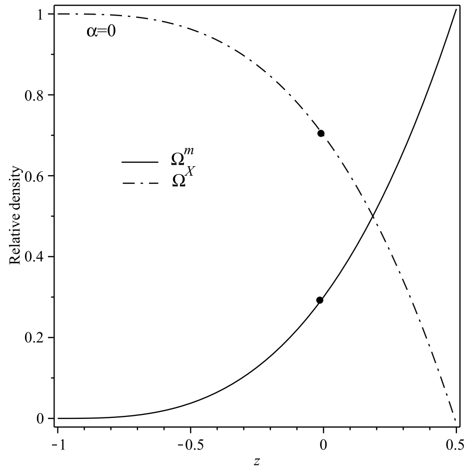


**Fig. 4** The plot of energy $\Omega^m$ and $\Omega^X$ versus redshift ($z$) for $\Omega_0^m = 0.3$ and $\ell_0 = n = 1$. The dots signify the current values of $\Omega^X$ and $\Omega^m$.

## 5 VISCOUS DE (INTERACTING CASE)

In this section we consider the interaction between dark and barotropic fluids. For this purpose, we can write the continuity equations for barotropic and dark fluids as

$$\dot{\rho}^m + 3\frac{\dot{a}}{a}\left(\rho^m + p^m\right) = \dot{\rho}^m + (1+\omega^m)\rho^m(2n+1)\frac{\dot{C}}{C} = Q, \tag{40}$$

and

$$\dot{\rho}^X + 3\frac{\dot{a}}{a}\left(\rho^X + p^X_{\text{eff}}\right) = \dot{\rho}^X + (1+\omega^X_{\text{eff}})\rho^X(2n+1)\frac{\dot{C}}{C} = -Q. \tag{41}$$

The quantity $Q$ expresses the interaction between the dark components. Since we are interested in an energy transfer from DE to DM, we consider $Q > 0$. $Q > 0$ ensures that the second law of thermodynamics is fulfilled (Pavón & Wang 2009). Here we emphasize that the continuity Equations (40) and (41) imply that the interaction term ($Q$) should be proportional to a quantity with units of inverse time; that is, $Q \propto \frac{1}{t}$. Therefore, a first, natural candidate can be the Hubble factor $H$ multiplied by the energy density. Following Amendola et al. (2007) and Guo et al. (2007), we consider

$$Q = H\sigma\rho^m, \tag{42}$$

where $\sigma$ is a coupling constant. Using Equation (42) in Equation (40) and after integrating, we obtain

$$\rho^m = \rho_0 C^{-(2n+1)(1+\omega^m-\sigma)} = \rho_0 l \sinh^{-3(1+\omega^m-\sigma)}(T), \tag{43}$$

where $l = \ell_3^{-(2n+1)(1+\omega^{(m)}-\sigma)}$.

By using Equations (20), (21) and (43) in Equations (13) and (16), we obtain

$$\rho^X = n(n+2)\frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{C^{2n}} - \rho_0 C^{-(2n+1)(1+\omega^m-\sigma)}, \tag{44}$$

and

$$p^X = -\left[2n\frac{\ddot{C}}{C} + n(3n-2)\frac{\dot{C}^2}{C^2} - \frac{\alpha^2}{C^{2n}}\right] - \rho_0(\omega^m - \sigma)C^{-(2n+1)(1+\omega^{(m)}-\sigma)}. \quad (45)$$

Using Equation (26) in Equations (44) and (45), we obtain the values of $\rho^X$ and $p^X_{\rm eff}$ as

$$\begin{aligned}\rho^X &= \frac{9n(n+2)}{(2n+1)^2}\coth^2(t) - \alpha^2\ell_3^{-2n}\sinh^{-\frac{6n}{(2n+1)}}(t) - \rho_0 l_0 \sinh^{-3(1+\omega^m-\sigma)}(t) \\ &= \frac{9n(n+2)}{(2n+1)^2}\left[1+(1+z)^2\right] - \alpha^2\ell_3^{-2n}(1+z)^{\frac{6n}{(2n+1)}} - \rho_0 l_0(1+z)^{3(1+\omega^m-\sigma)} \quad (46)\end{aligned}$$

