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Spacecraft Doppler tracking with possible violations of LLI and LPI: preliminary bounds on LLI from Mars Express *

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Abstract Three-way spacecraft Doppler tracking is currently widely used and it plays an important role in the control and navigation of deep space missions. Using the theory of three-way Doppler tracking, including possible violations of the local Lorentz invariance (LLI) and the local position invariance (LPI), we analyze the post-fit residuals of three-way Doppler tracking data of Mars Express. These Doppler observations were carried out from August 7th to 8th in 2009, with an uplink station administered by the European Space Agency at New Norcia in Australia and three downlink stations at Shanghai, Kunming and Urumqi in China. We find that, although these observations impose preliminary bounds on LLI at the level of 10^{-2}, they are not suitable for testing LPI because of the configuration of these stations and the accuracy of the observations. To our knowledge, this is one of the first attempts in China to apply radio science to the field of fundamental physics.

Key words: space vehicles — techniques: radial velocities — gravitation

1 INTRODUCTION

As one of the most important current methods for determining the motion of a spacecraft, the Doppler tracking technique has been successfully implemented in many deep space missions for control and navigation (Kruger 1965; Moyer & Yuen 2000). It can also be used for a variety of scientific applications, such as fundamental physics (e.g. chapter 7.8 of Kopeikin et al. 2011). In this work, we focus on testing the Einstein equivalence principle (EEP).

EEP is the cornerstone for building general relativity (GR) and all other metric theories of gravity. It states that: (1) in a homogeneous gravitational field, the acceleration of a freely-falling and structureless test particle is independent of its properties — its mass, composition or thermodynamic state, which is the so-called weak equivalence principle (WEP); (2) the outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling measuring apparatus where it is performed, which is the so-called local Lorentz invariance (LLI); and (3) the outcome of any local non-gravitational experiment is independent of where and when it is performed, which is

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the so-called local position invariance (LPI) (see Will 1993, 2006; Kopeikin et al. 2011, for more details). The second and third pieces of EEP (i.e. LLI and LPI) can be tested by measuring the frequency of a signal transmitted from a clock as it moves in the gravitational field of a massive body (e.g. Krisher 1990). Some experiments have been conducted in the vicinity of Earth (Vessot et al. 1980) and in interplanetary space (e.g. Krisher et al. 1990, 1993), confirming EEP at the level of $\sim 10^{-4} - \sim 10^{-2}$.

All these experiments relied on a one-way radio signal transmitted from the spacecraft to ground stations. The transmitted frequency was referred to as the onboard clock or frequency standard, while the received signal was referred to as these standards at the stations. However, onboard frequency standards are significantly less stable than ground-based standards and they are limited by their own noise. One solution for this is to use two-way or three-way Doppler tracking. Considering these advantages, Deng & Xie (2014) extended relativistic theories of two-way and three-way Doppler tracking by including possible violations of LLI and LPI.

In this work, we will analyze the post-fit residuals of three-way Doppler tracking data of Mars Express (MEX) with this model and try to find preliminary bounds on LLI and LPI. These Doppler observations were carried out from August 7th to 8th in 2009, with an uplink station administered by the European Space Agency (ESA) at New Norcia (NN) in Australia and three downlink stations at Shanghai (SH), Kunming (KM) and Urumqi (UR) in China.

The rest of the paper is organized as follows. Section 2 is devoted to describing three-way Doppler tracking with possible violations of LLI and LPI and their detectability. In Section 3, we present three-way Doppler observation of MEX and its data reduction. The post-fit residuals are taken to estimate the bounds on LLI in Section 4. Finally, in Section 5, we summarize our results.

2 THREE-WAY DOPPLER TRACKING WITH VIOLATIONS OF LLI AND LPI

Starting from the one-way Doppler tracking model (Krisher et al. 1993), Deng & Xie (2014) extended the theories of two-way and three-way Doppler tracking by making them include possible violations of LLI and LPI in order to test EEP. Here, we only brief the primary results that resulted from this work. More details can be found in Deng & Xie (2014).