and

$$\begin{aligned}p^X_{\rm eff} &= -\left[\frac{9(n^2+n+1)}{(2n+1)^2}\coth^2(t) - \frac{3(n+1)}{(2n+1)}\cosh^2(t)\right] \\ &\quad -(\omega^m-\sigma)\rho_0 l_0\sinh^{-3(1+\omega^m-\sigma)}(t) - 3\xi_0 H(\rho^X)^\tau \\ &= -\left[\frac{9(n^2+n+1)}{(2n+1)^2}\left[1+(1+z)^2\right] - \frac{3(n+1)}{(2n+1)}\left[1+(1+z)^{-2}\right]\right] \\ &\quad -(\omega^m-\sigma)\rho_0 l_0(1+z)^{3(1+\omega^m-\sigma)} - 3\xi_0 H(\rho^X)^\tau . \quad (47)\end{aligned}$$

Also the EoS parameter for DE ($\omega^X_{\rm eff}$) is obtained as

$$\begin{aligned}\omega^X_{\rm eff} &= -\left[\frac{\frac{9(n^2+n+1)}{(2n+1)^2}\coth^2(t) - \frac{3(n+1)}{(2n+1)}\cosh^2(t) + 3\Omega^m_0 l_0(\omega^m-\sigma)\sinh^{-3(1-\omega^m)}(t)}{\frac{9n(n+2)}{(2n+1)^2}\coth^2(t) - \alpha^2\ell_3^{-2n}\sinh^{-\frac{6n}{(2n+1)}}(t) - 3\Omega^m_0 l_0\sinh^{-3(1+\omega^m)}(t)}\right] \\ &\quad -\zeta_0 H^\tau(\Omega^X)^{\tau-1} \\ &= -\left[\frac{\frac{9(n^2+n+1)}{(2n+1)^2}\left[1+(1+z)^2\right] - \frac{3(n+1)}{(2n+1)}\left[1+(1+z)^{-2}\right] + 3\Omega^m_0 l_0(\omega^m-\sigma)(1+z)^{3(1-\omega^m)}}{\frac{9n(n+2)}{(2n+1)^2}\left[1+(1+z)^2\right] - \alpha^2\ell_3^{-2n}(1+z)^{\frac{6n}{(2n+1)}} - 3\Omega^m_0 l_0(1+z)^{3(1+\omega^m)}}\right] \\ &\quad -\zeta_0 H^\tau(\Omega^X)^{\tau-1}. \quad (48)\end{aligned}$$

The behavior of EoS ($\omega^X_{\rm eff}$) parameter for DE in terms of redshift $z$ is shown in Figures 5 and 6. Again, since we are interested in the late time and future evolution of DE, we plot the range of redshift $z$ from $-1$ to $z = 5$. Here the parameter $\omega^m$ is taken to be 0. In Figure 5 we fix the parameter $\zeta_0 = 0$ and vary $\sigma$ as 0, 0.3 and 0.5. In Figure 6 we fix $\sigma = 0.3$ and vary $\zeta_0$ as 0.1, 0.3 and 0.5. The plots show that the evolution of $\omega^X_{\rm eff}$ apparently depends on the parameters $\sigma$ and $\zeta_0$. It is clear (from Fig. 5) that the interaction prevents the EoS parameter of DE from moving to darker regions as in the non-interacting case (Fig. 1). But considering the bulk viscosity in the cosmic fluid, this compensates the effect of interaction (see Fig. 6).

The expressions for the matter-energy density $\Omega^m$ and dark-energy density $\Omega^X$ are given by

$$\Omega^m = \frac{\rho^m}{3H^2} = \frac{\rho_0 l_0\sinh^{-3(1+\omega^m-\sigma)}(t)}{3\coth^2(t)} = \Omega^m_0 l_0(1+z)^{3(1-\omega^m-\sigma)}, \quad (49)$$

and

$$\begin{aligned}\Omega^X &= \frac{\rho^X}{3H^2} = \frac{3n(n+2)}{(2n+1)^2} - \frac{\alpha^2\ell_3^{-2n}\sinh^{-\frac{6n}{(2n+1)}}(t) + \rho_0 l_0\sinh^{-3(1+\omega^m-\sigma)}(t)}{3\coth^2(t)} \\ &= \frac{3n(n+2)}{(2n+1)^2} - \frac{\alpha^2\ell_3^{-2n}(1+z)^{\frac{6n}{(2n+1)}}}{3\left[1+(1+z)^2\right]} - \Omega^m_0 l_0(1+z)^{3(1-\omega^m-\sigma)} . \quad (50)\end{aligned}$$

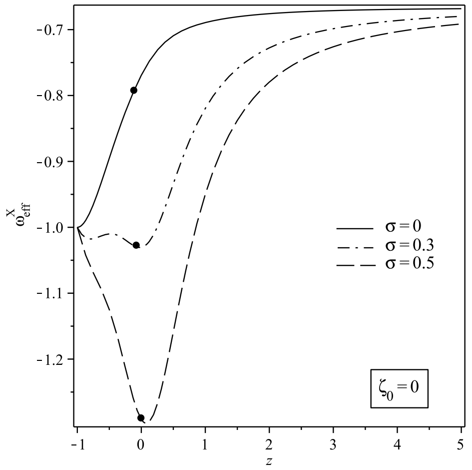


**Fig. 5** The EoS parameter $\omega^X_{\rm eff}$ versus $z$ for $n = \beta = \alpha = \ell_3 = l_0 = 1$ and $\Omega^m_0 = 0.3$. The dots signify the current values of $\omega^X_{\rm eff}$. In this case, we fix $\zeta_0 = 0$ and vary $\sigma$.

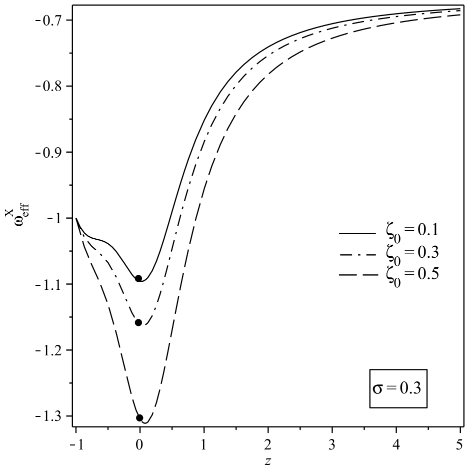


**Fig. 6** The EoS parameter $\omega^X_{\rm eff}$ versus $z$ for $n = \beta = \alpha = \ell_3 = l_0 = 1$ and $\Omega^m_0 = 0.3$. The dots signify the current values of $\omega^X_{\rm eff}$. In this case, we fix $\sigma = 0.3$ and vary $\zeta_0$.

Adding Equations (49) and (50), we obtain total energy ($\Omega$)

$$\begin{aligned}\Omega &= \Omega^m + \Omega^X = \frac{9n(n+2)}{(2n+1)^2} - \frac{\alpha^2\ell_3^{-2n}\sinh^{-\frac{6n}{2n+1}}(t)}{3\coth^2(t)} \\ &= \frac{9n(n+2)}{(2n+1)^2} - \frac{\alpha^2\ell_3^{-2n}(1+z)^{\frac{6n}{2n+1}}}{3\left[1+(1+z)^2\right]}\,, \end{aligned} \tag{51}$$

which is the same as Equation (38). Therefore, we observe that in the interacting case the density parameter has the same properties as in the non-interacting case.

The values of $\Omega^m$ versus $\Omega^X$, which are permitted by our models in the interacting case, are shown in Figures 7 and 8. In both figures the line $1 = \Omega^m + \Omega^X$ indicates a flat Universe separating open from closed Universes. In Figure 7 we fix the parameter $\alpha = 2$ and vary $\sigma$ as 0, 0.3 and 0.5. In Figure 8 we fix $\sigma = 0.5$ and vary $\alpha$ as 0, 0.5, 1 and 2. The plots show that the evolution of $\Omega^X$ versus $\Omega^m$ apparently depends on the parameters $\sigma$ and $\alpha$.

Figure 9 depicts the evolution of the relative densities. From this figure we observe that the interaction parameter $\sigma$ has an impact on the evolution of the densities depending on its value.