In three-way Doppler tracking, there are two stations. One ground station (S_1) emits a radio signal with frequency ν_E at time t_E and a spacecraft (P) receives the signal with frequency ν' at time t'. Spacecraft (P) then immediately transmits the radio signal $q\nu'$ back, where q is a known ratio between two integers. The other station (S_2) receives the signal with frequency ν_R at time t_R. The whole procedure can be decomposed into two one-way Doppler trackings and the shift in frequency in this open-loop can be easily and concisely expressed as

$$\left.\frac{\nu_\mathrm{R}}{q\nu_\mathrm{E}}\right|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} = \frac{\nu'}{\nu_\mathrm{E}}\cdot\frac{\nu_\mathrm{R}}{q\nu'} = \mathcal{F}_{\mathrm{S}_1\to\mathrm{P}}(t_\mathrm{E},t')\cdot\mathcal{F}_{\mathrm{P}\to\mathrm{S}_2}(t',t_\mathrm{R}) + \mathcal{O}(\epsilon^3)\,, \tag{1}$$

where $\epsilon \equiv c^{-1}$ and c is the speed of light. The function $\mathcal{F}$ contains two parts: $\hat{\mathcal{F}}$ and $\bar{\mathcal{F}}$. $\hat{\mathcal{F}}$ represents the shift in frequency as predicted by EEP and $\bar{\mathcal{F}}$ indicates the effects caused by possible violations of LLI and LPI. Their full expressions can be found in equations (8) and (9) in Deng & Xie (2014). With the linear approximation of the light-time solution (see chapter 8 in Moyer & Yuen 2000, for details), we can have a deviation in the redshift δz from the prediction by EEP in the Barycentric Celestial Reference System (BCRS) (Soffel et al. 2003), which is (Deng & Xie 2014)

$$\delta z \equiv \left.\frac{\nu_\mathrm{R}}{q\nu_\mathrm{E}}\right|_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} - \left.\frac{\nu_\mathrm{R}}{q\nu_\mathrm{E}}\right|^{\mathrm{EEP}}_{\mathrm{S}_1\to\mathrm{P}\to\mathrm{S}_2} = \delta z_\mathrm{LLI} + \delta z_\mathrm{LPI} + \mathcal{O}(\epsilon^3)\,, \tag{2}$$

where $\delta z_{\rm LLI}$ and $\delta z_{\rm LPI}$ are, respectively, caused by the possible violations of LLI and LPI, and they are

$$\delta z_{\rm LLI} \;=\; \frac{1}{2}\epsilon^2\bigg[\bar{\beta}_{\rm S_2}\boldsymbol{v}^2_{\rm S_2}(t_{\rm R}) - \bar{\beta}_{\rm S_1}\boldsymbol{v}^2_{\rm S_1}(t_{\rm R})\bigg], \tag{3}$$

$$\delta z_{\rm LPI} \;=\; \epsilon^2\bigg\{\sum_A \bar{\alpha}^{\rm A}_{\rm S_2}U_{\rm A}\Big[\boldsymbol{y}_{\rm S_2}(t_{\rm R})\Big] - \sum_A \bar{\alpha}^{\rm A}_{\rm S_1}U_{\rm A}\Big[\boldsymbol{y}_{\rm S_1}(t_{\rm R})\Big]\bigg\}. \tag{4}$$

Here, violations of LLI can be tested by fitting the dimensionless parameters $\bar{\beta}_{\rm S_1}$ and $\bar{\beta}_{\rm S_2}$, which are associated with $\rm S_1$ and $\rm S_2$, respectively. If LLI is valid, then $\bar{\beta}_{\rm S_{1/2}} = 0$. Violations of LPI can be tested by fitting the dimensionless parameters $\bar{\alpha}^{\rm A}_{\rm S_1}$ and $\bar{\alpha}^{\rm A}_{\rm S_2}$, which are associated with the gravitational field of body A and two of the stations, respectively. If LPI holds true, $\bar{\alpha}^{\rm A}_{\rm S_{1/2}} = 0$.

Before applying this model to Doppler observations, it is first necessary for us to investigate the detectability of these parameters. It is worth mentioning that this discussion on detectability is *not* a statistical estimation of the parameters, which will be statistically estimated by the method of weighted least squares in Section 4. Considering the two ground stations on Earth, we can rewrite $\delta z_{\rm LLI}$ as

$$\delta z_{\rm LLI} = \frac{1}{2}\epsilon^2\bigg[(\bar{\beta}_{\rm S_2} - \bar{\beta}_{\rm S_1})\boldsymbol{v}^2_{\oplus} + 2\boldsymbol{v}_{\oplus}\cdot(\bar{\beta}_{\rm S_2}\boldsymbol{V}_{\rm S_2} - \bar{\beta}_{\rm S_1}\boldsymbol{V}_{\rm S_1}) + \bar{\beta}_{\rm S_2}\boldsymbol{V}^2_{\rm S_2} - \bar{\beta}_{\rm S_1}\boldsymbol{V}^2_{\rm S_1}\bigg], \tag{5}$$