## 6 STATEFINDER DIAGNOSTIC

Since many models have been suggested to describe the current cosmic acceleration, it is very important to find a way to discriminate between the various contenders in a model-independent manner. For this purpose, Sahni et al. (2003) have introduced a new cosmological diagnostic pair $\{s, r\}$, which is called the statefinder. The parameters $s$ and $r$ are dimensionless and only depend on the scale factor $a$, therefore $\{s, r\}$ is a geometrical diagnostic. They are defined as

$$r \equiv \frac{\dddot{a}}{aH^3}, \quad s \equiv \frac{r-\Omega}{3(q-\frac{\Omega}{2})}. \tag{52}$$

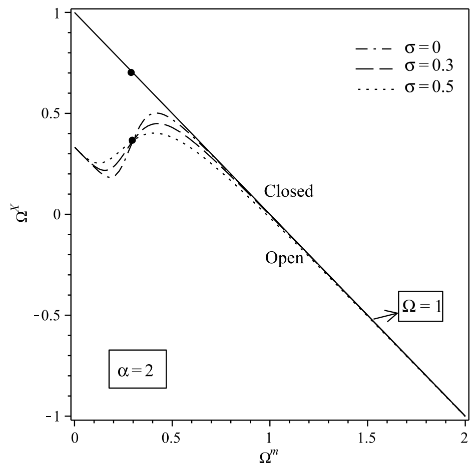


**Fig. 7** The plot of $\Omega^X$ versus $\Omega^m$ for $n = \ell_0 = \ell_3 = 1$ and $\Omega_0^m = 0.3$. The solid line indicates a flat Universe ($n = 1$, $\alpha = 0$). The dots signify the current values of $\Omega^X$ and $\Omega^m$. In this case, we fix $\alpha = 2$ and vary $\sigma$.

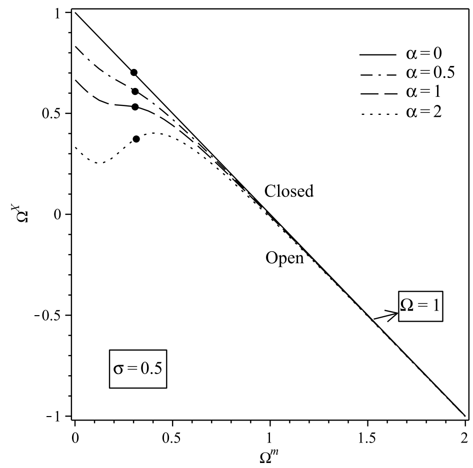


**Fig. 8** The plot of $\Omega^X$ versus $\Omega^m$ for $n = \ell_0 = \ell_3 = 1$ and $\Omega_0^m = 0.3$. The solid line indicates a flat Universe ($n = 1$, $\alpha = 0$). The dots signify the current values of $\Omega^X$ and $\Omega^m$. In this case, we fix $\sigma = 0.5$ and vary $\alpha$.

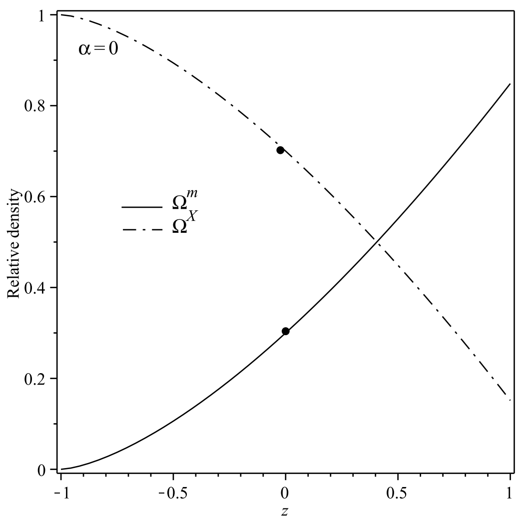


**Fig. 9** The plot of energy $\Omega^m$ and $\Omega^X$ versus redshift ($z$) for $\Omega_0^m = 0.3,\ \ell_0 = n = 1$ and $\sigma = 0.5$. The dots signify the current values of $\Omega^X$ and $\Omega^m$.