where $\boldsymbol{v}_{\oplus}$ is the velocity of the Earth in the BCRS and $\boldsymbol{V}_{\rm S_1}$ and $\boldsymbol{V}_{\rm S_2}$ are, respectively, the velocities of two stations in the Geocentric Celestial Reference System (GCRS) (Soffel et al. 2003). If we assume $\bar{\beta}_{\rm S_1} = \bar{\beta}_{\rm S_2} = \bar{\beta}$ for simplicity, $\delta z_{\rm LLI}$ can be simplified as

$$\delta z_{\rm LLI} = \frac{1}{2}\epsilon^2\bar{\beta}(2\boldsymbol{v}_{\oplus} + \boldsymbol{V}_{\rm S_1} + \boldsymbol{V}_{\rm S_2})\cdot(\boldsymbol{V}_{\rm S_2} - \boldsymbol{V}_{\rm S_1}), \tag{6}$$

which suggests that, in order to increase the detectability of $\bar{\beta}$, one needs to use two stations whose different velocities in the GCRS are as large as possible. Since $|\boldsymbol{v}_{\oplus}| \sim 3\times10^4$ m s^{-1} and $|\boldsymbol{V}_{\rm S_{1/2}}| \lesssim 4.5\times10^2$ m s^{-1}, we can obtain

$$\bar{\beta} \sim c^2\delta z_{\rm LLI}\Big[\boldsymbol{v}_{\oplus}\cdot(\boldsymbol{V}_{\rm S_2} - \boldsymbol{V}_{\rm S_1})\Big]^{-1} \gtrsim 6\times10^{-3}\bigg(\frac{\delta z_{\rm LLI}}{10^{-12}}\bigg). \tag{7}$$

This means that if residuals of the Doppler observation are at the level of 10^{-12}, then the parameter $\bar{\beta}$ can be determined down to the level of $\sim 10^{-3}$. For the detectability of violations of LPI, if the only monopole component of the gravitational field of the Sun is taken into account, we can rewrite $\delta z_{\rm LPI}$ as

$$\delta z_{\rm LPI} = \epsilon^2\bigg[\bar{\alpha}^{\odot}_{\rm S_2}\frac{GM_{\odot}}{|\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus} - \boldsymbol{Y}_{\rm S_2}|} - \bar{\alpha}^{\odot}_{\rm S_1}\frac{GM_{\odot}}{|\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus} - \boldsymbol{Y}_{\rm S_1}|}\bigg], \tag{8}$$

where $\boldsymbol{y}_{\odot}$ and $\boldsymbol{y}_{\oplus}$ are, respectively, the positions of the Sun and the Earth in the BCRS and $\boldsymbol{Y}_{\rm S_1}$ and $\boldsymbol{Y}_{\rm S_2}$ are, respectively, the positions of two stations in the GCRS. If we assume $\bar{\alpha}^{\odot}_{\rm S_1} \sim \bar{\alpha}^{\odot}_{\rm S_2} = \bar{\alpha}^{\odot}$ and make use of the condition that $|\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus}| \gg |\boldsymbol{Y}_{\rm S_{1/2}}|$, then $\delta z_{\rm LPI}$ can be simplified as

$$\delta z_{\rm LPI} = \epsilon^2\bar{\alpha}^{\odot}\frac{GM_{\odot}}{|\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus}|^3}(\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus})\cdot(\boldsymbol{Y}_{\rm S_2} - \boldsymbol{Y}_{\rm S_1}) + \mathcal{O}(\boldsymbol{Y}^2_{\rm S_{1/2}}). \tag{9}$$

Since $|\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus}| \sim 1$ au and $|\boldsymbol{Y}_{\rm S_2} - \boldsymbol{Y}_{\rm S_1}| \le 2R_{\oplus}$, where $R_{\oplus}$ is the radius of the Earth, we can estimate that

$$\bar{\alpha}^{\odot} \sim \delta z_{\rm LPI}\bigg(\frac{GM_{\odot}}{c^2|\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus}|}\bigg)^{-1}\bigg[\frac{(\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus})\cdot(\boldsymbol{Y}_{\rm S_2} - \boldsymbol{Y}_{\rm S_1})}{|\boldsymbol{y}_{\odot} - \boldsymbol{y}_{\oplus}|^2}\bigg]^{-1} \gtrsim \frac{\delta z_{\rm LPI}}{10^{-12}}. \tag{10}$$

This means that even with residuals of Doppler data at the level of 10^{-12}, the parameter $\bar{\alpha}^{\odot}$ can barely be determined. Thus, according to Equations (7) and (10), the detectability of a possible violation of LLI is higher than the possible violation of LPI by about two orders of magnitude for two stations on the Earth conducting three-way Doppler tracking. In addition, in this configuration, if and only if δz can be measured to much better than 10^{-12} will the observability of $\bar{\alpha}^{\odot}$ be significant.

Again, this is merely a short discussion about the detectability of these parameters instead of a robust statistical estimation. In the following parts of this work we will analyze the post-fit residuals of the three-way Doppler tracking data of MEX (see Sect. 3) and use these residuals to estimate the preliminary bounds on these violations by using the method of weighted least squares (see Sect. 4).

3 THREE-WAY DOPPLER TRACKING OF MEX

In a cooperation between China and ESA, MEX was tracked by three-way Doppler observations. This started at 20:00 on 2009 August 7 (UTC) and ended at 08:04 on 2009 August 8 (UTC). The uplink signals were sent by the station administered by ESA at NN in Australia and the downlink signals were received by three stations at SH, KM and UR in China. These observational data were processed by the software MarsODP (Huang et al. 2009) for orbit determination of MEX. MarsODP was developed by a group from Shanghai Astronomical Observatory, China. It can reduce data from two/three-way range measurements, one/two/three-way Doppler tracking, VLBI and other types of observations.

As the first step, we use the post-fit residuals of these three-way Doppler tracking in the X band to estimate the preliminary bounds on possible violations of LLI and LPI in this work. These residuals were obtained by fitting the observational data with the standard model built on Newton's law and Einstein's GR (see Huang et al. 2009, for details). Therefore, the effects of violations of LLI and LPI were not modeled in MarsODP and the parameters $\bar{\beta}_{\mathrm{S}_{1/2}}$ and $\bar{\alpha}^{\mathrm{A}}_{\mathrm{S}_{1/2}}$ were not determined in the fitting. In this sense, the results we obtain in the next section may not be considered to be genuine "constraints" (it would be so if one solved for them in a covariance analysis by reanalyzing the data with modified software including these effects) but as preliminary indications of acceptable values according to the best contemporary knowledge, we call them "preliminary bounds" (see Iorio 2014, for a further discussion).

Figure 1 shows the post-fit residuals δv (left panel) and their statistical histograms (right panel) after MarsODP was used to process the three-way Doppler tracking data obtained by three Chinese stations at SH, KM and UR (see Cao et al. 2011, for details). The time coordinates are represented by taking 2009-Aug-07 00:00:00.000 (UTC) as the zero point. The gaps around 24h are caused by the absence of uplink signals. The mean value of these residuals is $\sim 10^{-4}$ m s^{-1} and the standard deviation is about $\sim 3 \times 10^{-4}$ m s^{-1}. This means that the values of δz are at the level of 10^{-12} and these observations are not sufficiently sensitive to detect possible violations of LPI (see the discussion about the detectability in Sect. 2). Therefore, in the next section, we will only focus on determining the bounds on violation of LLI.

4 PRELIMINARY BOUNDS ON LLI

By using the method of weighted least squares, we estimate the preliminary bounds on violations of LLI in two different cases. In Case I, we *assume* $\bar{\beta}_{\mathrm{S}_1} = \bar{\beta}_{\mathrm{S}_2} = \bar{\beta}$ and make other parameters vanish. For comparison, we also consider another condition, Case I', which includes an additional contribution from violations of LPI due to the Sun. As in the special case investigated by Krisher et al. (1993), we *assume* $\bar{\beta}_{\mathrm{S}_1} = \bar{\beta}_{\mathrm{S}_2} = \bar{\alpha}^{\odot}_{\mathrm{S}_1} = \bar{\alpha}^{\odot}_{\mathrm{S}_2} = \bar{\beta}'$ in Case I'. The downlinks of SH, KM and UR yield the bounds on $\bar{\beta}$ of $(-8.497 \pm 0.010) \times 10^{-2}$, $(-1.450 \pm 0.004) \times 10^{-2}$ and $(-0.975 \pm 0.002) \times 10^{-2}$, respectively (see Table 1 for a summary). Since the stations at NN and UR have the largest difference in their velocities in GCRS, they have the highest sensitivity and obtain the