Here the formalism of Sahni and coworkers is extended to permit models of a curved Universe. By using these parameters one can differentiate between different forms of DE. For example, although the quintessence, phantom and Chaplygin gas models tend to approach the $\Lambda$CDM fixed point ($\{s, r\}_{\Lambda\mathrm{CDM}} = \{0, 1\}$), for quintessence and phantom models the trajectories lie in the region $s > 0,\ r < 1$ whereas for Chaplygin gas models trajectories lie in the region $s < 0,\ r > 1$. In

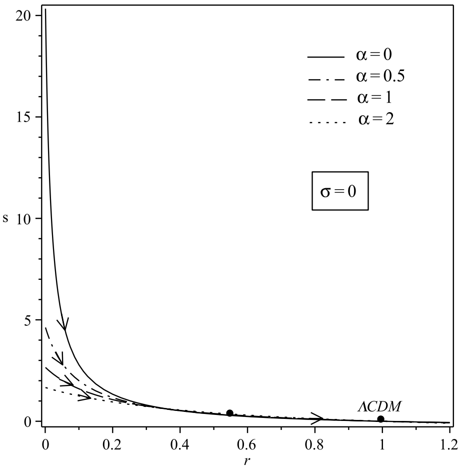


**Fig. 10** $s$ - $r$ evolution diagram. The dots signify the current values of the statefinder pair $\{s, r\}$. In this case, we fix $\sigma = 0$ (non-interacting case) and vary $\alpha$.

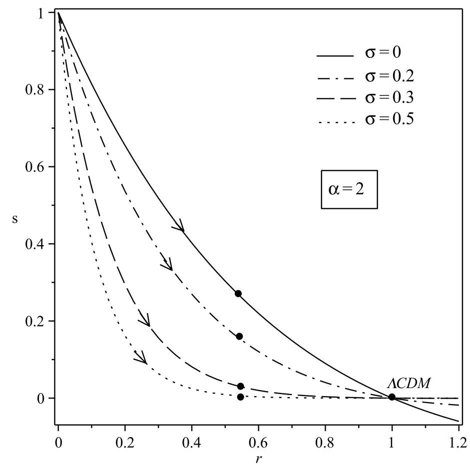


**Fig. 11** $s$ - $r$ evolution diagram. The dots signify the current values of the statefinder pair $\{s, r\}$. In this case, we fix $\alpha = 2$ (interacting case) and vary $\sigma$.

general, the statefinder parameters are given by

$$r = \Omega^m + \frac{9\omega^X}{2}\Omega^X(1+\omega^X) - \frac{3}{2}\Omega^X\frac{\dot{\omega}^X}{H}, \tag{53}$$

$$s = 1 + \omega^X - \frac{1}{3}\frac{\dot{\omega}^X}{\omega^X H}. \tag{54}$$

Since we have the analytical expression of $\omega^X_{\rm eff}$ in both non-interacting and interacting cases, we can easily obtain $\frac{\dot{\omega}^X_{\rm eff}}{H}$. Thus, we can calculate the statefinder parameters in this scenario.

The evolution of the statefinder pair $\{s, r\}$ is shown in Figures 10 and 11. In Figure 10 we fix the parameter $\sigma = 0$ and vary $\alpha$ as 0, 0.5, 1 and 2. In Figure 11 we fix $\alpha = 2$ and vary $\sigma$ as 0, 0.2, 0.3 and 0.5. The dots show the current values of the statefinder pair $\{s, r\}$ for different DE models. Here, we observe that the interaction parameter $\sigma$ makes the model evolve along different trajectories on the $s - r$ plane.

## 7 CONCLUDING REMARKS

In this paper we have studied DE in the scope of anisotropic Bianchi type III space-time. We considered two cases: (i) when DE and DM do not interact with each other and (ii) when there is an interaction between these two dark components. In non-interacting as well as weakly interacting ($\sigma \sim 0$) cases we observed that in the absence of viscosity, DE EoS parameter does not cross the PDL and, hence, always varies in the quintessence region. However, in both cases when DE is considered to be viscous rather than perfect, its EoS parameter could cross the PDL depending on the values of coupling constant $\sigma$ and bulk viscosity coefficient $\zeta_0$. But in this case, although the DE EoS parameter could cross the PDL and vary in the phantom region, it ultimately tends to the cosmological constant region $\omega^{\rm de} = -1$. This special behavior of the EoS parameter is because of our choice of bulk viscosity, which is a decreasing function of time (redshift) in an expanding Universe. It has also been shown that in both cases, according to the $\Omega^X$–$\Omega^m$ phase diagram (see Figs. 3, 7 and 8), deviation from the flat Universe ($\Omega = 1$) only depends on the geometric parameter $\alpha$ and not on the interaction parameter $\sigma$.

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