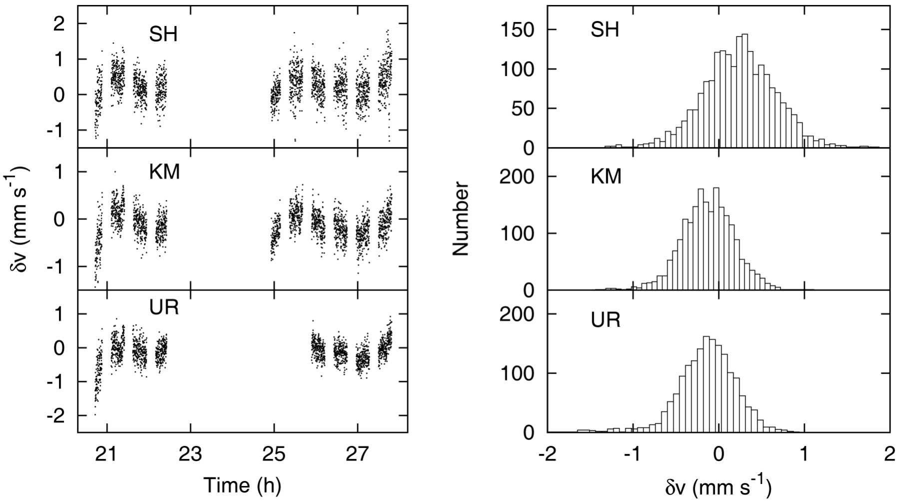

Fig. 1 The post-fit residuals δv (*left panel*) and their statistical histograms (*right panel*) after MarsODP was used to process the three-way Doppler tracking data obtained by three Chinese stations in SH, KM and UR. The time coordinates are represented by taking 2009-Aug-07 00:00:00.000 (UTC) as the zero point.

Table 1 Summary of Preliminary Bounds on LLI for Case I and Case II

Uplink	Downlink	Case I $\bar{\beta}$ (10^{-2})	Case I' $\bar{\beta}'$ (10^{-2})	Case II $\bar{\beta}_{\rm U}$ (10^{-2})	Case II $\bar{\beta}_{\rm D}$ (10^{-2})
NN	SH	-8.497 ± 0.010	-8.586 ± 0.010	3.091 ± 0.021	3.111 ± 0.021
NN	KM	-1.450 ± 0.004	-1.452 ± 0.003	0.902 ± 0.006	0.889 ± 0.006
NN	UR	-0.975 ± 0.002	-0.975 ± 0.002	2.732 ± 0.015	2.696 ± 0.015

NN: New Norcia, SH: Shanghai, KM: Kunming, and UR: Urumqi; $\bar{\beta}_{\rm U/D}$ is the $\bar{\beta}$ associated with the uplink/downlink.

tightest bound on $\bar{\beta}$. As we discussed detectability in Section 2, the contribution of the violation in LPI associated with the Sun is at least two orders of magnitude less than that of violation in LLI, which makes $\bar{\beta}'$ very close $\bar{\beta}$ (also see Table 1).

In Case II, we treat $\bar{\beta}_{\rm S_1}$ and $\bar{\beta}_{\rm S_2}$ as *free* parameters and they are simultaneously estimated. In the configuration that NN was the uplink and SH was the downlink, both $\bar{\beta}_{\rm NN}$ and $\bar{\beta}_{\rm SH}$ are at the level of 3%. The levels of $\sim 10^{-2}$ are obtained as well when KM and UR were the downlinks respectively (see Table 1).

5 CONCLUSIONS

Using the theory of three-way Doppler tracking, including possible violations of LLI and LPI, we analyze the post-fit residuals of three-way Doppler tracking of MEX. These Doppler observations were carried out from August 7th to 8th in 2009, with an uplink station administered by ESA at NN in Australia and three downlink stations at SH, KM and UR in China. We find that these observations impose preliminary bounds on LLI at the level of 10^{-2}, but they are not suitable for testing LPI because of the configuration of these stations and the accuracy of the observations.

To our knowledge, this is one of the first attempts in China to do radio science in the field of fundamental physics. With the development of techniques used for tracking spacecraft, we wish to obtain better bounds on possible violations of LLI and LPI in the future.

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